MATHEMATICS

STUDY MATERIAL

PROBABILITY

IIT-JEE

THE NARAYANA GROUP

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PREFACE

Dear Student,

Heartiest congratulations on making up your mind and deciding to be an engineer to serve the society.

As you are planning to take various Engineering Entrance Examinations, we are sure that this **STUDY PACKAGE** is going to be of immense help to you.

At NARAYANA we have taken special care to design this package according to the **Latest Pattern of IIT-JEE,** which will not only help but also guide you to compete for IIT-JEE, AIEEE & other State Level Engineering Entrance Examinations.

The salient features of this package include :

- Power packed division of units and chapters in a scientific way, with a correlation being there.
- Sufficient number of solved examples in Physics, Chemistry & Mathematics in all the chapters to motivate the students attempt all the questions.
- All the chapters are followed by various types of exercises, including Objective - Single Choice Questions, Objective - Multiple Choice Questions, Comprehension Type Questions, Match the Following, Assertion-Reasoning & Subjective Questions.

These exercises are followed by answers in the last section of the chapter including Hints & Solutions wherever required. *This package will help you to know* what to study, how to study, time management, your weaknesses and improve your performance.

We, at NARAYANA, strongly believe that quality of our package is such that the students who are not fortunate enough to attend to our Regular Classroom Programs, can still get the best of our quality through these packages.

We feel that there is always a scope for improvement. We would welcome your suggestions & feedback.

Wish you success in your future endeavours.

THE NARAYANA TEAM

ACKNOWLEDGEMENT

While preparing the study package, it has become a wonderful feeling for the NARAYANA TEAM to get the wholehearted support of our Staff Members including our Designers. They have made our job really easy through their untiring efforts and constant help at every stage.

We are thankful to all of them.

THE NARAYANA TEAM

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PROBABILITY

PROBABILITY

IIT- JEE SYLLABUS

Addition and multiplication rules of Probability, Conditional probability, Independence of events, Computation of probability of events using permutations and combinations.

CONTENTS

- Basic Definition
- Probability of an event
- Odds in favour and against an event
- Addition theorem of probability
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- Baye's theorem
- Binomial distribution for repeated experiments
- Geometrical probability

INTRODUCTION

From the time immemorial, Human Life has been full of uncertainties. In our everyday life, we very often make guesses and use statements like - probably it will rain today, possibility of India to win this Cricket World cup is more than any other team, chances of congress coming to power this year are strong. In the above statements, the words probably, certainly, possibility, chances etc. convey the sense of uncertainty or certainty about the occurrence of some event. The word 'Probability' or 'chance' of any event measures one's degree of belief about the occurrence of that event. Ordinarily, it may appear that there cannot be any exact measurement for these uncertainties but in probability theory, we do have methods for measuring the degree of certainty or uncertainty of events in terms of numbers lying between 0 and 1, provided certain conditions are satisfied. A probability of 1 means 100% chance of occurrence of an event which obviously is the maximum chance.

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1. BASIC DEFINITIONS

RANDOM EXPERIMENT

An experiment which can result in more than one outcome and so, whose outcome cannot be predicted with certainty, is called ***Random experiment***.

Point to be Noted : An experiment whose outcome is known in advance, is not a random experiment. For example, when a ball is thrown upward, it will surely fall downward. So it is not a random experiment.

Examples:

(i) Tossing of a coin may result in head or tail.

(ii) Throwing of a die may result in anyone of six outcomes {1, 2, 3, 4, 5, 6}

SAMPLE SPACE

The sample space of a random experiment is defined as the set of all possible outcomes of the experiment. Sample space is usually represented by 'S'. So since sample space consists of all the possibilities, sample space will surely occur in any experiment.

Examples:

(i) Tossing of a coin results in either a head or a tail turning up. Let H denote the occurrence of head and T denote the occurrence of tail.
Then, sample space S = {H, T}

(ii) Throwing of a die will result in anyone of six outcomes 1, 2, 3, 4, 5 or 6.
So sample space = {1, 2, 3, 4, 5, 6}

(iii) Tossing of two coins results in the sample space S = {HH, HT, TH, TT}

DISCRETE SAMPLE SPACE

A sample space having finite number of sample points, is said to be 'Discrete Sample Space'.

SAMPLE POINT

Each possible outcome of a random experiment is called a sample point. For a point H & T are sample point or sample elements.

EVENT

The event is a subset of a sample space.

Points to be Noted

(i) ϕ is also a subset of S which is called an impossible event.

(ii) Sample space plays the same role in 'probability' as does the universal set in 'Set Theory'.

Example:

In the throwing of a die, sample space is S = {1, 2, 3, 4, 5, 6}

A = {1, 3, 5}, B = {2, 4, 6}, C = {1, 2, 3, 4} , all being the subsets of S, are called events. Here A is the event of occurrence of an odd number, B is the event of occurrence of an even number, and C is the event of occurrence of a number less than 5.

SIMPLE EVENT OR ELEMENTARY EVENT

An event which cannot be further split is a simple event.

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COMPOUND EVENT

An event consisting of more than one sample points is called Compound Event.

Examples:

(i) If a die is throwing, then sample space is S = {1, 2, 3, 4, 5, 6}
A = {1, 3, 5}, B = {2, 4, 6} and C = {1, 2, 3, 4} are compound events.

TRIAL

Each performance of the random experiment is called a Trial.

EQUALLY LIKELY EVENTS

A set of events is said to be equally likely if taking into consideration all the relevent factors, there is no reason to expect one of them in preference to the others i.e. in simple words, the events are equally likely to occur. Obviously, the chances of occurrence of equally likely events are equal and so their probabilities are equal.

Mathematically:

Examples :

(i) In case of tossing a fair coin, occurrence of head and tail are equally likely events

(ii) In the throwing of an unbiased die, all the six faces 1, 2, 3, 4, 5 and 6 are equally likely to come up.

MUTUALLY EXCLUSIVE EVENTS

A set of events is said to be mutually exclusive if the occurrence of one of them rules out the occurrence of any of the remaining events i.e. in simple words, no two out of the set of events can occur simultaneously.

In set theoretic notation, two events A_1 and A_2 are mutually exclusive if

$$A_1 \cap A_2 = \phi$$

For more than two events,

In set theoretic notation, events A_1, A_2, ..., A_m are mutually exclusive if

$A_i \cap A_j = \phi$ for all pairs (i, j) satisfying $1 \le i, j \le m$, where $i \ne j$

Examples:

(i) In the tossing of a coin, the events {H} and {T} are mutually exclusive events since head and tail can't occur simultaneously in any performance of the experiment.

(ii) In the throwing of a die, the events E_1 = {1, 3, 5} and E_2 = {2, 4, 6} are mutually exclusive events since an odd number and an even number can't occur simultaneously in any performance of the experiment.

EXHAUSTIVE EVENTS

A set of events is said to be exhaustive if atleast one of them must necessarily occur on each performance of the experiment i.e. in simple words, if all the events collectively, cover all the possible outcomes of the experiment, then they are called exhaustive events.

In set theoretic notation, events A_1, A_2, ..., A_m are called exhaustive

If $A_1 \cup A_2 \cup ..... \cup A_m = S$ i.e. $\bigcup_{i=1}^{m} A_i = S$

Mathematically, events A_1, A_2, ..., A_m are called exhaustive.

Example:

In the throwing of a die, the events {1, 2}, {2, 3, 4}, {3, 4, 5, 6} and {1, 3, 6} are exhaustive system of events.

Since {1, 2} $\cup$ {2, 3, 4} $\cup$ {3, 4, 5, 6} $\cup$ {1, 3, 6} = {1, 2, 3, 4, 5, 6} = S

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INDEPENDENT EVENTS

Two or more events are said to be independent if occurrence or non-occurrence of any of them does not affect the probability of occurrence or non-occurrence of other events.

Examples:

(i) Consider the drawing of two balls one after the other, with replacement from an urn containing two or more balls of varying colours. Then, the two draws are independent of each other and any two events defined on two draws (one on each draw) will be independent.

But if the two balls were drawn one by one, without replacement, then probability of occurrence of any event defined on second draw would depend upon the result of first draw. So any two events defined on two draws (one on each draw) would have been dependent.

(ii) Consider the drawing of two cards one after the other, with replacement from a pack of 52 playing cards. Then, the two draws are independent of each other and any two events defined on two draws (one on each draw) will be independent.

But if the two cards were drawn one by one, without replacement, then probability of occurrence of any event defined on 2nd draw would depend upon the result of first draw. So any two events defined on two draws (one on each draw) would have been dependent.

RELATION BETWEEN MUTUALLY EXCLUSIVE AND INDEPENDENT EVENTS

Mutually exclusive events will never be independent and independent events will never be mutually exclusive as explained below.

Mutually exclusive events are those events in which occurrence of one of them rules out the occurrence of other. It shows clearly that the chances of their occurrences or non-occurrences depend upon each other and so the events cannot be independent. Similarly independent events are the events which don’t depend upon each other but mutually exclusive events affect each other. So independent events can’t be mutually exclusive.

2. PROBABILITY OF AN EVENT

CLASSICAL OR MATHEMATICAL DEFINITION OF PROBABILITY

If there are n equally likely, mutually exclusive and exhaustive events and m of which are favourable to the event E,

then probability of occurrence of event E $= P(E) = \dfrac{n(E)}{n(S)}$

$$= \frac{\text{number of cases favourable to event E}}{\text{Total number of cases}} = \frac{m}{n}$$

It may be observed here that $0 \le m \le n$

$$\Rightarrow \quad 0 \le \frac{m}{n} \le 1 \quad \Rightarrow \quad 0 \le P(E) \le 1$$

The number of cases unfavourable to event E = n – m

$$\Rightarrow \quad P(E^c) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(E)$$

So $\quad P(E) + P(E^C) = 1$

If E is sure event, $P(E) = \dfrac{n}{n} = 1$

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If E is impossible event, $P(E) = \frac{0}{n} = 0$

So $0 \le P(E) \le 1$

Example :

In the tossing of a coin, sample space S = {H, T} $\Rightarrow$ n(S) = 2.

Then, $P(H) = \frac{n(H)}{n(S)} = \frac{1}{2}$

Similarly, $P(T) = \frac{n(T)}{n(S)} = \frac{1}{2}$

So occurrence of head and tail are equally likely events in the tossing of a coin.

3. ODDS IN FAVOUR AND AGAINST AN EVENT

If a cases are favourable to the event A and b cases are not favourable to the event A that is favourable to the $\bar{A}$, then P(A) = $\frac{\text{number of favourable cases}}{\text{number of total cases}} = \frac{a}{a+b}$

and $P(\bar{A}) = \frac{\text{number of non favourable cases}}{\text{number of total cases}} = \frac{b}{a+b}$

Odds in favour of event A = $\frac{P(A)}{P(\bar{A})} = a : b$

Odds in against of event A = $\frac{P(\bar{A})}{P(A)} = b : a$ [P(A) + P$(\bar{A})$ = 1]

Illustration 1: ***Find the probability of the event A if (i) odds in favour of event A are 5 : 7 (ii) odds against A are 3 : 4.***

Solution : (i) Odds in favour of event A are 5 : 7. Let P(A) = p

$\therefore$ p : 1 – p = 5 : 7

$\Rightarrow \frac{p}{1-p} = \frac{5}{7}$ $\Rightarrow 7p = 5 - 5p$

$\Rightarrow 12p = 5$ $\Rightarrow p = \frac{5}{12}$

$\therefore P(A) = \frac{5}{12}$

(ii) Odds against event A are 3 : 4. Let P(A) = p.

$\therefore$ 1 – p : p = 3 : 4

$\Rightarrow \frac{1-p}{p} = \frac{3}{4}$ $\Rightarrow$ 4 – 4p = 3p

$\Rightarrow$ 7p = 4 $\Rightarrow p = \frac{4}{7}$

$\therefore P(A) = \frac{4}{7}$

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Illustration 2: ***A bag contains 5 white, 7 black and 8 red balls. A ball is drawn at random. Find the probability of getting:***
(i) red ball (ii) non-white ball

Solution : Number of white balls = 5
Number of black balls = 7
Number of red balls = 8
$\therefore$ Total number of balls = 5 + 7 + 8 = 20.
(i) Let R = event of getting red ball

$$\therefore \quad \text{P(red ball)} = P(R) = \frac{n(R)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

(ii) Let W = event of getting white ball

$$\therefore \quad \text{P(non-white ball)} = P(\bar{W}) = \frac{\text{number of non-white balls}}{\text{total number of balls}} = \frac{7+8}{20} = \frac{15}{20} = \frac{3}{4}$$

4. ADDITION THEOREM OF PROBABILITY

If A and B are two events related with an experiment then the probability that either of the events will occur (or at least one of the events will occur) is

$\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$ where $(A \cap B)$ is the event defined as the event that both A and B are occuring.

Note : If A and B are mutually exclusive events then $P(A \cap B) = 0$

$\therefore$ for mutually exclusive events

$\mathbf{P(A \cup B) = P(A) + P(B)}$

Illustration 3: ***A pair of dice is rolled once. Find the probability of throwing a total of 8 or 10.***

Solution : In this case, sample space S consists for $6 \times 6 = 36$ equally likely simple events of the type (x, y) where x, y $\in$ {1, 2, 3, 4, 5, 6}
i.e. S = {(x, y) : x, y $\in$ {1, 2, 3, 4, 5, 6}}.
Let E_1: 'a total of 8' and E_2: 'a total of 10', then E_1 and E_2 are mutually exclusive events as the sum of number on the uppermost faces of the two dice in a single throw cannot be 8 and 10 simultaneously.
Now E_1 = {(2, 6), (4, 4), (6, 2), (3, 5), (5, 3)} and E_2 = {(4, 6), (5, 5), (6, 4)}.
$\therefore$ Required probability = $P(E_1 \cup E_2)$

$$= P(E_1) + P(E_2) = \frac{5}{36} + \frac{3}{36} = \frac{8}{36} = \frac{2}{9}$$

Illustration 4 : ***Find the probability of 4 turning up for at least once in two tosses of a fair die.***

Solution : Here S = {(1, 1), (1, 2), ..., (6, 5), (6, 6)}.
Let A = event of getting 4 on the first die
and B = event of getting 4 on the second die.
$\therefore$ A = {(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)}
and B = {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)}.

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$\therefore$ $P(A)=\dfrac{n(A)}{n(S)}=\dfrac{6}{36}=\dfrac{1}{6}$ and $P(B)=\dfrac{n(B)}{n(S)}=\dfrac{6}{36}=\dfrac{1}{6}$.

The events A and B are not m.e. because the sample (4, 4) is common to both.

$\therefore$ $A\cap B=\{(4,\ 4)\}$ $\therefore$ $P(A\cap B)=\dfrac{1}{36}$

By addition theorem, the required probability of getting four at least once is

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{1}{6}+\frac{1}{6}-\frac{1}{36}=\frac{11}{36}.$$

Note : Probabilities of mutually exclusive, exhaustive and equally likely events:

Let $E_1, E_2, ..., E_n$ be n mutually exclusive, exhaustive and equally likely events.

Then $P(E_1\cup E_2....\cup E_n)=1$ (since exhaustive events)

But $P(E_1\cup E_2....\cup E_n)=P(E_1)+P(E_2)+\ .....+P(E_n)$ (since mutually exclusive events)

$\Rightarrow$ $P(E_1) + P(E_2) + ... + P(E_n) = 1$

Also $P(E_1) = P(E_2) = ... = P(E_n)$ (Since equally likely events)

$\Rightarrow$ $n\,P(E_i) = 1,\ i\in\{1, 2, 3, ...n\}$ $\Rightarrow$ $P(E_i)=\dfrac{1}{n}$

So $P(E_1) = P(E_2) = ... = P(E_n) = 1/n$

So probability of each of n mutually exclusive, exhaustive and equally likely events is $\dfrac{1}{n}$.

EXERCISE – 1

1. An honest die is thrown randomly. A person requires either multiple of 2 or multiple of 3. What is the probability in his favour?
2. A and B are solving a problem of mathematics. The probability of A to solve is 1/2 and probability of B to solve is 1/3, respectively. What is the probability that problem is solved?

5. ALGEBRA OF EVENTS

(i) $\mathbf{P(A)+P(A^c)=1}$

(ii) $\mathbf{P(A-B)=P(A\cap\overline{B})=P(A\cup B)-P(B)=P(A)-P(A\cap B)}$

(iii) $\mathbf{P(B-A)=P(B\cap\overline{A})=P(A\cup B)-P(A)=P(B)-P(A\cap B)}$

(iv) $\mathbf{P(A\Delta B)=P(A\cup B)-P(A\cap B)}$

(v) $\mathbf{P(A\Delta B)=2P(A\cup B)-\left(P(A)+P(B)\right)}$

(vi) $\mathbf{P(A\Delta B)=P(A)+P(B)-2P(A\cap B)}$

(vii) $\mathbf{P\left(\overline{A}\cap\overline{B}\right)=P(S)-P(A\cup B)=1-P(A\cup B)}$

(viii) $\mathbf{A\subseteq B\ \Rightarrow\ P(A)\le P(B)}$

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(ix) **P(exactly two of A, B, C occur)**

$$\mathbf{= P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)}$$

(x) **P(at least two of A, B, C occur)**

$$\mathbf{= P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)}$$

Illustration 5: ***Three critics review a book. Odds in favour of the book are 5 : 2, 4 : 3 and 3 : 4 respectively for the three critics. Find the probability that majority are in favour of the book.***

Solution : Let the critics be E_1, E_2 and E_3. Let $P(E_1)$, $P(E_2)$ and $P(E_3)$ denotes the probabilities of the critics E_1, E_2 & E_3 to be in favour of the book. Since the odds in favour of the book for the critics E_1, E_2 & E_3 are 5 : 2, 4 : 3 and 3 : 4 respectively,

$$\therefore \quad P(E_1) = \frac{5}{5+2} = \frac{5}{7};\ P(E_2) = \frac{4}{4+3} = \frac{4}{7} \text{ and } P(E_3) = \frac{3}{3+4} = \frac{3}{7}$$

Clearly, the event of majority being in favour = the event of at least two critics being in favour.

$\therefore$ The required probability

$$= P(E_1E_2\bar{E}_3) + P(\bar{E}_1E_2E_3) + P(E_1\bar{E}_2E_3) + P(E_1E_2E_3)$$

$$= P(E_1).P(E_2).P(\overline{E_3}) + P(\bar{E}_1)\ P(E_2)\ P(E_3) + P(E_1)\ P(\bar{E}_2)\ P(E_3) + P(E_1)P(E_2)P(E_3)$$

{ $\because$ E_1, E_2 & E_3 are independent}

$$= \frac{5}{7}.\frac{4}{7}.\left(1-\frac{3}{7}\right)+\left(1-\frac{5}{7}\right).\frac{4}{7}.\frac{3}{7}.\frac{5}{7}.\left(1-\frac{4}{7}\right).\frac{3}{7}+\frac{5}{7}.\frac{4}{7}.\frac{3}{7} \quad = \frac{1}{7^3}[80+24+45+60]$$

$$= \frac{209}{343}$$

Illustration 6: ***The probability that atleast one of A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2. Find*** $P(\bar{A}) + P(\bar{B})$**.**

Solution : Given $P(A \cup B) = 0.6$

and $P(A \cap B) = 0.2$

Now $P(\bar{A}) + P(\bar{B}) = 1 - P(A) + 1 - P(B)$

= 2 – [P(A) + P(B)] = 2 – [P (A) + P(B) – $P(A \cap B)$] – $P(A \cap B)$

$= 2 - P(A \cup B) - P(A \cap B)$ = 2 – 0.6 – 0.2 = 1.2.

Illustration 7: ***The probability of two events A and B are 0.25 and 0.50 respectively. The probability of their simultaneous occurrence is 0.14. Find the probability that neither A nor B occurs.***

Solution : We have P(A) = 0.25, P(B) = 0.50 and $P(A \cap B) = 0.14$

By addition theorem,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.25 + 0.50 - 0.14 = 0.61$$

Now, P(neither A nor B) = $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})^* = 1 - P(A \cup B) = 1 - 0.61 = 0.39$

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Illustration 8: ***A and B are two non-mutually exclusive events. If $P(A)=\frac{1}{4}$, $P(B)=\frac{2}{5}$ and $P(A\cup B)=\frac{1}{2}$, find the values of $P(A\cap B)$ and $P(A\cap B^c)$.***

Solution : We have $P(A)=\frac{1}{4}$, $P(B)=\frac{2}{5}$, $P(A\cup B)=\frac{1}{2}$

By addition theorem, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\therefore \quad \frac{1}{2}=\frac{1}{4}+\frac{2}{5}-P(A\cap B)$

$\therefore \quad P(A\cap B)=\frac{1}{4}+\frac{2}{5}-\frac{1}{2}=\frac{2+8-10}{20}=\frac{3}{20}$

We have, $(A\cap B^c)\cap(A\cap B)=A\cap(B^c\cap B)=A\cap\phi=\phi$

$\therefore$ The event $A\cap B^c$ and $A\cap B$ are mutually exclusive and

$(A\cap B^c)\cup(A\cap B)=A\cap(B^c\cup B)=A\cap S=A$ (S is the sample space)

$\therefore$ By addition theorem, $P(A)=P(A\cap B^c)+P(A\cap B)$

$\therefore \quad \frac{1}{4}=P(A\cap B^c)+\frac{3}{20}$ or $P(A\cap B^c)=\frac{1}{4}-\frac{3}{20}=\frac{1}{10}$

EXERCISE – 2

1. Let A and B be two events defined on a sample space. Given P(A) = 0.4, P(B) = 0.80 and $P\left(\bar{A}\cap\bar{B}\right)$ = 0.10. Find (i) $P\left(\bar{A}\cup B\right)$ (ii) $P\left[\left(\bar{A}\cup B\right)\cup\left(A\cap\bar{B}\right)\right]$
2. Given $P\left(A\cup B\right)=\frac{5}{6}$, $P\left(A\cap B\right)=\frac{1}{3}$, $P\left(\bar{B}\right)=\frac{1}{2}$. Determine P(A) and P(B).

6. CONDITIONAL PROBABILITY

If A and B are two events, then the conditional probability of event A given that event B has already occured $P\left(A/B\right)$ is defined as $\mathbf{P\left(A/B\right)=\frac{P(A\cap B)}{P(B)}}$

Similarly, the probability of B given that A has already occured will be $\mathbf{P\left(B/A\right)=\frac{P(A\cap B)}{P(A)}}$

Therefore, $\boxed{\mathbf{P(A\cap B)=P(A)P\left(B/A\right)=P(B)\cdot P\left(A/B\right)}}$

Illustration 9: If a pair of dice is thrown and it is known that sum of the numbers is even, then find the probability that the sum is less than 6.

Solution : Let A be the given event and let B be the event, whose probability is to be found. Then

Required probability $P\left(\frac{B}{A}\right)=\frac{P\left(B\cap A\right)}{P(A)}=\frac{4/36}{18/36}=\frac{2}{9}$.

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7. MULTIPLICATION THEOREM OF PROBABILITY

If A and B are any two events, then

$$\mathbf{P(A \cap B) = P(B).P\left(\frac{A}{B}\right)} \qquad \textbf{if } \mathbf{P(B) \neq 0}$$

Similarly, $\mathbf{P(A \cap B) = P(A).P\left(\frac{B}{A}\right)}$ $\qquad$ **if** $\mathbf{P(A) \neq 0}$

Generalized form of Multiplication Theorem

Suppose A_1, A_2, ..., A_m be m events such that $\mathbf{P(A_1 \cap A_2 \cap ..... \cap A_m) \neq 0}$, then

$$\mathbf{P(A_1 \cap A_2 \cap ..... \cap A_m) = P(A_1)\,P\left(\frac{A_2}{A_1}\right)P\left(\frac{A_3}{A_1A_2}\right)P\left(\frac{A_4}{A_1A_2A_3}\right)....}$$

$$\mathbf{...P\left(\frac{A_m}{A_1A_2....A_{m-1}}\right)}$$

Illustration 10: ***P_1, P_2, . . . , P_8 are eight players participating in a tournament. If i < j, then P_i will win the match against P_j. Players are paired up randomly for first round and winners of this round again paired up for the second round and so on. Find the probability that P_4 reaches in the final.***

Solution : Let A_1 be the event that in the first round the four winners are P_1, P_4, P_i, P_j, where i $\in$ {2, 3}, j $\in$ {5, 6, 7}and let A_2 be the event that out of the four winners in the first round, P_1 and P_4 reaches in the final.

The event A_1 will occur, if P_4 plays with any of P_5, P_6, P_7 or P_8 (say with P_6) and P_1, P_2 and P_3 are not paired with P_5, P_7 and P_8. Further A_2 will occur if P_1 plays with P_j.

The required probability = $P\left(A_1 \cap A_2\right)$ = $P(A_1)$. $P\left(\frac{A_2}{A_1}\right)$.

$$= \frac{4\times\left(\frac{\lfloor 6}{(\lfloor 2)^3\lfloor 3} - \lfloor 3\right)}{\frac{\lfloor 8}{(\lfloor 2)^4\lfloor 4}} \times \frac{1}{\frac{\lfloor 4}{(\lfloor 2)^2\lfloor 2}} = \frac{4}{35}.$$

(Here we have used the concept of division into groups).

Note : For mutually independent and pairwise independent events.

Let A and B be events associated with a random experiment. The events A and B are independent if and only if $P(A \cap B) = P(A)\cdot P(B)$**.** Since A and B are mutually independent events $P\left(B/A\right) = P(B)$ or $P\left(A/B\right) = P(A)$

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Let A and B be independent events.

$$\therefore \qquad P(A\cap B)=\left(\frac{P(A\cap B)}{P(B)}\right)P(B)=P(A/B)P(B) \qquad \left(\because \quad P(A/B)=\frac{P(A\cap B)}{P(B)}\right)$$

$$= P(A)\ P(B) \qquad (\because \quad P(A/B)=P(A))$$

$$\therefore \qquad P(A\cap B)=P(A)\ P(B).$$

Conversely, let $P(A\cap B)=P(A)\ P(B)$.

$$\therefore \qquad P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)\,P(B)}{P(B)}=P(A)$$

and $$P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{P(A\cap B)}{P(A)}=\frac{P(A)\,P(B)}{P(A)}=P(B)$$

$\therefore$ P(B/A) = P(A) and P(B/A) = P(B).

$\therefore$ A and B are independent events.

So $P(A\cap B)=P(A)P(B)$ is the necessary and sufficient condition for the events A and B to be independent.

Illustration 11: ***A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that neither of them occurs is 1/3. Find the probability of the occurrence of A.***

Solution : Let P(A) = x and P(B) = y. Since A and B are independent,

$$P(A\cap B)=P(A)P(B)=xy=1/6, \qquad \text{...(i)}$$

and $P(A'\cap B')=P(A')P(B')\ =(1-x)(1-y)=1/3$ ...(ii)

It should be noted that x > 0, y > 0 ...(iii)

Solving (i) and (ii), we get x = 1/3 or x = 1/2.

REMARK

Three events A, B and C are said to be mutually independent if,

$P(A\cap B)$ = P(A). P(B), $P(A\cap C)$ = P(A). P(C), $P(B\cap C)$ = P(B). P(C) and

$P(A\cap B\cap C)$ = P(A). P(B). P(C)

These events would be said to be pair–wise independent if,

$P(A\cap B)$ = P(A). P(B), $P(B\cap C)$ = P(B). P(C) and $P(A\cap C)$ = P(A). P(C).

Illustration 12: ***A lot contains 50 defective and 50 non-defective bulbs. Two bulbs are drawn at random, one at a time with replacement. The events A, B and C are defined as under:***
A = {The first bulb is defective}, B = {The second bulb is non-defective}
C = {The two bulbs are either both defective or both non-defective}
Catogorize the events A, B and C to be pairwise independent or mutually independent.

Solution : P(A) = $\frac{1}{2}\times 1=\frac{1}{2}$, P(B) = $1\times\frac{1}{2}=\frac{1}{2}$ and P(C) = $\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}=\frac{1}{2}$

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$P(A \cap B) = P$ (the first bulb in defective and the second bulb is non-defective)

$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

$P(B \cap C)$ = P(both the bulbs are non-defective) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(C \cap A)$ = P (both the bulbs are defective) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(A \cap B \cap C) = P(\phi) = 0$

As $P(A \cap B) = P(A).P(B), P(B \cap C) = P(B).P(C)$ and $P(C \cap A) = P(C).P(A)$, the events
A, B and C are pairwise independent.

Since $P(A \cap B \cap C) = 0 \neq P(A).P(B).P(C)$, A, B and C are not mutually independent.

If an event is independent of itself, then identify the event. Think about it mathematically as well as logically also.

8. TOTAL PROBABILITY THEOREM

Let $E_1, E_2, ..., E_n$ be n mutually exclusive and exhaustive events and event A is such that it can occur with any of the events $E_1, E_2, E_3, ...\ E_n$ then the probability of the occurrence of event A can be given as P(A) = $P(A \cap E_1) + P(A \cap E_2) + ... + P(A \cap E_n)$

$\therefore\ P(A) = P(E_1) \cdot P\left(A/E_1\right) + P(E_2) \cdot P\left(A/E_2\right) + ... + P(E_n) \cdot P\left(A/E_n\right)$ or ;

$$\boxed{\mathbf{P(A) = \sum_{i=1}^{n} P(E_i) P\left(A/E_i\right)}}$$

Illustration 13: ***In a certain city only 2 newspapers A and B are published. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. It is also known that 30% of those who read A but not B look into advertisement and 40% of those who read B but not A look into advertisements while 50% of those who read both A and B look into advertisements. What is the percentage of the population who read an advertisement?***

Solution : Let P(A) and P(B) be the percentage of the population in a city who read newspapers A and B respectively.

$\therefore \quad P(A) = \frac{25}{100} = \frac{1}{4}$, $P(B) = \frac{20}{100} = \frac{1}{5}$ and $P(A \cap B) = \frac{8}{100} = \frac{2}{25}$

$\therefore$ Percentage of those who read A but not B

$= P(A \cap \bar{B}) = P(A) - P(A \cap B) = \frac{1}{4} - \frac{2}{25} = \frac{17}{100} = 17\%$

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Similarly,

$$P(B\cap\bar{A})=P(B)-P(A\cap B)=\frac{3}{25}=12\%$$

$\therefore$ Percentage of those who read advertisements

$$=30\% \text{ of } P(A\cap\bar{B})+40\% \text{ of } P(B\cap\bar{A})+50\% \text{ of } P(A\cap B)$$

$$=\frac{30}{100}\times\frac{17}{100}+\frac{40}{100}\times\frac{3}{25}\times\frac{50}{100}\times\frac{2}{25}=\frac{139}{1000}=13.9\%$$

Hence the percentage of the population who read an advertisement is 13.9%.

Illustration 14: ***Two sets of candidates are competing for the positions on the board of directors of a company. The probabilities that the first and second sets will win are 0.6 and 0.4 respectively. If the first set wins, the probability of introducing a new product is 0.8, and the corresponding probability, if the second set wins is 0.3. What is the probability that the new product will be introduced?***

Solution : Let A_1 (A_2) denotes the event that first (second) set wins and let B be the event that a new product is introduced.

$\therefore$ $P(A_1) = 0.6$, $P(A_2) = 0.4$

$$P\left(\frac{B}{A_1}\right)=0.8,\ P\left(\frac{B}{A_2}\right)=0.3$$

$$P(B)=P(B\cap A_1)+P(B\cap A_2)=P(A_1).P\left(\frac{B}{A_1}\right)+P(A_2)P\left(\frac{B}{A_2}\right).$$

$$=0.6\times0.8+0.4\times0.3=0.6.$$

Illustration 15: ***In a multiple choice question there are four alternative answers of which one or more than one is correct. A candidate will get marks on the question only if he ticks the correct answers. The candidate decides to tick answers at random. If he is allowed up to three chances to answer the question, find the probability that he will get marks on it.***

Solution : The total number of ways of ticking one or more alternatives out of 4 is ${}^4C_1+{}^4C_2+{}^4C_3+{}^4C_4=15$.

Out of these 15 combinations only one combination is correct. The probability of ticking the alternative correctly at the first trial is $\frac{1}{15}$ that at the second trial is $\left(\frac{14}{15}\right)\left(\frac{1}{14}\right)=\frac{1}{15}$ and that at the third trial is $\left(\frac{14}{15}\right)\left(\frac{13}{14}\right)\left(\frac{1}{13}\right)=\frac{1}{15}$. Thus the probability that the candidate will get marks on the question if he is allowed upto three trials is $\frac{1}{15}+\frac{1}{15}+\frac{1}{15}=\frac{1}{5}$.

Illustration 16: ***Three groups A, B and C are competing for positions on the Board of Directors of a company. The probabilities of their winning are 0.5, 0.3, 0.2 respectively. If the Group A wins, the probability of introducing a new product is 0.7 and the corresponding probabilities for groups B and C are 0.6 and 0.5 respectively. Find the probability that the new product will be introduced.***

Solution : Let A, B, C denote the events of getting positions on the Board of directors by groups A, B and C respectively.

Since P(A) + P(B) + P(C) = 0.5 + 0.3 + 0.2 = 1, so the events A, B and C are exhaustive.

Let the events of introducing the new product be denoted by D.

Then given that

$$P(D/A) = 0.7,\ P(D/B) = 0.6 \text{ and } P(D/C) = 0.5.$$

$$P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C)$$

$$= P(A)\ P(D/A) + P(B)\ P(D/B) + P(C)\ P(D/C)$$

$$= 0.5 \times 0.7 + 0.3 \times 0.6 + 0.2 \times 0.5$$

Illustration 17: ***A speaks truth in 75% of the cases and B in 80% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?***

Solution : Let E_1 denote the event that A speaks the truth and E_2 the event that B speaks the truth, Then

$$P(E_1) = \frac{75}{100} = \frac{3}{4} \text{ so that } P(\overline{E}_1) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$P(E_2) = \frac{80}{100} = \frac{4}{5} \text{ so that } P(\overline{E}_2) = 1 - \frac{4}{5} = \frac{1}{5}$$

Let E be the event that A and B contradict each other. Obviously A and B will contradict each other if one of them speaks the truth and the other does not.

$\therefore$ $E = E_1\overline{E}_2 + \overline{E}_1E_2$ when $E_1\overline{E}_2$ and $\overline{E}_1E_2$ are mutually exclusive events.

Now $P(E_1\overline{E}_2)$ = the probability that A speaks the truth and B does not.

$$= P(E_1).\ P(\overline{E}_2) = \frac{3}{4} \times \frac{1}{5} = \frac{3}{20}.$$

Similarly,

$$P(\overline{E}_1 E_2) = P(\overline{E}_1).\ P(E2) = \frac{1}{4} \times \frac{4}{5} = \frac{1}{5}$$

Since $E_1\overline{E}_2$ and $\overline{E}_1E_2$ are mutually exclusive events, $\therefore$ we have

$$P(E) = P(E_1\overline{E}_2 + \overline{E}_1E_2) = P(E_1\overline{E}_2) + P(\overline{E}_1E_2)$$

$$= \frac{3}{20} + \frac{1}{5} = \frac{35}{100}.$$

Hence in 35% cases A and B will contradict each other.

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EXERCISE – 3

1. There are 2 groups of subjects, one of which consists of 5 science subjects and 3 engineering subjects and other consists of 3 science and 5 engineering subjects. An unbiased die is cast. If the number 3 or 5 turns up a subject is selected at random from first group, otherwise the subject is selected from second group. Find the probability that an engineering subject is selected.
2. An anti aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probability of hitting the plane at first, second, third and fourth shotes are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that the gun hits the plane.
3. A, B and C in order toss a coin. The first one to throw a head wins. What are their respective chances of winning? Assume that the game continue indefinitely.

9. BAYE'S THEOREM

Let $E_1, E_2, ..., E_n$ be n mutually exclusive and exhaustive events and event A is such that it can occur with any of the events $E_1, E_2, E_3, ... E_n$ then given that event A has already occurred then the probability that it has occurred with event E_i is actually the conditional probability $P\left(E_i/A\right)$.

$$P\left(\frac{E_i}{A}\right) = \frac{P(E_i \cap A)}{P(A)} \quad \text{(from conditional probability)}$$

P(A), by total probability theorem, is given as

$$P(A) = P(E_1)\ P\left(\frac{A}{E_1}\right) + P(E_2)\ P\left(\frac{A}{E_2}\right) + ... + P(E_n)\ P\left(\frac{A}{E_n}\right)$$

So $$P\left(\frac{E_i}{A}\right) = \frac{P(E_i \cap A)}{P(A)} = \frac{P(E_i)P(A/E_i)}{P(A)} \quad \text{(using multiplication theorem)}$$

$$= \frac{P(E_i)\,P(A/E_i)}{P(E_1)\ P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + ... + P(E_n)P\left(\frac{A}{E_n}\right)} \quad \text{(using total probability theorem)}$$

$$\therefore \quad \mathbf{P\left(\frac{E_i}{A}\right) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P\left(\frac{A}{E_i}\right)}}$$

Illustration 18: ***Box I contains 2 white and 3 red balls and box II contains 4 white and 5 red balls. One ball is drawn at random from one of the boxes and is found to be red. Find the probability that it was from box II.***

Solution : Let A denote the event that the drawn ball is red

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Let $A_1 \equiv$ The event that box I is selected and let $A_2 \equiv$ The event that box II is selected

I	II
2w 3R	4w 5R

$$\therefore \qquad P\left(\frac{A_2}{A}\right)=\frac{P(A_2).P\left(\frac{A}{A_2}\right)}{P(A_1)P\left(\frac{A}{A_1}\right)+P(A_2).P\left(\frac{A}{A_2}\right)} = \frac{\frac{1}{2}.\frac{5}{9}}{\frac{1}{2}.\frac{5}{9}+\frac{1}{2}.\frac{3}{5}} = \frac{25}{32}$$

Illustration 19: ***Let A and B be two independent witnesses in a case. The probability that A will speak the truth is x and the probability that B will speak the truth is y. A and B agree in a certain statement. Show that the probability that the statement is true is $\frac{xy}{1-x-y+2xy}$.***

Solution : Let E_1 be the event that both A and B speak the truth, E_2 be the event that both A and B tell a lie and E be the event that A and B Agree in a certain statement. Let C be the event that A speaks the truth and D be the event that B speaks the truth.

$\therefore$ $E_1 = C \cap D$ and $E_2 = C' \cap D'$. $P(E_1) = P(C \cap D) = P(C)\,P(D) = xy$ and

$P(E_2) = P(C' \cap D') = P(C')\,P(D') = (1 - x)(1 - y) = 1 - x - y + xy$

Now $P\left(\frac{E}{E_1}\right)$ = probability that A and B will agree when both of them speak the truth = 1

and $P\left(\frac{E}{E_2}\right)$ = probability that A and B will agree when both of them tell a lie = 1.

Clearly, $\left(\frac{E_1}{E}\right)$ be the event that the statement is true.

$$\therefore P\left(\frac{E_1}{E}\right)=\frac{P(E_1).P(E/E_1)}{P(E_1).P(E/E_1)+P(E_2)P(E/E_2)} = \frac{xy.1}{xy/1+(1-x-y+xy).1}=\frac{xy}{1-x-y+2xy}$$

Illustration 20: ***In a factory machines A, B, C manufacture 15%, 25%, 60% of the total production of bolts respectively. Of the bolts manufactured by machine A, 4%; by B, 2% and by C, 3% are defective respectively. A bolt is drawn at random and is found to be defective. What is the probability that it was produced by (i) machine A, (ii) machine B, (iii) machine C?***

Solution : Let A, B and C be the events of production of bolt on machines A, B and C respectively and D be the event for a bolt to be defective. Then

$$P(A \cap D)=\frac{15}{100}\times\frac{4}{100}$$

$$P(B \cap D)=\frac{25}{100}\times\frac{2}{100}$$

$$P(C \cap D)=\frac{60}{100}\times\frac{3}{100}$$

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$$\therefore \qquad P(D) = P(A \cap D) + P(B \cap D) + P(C \cap D) = \frac{290}{100 \times 100}$$

We have to find the probabilities

$$P\left(\frac{A}{D}\right), P\left(\frac{B}{D}\right), P\left(\frac{C}{D}\right)$$

$$\text{Now } P\left(\frac{A}{D}\right) = \frac{P(A \cap D)}{P(D)} = \frac{\frac{15}{100} \times \frac{4}{100}}{\frac{290}{100 \times 100}} = \frac{6}{29}$$

$$P\left(\frac{B}{D}\right) = \frac{P(B \cap D)}{P(D)} = \frac{\left(\frac{50}{100 \times 100}\right)}{\left(\frac{290}{100 \times 100}\right)} = \frac{5}{29}$$

$$P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{\left(\frac{180}{100 \times 100}\right)}{\left(\frac{290}{100 \times 100}\right)} = \frac{18}{29}$$

10. BINOMIAL DISTRIBUTION FOR REPEATED EXPERIMENTS

If the probability of success of an event in one trial is p, and that of its failure is q so that p + q = 1, then the probability of exactly r successes in n trials of the concerned experiment is

${}^nC_r\, p^r\, q^{n-r}$. i.e. $(r + 1)^{th}$ term in the expansion of $(q + p)^n$.

Case – I

Probability of success r times out of n total trials

$$= P(r) = {}^nC_r\, p^r\, q^{n-r}$$

Case – II

Probability of success at least r times out of total n trials

$$= P(\geq r) = {}^nC_r\, p^r\, q^{n-r} + {}^nC_{r+1}\, p^{r+1}\, q^{n-r-1} + ... + {}^nC_n\, p^n$$

Case – III

Probability of success at most ‘r’ times out of total n tirals

$$= P(\leq r) = {}^nC_0\, q^n + {}^nC_1\, p^1\, q^{n-1} + {}^nC_2\, p^2\, q^{n-2} + ... + {}^nC_r\, p^r\, q^{n-r}$$

Illustration 21: ***Find the minimum number of tosses of a pair of dice, so that the probability of getting the sum of the numbers on the dice equal to 7 on atleast one toss, is greater than 0.95. (Given $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$).***

Solution : n(S) = 36

Let E be the event of getting the sum of digits on the dice equal to 7, then n(E) = 6.

$P(E) = \frac{6}{36} = \frac{1}{6} = p$, then $P(E') = q = \frac{5}{6}$

probability of not throwing the sum 7 in first m trails = q^m .

$\therefore$ P(at least one 7 in m throws) = $1 - q^m = 1 - \left(\frac{5}{6}\right)^m$.

According to the question $1-\left(\frac{5}{6}\right)^m > 0.95$

$\Rightarrow \quad \left(\frac{5}{6}\right)^m > 0.05$

$\Rightarrow \quad m\{\log_{10} 5 - \log_{10} 6\} < \log_{10} 1 - \log_{10} 20$

$\therefore$ m > 16.44

Hence, the least number of trails = 17.

Illustration 22: ***A coin is tossed n times: What is the chance that the head will present itself an odd number of times?***

Solution : In one throw of a coin, the number of possible ways is 2 since either head (H) or tail (T) may appear. In two throws of a coin, the total no. of ways in $2 \times 2 = 2^2$, since corresponding to each way of the first coin there are 2 ways of second. Similarly in three throws of a coin, the number of ways is 2^3 and thus in n throws, the number of total ways = 2^n.

The favourable no. of ways = the no. of ways in which head will occur once or thrice or 5 times, and so on.

$$= {}^nC_1 + {}^nC_3 + {}^nC_5 ...... = 2^{n-1}$$

Hence required probability $P = \frac{2^{n-1}}{2^n} = \frac{1}{2}$.

Illustration 23: ***In five throws with a single die, what is the chance of throwing (a) three aces exactly (b) three aces at least?***

Solution : Let p be the chance of throwing an ace and q be the chance of not throwing an ace in a single throw with one die then p = 1/6 and q = 5/6.

Now the chances of throwing no ace, one ace, two ace etc. in five throws with a single die are the first, second, third terms etc. in the binomial expansion

$$(p+q)^5 = q^5 + {}^5C_1 q^4 p + {}^4C_2 q^3 p^2 + {}^5C_3 q^2 p^3 + {}^5C_4 q^1 p^4 + p^5$$

Hence (a) chance of 3 aces exactly

$= {}^5C_3 q^2 p^3 = 10(5/6)^2 (1/6)^3 = 125/3888$.

(b) Chance of throwing three aces at least

$= {}^5C_3 q^2 p^3 + {}^5C_4 q p^4 + p^5$

$= 10(5/6)^2 (1/6)^3 + 5(5/6) (1/6)^4 + (1/6)^5$

$= (1/6)^5 [250 + 25 + 1] = 23/648$.

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EXERCISE – 4

1. Numbers are selected at random, one at a time, from the two digit numbers 00, 01, 02, . . . , 99 with replacement. An event E occurs if and only if the product of the two digits of a selected number is 18. If four numbers are selected, find the probability that the event E occurs at least 3 times.
2. Find the least number of times must a fair die be tossed, in order to have a probability of at least $\frac{91}{216}$, of getting atleast one six.
3. A die is thrown 7 times. What is the probability that an odd number turns up (i) exactly 4 times (ii) atleast 4 times.

11. GEOMETRICAL PROBABILITY

If the number of points in the sample space is infinite, then we can not apply the classical definition of probability. For instance, if we are interested to find the probability that a point selected at random in a circle of radius r, is nearer to the centre then the circumference, we can not apply the classical definition of probability. In this case we define the probability as follows :

$$\mathbf{P = \frac{Measure\ of\ the\ favourable\ region}{Measure\ of\ the\ sample\ space}},$$

where measure stands for length, area or volume depending upon whether S is one-dimensional, two – dimensional or three – dimensional region.

Thus the probability, that the chosen point is nearer to the centre than the circumference =

$$= \frac{\text{area of the region of the favourable point}}{\text{area of the circle}} = \frac{\pi(r/2)^2}{\pi r^2} = \frac{1}{4}.$$

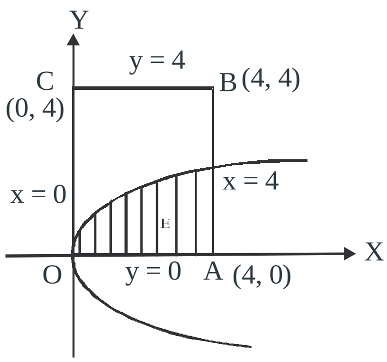
Illustration 24: ***Two numbers $x \in R$ and $y \in R$ are selected such that $x \in [0, 4]$ and $y \in [0, 4]$. Find the probability that the selected numbers satisfy $y^2 \le x$.***

Solution : n(S) = area of the square OABC = $4 \times 4 = 16$.

The selected numbers x, y will satisfy $y^2 \le x$ if the point (x, y) is an interior point of the parabola $y^2 = x$.

$\therefore$ n(E) = The area of that portion of the square which falls in the interior of the parabola $y^2 = x$

$$= \int_0^4 \sqrt{x}\,dx = \frac{16}{3}.$$



Required probability $= \frac{n(E)}{n(S)} = \frac{16/3}{16} = \frac{1}{3}$.

Illustration 25: ***Two point P, Q are taken at random on a straight line OA of lenght a, show that the chance that PQ > b, where b > a is $\left(\frac{a-b}{a}\right)^2$.***

Solution : The points are as likely to fall in the order O, A, C, P as in the order O, C, A, P. We may therefore suppose that C is to the right of A.

Draw OP' at right angles to OP and equal to it. Complete the figure as in the diagram, where

$$OL = AB' = b$$

If δx is small, the number of cases in which the distance of A from O lies between x and $x + \delta x$ and C is in AP, is represented by δx . AP i.e. by the area of the shaded rectangle. Of these, the favourable cases are those in which C lies in BP, and their number is represented by the upper part of the shaded rectangle cut off by LM. Hence the total number of cases is represented by area of the triangle of OPP', and the total number of favourable cases by the of the triangle LMP',

$\therefore$ the required chance $= \frac{\Delta\ LMP'}{\Delta\ OPP'} = \left(\frac{a-b}{a}\right)^2$

■■

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SOLVED EXAMPLES

SECTION - I

SUBJECTIVE QUESTIONS

Problem 1: A coin is tossed m + n times (m > n). Show that the probability of at least m consecutive heads come up is $\frac{n+2}{2^{m+2}}$.

Solution : Let H, T and S be the events "head turns up", "tail turns up" and "head or tail turns up"

Then P(H) = P(T) = $\frac{1}{2}$ and P(S) = 1

Since the given event is "at least m consecutive heads turn up", therefore in any favorable out come there are m consecutive heads and the rest are any of head or tail

Consider the events

$A_1 = \left\{\underbrace{H, H, H, \ldots, H}_{m\text{ times}}, \underbrace{S, S, S, \ldots, S}_{n\text{ times}}\right\}$ with $P(A_1) = \frac{1}{2^m}.1^n = \frac{1}{2^m}$

$A_2 = \left\{T, \underbrace{H, H, H, \ldots, H}_{m\text{ times}}, \underbrace{S, S, S, \ldots, S}_{n-1\text{ times}}\right\}$ with $P(A_2) = \frac{1}{2}.\frac{1}{2^m}.1^{n-1} = \frac{1}{2^{m+1}}$

$A_3 = \left\{S, T, \underbrace{H, H, H, \ldots, H}_{m\text{ times}}, \underbrace{S, S, S, \ldots, S}_{n-2\text{ times}}\right\}$ with $P(A_3) = 1.\frac{1}{2}.\frac{1}{2^m}.1^{n-2} = \frac{1}{2^{m+1}}$

. . . $A_{n+1} = \left\{\underbrace{S, S, S, \ldots, S}_{n-1\text{ times}}, T, \underbrace{H, H, H, \ldots, H}_{m\text{ times}}\right\}$ With $P(A_{n+1}) = 1^{n-1}.\frac{1}{2}.\frac{1}{2^m} = \frac{1}{2^{m+1}}$

The given event is $A_1 \cup A_2 \cup A_3 \cup A_{n+1}$. As A_1, A_2, A_3, . . ., A_{n+1} are pair – wise mutually exclusive.

The required probability

$$= P(A_1) + P(A_2) + P(A_3) + \ldots + P(A_{n+1}) = \frac{1}{2^m} + \underbrace{\frac{1}{2^{m+1}} + \frac{1}{2^{m+1}} + \ldots + \frac{1}{2^{m+1}}}_{n-\text{times}}$$

$$= \frac{1}{2^m} + \frac{n}{2^{m+1}} = \frac{2+n}{2^{m+1}}.$$

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Problem 2: There are four six faced dice such that each of two dice bears the numbers 0, 1, 2, 3, 4 and 5 and the other two dice are ordinary dice bearing numbers 1, 2, 3, 4, 5 and 6. If all the four dice are thrown, find the probability that the total of numbers coming up on all the dice is 10.

Solution : Total number of sample points in the sample space = $6^4 = 1296$

Number of sample points in favour of the event

= Coefficient of x^{10} in the expansion of $(1 + x + x^2 + \ldots + x^5)^2 (x + x^2 + \ldots + x^6)^2$

= Coefficient of x^{10} in the expansion of $x^2(1 + x + x^2 + \ldots + x^5)^4$

= Coefficient of x^8 in the expansion of $(1 + x + x^2 + \ldots + x^5)^4$

= Coefficient of x^8 in the expansionof $\left(\frac{1-x^6}{1-x}\right)^4$

= Coefficient of x^8 in the expansion of $(1 - x^6)^4 (1 - x)^{-4}$

= Coefficient of x^8 in the expansion of $(1 - 4x^6)\left(1+4x+\frac{4\times5}{2!}x^2+\frac{4\times5\times6}{3!}x^3+\ldots\right)$

$= 1 \times {}^{11}C_8 - 4\times {}^5C_2 = 125$.

$\therefore$ Required probability = $\frac{125}{1296}$.

Problem 3: If m things are distributed among 'a' men and 'b' women, show that the probability that the number of things received by men is odd, is $\frac{1}{2}\left\{\frac{(b+a)^m-(b-a)^m}{(b+a)^m}\right\}$.

Solution : A particular thing is received by a man with probability p = $\frac{a}{a+b}$ and by a woman with probability q = $\frac{b}{a+b}$. If distributing a single object is an experiment, then this experiment is repeated m time. The required probability = ${}^mC_1.p.q^{m-1} + {}^mC_3.p^3.q^{m-3} + {}^mC_5.p^5.q^{m-5} + \ldots$

$$= \frac{(q+p)^m-(q-p)^m}{2} = \frac{1}{2}\left[1-\left(\frac{b-a}{b+a}\right)^m\right] = \frac{1}{2}\left\{\frac{(b+a)^m-(b-a)^m}{(b+a)^m}\right\}.$$

Problem 4: Let p be the probability that a man aged x years will die within a year. Let $A_1, A_2, \ldots, A_n$ be n men each aged x years. Find the probability that out of these n men A_1 will die with in a year and is first to die.

Solution : P(no one among $A_1, A_2 \ldots, A_n$ dies within a year) = $(1 - p)^n$

P (at least one among $A_1, A_2, \ldots, A_n$ dies within a year) = $1 - (1 - p)^n$

P(A_1dies within a year and is first to die) = $\frac{1}{n}\left[1-(1-p)^n\right]$.

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Problem 5 : A box contains 2 fifty paise coins, 5 twenty five paise coins and a certain fixed number $N(\geq 2)$ of ten and five paise coins. Five the probability that the total value of these 5 coins is less than one rupee and fifty paise.

Solution : Total number of ways of drawing 5 coins from

$N + 7$ coins $= {}^{N+7}C_5$. ...(i)

Let E be the event that the value of 5 coins is less one rupee & fifty paisa. Then E'(value of 5 coins ≥ 1.50) has the following cases

	50 p. (2)	25 p. (5)	5 or 10 p. (N > 2)	ways
I	2	3	–	$1.{}^5C_3$
II	2	2	1	${}^5C_3.{}^NC_1$
III	1	4	–	${}^2C_1.{}^5C_4$

Total ways = 20 + 10N = 10(N + 2) ...(ii)

$\therefore \quad P(E) = 1 - \frac{10(N+2)}{{}^{(N+7)}C_5}$

Problem 6: 5 girls and 10 boys sit at random in a row having 15 chairs numbered as 1 to 15, Find the probability that end seats are occupied by the girls and between any two girls odd number of boys sit.

Solution : There are four gaps in between the girls where the boys can sit. Let the number of boys in these gaps be 2a + 1, 2b + 1, 2c + 1, 2d + 1, then

$2a + 1 + 2b + 1 + 2c + 1 + 2d + 1 = 10$

or, $a + b + c + d = 3$

The number of solutions of above equation

= coefficient of x^3 in $(1 - x)^{-4} = {}^6C_3 = 20$

Thus boys and girls can sit in $20 \times 10! \times 5$ ways.

Hence the required probability = $\frac{20 \times 10! \times 5!}{15!}$

Problem 7: A chess game between two grandmasters A and B is won by whoever first wins A total of 2 games. A's chances of winning, drawing or losing a particular game are p, q and r, respectively. The games are independent and p + q + r = 1. Show that the probability that A wins the match after (n + 1) games ($n \geq 1$) is

$p^2[nq^{n-1} + n(n-1)\, rq^{n-2}]$

Use this result to show that the probability that A wins the match is

$$\frac{p^2(p+3r)}{(p+r)^3}$$

Find the probability that there is no winner.

Solution : A can win the match after (n + 1) games $(n \geq 1)$ in the following two mutually exclusive ways:

(i) A wins exactly one of the first n games and draws the remaining (n – 1), or

(ii) A wins exactly one of the first n games, loses exactly one of the first n games and draws the remaining (n – 2).

We have P(i) = $^nP_1pq^{n-1}$ and P(ii) = $^nP_2pq^{n-2}r$. Thus, the probability that A wins at the end of the (n + 1)th game is

$p(^nP_1pq^{n-1} + {}^nP_2pq^{n-2}r) = p^2[nq^{n-1} + n(n-1)\,rq^{n-2}]$

The probability that A wins the match is

$$\sum_{n=1}^{\infty} p^2[nq^{n-1} + n(n-1)rq^{n-2}] = p^2 \sum_{n=1}^{\infty} nq^{n-1} + p^2 r \sum_{n=1}^{\infty} n(n-1)q^{n-2}$$

We know that

$$\sum_{n=0}^{\infty} q^n = \frac{1}{1-q} \quad (0 < p < 1)$$

Differentiating both sides w.r.t q, we get

$$\sum_{n=1}^{\infty} nq^{n-1} = \frac{1}{(1-q)^2} \text{ and } \sum_{n=1}^{\infty} n(n-1)q^{n-2} = \frac{2}{(1-q)^3}$$

Thus, the probability that A wins the match is

$$p^2.\frac{1}{(1-q)^2} + \frac{2p^2r}{(1-q)^3} = \frac{p^2}{(p+r)^2} + \frac{2p^2r}{(p+r)^3} \qquad [\because\ p + q + r = 1]$$

$$= \frac{p^2(p+r) + 2p^2r}{(p+r)^3} = \frac{p^2(p+3r)}{(p+r)^3}$$

Similarly, the probability that B wins the match is

$$\frac{r^2(r+3p)}{(r+p)^3}$$

$$\therefore \qquad P(\text{A wins}) + P(\text{B wins}) = \frac{p^2(p+3r)}{(p+r)^3} + \frac{r^2(r+3p)}{(r+p)^3}$$

$$= \frac{p^3 + 3p^2r + 3pr^2 + r^3}{(p+r)^3} = \frac{(p+r)^3}{(p+r)^3} = 1$$

Problem 8 : (i) If four squares are chosen at random on a chess board, find the probability that they lie on a diagonal line.

(ii) If two squares are chosen at random on a chess board, what is the probability that they have exactly one corner in common?

(iii) If nine squares are chosen at random on a chess board, what is the probability that they form a square of size 3×3?

Solution : (i) Total no. of ways = $^{64}C_4$

The chess board can be divided into two parts by a diagonal line BD. Now, if we begin to select four squares from the diagonal P_1Q_1, P_2Q_2,...., BD, then we can find no. of squares selected

= $2(^4C_4 + {}^5C_4 + {}^6C_4) = 182$

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and similarly no. of squares for the diagonals chosen parallel to AC = 182

$\therefore$ Total favourable ways = 364

$\therefore$ required probability = $\dfrac{364}{{}^{64}C_4}$

(ii) Total ways = 64×63

Now if first square is in one of the four corners, the second square can be chosen in just one way = (4) (1) = 4

If the first square is one of the 24 non-corner squares along the sides of the chess board, the second square can be chosen in two ways = (24) (2) = 48.

Now, if the first square is amy of the 36 remaining squares, the second square can be chosen in four ways = (36) (4) = 144

$\therefore$ favourable ways = 4 + 48 + 144 = 196

$\therefore$ required probability = $\dfrac{196}{64\times63}=\dfrac{7}{144}$

(iii) Total ways = ${}^{64}C_9$

A chess board has 9 horizontal and 9 vertical lines. We see that a square of size 3×3 can be formed by choosing four consecutive horizontal and vertical lines.

Hence favourable ways = $({}^6C_1)$ $({}^6C_1)$ = 36

$\therefore$ **Required probability =** $\dfrac{36}{{}^{64}C_9}$

Problem 9: If n positive integers taken at random are multiplied together, show that the probability that the last digit of the product is 5 is $\dfrac{5^n-4^n}{10^n}$

and that the probability of the last digit being 0 is $\dfrac{10^n-8^n-5^n+4^n}{10^n}$

Solution : Let n positive integers be x_1, x_2, ..., x_n. Let a = $x_1.x_2....x_n$.

Let S be the sample space, since the last digit in each of the numbers, x_1, x_2,..., x_n can be any one of the digits 0, 1, 3, ..., 9 (total 10)

$\therefore$ n(S) = 10^n

Let E_2 and E_2 be the events when the last digit in a is 1, 3, 5, 7 or 9 and 1, 3, 7 or 9 respectively

$\therefore$ $n(E_1) = 5^n$ and $n(E_2) = 4^n$

and let E be the event that the last digit in a is 5.

$\therefore$ $n(E) = n(E_1) - n(E_2) = 5^n - 4^n$

Hence required probability

$$P(E)=\frac{n(E_1)}{n(S)}=\frac{5^n-4^n}{10^n}$$

Second part. Let E_3 and E4 be the events when the last digit in a is 1, 2, 3, 4, 6, 7, 8 or 9 and 0 respectively.

Then $n(E_4)$ $n(S) - n(E_3) - n(E)$

$= 10^n - 8^n - (5^n - 4^n) = 10^n - 8^n - 5^n + 4^n$

$\therefore$ Required probability,

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{10^n - 8^n - 5^n + 4^n}{10^n}$$

Problem 10: A is a set containing n elements. A subset P_1 of A is chosen at random and the set A is then reconstructed by replacing the elements of P_1. A subset P_2 of A is now chosen at random and again the set A is reconstructed by replacing the elements of P_2. This process is continued by choosing subsets $P_2, P_3, ..., P_m$, with $m \geq 2$. Find the probability that

(1) $P_i \cap P_j = \phi$ for $i \neq j$ and $i, j = 1, 2, ..., m$;

(2) $P_1 \cap P_2 \cap ... \cap P_m = \phi$

Solution : Let $A = \{a_1, a_2, ..., a_n\}$

Let S be the sample space and E_1 be the event that $P_i \cap P_j = \phi$ for $i \neq j$ and E_2 be the event that $P_1 \cap P_2 \cap ... \cap P_m = \phi$.

$\therefore$ Number of subsets of A = 2^n

$\therefore$ each $P_1, P_2, ..., P_m$ can be selected in 2^n ways.

$\therefore$ n(S) = Total of selections of $P_1, P_2, ..., P_m$

$= (2^n)^m = 2^{mn}$

(1) When $P_i \cap P_j = \phi$ for $i \neq j$, element of A either does not belong to any of subsets, or it belongs to at most one of them. Therefore, there are m + 1 choices for each element

$\therefore$ $n(E_1) = (m + 1)^n$

$\therefore$ Required probability $P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{(m+1)^m}{2^{mn}}$

(2) When $P_1 \cap P_2 \cap ... \cap P_m = \phi$

i.e., an element of A does not belong to all the subsets. There are 2^m ways an element does not belong to a subset. On the other hand, there is only one way the element can belong to the intersection. Therefore $(2m - 1)$ elements does not belong to the intersection.

$n(E_2)$ = Number of favourable ways for all n elements

$= (2^m - 1)^n$.

Hence the required probability

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{(2^m - 1)^n}{2^{mn}}$$

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SECTION - II

OBJECTIVE QUESTIONS (SINGLE AND MULTIPLE CHOICE)

Problem 1: A natural number is chosen at random from the first one hundred natural numbers. The probability that $\frac{(x-20)(x-40)}{x-30} < 0$ is

(a) $\frac{1}{50}$ (b) $\frac{3}{50}$

(c) $\frac{3}{25}$ (d) $\frac{7}{25}$

Solution : **Ans. (d)**

From the wavy curve method, given inequality is satisfied for x < 20 or 30 < x < 40.

$\therefore$ Number of favourable outcomes = 28

Required probability = $\frac{28}{100} = \frac{7}{25}$.

Problem 2: If $\frac{1+3p}{3}, \frac{1-p}{2}$ and $\frac{1-p}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is

(a) ϕ (b) $\left[\frac{1}{2}, \frac{1}{3}\right]$

(c) [0, 1] (d) none of these

Solution : **Ans. (a)**

We have

$0 \le \frac{1+3p}{3}, \frac{1-p}{2}$ and $\frac{1-2p}{2} \le 1 \Rightarrow p \in \left[-\frac{1}{3}, \frac{1}{2}\right]$. Further if the events

(say E_1, E_2 and E_3) are exclusive, then its necessary and sufficient condition is

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) \Rightarrow P(E_1 \cup E_2 \cup E_3) = \frac{8-3p}{6}$$

$$\Rightarrow 0 \le \frac{8-3p}{6} \le 1$$

$$\Rightarrow p \in \left[\frac{2}{3}, \frac{8}{3}\right]$$

Hence the required set is ϕ.

Problem 3: For independent events $A_1, \ldots, A_n$, $P(A_i) = \frac{1}{i+1}$, $i = 1, 2, \ldots, n$. Then the probability that none of the events will occur is

(a) $n/(n + 1)$ (b) $n - 1/(n + 1)$

(c) $1/(n + 1)$ (d) none of these

Solution : **Ans. (c)**

$$P(\text{non occurrence of } (A_i)) = 1 - \frac{1}{(i+1)} = \frac{i}{(i+1)}$$

$$\therefore \quad P(\text{non occurrence of any of events}) = \left(\frac{1}{2}\right).\left(\frac{2}{3}\right)\ldots\left\{\frac{n}{(n+1)}\right\} = \frac{1}{(n+1)}.$$

Problem 4: A bag contains a large number of white and black marbles in equal proportions. Two samples of 5 marbles are selected (with replacement) at random. The probability that the first sample contains exactly 1 black marble, and the second sample contains exactly 3 black marbles, is

(a) $\frac{25}{512}$ (b) $\frac{15}{32}$

(c) $\frac{15}{1024}$ (d) $\frac{35}{256}$

Solution : **Ans. (a)**

Let the number of marble be 2n (where n is large)

$$\text{Required probability} = \lim_{n\to\infty} \frac{n \times {}^{n}C_4}{{}^{2n}C_5} \times \frac{{}^{n}C_3 \times {}^{n}C_2}{{}^{2n}C_5}$$

$$= \lim_{n\to\infty} \frac{n \times n(n-1)(n-2)(n-3)}{4!} \times \frac{n(n-1)(n-2)}{3!} \times \frac{n(n-1)}{2!} \times \frac{(5)^2\left((2n-5)!\right)^2}{(2n!)^2}$$

$$= \lim_{n\to\infty} \frac{n^4(n-1)^3(n-2)^2(n-3)\left((2n-5)!\right)^2 \times 5\times5\times4\times3!}{3!2!(2n!)^2}$$

$$= \lim_{n\to\infty} \frac{50.n^4(n-1)^3(n-2)^2(n-3)}{\left(2n(2n-1)(2n-2)(2n-3)(2n-4)\right)^2} = \frac{50}{1024} = \frac{25}{512}.$$

Problem 5 : The probability that in a group of N (< 365)people, at least two will have the same birthday is

(a) $1 - \frac{(365)!}{(365-N)!(365)!}$ (b) $\frac{(365)^N(365)!}{(365-N)!} - 1$

(c) $1 - \frac{(365)^N(365)!}{(365+N)!}$ (d) none of these

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Solution : **Ans. (d)**

Let $\overline{A}$ be the event of different birthdays. Each can have birthday in 365 ways, so N persons can have their birthdays in 365^N ways. Number of ways in which all have different birthdays $= {}^{365}P_N$

$$\therefore \quad P(A) = 1 - P(\overline{A}) = 1 - \frac{{}^{365}P_N}{(365)^N} = 1 - \frac{(365)!}{(365)^N (365-N)!}.$$

Problem 6: A draw two cards at random from a pack of 52 cards. After returning them to the pack and shuffling it, B draws two cards at random. The probability that there is exactly one common card, is

(a) $\frac{25}{546}$ (b) $\frac{50}{663}$

(c) $\frac{25}{663}$ (d) none of these

Solution : **Ans. (b)**

Let S be the sample space and let E be the required event, then n(S) = $\left({}^{52}C_2\right)^2$. For the number of elements in E, we first choose a card (which we want common) and then from the remaining cards (51 in numbers) we choose two cards and distribute them among A and B in 2! ways. Hence n (E) = ${}^{52}C_1 . {}^{51}C_2 . 2!$. Thus P(E) = $\frac{50}{663}$.

Problem 7: A company has two plants to manufacture televisions. Plant I manufacture 70% of televisions and plant II manufacture 30%. At plant I, 80% of the televisions are rated as of standard quality and at plant II, 90% of the televisions are rated as of standard quality. A television is chosen at random and is found to be of standard quality. The probability that it has come from plant II is

(a) $\frac{17}{50}$ (b) $\frac{27}{83}$

(c) $\frac{3}{5}$ (d) none of these

Solution : **Ans. (b)**

Let E be the event that a television chosen randomly is of standard quality. We have to find

$$P(II/E) = \frac{P(E/II).P(II)}{P(E/I).P(I) + P(E/II).P(II)}$$

$$= \frac{(9/10)(3/10)}{(4/5)(7/10) + (9/10)(3/10)} = 27/83$$

Problem 8 : $x_1, x_2, x_3, \ldots, x_{50}$ are fifty real numbers such that $x_r < x_{r+1}$ for r = 1, 2, 3, . . ., 49. Five numbers out of these are picked up at random. The probability that the five numbers have x_{20} as the middle number is

(a) $\dfrac{{}^{20}C_2 \times {}^{30}C_2}{{}^{50}C_5}$ (b) $\dfrac{{}^{30}C_2 \times {}^{19}C_2}{{}^{50}C_5}$

(c) $\dfrac{{}^{19}C_2 \times {}^{31}C_3}{{}^{50}C_5}$ (d) none of these

Solution : **Ans. (b)**

$n(S) = {}^{50}C_5$ = Total number of ways

$n(E) = {}^{30}C_2 \times {}^{19}C_2$ = Number of favourable ways

$$P(E) = \frac{{}^{30}C_2 \times {}^{19}C_2}{{}^{50}C_5}.$$

Problem 9 : The probability that a man can hit a target is $\frac{3}{4}$. He tries 5 times. The probability that he will hit the target at least three times is

(a) $\dfrac{291}{364}$ (b) $\dfrac{371}{461}$

(c) $\dfrac{471}{502}$ (d) $\dfrac{459}{512}$

Solution : **Ans. (d)**

$$P = \frac{3}{4}, q = \frac{1}{4},\ n = 5$$

$$\text{Required probability} = {}^5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 + {}^5C_4\left(\frac{3}{4}\right)^4\cdot\left(\frac{1}{4}\right) + {}^5C_5\left(\frac{3}{4}\right)^5 = \frac{459}{512}.$$

Problem 10 : A die is thrown 7 times. The chance that an odd number turns up at least 4 times, is

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{2}$

(c) $\dfrac{1}{8}$ (d) none of these

Solution : **Ans. (b)**

For at least 4 successes, required probability

$$= {}^7C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 + {}^7C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2 + {}^7C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^1 + {}^7C_7\left(\frac{1^7}{2}\right) = \frac{1}{2}.$$

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Problem 11 : Let p be the probability that in a pack of cards two kings are adjacent and q be the probability that no two kings are together then

(a) p = q (b) p < q

(c) p + q = 1 (d) $q = \dfrac{48 \times 47 \times 46}{52 \times 51 \times 50}$

Solution : **Ans. (c), (d)**

We will first find q (it is difficult to find p !)

q = p (no two kings are together)

Now a pack of cards can be kept in 52! ways. The number of ways in which no two kings are adjacent must be

$^{49}C_4 \times 4\,! \times 48\,!$ (there are 49 empty spades of 48 non-kings)

$$\Rightarrow Q = \frac{^{49}C_4 \times 4! \times 48!}{52!} = \frac{48 \times 47 \times 46}{52 \times 51 \times 50}$$

$\Rightarrow$ choice (d) is correct

Now p and q should be complementary

$\Rightarrow$ p + q = 1 $\Rightarrow$ Choice (c) is true.

Problem 12 : If A and B are two events such that $P(A \cup B) \geq \frac{3}{4}$ and $\frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$ then

(a) $P(A) + P(B) \leq \frac{11}{8}$ (b) $P(A) \cdot P(B) \leq \frac{3}{8}$

(c) $P(A) + P(B) \geq \frac{7}{8}$ (d) none of these

Solution : **Ans. (a), (c)**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore\ 1 \geq P(A) + P(B) - P(A \cap B) \geq \frac{3}{4}.$$

As the minimum value of $P(A \cap B) = \frac{1}{8}$, we get

$$P(A) + P(B) - \frac{1}{8} \geq \frac{3}{4} \quad \Rightarrow \quad P(A) + P(B) \geq \frac{1}{8} + \frac{3}{4} = \frac{7}{8}$$

Problem 13 : If A and B are two independent events such that P(A) = 1/2 and P(B) = 1/5, then

(a) $P(A \cup B) = 3/5$ (b) $P(A \mid B) = 1/2$

(c) $P(A \mid A \cup B) = 5/6$ (d) $P(A \cap B \mid A' \cup B')] = 0$

Solution : **Ans. (a), (b), (c), (d)**

We have

$P(A \cup B) = 1 - P(A' \cap B') = 1 - P(A')P(B')$ [$\because$ A and B are independent]

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$$=1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right)=1-\frac{2}{5}=\frac{3}{5}$$

Next, $P(A|B)=P(A)=1/2$, because A and B are independent. Also,

$$P(A|A\cup B)=\frac{P[A\cap(A\cup B)]}{P(A\cup B)}=\frac{P(A)}{P(A\cup B)}=\frac{1/2}{3/5}=\frac{5}{6}$$

Lastly, since $A'\cup B'=(A\cup B)'$,

$P[(A\cap B)|(A'\cup B')]=0$.

Problem 14 : For two events A and B, if P(A) = P(A|B) = 1/4 and P(B|A) = 1/2, then

(a) A and B are independent

(b) A and B are mutually exclusive

(c) $P(A'|B)=3/4$

(d) $P(B'|A')=1/2$

Solution : **Ans. (a), (c), (d)**

We have $P(A)=P(A|B)=\dfrac{P(A\cap B)}{P(B)} \Rightarrow P(A\cap B)=P(A)P(B)$

Therefore, A and B are independent. Since

$P(A\cap B)=P(A)P(B|A)=(1/4)(1/2)=1/8\neq 0$

A and B cannot be mutually exclusive. As A and B are independent

$P(A'|B)=P(A')=1-P(A)=1-1/4=3/4$

Since A and B are independent,

Problem 15 : A student appears for tests I, II, and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II, III are p, q and 1/2, respectively. If the probability that the student is successful is 1/2, then

(a) p = 1, q = 0

(b) p = 2/3, q = 1/2

(c) p = 3/5, q = 2/3

(d) there are infinitely many values of p and q.

Solution : **Ans. (a), (b), (c), (d)**

Let A, B and C be the events that the student is successful in tests, I, II and III, respectively. Then, P(the student is successful)

$=P[(A\cap B\cap C')\cup(A\cap B'\cap C)\cup(A\cap B\cap C)]$

$=P(A\cap B\cap C')+P(A\cap B'\cap C)+P(A\cap B\cap C)$

$=P(A)P(B)P(C')+P(A)P(B')P(C)+P(A)P(B)P(C)$ [$\because$ A, B and C are independent]

= pq(1 – 1/2) + p(1 – q) (1/2) + (pq) (1/2)

$$=\frac{1}{2}[pq+p(1-q)+pq]=\frac{1}{2}p(1+q)$$

$$\therefore\quad \frac{1}{2}=\frac{1}{2}p(1+q) \Rightarrow p(1+q)=1$$

This equation is satisfied for all pairs of values in (a), (b) and (c). Also, it is satisfied for infinitely many values of p and q. For instance, when p = n/(n + 1) and q = 1/n, where n is any positive, integer.

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SECTION - III

COMPREHENSIVE QUESTIONS

Passage – I.

If f and g are two differentiable functions $x \in R$ satisfying the relation

f(x) = 2x – 1, g(x) = 20 – 3x

and also | x + y | < | x | + | y |, inequality sign holds when x and y both have opposite sign. If | x – y | > | x | – | y |, inequality sign holds when (x – y) and y have opposite sign

1. If a natural number ‘x’ is chosen at random from the first twenty five natural numbers. Find the probability that f(x).g(x) > 0.

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{6}{25}$ (d) $\frac{7}{25}$

2. If a natural number ‘x’ is selected at random from the first twenty five natural numbers. Find the probability that $\frac{f(x-1).g(x-1)}{f(x)-1} > 0$

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{6}{25}$ (d) $\frac{7}{25}$

3. If a natural number is chosen at random from the first twenty five natural numbers. Find the probability that domain of the function ‘F’ is defined where

$F(x) = \sqrt{f(x)-1} + \sqrt{g(x)-x}$,

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$

(c) $\frac{1}{8}$ (d) $\frac{1}{5}$

Sol. Ans. 1 – (c), 2 – (c), 3 – (d)

1. f(x) . g(x) = (2x – 1)(20 – 3x) > 0

By wavy curve method,

E = event that g(x) . f(x) > 0 = {1, 2, 3, 4, 5, 6}

$\therefore$ n(E) = 6 $\Rightarrow$ n(S) = 25

$\therefore \quad P(E) = \frac{6}{25} \quad \therefore$ Ans. (c)

2. $\frac{f(x-1)g(x-1)}{f(x)-1} = \frac{\{2(x-1)-1\}\{20-3(x-1)\}}{2x-1-1} > 0$

$\frac{(2x-3)(23-3x)}{2(x-1)} > 0$

By wavy curve method:
E = {2, 3, 4, 5, 6, 7} $\therefore$ n(E) = 6

$P(E) = \frac{6}{25}$ $\therefore$ Ans. (c)

3. $f(x) = \sqrt{2x-1-1} + \sqrt{20-3x-x}$

$= \sqrt{2(x-1)} + \sqrt{20-4x}$

f(x) is defined if $x - 1 \geq 0 \Rightarrow x \geq 1$

and $20 - 4x \geq 0 \Rightarrow x \leq 5$

$\therefore$ $x \in [1, 5]$
E = event for f(x) defined
E = {1, 2, 3, 4, 5} $\therefore$ n(E) = 5

$\therefore$ $P(E) = \frac{5}{25} = \frac{1}{5}$

Passage – II.

Statement I :

A number is chosen from each of the two sets {1, 2, 3, 4, 5, 6, 7, 8, 9} and {1, 2, 3, 4, 5, 6, 7, 8, 9}. If p_1 is one event that the sum of the two numbers be 10 and p_2 is another event that their sum be 8

Statement II :

One bag contains 6 blue balls and 5 green balls and another bag contains 7 blue and 4 green balls. Two balls are drawn, one from each bag. Then three events

$p_3 \to$ both balls are blue

$p_4 \to$ both balls are green

$p_5 \to$ one ball is blue and other green

Statement III :

An unbiased coin is tossed if head appears statement I is used and if tail appears statement II is used, then

4. $\boldsymbol{P}(p_1 + p_2)$ is equal to

(a) $\frac{16}{81}$ (b) $\frac{4}{81}$

(c) $\frac{8}{81}$ (d) none of these

5. $\boldsymbol{P}(\overline{p_3})$ is equal to

(1) $\frac{69}{121}$ (b) $\frac{79}{242}$

(c) $\frac{69}{242}$ (d) $\frac{79}{121}$

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6. If it is given tail appears, the probability of p_4 event is equal to

(a) $\frac{20}{121}$ (b) $\frac{21}{121}$

(c) $\frac{10}{121}$ (d) none of these

Sol. Ans. 4 – (c), 5 – (b), 6 – (a)

4. Probability that statement I is used = $1/2$

Now if I used then

$$p_1 = \frac{9}{{}^9C_1 \times {}^9C_1} = \frac{1}{9} (\text{sum being } 10)$$

Similarly $p_2 = \frac{7}{{}^9C_1 \times {}^9C_1} = \frac{7}{81} (\text{sum being } 8)$

So $p_1 + p_2 = 1/2\left(\frac{1}{9} + \frac{7}{81}\right)$

5. P_3 = both balls are blue

Probability of both balls are blue = $\frac{6}{11} \times \frac{7}{11} = \frac{42}{121}$

Probability that statement II is used = $\frac{1}{2}$

$$P(\bar{p}_3) = \frac{1}{2}(1 - p_3) = \frac{1}{2}\left(1 - \frac{42}{121}\right) = \frac{79}{242}$$

6. Given that tail appears

i.e. probability of statement II is used = 1.

P_4 = both balls are green

Probability of P_4 = $\frac{5}{11} \times \frac{4}{11} = \frac{20}{121}$

MATCH THE FOLLOWING

Problem:

	List – I	List – II
(a)	If $P(A_1 \cup A_2) = 1 - P(A'_1)P(A_2')$, then the events A_1 and A_2 are	(P) Mutually exclusive
(b)	Two fair dice are thrown. Let A be the event that the first die shows an even number and B be the event that the second die shows an odd number, The two events A and B are	(Q) dependent
(c)	A card is drawn from a pack of 52 cards. If A be the card of diamond, B be the card of an ace and $A \cap B$ be card is of ace of diamond. Then events A & B are	(R) independent
(d)	If $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{5}{6}$, then events A and B are	(S) independent and mutually inclusive

Sol. **Ans. (a) – (R), (b) – (R), (c) – (Q), (d) – (P)**

(a) $P(A_1 \cup A_2) = 1 - P(A_1')P(A_2')$

$$P(A_1) + P(A_2) - P(A_1 \cap A_2) = 1 - \{1 - P(A_1)\}\{1 - P(A_2)\}$$

$$= 1 - 1 + P(A_1) + P(A_2) - P(A_1) \cdot P(A_2)$$

$\therefore$ $P(A_1 \cap A_2) = P(A_1) \cdot P(A_2)$ $\quad$ $\therefore$ A_1 & A_2 are independent.

(b) Out come of first die does not effect the out come of 2nd die. hence A and B are independent.

(c) $n(A \cap B) = 1$, $\quad P(A \cap B) = \frac{1}{3}$

Here dimond cards are sample space

$$P(A) = \frac{13}{52} = \frac{1}{4} \quad \therefore\ P(A) \cdot P(B) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{52} \neq P(A \cap B)$$

$\therefore$ dependent

(d) $P(A) + P(B) = \frac{1}{3} + \frac{1}{2} = \frac{2+3}{6} = \frac{5}{6}$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{5}{6} - \frac{5}{6} = 0.$$

ASSERTION AND REASON TYPE QUESTIONS

In the following questions containing two statements viz. Assertion (A) & Reason (R). To choose the correct answer.

Mark (a) if both A and R are correct & R is the correct explanation for A.

Mark (b) if both A & R are correct but R is not correct explanation for A

Mark (c) if A is true but R is false

Mark (d) if A is false but R is true

1. **A :** A coin is tossed thrice. The probability that exactly two heads appear, is $\frac{3}{8}$.

R : Probability of success r times out of total n trials = P(r) = ${}^nC_r P^r q^{n-r}$ where p be the probability of success and q be the probability of failure.

Sol. **Ans. (a)**

$$P(H)=\frac{1}{2},\ P(T)=\frac{1}{2}$$

A = event that getting exactly two heads.

$$P(A)={}^3C_2\left(\frac{1}{2}\right)^2\cdot\left(\frac{1}{2}\right)=\frac{3!}{2!1!}\cdot\frac{1}{2^3}=\frac{3}{2^3}=\frac{3}{8}$$

R is also correct and is correct explanation for A.

2. **A :** The probability of getting a sum 5 or 6 in a single throw of a pair of fair dice is $\frac{1}{4}$.

R : If A and B are two mutually exclusive events, then $P(A\cup B)=P(A)+P(B)$.

Sol. **Ans. (a)**

A = sum is 5 = {(1, 4), (2, 3), (3, 2), (4, 1)}

B = sum is 6 = {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)}

Let X = event that getting sum is 5 or 6.

n(X) = n(A) + n(B) = 9, $P(X)=\frac{9}{36}=\frac{1}{4}$

R is correct and is correct explanation for A.

3. **A :** The probability to draw either a king or a heart card is $\frac{4}{13}$.

R : If A and B be any two events and 0 < P(A) < 1, 0 < P(B) < 1, then

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

Sol. **Ans. (a)**

k = event that out come is king

h = event that out come is heart

$$n(k\cap h)=1,\ P(k\cap h)=\frac{1}{56}$$

X = event that out come is either king or diamond = $k\cup h$

R is correct explanation for A.

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ASSIGNMENTS

SECTION - I

SUBJECTIVE QUESTIONS

LEVEL - I

1. A determinant of the second order is made with the element 0 and 1. What is probability that the determinant made is non-negative?

2. Three faces of a fair die are yellow, two faces red & one blue. The die is tossed 3 times. Find the probability that the colours yellow, red & blue appear in the first, second & the third tosses respectively.

3. If X and Y are independent binomial variates B(5, 1/2) and B(7, 1/2) then find the value of P(X + Y = 3).

4. Three persons A, B and C in order cut a pack of playing cards, replacing them after each cut, on the condition that the first who cuts a card of spade shall win a prize. Find their respective chances.

5. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Find the probability that out of four face values obtained, the minimum face value is not less than 2 & the maximum face value is not greater than 5.

6. Suppose the probability for A to win a game against B is 0.4. If A has the option of playing either a ‘best of 3 games’ or a ‘best of 5 games’ match against B, Which option should A choose so that the probability of his winning the match is higher? No game ends in a draw.

7. A factory A produces 10% defective valves and another factor B produces 20% defective. A bag contains 4 valves of factory A and 5 valves of factory B. If two valves are drawn at random from the bag , find the probability that atleast one valve is defective. Give your answer upto two places of decimals.

8. Let X be a set containing n elements. Two subsets A and B of X are chosen at random. Find the probability that $A \cup B = X$.

9. A man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point.

10. There is 30% chance that it rains on any particular day. What is the probability that there is at least one rainy day within a period of 7 – days? Given that there is at least one rainy day, what is the probability that there are at least two rainy days?

LEVEL - II

1. A die is rolled three times, find the probability of getting a larger number than the previous number.

2. If $a \in [-20, 0]$, then find the probability that the graph of the function $y = 16x^2 + 8(a + 5)x - 7a - 5$ is strictly above the x-axis.

3. If two squares are chosen at random on a chess board, find the chance that they have

   (i) a side in common (ii) a point in common

4. A box contains three coins. Two of them are fair and one two – headed. A coin is selected at random and tossed. If the head appears the coin is tossed again, if a tail appears, then another coin is selected from the remaining coins and tossed.

   (i) Find the probability that head appears twice.

   (ii) If the same coin is tossed twice, find the probability that it was two headed coin.

   (iii) Find the probability that tail appears twice.

5. Two persons each make a single throw with a pair of dice. Then find the probability that the throws are unequal.

6. A contest consists of predicting the results (win, draw, or loss) of 10 football matches. What is the probability that an entry contains at least 5 correct answers?

7. A tosses 2 fair coins and B tosses 3 fair coins. The game is won by the person who throw greater number of heads. In case of tie, the game is continued under the identical rules until some one wins the game. Find the probability of A winning the game.

8. A coin is tossed 7 times. Find the probability of at least 4 consecutive heads.

9. An author writes a good book with a probability of $\frac{1}{2}$. If it is good it is published with a probability of $\frac{2}{3}$. If it is not good, it is published with a probability of $\frac{1}{4}$. Find the probability that the author will get atleast one book published if he writes two books.

10. Three critics review a book. Odds in favour of the book are 5 : 2, 4 : 3 and 3 : 4 respectively for the three critics. Find the probability that majority are in favour of the book.

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LEVEL - III

1. Out of m persons sitting at a round table, the persons, A, B and C are selected at random. Prove that the chance that no two of these are sitting together is (m – 4) (m – 5)/(m – 1) (m – 2).

2. There are 6 red balls and 8 green balls in a bag. 5 balls are drawn out at random and placed in a red box. The remaining 9 balls are put in a green box. What is the probability that the number of red balls in the green box plus the number of green balls in the red box is not a prime number?

3. If A has (n + 1) and B has n fair coins, which they flip, show that the probability that A gets more heads than B is 1/2.

4. On the real number line points a and b are selected at random such that $-2 \le b \le 0$ and $0 \le a \le 3$. Find the probability that the distance d between a and b is greater than 3.

5. Two persons A and B agree to meet at a place between 11 noon to 12 noon. The first one to come waits for 20 minutes and then leaves. If the time of their arrival are independent and at random, then what is the probability of a meeting ?

6. Sixteen players $S_1, S_2, \ldots, S_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength.

   (i) Find the probability that the player S_1 is among the eight winners.

   (ii) Find the probability that exactly one of the two players S_1 and S_2 is among the eight winners.

7. Probability that each of the four men A, B, C and D tells the truth is 1/3. A makes a statement. D says that C says that B says that A's statement is true. What is the probability that A actually made the true statement.

8. Three points P, Q and R are seleted at random from the circumference of a circle. Find the probability that the points lie on a semicircle.

9. If n distinct biscuits are distributed among N beggars, find the chance that a particular begar will get exactly r(< n) biscuits.

10. Three players, A, B and C, toss a coin cyclically in that order (that is A, B, C, A, B, C, A, B,...) till a head shows. Let p be the probability that the coin shows a head. Let a, b, and l be, respectively, the probabilities that A, B and C gets the first head. Prove that

    b = (1 – p) a.

    Determine a, b and l (in terms of p).

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SECTION - II

SINGLE CHOICE QUESTIONS

1. The probability of getting the sum as a prime number when two dice are thrown together, is

(a) $\frac{1}{2}$ (b) $\frac{7}{12}$

(c) $\frac{5}{12}$ (d) none of these

2. A police-man fires six bullets on a dacoit. The probability that the dacoit will be killed by one bullet is 0.6. The probability that dacoit is still alive is

(a) 0.04096 (b) 0.004096

(c) 0.4096 (d) none of these

3. Two dice are thrown. The probability that the number appeared have a sum of 8, if it is known that the second dice always exhibits 4, is

(a) $\frac{5}{6}$ (b) $\frac{1}{6}$

(c) $\frac{2}{3}$ (d) none of these

4. The probability that Krishna will be alive 10 year hence is $\frac{7}{15}$ and Hari will be alive is $\frac{7}{10}$. The probability that both Krishna and Hari will be dead 10 years hence is

(a) $\frac{21}{150}$ (b) $\frac{24}{150}$

(c) $\frac{49}{150}$ (d) $\frac{56}{150}$

5. In a certain town, 40% of the people have brown hair, 25% have brown eyes and 15% have both brown hair and brown eyes. If a person selected at random from the town, having brown hair, the probability that he also has brown eye, is

(a) $\frac{1}{5}$ (b) $\frac{3}{8}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

6. There are 4 envelopes corresponding to 4 letters. If the letters are placed in the envelopes at random, the probability that all the letters are not placed in the right envelopes,is

(a) $\frac{18}{24}$ (b) $\frac{23}{24}$

(c) $\frac{17}{24}$ (d) none of these

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7. A book contains 1000 pages. A page is chosen at random. The probability that the sum of the digits of the marked number on the page is equal to 9, is
(a) 23/500 (b) 11/200
(c) 7/100 (d) none of these

8. Two unbiased dice are rolled. The probability that the sum of the numbers on the two faces is either divisible by 3 or divisible by 4, is
(a) 5/9 (b) 7/17
(c) 9/17 (d) none of these

9. A student takes his examination in four subjects α, β, γ and δ. He estimates his chance of passing in α as $\frac{4}{5}$, in β as $\frac{3}{4}$, in γ as $\frac{5}{6}$ and in δ as $\frac{2}{3}$. To qualify he must pass in α and at least two other subjects. The probability that he qualifies is
(a) 34/90 (b) 61/90
(c) 53/90 (d) none of these

10. 15 coupons are numbered 1, 2, 3, . . . 14, 15. Seven coupons are selected at random, one at a time with replacement. The probabioity that 9 would be the largest number appearing on the selected coupons, is
(a) $\left(\frac{1}{6}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$
(c) $\left(\frac{3}{5}\right)^7$ (d) none of these

11. One mapping is selected at random from all the mappings from the set S = {1, 2, 3, . . ., n} into itself. The probability that the selected mapping is one-to-one is
(a) $1/n^2$ (b) $1/n!$
(c) $(n - 1)!/n^{n-1}$ (d) none of these

12. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 and 1 only. The probability that the value of the determinant chosen is non-zero, is
(a) $\frac{3}{16}$ (b) $\frac{3}{8}$
(c) $\frac{1}{4}$ (d) none of these

13. A box contains 24 identical balls of which 12 are white and 12 are black. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4th time on the 7th draw, is
(a) $\frac{5}{64}$ (b) $\frac{27}{32}$
(c) $\frac{5}{32}$ (d) $\frac{1}{2}$

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14. Two numbers b and c are chosen at random (with replacement) from the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9. The probability that $x^2 + bx + c > 0$ for all $x \in R$, is

(a) 32/81 (b) 44/81

(c) 31/81 (d) none of these

15. An elevator starts with m passengers and stops at n floors $(m \le n)$. The probability that no two passengers alight at the same floor, is

(a) $\frac{{}^nP_m}{m^n}$ (b) $\frac{{}^nP_m}{n^m}$

(c) $\frac{{}^nC_m}{m^n}$ (d) $\frac{{}^nC_m}{n^m}$

16. A bag contains a large number of white and black marbles in equal proportions. Two samples of 5 marbles are selected (with replacement) at random. The probability that the first sample contains exactly 1 black marble, and the second sample contains exactly 3 black marbles, is

(a) $\frac{25}{512}$ (b) $\frac{15}{32}$

(c) $\frac{15}{1024}$ (d) $\frac{35}{256}$

17. If two events A and B are such that $P(A') = 0.3$, $P(B) = 0.4$ and $P(A \cap B') = 0.5$, then $P\left(\frac{B}{A \cup B'}\right) =$

(a) 1/4 (b) 1/5

(c) 3/5 (d) 2/5

18. If the integers m and n are chosen at random from 1 to 100, then the probability that a number of the form $7^n + 7^m$ is divisible by 5 equals

(a) 1/4 (b) 1/2

(c) 1/8 (d) none of these

19. A letter is known to have come either from LONDON or CLIFTON; on the postmark only the two consecutive letters ON are legible. The probability that it came from LONDON, is

(a) $\frac{5}{17}$ (b) $\frac{12}{17}$

(c) $\frac{17}{30}$ (d) $\frac{3}{5}$

20. In an entrance test there are multiple choice questions. There are four possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then the probability that he was guessing, is

(a) $\frac{37}{40}$ (b) $\frac{1}{37}$

(c) $\frac{36}{37}$ (d) $\frac{1}{9}$

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SECTION - III

MULTIPLE CHOICE QUESTIONS

1. A throws n + 1 coins and B throws n coins. Let P(m, k) be the probability that A throws m heads and B throws x heads where $0 \le m \le n+1, 0 \le k \le n$ then

(a) $P(m,k) = \left(\frac{1}{2}\right)^{2n+1}$
(b) $P(m,k) = {}^{n+1}C_m \; {}^{n}C_k \left(\frac{1}{2}\right)^{2n+1}$
(c) $\sum\limits_{0<k<m\le n} \sum P(m,k) = \frac{1}{2}$
(d) $\sum\limits_{0<k<m\le n} \sum P(m,k) = \left(\frac{1}{2}\right)^{n+1}$

2. Let 0 < P(A) < 1, 0 < P(B) < 1 and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$

(a) $P(B/A) = P(B) - P(A)$
(b) $P(A' \cup B') = P(A') + P(B')$
(c) $P(A \cup B)' = P(A')P(B')$
(d) $P(A/B) = P(A)$

3. Cards are drawn one by one without replacement until two aces are drawn. Let P(m) be the probability that the event occurs in exactly m trials then P(m) must be zero at

(a) m = 2
(b) m = 50
(c) m = 51
(d) m = 52

4. If A and B are two events then the value of the determinant chosen at random from all the determinants of order 2 with entries 0 or 1 only, is positive or negative respectively. Then

(a) $P(A) \ge P(B)$
(b) $P(A) \le P(B)$
(c) $P(A) = P(B) = 1/2$
(d) none of these

5. $A_1, A_2, ..., A_n$ are n independent events with $P(A_i) = \frac{1}{1+i}$ $(1 \le i \le n)$. The probability that none of $A_1, A_2, ..., A_n$ occur is

(a) $\frac{n!}{(n+1)!}$
(b) $\frac{n}{(n+1)}$
(c) $\frac{1}{(n+1)!}$
(d) $\frac{1}{n+1}$

6. If A and B are any two events then P (A – B) equals

(a) $P(A \cup B) - P(B)$
(b) $P(A) - P(A \cap B)$
(c) $P(B') + P(A \cup B) - 1$
(d) $P(B) - P(A \cap B)$

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7. If A and B are any two events then probability that exactly one of A and B occurs be

(a) $P(A \cup B) - P(A \cap B)$ (b) $P(A) + P(B) - 2P(A \cap B)$

(c) $P(\bar{A} \cup \bar{B}) - P(\bar{A} \cap \bar{B})$ (d) $P(\bar{A}) + P(\bar{B}) - 2P(\bar{A} \cap \bar{B})$

8. Given that $x \in [0,1]$ and $y \in [0,1]$. Let A be the event of (x, y) satisfying $y^2 \le x$ and B be the event of (x, y) satisfying $x^2 \le y$. Then

(a) $P(A \cap B) = \frac{1}{3}$ (b) P(A) > P(B)

(c) A and B are compatible events (d) P(A) = P(B)

9. Let E and F be two independent events. The probability that both E and F happen is 1/12 and the probability that neither E nor F happens is 1/2. Then

(a) P(E) = 1/3, P(F) = 1/4 (b) P(E) = 1/2, P(F) = 1/6

(c) P(E) = 1/6, P(F) = 1/2 (d) P(E) = 1/4, P(F) = 1/3

10. If A and B are two sets such that A < B, then which of the following is correct ?

(a) P(A) < P (B) (b) P (B – A) = P(B) – P(A)

(c) $P(A \cup B) = P(A) + P(B)$ (d) none of these

SECTION - IV

COMPREHENSION TYPE QUESTIONS

Passage – I.

If f(x) be a polynomial function satisfying the condition f(x).f(1/x) = f(x) + f(1/x) and f(10) = 101, $\forall$ x $\in$ R^+

1. Two real number x and y are selecting out at random such that 0 < x < 5, 0 < y < 5. Find the probability that y > f(x)

(a) $\frac{f(4)-f(0)}{f(5)-f(0)}$ (b) $\frac{f(3)-f(0)}{f(5)-f(0)}$

(c) $\frac{1}{3}\left[\frac{f(4)-f(0)}{f(5)-f(0)}\right]$ (d) $\frac{1}{3}\left[\frac{f(3)-f(0)}{f(4)+f(1)}\right]$

2. If a point (x, y) is selecting out at randon such that 0 < x < 6, 0 < y < 6, find the probability that $\frac{f(x)+f(y)}{x+y} \geq 6$

(a) $1-\left(\frac{4\pi}{27}-\frac{\sqrt{7}}{3}\right)$ (b) $\frac{\sqrt{7}}{3}-\frac{4\pi}{27}$

(c) $1+\frac{4\pi}{27}-\frac{\sqrt{7}}{3}$ (d) none of these

3. If two real numbers x and y are selected at random such that 0 < x < 4, 0 < y < 4, find the probability that f(x) + f(y) < 4y + 2

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$

(c) $\frac{\pi}{16}$ (d) $1-\frac{\pi}{4}$

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Passage – II.

If E, E_1, E_2 are any three events in a sample space, then P(E) is know as probability of getting event E, $P(\bar{E})$ is known as probability of not getting event E, $P(E_1 \cup E_2)$ or $P(E_1 + E_2)$ stands for probability of getting atleast one event E_1 or E_2, $P(E \cap E_2)$ or $P(E_1\, E_2)$ probability of getting both the events E_1, E_2 , $P(E_1/E_2)$ stands for probability of occurrence of E_1 after the occurrence of E_2 where $P(E_2) \neq 0$. Events E_1 & E_2 are said to be independent, if the occurrence of one does not influence other.

4. $P\left(\dfrac{\bar{E}_1}{\bar{E}_2}\right)\left(P(E_2) \neq 0\right) =$

(a) $\dfrac{1-P(E_1 \cup E_2)}{1-P(E_2)}$ (b) $\dfrac{1-P(E_1 \cap E_2)}{1-P(E_2)}$

(c) $\dfrac{1-P(\bar{E}_1 \cap \bar{E}_2)}{1-P(E_2)}$ (d) $\dfrac{1-P(\bar{E}_1 \cup \bar{E}_2)}{1-P(E_2)}$

5. The probability of getting exactly one event E_1 or E_2 is

(a) $P(E_1) + P(E_2) - P(E_1 \cap E_2)$ (b) $P(E_1) + P(E_2) - 2P(E_1 \cap E_2)$

(c) $P(E_1) + P(E_2)$ (d) none of these

6. E is any event in a sample space which is not impossible. If $a\sqrt{P(E)} + b\sqrt{P(\bar{E})} = 3$, then least value of $a^2 + b^2$ is

(a) 4 (b) 6

(c) 9 (d) 12

MATCHING TYPE QUESTIONS

In the following questions match the statements of List-I to their answers in List-II.

1. **List – I** | **List – II**

	List – I	List – II
(a)	A coin is tossed 6 times. The probability of obtaining four or more heads, is	(P) $\frac{989}{1000}$
(b)	If a die is thrown 5 times. The probability that an even number will come up exactly 3 times, is	(Q) $\frac{64}{81}$
(c)	Five shots are fired at a target. If each shot has a probability 0.6 of hitting the target, the probability that the target will be hit at least once, is	(R) $\frac{11}{32}$
(d)	An event succeeds twice as many times as it fails, then the probability that in the next 5 trials it will succeed at least 3 times, is	(S) $\frac{39}{125}$

2. **List – I** | **List – II**

List – I	List – II
(a) $P(\overline{A} \cap B)$	(P) $P(A) - P(B)$
(b) $P(A \cap \overline{B})$	(Q) $P(B) - P(A)$
(c) If $B \subset A$, $P(A \cap \overline{B})$	(R) $P(B) - P(A \cap B)$
(d) If $A \subset B$, $P(\overline{A} \cap B)$	(S) $P(A) - P(A \cap B)$

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ASSERTION-REASON TYPE QUESTIONS

In the following questions containing two statements viz. **Assertion (A)** & **Reason (R)**. To choose the correct answer.

Mark (a) if both A and R are correct & R is the correct explanation for A.
Mark (b) if both A & R are correct but R is not correct explanation for A
Mark (c) if A is true but R is false
Mark (d) if A is false but R is true

1. **Assertion (A) :** A binary number is made up of 8 digits. Suppose that the probability of an incorrect digit appearing is p and that errors in different digits are independent of each other. Then the probability of forming an incorrect answer is $1-(1-p)^8$.

   Reason (R) : Probability of success at least r times out of total n trials

   $$= P(\geq r) = {}^nC_r\, p^r q^{n-r} + {}^nC_{r+1}\, p^{r+1} q^{n-r-1} + ... + {}^nC_n P^n$$

2. **Assertion (A) :** In a horse race, the odds in favour of three horses are 1 : 2, 1 : 3 and 1 : 4. The probability that one of the horse will win the race is $\frac{47}{60}$.

   Reason (R) : For any three events A, B and C, which are mutually exclusive

   $$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

3. **Assertion (A) :** If two coins are tossed five times, then the chance that an odd number of heads is obtained is 1/2.

   Reason (R) : If a coin is tossed n times. Then the probability that head will appear an odd number of times is 1/2.

4. **Assertion (A) :** $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(CA) + P(ABC)$

   Reason (R) : $P(A \cup B) = P(A) + P(B) - P(AB)$

5. **Assertion (A) :** $P(A \cap \overline{B}) = P(A) - P(A \cap B)$

   Reason (R) : If $B \subset A$, then $P(A \cap \overline{B}) = P(A) - P(B)$

6. **Assertion (A) :** In a throw of die the probability of getting one in even number of throw is 5/11.

   Reason (R) : If P be the probability of success and q be the probability of failure, then p + q = 1

   $$P + P^2 + P^3 + ... \text{ to } \infty = \frac{P}{1-P},\ q \neq 0$$

7. **Assertion (A) :** The odds in favour of getting one ace after throwing a die, is 1 : 5.

   Reason (R) : If a cases are favourable to the event A and b cases are not favourable to the event A that is $\overline{A}$ then $P(A) = \frac{a}{a+b}$, $P(\overline{A}) = \frac{b}{a+b}$ odds in favour

   $$A = \frac{P(A)}{P(\overline{A})} = \frac{a}{b} = a : b$$

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SECTION - V

QUESTIONS ASKED IN IIT-JEE

SUBJECTIVE PROBLEMS

1. A person goes to office either by Car, Scooter, Bus or Train probability of which being $\frac{1}{7},\frac{3}{7},\frac{2}{7}$ and $\frac{1}{7}$, respectively. Probability that he reaches office late, if he takes Car, Scooter, Bus or Train is $\frac{2}{9},\frac{1}{9},\frac{4}{9}$ and $\frac{1}{9}$, respectively. Given that he reached office in time, then what is the probability that he travelled by a Car. **[2005]**

2. A is targeting B, B and C are targeting A. Probability of hitting the target by A, B and C are 2/3, $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If A is hit, then find the probability that B hits the target and C does not. **[2003]**

3. For a student to qualify, he must pass at least two out of three exams. The probability that he will pass the 1st exam is p. If he fails in one of the exams, then the probability of his passing in the next exam is p/2, otherwise it remains the same. Find the probability that he will qualify. **[2003]**

4. A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair? **[2002]**

5. An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6 is thrown n times and the list of n numbers showing up is noted. What is the probability that, among the numbers 1, 2, 3, 4, 5, 6 only three numbers appear in this list? **[2001]**

6. A coin has probability p of showing head when tossed. It is tossed n times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that $p_1 = 1$, $p_2 = 1 - p^2$ and $p_n = (1 - p)p_{n-1} + p(1 - p)p_{n-2}$ for all $n \geq 3$. **[2000]**

7. Three players A, B and C, toss a coin cyclically in that order (i.e., A, B, C, A, B, C, A, B, . . .) till a head shows.Let p be the probability that the coin shows a head. Let α,β and γ be respectively the probabilities that A, B and C gets the first head. Prove that $\beta=(1-p)\alpha$. Determine α, β and γ (in terms of p). **[1998]**

8. Three numbers are chosen at random without replacement from {1, 2, ..., 10}. The probability that the minimum of the chosen numbers is 3, or their maximum is 7, is.... **(I.I.T. 97, 2)**

9. Sixteen players S_1, S_2, ..., S_{16} play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength. **(I.I.T. 97, 5)**

   (a) Find the probability that the player S_1 is among the eight winners.

   (b) Find the probability that exactly one of the two players S_1 and S_2 is among the eight winners.

10. In how many ways three girls & nine boys can be seated in two vans, each having numbered seats, 3 in the front & 4 at the back? How many seating arrangements are possible if three girls should sit together in a back row on adjacent seats? Now, if all the seating arrangements are equally likely, what is the probability of 3 girls sitting together in a back row on deficient seats? **(I.I.T. 96, 5)**

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OBJECTIVE PROBLEMS

11. Let EC denote the complement of an E. Let E, F, G be pairwise independent events with P(G) > 0 and $P(E^C \cap F^C / G)$ equals

(a) $P(E^C) + P(F^C)$ (b) $P(E^C) - P(F^C)$

(c) $P(E^C) - P(F)$ (d) $P(E) - P(F^C)$ **[IIT-2007]**

12. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

(a) 1/2 (b) 1/3

(c) 2/5 (d) 1/5 **[IIT-2007]**

13. A six faced fair dice is thrown until 1 comes, then the probability that 1 comes in even number of trials is

(a) 5/11 (b) 5/6

(c) 6/11 (d) 1/6 **[IIT-2005]**

14. Two numbers are selected randomly from the set S = {1, 2, 3, 4, 5, 6} without replacement one by one. The probability that minimum of the two numbers is less than 4 is

(a) 1/15 (b) 14/15

(c) 1/5 (d) 4/5 **(I.I.T. 2k3, 3) Screening exam**

15. If $P(B) = \frac{3}{4}$, $P(A \cap B \cap \overline{C}) = \frac{1}{3}$ and $P(\overline{A} \cap B \cap \overline{C}) = \frac{1}{3}$, then $P(B \cap C)$ is

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{15}$ (d) $\frac{1}{9}$ **(I.I.T. 2k3, 3) Screening exam**

16. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in at least two and a 40% chance of passing in exactly two. Which of the following relations are true?

(a) p + m + c = 19/20 (b) p + m + c = 27/20

(c) pmc = 1/10 (d) pmc = 1/4 **(I.I.T. 99, 3)**

17. If the intergers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals

(a) 1/4 (b) 1/7

(c) 1/8 (d) 1/49 **(I.I.T. 99, 2)**

18. A fair coin is tossed repeatedly. If the tail appears on first four tosses, then the probability of the head appearing on the fifth toss equals:

(a) 1/2 (b) 1/32

(c) 31/32 (d) 1/5 **(I.I.T. 98, 2)**

19. If E and F are events with $P(E) \le P(F)$ and $P(E \cap F) > 0$, then

(a) occurrence of E $\Rightarrow$ occurrence of F

(b) occurrence of F $\Rightarrow$ occurrence of E

(c) non occurrence of E $\Rightarrow$ non occurrence of F

(d) none of the above implications holds **(I.I.T. 98, 2)**

20. There are four machines and it is known that exactly two of them are faulty machines are identified. Then the probability that only two tests are needed is

(a) 1/3 (b) 1/6

(c) 1/2 (d) 1/4 **(I.I.T. 98, 2)**

21. If $\overline{E}$ and $\overline{F}$ are the complementary events of events E and F respectively and if 0 < P(F) < 1, then:

(a) $P(E/F) + P(\overline{E}/F) = 1$ (b) $P(E/F) + P(E/\overline{F}) = 1$

(c) $P(\overline{E}/F) + P(E/\overline{F}) = 1$ (d) $P(E/\overline{F}) + P(\overline{E}/\overline{F}) = 1$ **(I.I.T. 98, 2)**

22. If from each of the three boxes containing 3 white & 1 black, 2 which and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is

(a) 13/32 (b) 1/4

(c) 1/32 (d) 3/16

23. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals

(a) 1/2 (b) 7/15

(c) 2/15 (d) 1/3 **(I.I.T. 98, 2)**

24. For the three events A, B & C, P (exactly one of the events A or B occurs) = P (exactly one of the events B or C occurs) = P(exactly one of the events C or A occurs) = p & P (all the three events occur simultaneously) = p^2, where occurring is:

(a) $\frac{3p+2p^2}{2}$ (b) $\frac{p+3p^2}{4}$

(c) $\frac{p+3p^2}{2}$ (d) $\frac{3p+2p^2}{4}$ **(I.I.T. 96, 2)**

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PASSAGE :

There are n urns each containing n + 1 balls such that the ith urn contains i white balls and (n + 1 – i) red balls. Let u_i be the event of selecting ith urn, i = 1, 2, 3, … n and w denotes the event of getting a white balls. [2006]

25. If $P(u_i) \propto i$, where i = 1, 2, 3, … n, then $\lim_{n\to\infty} P(w)$ is equal to

(a) 1 (b) 2/3

(c) 3/4 (d) 1/4

26. If $P(u_i)$ = c, where c is a constant then $P(u_n/w)$ is equal to

(a) $\frac{2}{n+1}$ (b) $\frac{1}{n+1}$

(c) $\frac{n}{n+1}$ (d) $\frac{1}{2}$

27. If n is even and E denotes the event of choosing even numbered urn ($P(u_i) = \frac{1}{n}$), then the value of P(w/E) is

(a) $\frac{n+2}{2n+1}$ (b) $\frac{n+2}{2(n+1)}$

(c) $\frac{n}{n+1}$ (d) $\frac{1}{n+1}$

ASSERTION-REASON :

28. Let $H_1, H_2, ..., H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, i = 1, 2, ...n. Let E be any other event with 0 < P(E) < 1

A : $P(H_i/E) > P(E/H_i) \,.\, P(H_i)$ for i = 1, 2, ..., n

R : $\sum_{i=1}^{n} P(H_i) = 1$

■■

ANSWERS

EXERCISE - 1

1. $\frac{2}{3}$ 2. $\frac{2}{3}$

EXERCISE - 2

1. (i) 0.82 (ii) 0.76 2. $P(A)=\frac{2}{3},\ P(B)=\frac{1}{2}$

EXERCISE - 3

1. $\frac{13}{24}$ 2. 0.6976 3. $\frac{4}{7},\frac{2}{7},\frac{1}{7}$

EXERCISE - 4

1. $\frac{97}{(25)^4}$ 2. 3 3. (i) $\frac{35}{128}$ (ii) $\frac{1}{2}$

SECTION - I

Subjective Questions

LEVEL - I

1. $\frac{13}{16}$ 2. $\frac{1}{36}$ 3. $\frac{55}{1024}$ 4. $\frac{16}{37},\frac{12}{37},\frac{9}{37}$

5. $\frac{16}{81}$ 6. best of 3 games 7. $\frac{517}{1800}$ 8. $\frac{3^n}{4^n}$

9. $462\,(0.24)^5$ 10. $\dfrac{1-\left(\frac{7}{10}\right)^7-{}^7C_1\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)^6}{1-\left(\frac{7}{10}\right)^7}$

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LEVEL - II

1. 5/54 2. 13/20 3. (i) 1/18, (ii) 7/144

4. (i) 1/2, (ii) 1/2, (iii) 1/12 5. $\frac{575}{648}$

6. $\frac{12585}{3^{10}}$ 7. $\frac{3}{11}$ 8. $\frac{5}{32}$ 9. $\frac{407}{576}$ 10. $\frac{209}{243}$

LEVEL - III

2. $\frac{213}{1001}$ 4. 1/3 5. 5/9

6. 1/2, 8/15 7. 1/5 8. 3/4 9. ${}^nC_r(N - 1)^{n-r}/N^r$

10. $\frac{P}{1-(1-P)^3}, \frac{P(1-P)^2}{1-(1-P)^3}$

SECTION - II

Single Choice Questions

1.	(c)	2.	(b)	3.	(b)	4.	(b)	5.	(b)
6.	(b)	7.	(b)	8.	(a)	9.	(b)	10.	(c)
11.	(c)	12.	(b)	13.	(c)	14.	(a)	15.	(b)
16.	(c)	17.	(a)	18.	(c)	19.	(b)	20.	(b)

SECTION - III

Multiple Choice Questions

1.	(b), (c)	2.	(c), (d)	3.	(c), (d)
4.	(a), (b)	5.	(a), (d)	6.	(a), (b), (c)
7.	(a), (b), (c), (d)	8.	(a), (c), (d)	9.	(a), (d)
10.	(a), (b)				

SECTION - IV

Comprehension Type Questions

1.	(c)	2.	(c)	3.	(a)
4.	(a)	5.	(b)	6.	(c)

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Matching Type Questions

1. (a) – (R), (b) – (S), (c) – (P), (d) – (Q)
2. (a) – (R), (b) – (S), (c) – (P), (d) – (Q)

Assertion Reason Type Questions

1. (b) 2. (a) 3. (b) 4. (a) 5. (b)
6. (a) 7. (a)

SECTION - V

Question Asked in IIT-JEE

1. 1/7 2. 1/2 3. $2p^2 - p^3$ 4. $\frac{9m}{8N+m}$

5. $\frac{{}^6C_3[3^n - 3.2^n + 3]}{6^n}$ 7. $\alpha = \frac{P}{1-(1-P)^3}, \beta = \frac{(1-P)P}{1-(1-P)^3}, \gamma = \frac{(1-P)^2 P}{1-(1-P)^3}$

8. 11/40 9. (a) 1/2, (b) 8/15 10. $713!,\ 12!\frac{1}{91}$

11. (c) 12. (c) 13. (a) 14. (d) 15. (a)
16. (b),(c) 17. (a) 18. (a) 19. (d) 20. (b)
21. (a) 22. (a) 23. (b) 24. (a) 25. (b)
26. (a) 27. (b) 28. (b)

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