# AREA & DIFFERENTIAL EQUATIONS

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# AREA & DIFFERENTIAL EQUATION

**Area under the curves**

- $y^2 = 4ax$ & $x^2 = 4ay$ **is** $\frac{16}{3}ab$
- $y^2 = 4ax$ & $y = mx$ **is** $\frac{8a^2}{3m^3}$
- $y^2 = 4ax$ & **its last rectum** $\frac{8}{3}a^2$
- Sin px or Cos px & X-axis (one loop or arch) is 2/p
- $\mathbf{x^2 + y^2} \le 2ax, \quad y^2 \ge ax,$ $\frac{a^2}{12}(3\pi - 8)$
- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\pi ab$
- $\frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1 \le \frac{x}{a} + \frac{y}{b} = 1$ $\frac{ab}{4}(\pi - 2)$



**1. Curve Tracing**

In order to find the area bounded by several curves, sometimes it is necessary to have an idea of the rough sketches of these curves. To find the approximate shape of a curve represented by the cartesian equation, the following steps are very useful.

**1. Symmetry**

*(i)* If curve remains unaltered on replacing x by –x, then it is symmetrical about y-axis.

*(ii)* If curve remains unaltered on replacing y by –y, then it is symmetrical about x-axis.

**2. Intersection with axes**

*(i)* To find points of intersection of the curve with x-axis, replace y = 0 in the curve and get corresponding values of x.

*(ii)* To find points of intersection of the curve with y-axis, replace x = 0 in the curve and get corresponding values of y.

**3. The regions where curves does not exist**

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*(i)* Find those values of x for which corresponding values of y do not exist.
*(ii)* Find intervals where $f$(x) is positive.

**4. Asymptotes**

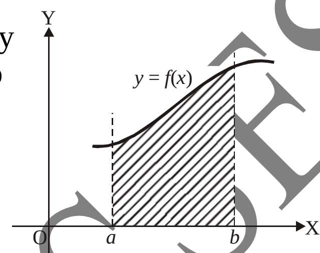
*(i)* Observe where y approaches as x approaches $\pm\infty$.
*(ii)* If necessary, observe where x approaches as y approaches $\pm\infty$.

**5. Find points of local minimum**

Put $f'(x) = 0$ and find points of local maximum and minimum.

**2. Important Results**

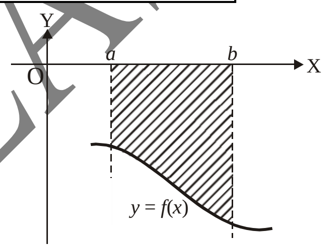
**I.** If $f$(x) $\leq 0$ for all x $\in$ [a, b], then Area bounded by the curve y = $f$(x), X-axis and the x = a and x = b is given by




$$A = \int_a^b f(x)dx$$

**Note :** The whole of the curve in the internal [a, b] lies above X-axis.

**II.** If $f$(x) $\leq 0$ for all x $\in$ [a, b], then Area bounded by the curve y = $f$(x), X-axis and the x = a and x = b is given by

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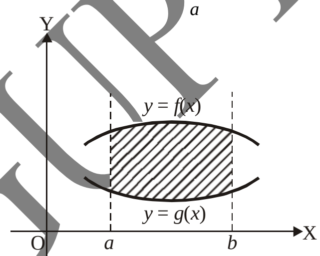
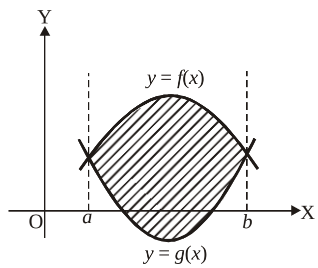
$$\text{Area} = \left|\int_a^b f(x)dx\right|$$

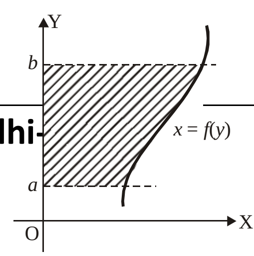
**III.** Area bounded by two curves , y = $f$(x) and above and below is given by :

$$\text{Shaded area} = \int_a^b [f(x) - g(x)]dx$$




**IV.** If $f$(y) $\leq 0$ for all y $\in$ [a, b], then Area bounded by the curve x = $f$(y), Y-axis and the y = a



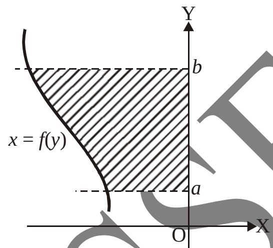
and y = b is given by

$$\text{Area} = \int_a^b f(y)dx$$

**Note :** The whole of the curve in the internal [a, b] lies above Y-axis.

**V.** If $f$(y) $\leq 0$ for all y $\in$ [a, b], then Area bounded by the curve x = $f$(y), Y-axis and the y = a and y = b is given by

$$\text{Area} = \left|\int_a^b f(y)dy\right|$$






**ASSIGNMENT**
**AREA & DIFFERENTIAL EQUATION**

1. Area common to y = √x & x = √y is

(a) 1 (b) 2/3
(c) 1/3 (d) N/Ts

2. Area bounded by $y^2 = x$ & $2y = x$ is

(a) 1/3 (b) 2/3

(c) 1 (d) 4/3

3. Area bounded by $y^2 = 16x$ & $y = mx$ is 2/3 then m is

(a) 1 (b) 2

(c) 3 (d) 4

4. Area of region $\{(x, y): x^2 + y^2 \le 1 \le x + y\}$ is:

(a) $\pi^2/5$ (b) $\pi^2/2$

(c) $\pi^2/5$ (d) $\pi/4 - 1/2$

5. Area bounded by the lines $y = 2 + x$, $y = 2 - x$ and $x = 2$ is :

(a) 3 (b) 4

(c) 8 (d) 16.

6. Area bounded by $y = |x-1|$ & $y = 1$ is

(a) 1 (b) 2

(c) 1/2 (d) N/T

7. Area bdd. by $y = |x| - 1$ & $y = -|x| + 1$ is

(a) 1 (b) 2

(c) $2\sqrt{2}$ (d) 4

8. Area common to $y = 2x^2$, $y = x^2 + 4$

(a) 2/3 (b) 3/2

(c) 32/3 (d) N/T

9. Area bounded by $y = 2 - x^2$ & $x + y = 0$ is

(a) 1/2 (b) 1/3

(c) 2/9 (d) 9/2

10. Area bounded by the curve $y = x^3$, $y = x^2$ and the ordinates $x = 1$, $x = 2$ is:

(a) 17/2 (b) 12/17

(c) 2/7 (d) 7/2

11. Let $A_1$ be the area of the parabola $y^2 = 4ax$ lying between vertex and latus rectum and $A_2$ be the area between latus rectum and double ordinate $x = 2a$. Then $\frac{A_1}{A_2} =$

(a) $2\sqrt{2} - 1$ (b) $\frac{2\sqrt{2}+1}{7}$

(c) $\frac{2\sqrt{2}-1}{7}$ (d) None of these

12. Area by $y = x$ & $y = x^3$ is

(a) 1/2 (b) 3/2

(c) 2 (d) 5/2

13. Area bounded by $y = x^2$ on left hand of y –axis ,y-axis & lines $y = 1$, $y = 4$

(a) 14 (b) 28/3

(c) 14/3 (d) N/T

14. Area of triangular region formed by $y = \sin x$, $y = \cos x$ & $x = 0$ is

(a) $1+\sqrt{2}$ (b) $\sqrt{2}$

(c) $\sqrt{2} - 1$ (d) 1

15. Area bounded between $x \sin x$, x- axis $x \in [0, 2\pi]$

(a) π (b) 2π

(c) 4π (d) N/T

16. Area enclosed by parabola $(y - 2)^2 = x - 1$, the tangent to parabola at (2, 3) and x – axis is :

(a) 4 sq. units (b) 5 sq. units

(c) 3 sq. units (d) none of these.

17. Area bounded by $x^2 + y^2 = 4$, $\sqrt{3}\,y = x$ & X- axis is

(a) $3\pi/4$ (b) $\pi - \pi/\sqrt{3}$

(c) $\pi/3$ (d) N/T

18. For $b > a > l$, the area enclosed by the curve $y = \text{In } x$ ,y axis and the straight lines $y = \text{In } a$ and $y = \text{In } b$, is

(a) b-a (b) b(ln b – 1 )- a(ln a-1 )

(c) In a) (b-a) (d) (ln b) On a)

19. Area bounded by $y = \sin x$ and $y = \cos x$ and y-axis is $A_1$ and area bounded $y = \sin x$, $y = \cos x$ and x-axis is $A_2$ where $0 \le x \le \frac{\pi}{2}$ Then, $A_1 : A_2$

(a) 1:2 (b) 2 : 3

(c) 2 : 1 (d) 1: 2

20. Area bounded by [ sinx ] with X – axis , $0 < x < 2000\pi$

(a) 1000 (b) 2000π

(c) 1000π (d) N/T

21. The order of the differential equation of all circles of all circles of radius r, having center on y – axis and passing through the origin is

(a) 1 (b) 2

(c) 3 (d) 4.

22. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter , is of

(a) order 1 (b) order 2

(c) degree 3 (d) degree 4

23. The order & degree of differential equation $\left(\frac{d^3y}{dx^3}\right)^{3/2} + \left(\frac{d^3y}{dx^3}\right)^{2/3} = 0$ are resp

(a) 2 ,9 (b) 3, 6

(c) 3 , 4 (d) 3 , 9

24. Order of differential equation whose general solution is $y = c_1e^x + c_2e^{2x} + c_3e^{3x} + c_4e^{x+c_5}$ where $c_1, c_2, c_3, c_4, c_5$, are arbitrary constants

(a) 5 (b) 4

(c) 3 (d) none of these.

25. Order of differential equation of all parabolas whose axis of symmetry is parallel to x- axis

(a) 1 (b) 3

(c) 2 (d) N/T.

26. Solution of $\frac{x\,dy}{x^2+y^2} = \left(\frac{y}{x^2+y^2} - 1\right) dx$ is

(a) $\tan^{-1}y - \tan^{-1}x = c$ (b) $\tan^{-1}y + \tan^{-1}x = c$

(c) $\tan^{-1}\frac{y}{x} = y + c$ (d) $\tan^{-1}\frac{y}{x} + x = +c$

27. Solution of the differential equation $x = 1 + xy\frac{dy}{dx} + \frac{(xy)^2}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{(xy)^3}{3!}\left(\frac{dy}{dx}\right)^3 + \ldots\ldots$ is

(a) $y = \log x + c$

(b) $y = (\log x)^2 + c$

(c) $y = \pm\sqrt{(\log x)^2 + 2c}$

(d) $xy = x^y + c$

28. The solution of the differential equation $\frac{dy}{dx} + \frac{3y}{x} = \frac{1}{x^2}$ given that y = 2 when x = 1 is

(a) $3x^3y = x^2 + 5$ (b) $2x^3y + x^2 = 3$

(c) $2x^3y - x^2 = 3$ (d) $x^3y - x^2 = 1$

29. Order of Diff. $eq^n$ with general sol

$y = (c_1 + c_2)\sin(x + c_3) - c_4e^{x+c_5}$

(a) 5 (b) 4

(c) 2 (d) 3

30. The order and degree of the difrential equation whose solution is $y = cx + c^2 - 3c^{3/2} + 2$, where c is a parameter, are respectively

(a) 1 and 4 (b) 1 and 3

(c) 1 and 2 (d) none of these.

31. The order of the differential equation of a family of ellipses with fixed directrix and fixed eccentricity is:

(a) one (b) two

(c) three (d) four

32. The differential equation of all conics whose centres lie at the origin is of order

(a) 2 (b) 3

(c) 4 (d) N/T

33. The differential equation of all conics whose axes coincide with the axes of coordinates is of order

(a) 2 (b) 3

(c) 4 (d) 1

34. The general solution of the differentiating equation $\frac{dy}{dx} + y\tan x = \sec x$ is

(a) y = sinx + c cosx (b) y = tanx + cotx + c

(c) y = sinx - c cosx (d) N/T

35. The solution of the differential equation $3e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$ is

(a) $e^x \tan y = C$

(b) $Ce^x = (1 - \tan y)^3$

(c) $C \tan y = (1 - e^x)^2$

(d) $\tan y = C(1 - e^x)^3$

36. The general solution of the differentiating equation $(x + 2y^3)\frac{dy}{dx} = y$ is

(a) $x - y^3$ (b) $y = x(x^2 + c)$

(c) $y - x^3 = c$ (d) $x = y(y^2 + c)$

37. Solution of $y\,dx + (x - y^3)\,dy = 0$ is

(a) $xy = y^3/3 + c$ (b) $xy = y^4 + c$

(c) $4xy = y^4 + c$ (d) $4y = y^3 + c$

38. The differential equation of all circles which pass through the origin and whose centers are on the x – axis is

(a) $y^2 = xy' + xy$ (b) $y^2 = x^2 + 2xyy'$

(c) $yy' = 2xy + x^2$ (d) none of these.

39. The differential equation $(1 + y^2)\,x\,dx - (1 + x^2)\,y\,dy = 0$ represents a family of :

(a) ellipses of constant eccentricity

(b) ellipses of variable eccentricity

(c) hyperbolas of constant eccentricity

(d) hyperbolas of variable eccentricity.

40. The curve in which the slope of the tangent at any point equals the ratio of the abscissa to the ordinate of the point is

(a) an ellipse

(b) a rectangular hyperbola

(c) a circle

(d) none of these.

41. Integrating factor of $(xy-1)\frac{dy}{dx} + y^2 = 0$ is

(a) $1/x$ (b) $1/y$

(c) $1/xy$ (d) $xy$

42. The solution of the differential equation $2x + \frac{dy}{dx} - y = 3$ given that $y(0) = -1$ represent

(a) straight line (b) circle

(c) parabola (d) ellipse

43. The differential equation of the family given by $e^{2y} + 2cxe^{x} + c^2 = 0$, where c is a parameter is

(a) $(1 + x^2)\left(\frac{dy}{dx}\right)^2 - 1 = 0$

(b) $(1 - x^2)\left(\frac{dy}{dx}\right)^2 - 1 = 0$

(c) $(1 - x^2)\left(\frac{dy}{dx}\right)^2 - 1 = 0$

(d) $(1 + x^2)\left(\frac{dy}{dx}\right)^2 + 1 = 0$

44. The solution of differential equation $2x\frac{dy}{dx} - y = 3$ represents

(a) Circles (b) st. lines

(c) Ellipses (d) Parabolas

45. Integrating factor of $\frac{dy}{dx}(x\log x) + y = 2\log x$

(a) $e^x$ (b) $\log x$

(c) $\log(\log x)$ (d) $x$

46. Equation of all curves sub normal is constant is

(a) $y = ax + b$ (b) $y^2 = 2ax + b$

(c) $ay^2 - x^2 = a$ (d) N/T.

47. The slope of the tangent at (x, y) to a curve y = f(x) passing through $\left(1, \frac{1}{4}\right)$ is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$. Then the equation of the curve

(a) $\tan\frac{y}{x} - 2 + x = 0$ (b) $\tan y = x$

(c) $\tan\left(\frac{y}{x}\right) = \log\left(\frac{e}{x}\right)$ (d) $\tan\left(\frac{y}{x}\right) = \log\frac{x}{e}$

48. General solution of $x^3\frac{dy}{dx} - x^2y + y^4\cos x = 0$ is

(a) $x^3 = y^3(C + 3\sin x)$

(b) $y^3 = x^3(C - \sin x)$

(c) $x^3 = y^3(3\sin x + y) + C$

(d) $x^3 + y^3 = 3x^3y^3\sin x$

49. The general solution of the differential equation $\frac{y}{\left(\frac{dy}{dx}\right)} + x = 2a$, where a is a constant which passes through point (2a, a) is

(a) a hyperbola

(b) an ellipse

(c) a parabola

(d) a pair of straight lines

50. cThe solution of the differential equation $\frac{dy}{dx} + \frac{3y}{x} = \frac{1}{x^2}$ given that y = 2 when x = 1 is

(a) $3x^3y = x^2 + 5$
(b) $2x^3y + x^2 = 3$
(c) $2x^3y - x^2 = 3$
(d) $x^3y - x^2 = 1$



**ANSWER (AREA & DIFFERENTIAL EQUATION)**

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| c | d | d | d | b | a | b | c | d | a | b | a | c | c | d |
| 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| b | c | a | d | c | a | ac | d | c | b | d | c | c | d | a |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 |
| b | b | a | a | d | d | a | b | d | b | b | a | b | d | b |
| 46 | 47 | 48 | 49 | | | | | | | | | | | |
| b | c | a | d | | | | | | | | | | | |