FIITJEE Solutions to IIT-JEE-2011

CODE 8

PAPER 1

Time: 3 Hours **Maximum Marks: 240**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. The **question paper CODE** is printed on the right hand top corner of this sheet and on the back page (page No. 36) of this booklet.
2. No additional sheets will be provided for rough work.
3. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets are NOT allowed.
4. Write your name and registration number in the space provided on the back page of this booklet.
5. The answer sheet, a machine-gradable Optical Response Sheet (ORS), is provided separately.
6. DO NOT TAMPER WITH/MULTILATE THE ORS OR THE BOOKLET.
7. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilators.
8. This question Paper contains 36 pages having 69 questions.
9. On breaking the seals, please check that all the questions are legible.

B. Filling the Right Part of the ORS:

10. The ORS also has a CODE printed on its Left and Right parts.
11. Make sure the CODE on the ORS is the same as that on this booklet. **If the codes do not match ask for a change of the booklet.**
12. Write your Name, Registration No. and the name of centre and sign with pen in the boxes provided**. Do not write them anywhere else. Darken the appropriate bubble UNDER** each digit of your Registration No. with a **good quality HB pencil.**

C. Question paper format and Marking scheme:

13. The question paper consists of **3 parts** (Chemistry, Physics and Mathematics). Each part consists of **four sections.**
14. In **Section I** (Total Marks: 21), for each question you will be awarded 3 marks if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** if no bubble is darkened. In all other cases, **minus one (–1)** mark will be awarded.
15. In **Section II** (Total Marks: 16), for each question you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero** marks other wise. There are **no negative marks** in this section.
16. In **Section III** (Total Marks: 15), for each question you will be awarded **3 marks** if you darken **ONLY** the bubble corresponding to the correct answer and zero marks if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
17. In **Section IV** (Total Marks: 28), for each question you will be awarded 4 marks if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** otherwise. There are **no negative marks** in this section.

Write your name, registration number and sign in the space provided on the back of this booklet.

PAPER-1 [Code – 8]
IITJEE 2011
PART - I: CHEMISTRY

SECTION – I (Total Marks : 21)
(Single Correct Answer Type)

This section contains **7 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. Extra pure N_2 can be obtained by heating
(A) NH_3 with CuO
(B) NH_4NO_3
(C) $(NH_4)_2Cr_2O_7$
(D) $Ba(N_3)_2$

Sol. **(D)**
$$Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$$

2. Dissolving 120 g of urea (mol. wt. 60) in 1000 g of water gave a solution of density 1.15 g/mL. The molarity of the solution is
(A) 1.78 M
(B) 2.00 M
(C) 2.05 M
(D) 2.22 M

Sol. **(C)**
Total mass of solution = 1000 + 120 = 1120 g
Total volume of solution in (L) $= \frac{1120}{1.15} \times 10^3$
$$M = \frac{W}{M} \times \frac{1}{V(\text{in L})} = \frac{120}{60} \times \frac{1.15 \times 10^3}{1120} = 2.05\ M$$

3. Bombardment of aluminium by α-particle leads to its artificial disintegration in two ways, (i) and (ii) as shown. Products X, Y and Z respectively are,
$$\begin{tikzcd} {}^{27}_{13}Al \arrow[r, "(ii)"] \arrow[d, "(i)"'] & {}^{30}_{15}P + Y \arrow[d] \\ {}^{30}_{14}Si + X & {}^{30}_{14}Si + Z \end{tikzcd}$$
(A) proton, neutron, positron
(B) neutron, positron, proton
(C) proton, positron, neutron
(D) positron, proton, neutron

Sol. **(A)**
$${}^{27}_{13}Al + {}_2\alpha^4 \rightarrow {}^{30}_{14}Si + {}_1p^1 (X)$$
$${}^{27}_{13}Al + {}_2\alpha^4 \rightarrow {}^{30}_{15}P + {}_0n^1 (Y)$$
$${}^{30}_{15}P \rightarrow {}^{30}_{14}Si + {}_{+1}\beta^0 (Z)$$

4. Geometrical shapes of the complexes formed by the reaction of Ni^{2+} with Cl^-, CN^- and H_2O, respectively, are
(A) octahedral, tetrahedral and square planar
(B) tetrahedral, square planar and octahedral
(C) square planar, tetrahedral and octahedral
(D) octahedral, square planar and octahedral

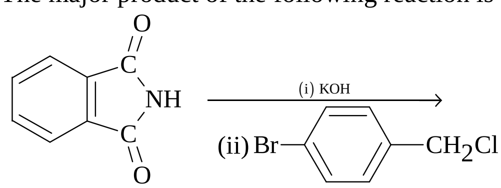
Sol. **(B)**

$[NiCl_4]^{2-} \rightarrow$ Tetrahedral

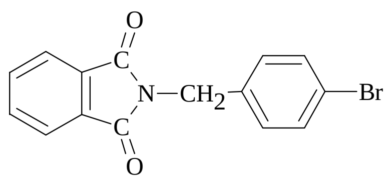
$[Ni(CN)_4]^{2-} \rightarrow$ Square Planar

$[Ni(H_2O)_6]^{2+} \rightarrow$ Octahedral

5. The major product of the following reaction is



(A)



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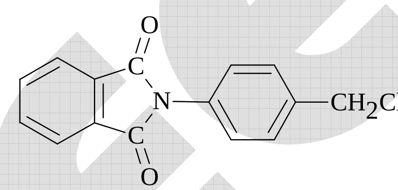
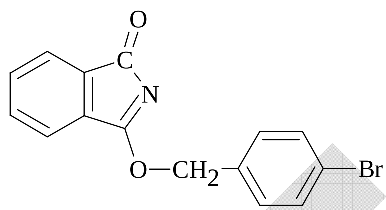
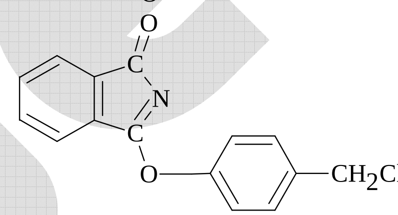


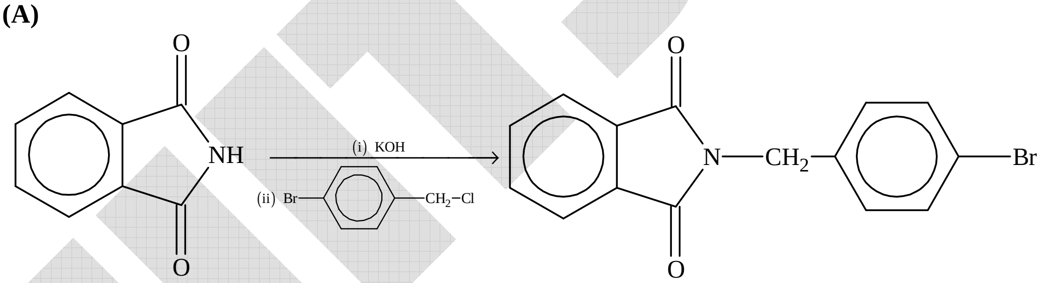
(C)



(D)



Sol. **(A)**



[**Reason:** Due to partial double bond character along C–Br bond prevents the attack of nucleophile at phenylic position]

6. Among the following compounds, the most acidic is

(A) p-nitrophenol

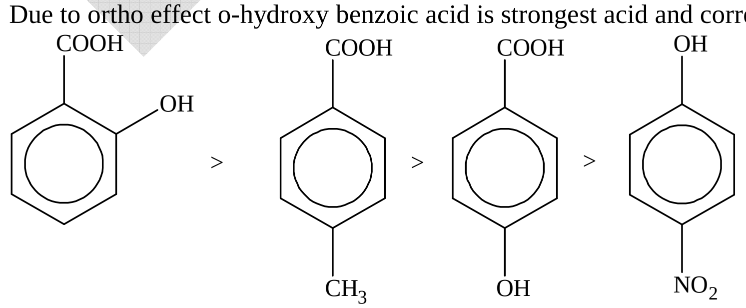
(B) p-hydroxybenzoic acid

(C) o-hydroxybenzoic acid

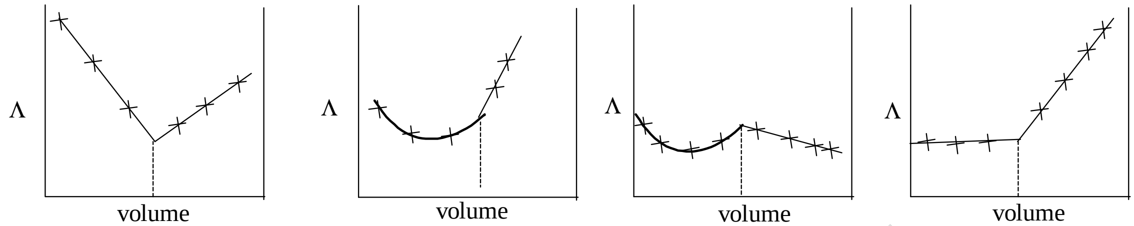
(D) p-toluic acid

Sol. **(C)**

Due to ortho effect o-hydroxy benzoic acid is strongest acid and correct order of decreasing K_a is



7. $AgNO_3$(aq.) was added to an aqueous KCl solution gradually and the conductivity of the solution was measured. The plot of conductance (Λ) versus the volume of $AgNO_3$ is



(P) **(Q)** **(R)** **(S)**

(A) (P)
(B) (Q)
(C) (R)
(D) (S)

Sol. **(D)**

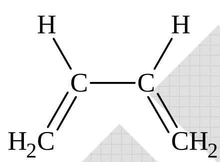
$$AgNO_3 + KCl(aq) \rightarrow AgCl(s) + KNO_3(aq)$$

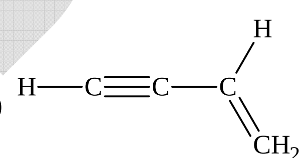
Initially there is aq. KCl solution now as solution of $AgNO_3$ is added, AgCl(s) is formed. Hence conductivity of solution is almost compensated (or slightly increase) by the formation of KNO_3. After end point conductivity increases more rapidly because addition of excess $AgNO_3$ solution.

SECTION – II (Total Marks : 16)
(Multiple Correct Answers Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

8. Amongst the given options, the compound(s) in which all the atoms are in one plane in all the possible conformations (if any), is (are)

(A)


(B)


(C) $H_2C{=}C{=}O$

(D) $H_2C{=}C{=}CH_2$

Sol. **(B, C)**

Along C–C single bond conformations are possible in butadiene in which all the atoms may not lie in the same plane.

9. Extraction of metal from the ore **cassiterite** involves

(A) carbon reduction of an oxide ore
(B) self-reduction of a sulphide ore
(C) removal of copper impurity
(D) removal of iron impurity

Sol. **(A, C, D)**

$$SnO_2 + 2C \rightarrow 2CO + Sn$$

The ore cassiterite contains the impurity of Fe, Mn, W and traces of Cu.

10. According to kinetic theory of gases

(A) collisions are always elastic
(B) heavier molecules transfer more momentum to the wall of the container
(C) only a small number of molecules have very high velocity
(D) between collisions, the molecules move in straight lines with constant velocities

Sol. **(A, B, C, D)**

11. The correct statement(s) pertaining to the adsorption of a gas on a solid surface is (are)
(A) Adsorption is always exothermic
(B) Physisorption may transform into chemisorption at high temperature
(C) Physiosorption increases with increasing temperature but chemisorption decreases with increasing temperature
(D) Chemisorption is more exothermic than physisorption, however it is very slow due to higher energy of activation

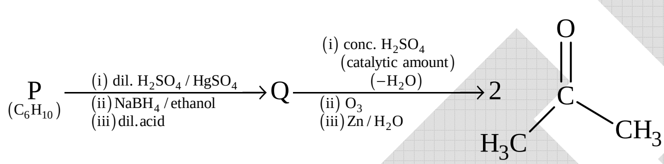
Sol. **(A, B, D)**

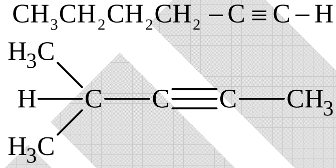
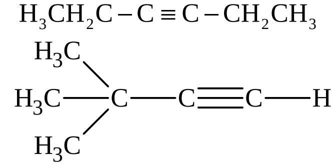
SECTION-III (Total Marls : 15)
(Paragraph Type)

This section contains **2 paragraphs.** Based upon one of paragraphs **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of **which ONLY ONE** is correct.

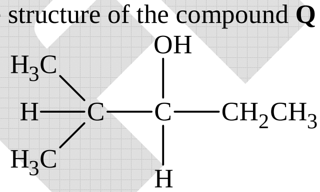
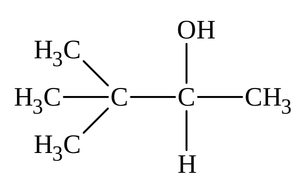
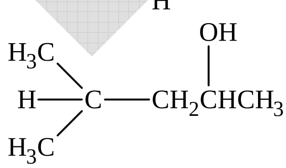
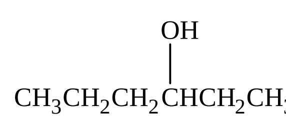
Paragraph for Question Nos. 12 and 13

An acyclic hydrocarbon **P**, having molecular formula C_6H_{10}, gave acetone as the only organic product through the following sequence of reaction, in which **Q** is an intermediate organic compound.



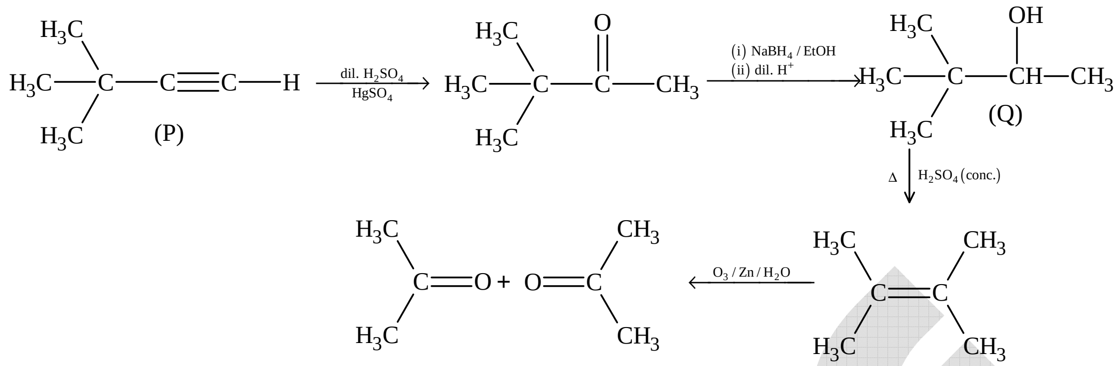
12. The structure of compound **P** is
(A) $CH_3CH_2CH_2CH_2-C\equiv C-H$
(B) $H_3CH_2C-C\equiv C-CH_2CH_3$
(C)

(D)


Sol. **(D)**

13. The structure of the compound **Q** is
(A)

(B)

(C)

(D)


Sol. **(B)**

Solution for the Q. No. 12 to 13



Paragraph for Question Nos. 14 to 16

When a metal rod **M** is dipped into an aqueous colourless concentrated solution of compound **N**, the solution turns light blue. Addition of aqueous NaCl to the blue solution gives a white precipitate **O**. Addition of aqueous NH_3 dissolves **O** and gives an intense blue solution.

14. The metal rod **M** is
(A) Fe
(B) Cu
(C) Ni
(D) Co

Sol. **(B)**

$$\underset{M}{Cu} + 2\underset{N}{AgNO_3} \rightarrow \underset{Blue}{Cu(NO_3)_2} + 2Ag$$

While Cu partially oxidizes to $Cu(NO_3)_2$ and remaining $AgNO_3$ reacts with NaCl.

15. The compound **N** is
(A) $AgNO_3$
(B) $Zn(NO_3)_2$
(C) $Al(NO_3)_3$
(D) $Pb(NO_3)_2$

Sol. **(A)**

16. The final solution contains
(A) $[Pb(NH_3)_4]^{2+}$ and $[CoCl_4]^{2-}$
(B) $[Al(NH_3)_4]^{3+}$ and $[Cu(NH_3)_4]^{2+}$
(C) $[Ag(NH_3)_2]^{+}$ and $[Cu(NH_3)_4]^{2+}$
(D) $[Ag(NH_3)_2]^{+}$ and $[Ni(NH_3)_6]^{2+}$

Sol. **(C)**

$$\underset{(N)}{AgNO_3} + NaCl \rightarrow \underset{(O)}{AgCl}\downarrow + NaNO_3$$

$$AgCl + 2NH_3 \rightarrow [Ag(NH_3)_2]^+ Cl^-$$

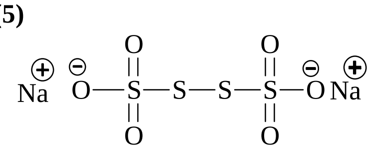
$$Cu(NO_3)_2 + 4NH_4OH \rightarrow [Cu(NH_3)_4]^{2+}$$

SECTION-IV (Total Marks : 28)
(Integer Answer Type)

This section contains **7** questions. The answer to each of the questions is a **single digit integer,** ranging from 0 to 9. The bubble corresponding to the correct is to be darkened in the ORS.

17. The difference in the oxidation numbers of the two types of sulphur atoms in $Na_2S_4O_6$ is

Sol. **(5)**



S will have oxidation number = +5, 0
Difference in oxidation number = 5

18. A decapeptide (Mol. Wt. 796) on complete hydrolysis gives glycine (Mol. Wt. 75), alanine and phenylalanine. Glycine contributes 47.0% to the total weight of the hydrolysed products. The number of glycine units present in the decapeptide is

Sol. **(6)**

For n-units of glycine,

$$\frac{n\times 75}{(796+9\times 18)}\times 100 = 47$$

$\Rightarrow n = 6$

19. The work function (ϕ) of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
ϕ (eV)	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

Sol. **(4)**

The energy associated with incident photon = $\frac{hc}{\lambda}$

$$\Rightarrow E = \frac{6.6\times 10^{-34}\times 3\times 10^{8}}{300\times 10^{-9}}\ J$$

$$E \text{ in eV} = \frac{6.6\times 10^{-34}\times 3\times 10^{8}}{300\times 10^{-9}\times 1.6\times 10^{-19}} = 4.16\ eV$$

So, number of metals showing photo-electric effects will be (4), i.e., Li, Na, K, Mg

20. The maximum number of electrons that can have principal quantum number, n = 3, and spin quantum number, $m_s = -\frac{1}{2}$, is

Sol. **(9)**

For principal quantum number (n = 3)
Number of orbitals = $n^2 = 9$

So, number of electrons with $m_s = -\frac{1}{2}$ will be 9.

21. Reaction of Br_2 with Na_2CO_3 in aqueous solution gives sodium bromide and sodium bromate with evolution of CO_2 gas. The number of sodium bromide molecules involved in the balanced chemical equation is

Sol. **(5)**

$$3Br_2 + 3Na_2CO_3 \longrightarrow 5NaBr + NaBrO_3 + 3CO_2$$

So, number of NaBr molecules = 5

22. To an evacuated vessel with movable piston under external pressure of 1 atm, 0.1 mol of He and 1.0 mol of an unknown compound (vapour pressure 0.68 atm. at 0^oC) are introduced. Considering the ideal gas behaviour, the total volume (in litre) of the gases at 0^oC is close to

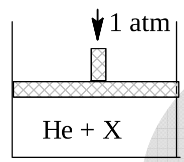
Sol. **(7)**

For any ideal gas, PV = nRT

$0.32 \times V = 0.1 \times 0.0821 \times 273$

V = 7 litre

(unknown compound X will not follow ideal gas equation)

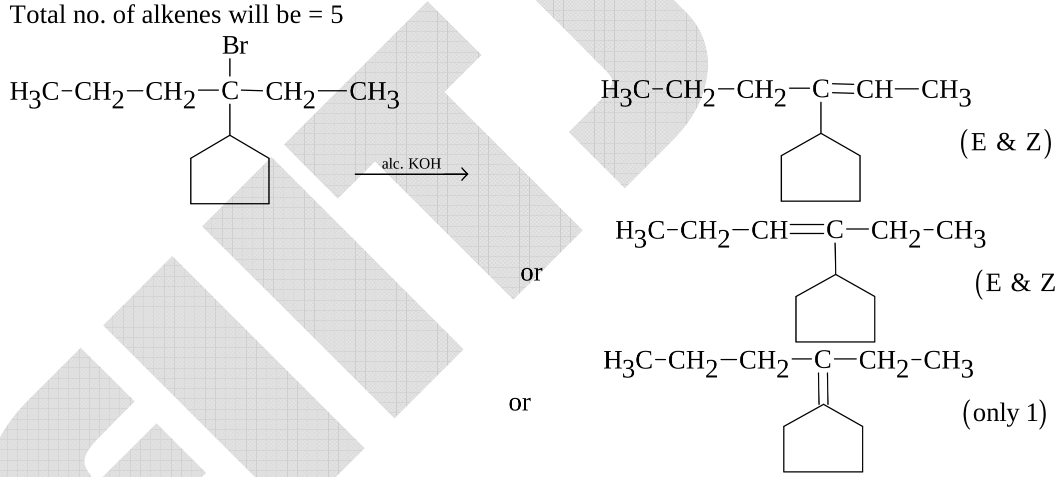


For He, n = 0.1, P = 0.32 atm., V = ?, T = 273

23. The total number of alkenes possible by dehydrobromination of 3-bromo-3-cyclopentylhexane using alcoholic KOH is

Sol. **(5)**

Total no. of alkenes will be = 5



PART - II: PHYSICS

SECTION – I (Total Marks : 21)
(Single Correct Answer Type)

This section contains **7 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

24. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is

(A) 8.50 kHz (B) 8.25 kHz
(C) 7.75 kHz (D) 7.50 kHZ

Sol. **(A)**

$$f = \frac{320}{320-10}\times 8\times 10^3 \times \frac{320+10}{320}$$

$= 8.5$ kHz

25. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is

(A) 1215 Å (B) 1640 Å
(C) 2430 Å (D) 4687 Å

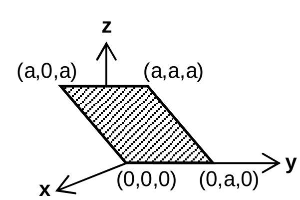
Sol. **(A)**

$$\frac{1}{6561} = R\left(\frac{1}{4}-\frac{1}{9}\right) = \frac{5R}{36}$$

$$\frac{1}{\lambda} = 4R\left(\frac{1}{4}-\frac{1}{16}\right) = \frac{3R\times 4}{16}$$

$\lambda = 1215$ Å.

26. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is



(A) $2E_0a^2$ (B) $\sqrt{2}E_0a^2$

(C) $E_0\,a^2$ (D) $\frac{E_0a^2}{\sqrt{2}}$

Sol. **(C)**

(E_0) (Projected area) $= E_0a^2$

27. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is

(A) $\frac{9}{8}RT_1$ (B) $\frac{3}{2}RT_1$

(C) $\frac{15}{8}RT_1$ (D) $\frac{9}{2}RT_1$

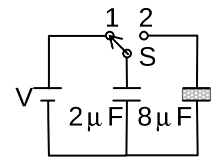
Sol. **(A)**

$TV^{\gamma-1} = C$

$T_1(5.6)^{2/3} = T_2\,(0.7)^{2/3} \quad \Rightarrow \quad T_2 = T_1(8)^{2/3} = 4T_1$

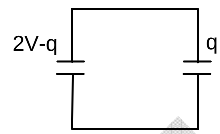
$\therefore \quad \Delta w(\text{work done on the system}) = \dfrac{nR\Delta T}{\gamma - 1} = \dfrac{9}{8}RT_1$

28. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



(A) 0 % (B) 20 %
(C) 75 % (D) 80 %

Sol. **(D)**



$U_i = \dfrac{1}{2}\times 2\times V^2 = V^2$

$q_i = 2V$

Now, switch S is turned to position 2

$\dfrac{2V - q}{2} = \dfrac{q}{8}$

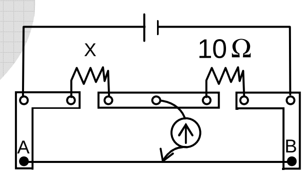
$8V - 4q = q$

$\Rightarrow q = \dfrac{8V}{5}$

$\Delta H = V^2 - \left(\dfrac{64V^2}{2\times 25\times 8} + \dfrac{4V^2}{2\times 25\times 2}\right)$

$= \dfrac{4V^2}{5}$

29. A meter bridge is set-up as shown, to determine an unknown resistance 'X' using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is



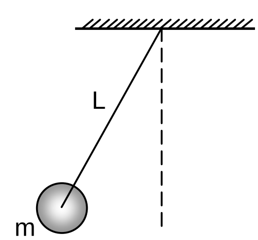
(A) 10.2 ohm (B) 10.6 ohm
(C) 10.8 ohm (D) 11.1 ohm

Sol. **(B)**

$X\,(48 + 2) = (10)\,(52 + 1)$

$X = \dfrac{530}{50} = 10.6\Omega$

30. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is



(A) 9 (B) 18
(C) 27 (D) 36

Sol. **(D)**

$324 = 0.5\,\omega^2\,(0.5)$

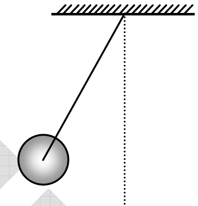
$\omega^2 = 324 \times 4$

$\omega = \sqrt{1296} = 36 \text{ rad/s}$

SECTION – II (Total Marks : 16)
(Multiple Correct Answers Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

31. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (<L) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its centre and (case B) the disc is free to rotate about its centre. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true?



(A) Restoring torque in case A = Restoring torque in case B
(B) restoring torque in case A < Restoring torque in case B
(C) Angular frequency for case A > Angular frequency for case B.
(D) Angular frequency for case A < Angular frequency for case B.

Sol. **(A, D)**
In case A

$$mg(\ell/2)\sin\theta + Mg\,\ell\sin\theta = \left(\frac{m\ell^2}{3} + \frac{MR^2}{2} + M\ell^2\right)\alpha_A$$

In case B

$$mg(\ell/2)\sin\theta + Mg\,\ell\sin\theta = \left(\frac{m\ell^2}{3} + M\ell^2\right)\alpha_B$$

$\tau_A = \tau_B$, $\omega_A < \omega_B$

32. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B (<R_A) are kept far apart and each is given charge '+Q'. Now they are connected by a thin metal wire. Then

(A) $E_A^{inside} = 0$
(B) $Q_A > Q_B$
(C) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$
(D) $E_A^{on\ surface} < E_B^{on\ surface}$

Sol. **(A, B, C, D)**
$R_B < R_A$
$Q_A + Q_B = 2Q$

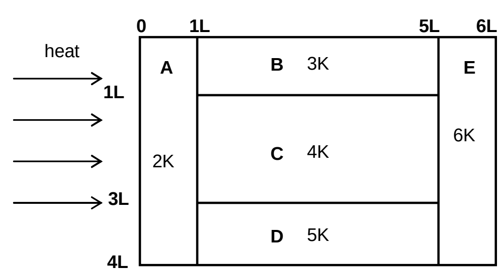
$$\frac{kQ_A}{R_A} = \frac{kQ_B}{R_B}$$

$\sigma_A R_A = \sigma_B R_B$

$$Q_A = \frac{2QR_A}{R_A + R_B}$$

$$Q_B = \frac{2QR_B}{R_A + R_B}$$

33. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state



(A) heat flow through A and E slabs are same.
(B) heat flow through slab E is maximum.
(C) temperature difference across slab E is smallest.
(D) heat flow through C = heat flow through B + heat flow through D.

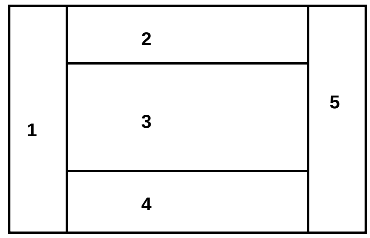
Sol. **(A, C, D) or (A, B, C, D)**

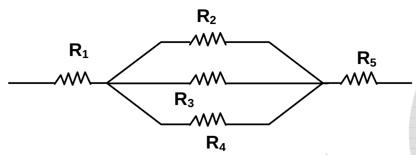
Let width of each rod is d

$R_1 = \dfrac{1}{8kd}$, $R_2 = \dfrac{4}{3kd}$

$R_3 = \dfrac{1}{2kd}$, $R_4 = \dfrac{4}{5kd}$

$R_5 = \dfrac{1}{24kd}$,





34. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is / are true?
(A) They will never come out of the magnetic field region.
(B) They will come out travelling along parallel paths.
(C) They will come out at the same time.
(D) They will come out at different times.

Sol. **(B, D)**

Both will travel in semicircular path

Since, m is different, hence time will be different

SECTION-III (Total Marls : 15)
(Paragraph Type)

This section contains **2 paragraphs.** Based upon one of paragraphs **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of **which ONLY ONE** is correct.

Paragraph for Question Nos. 35 and 36

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let ‘N’ be the number density of free electrons, each of mass ‘m’. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency ‘ω_p’ which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

35. Taking the electronic charge as ‘e’ and the permittivity as ‘ε_0’. Use dimensional analysis to determine the correct expression for ω_p.

(A) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (B) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$

(C) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (D) $\sqrt{\dfrac{m\varepsilon_0}{Ne^2}}$

Sol. **(C)**

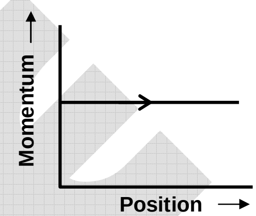
36. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27}\ m^{-3}$. Taking $\varepsilon_0 = 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units.

(A) 800 nm (B) 600 nm
(C) 300 nm (D) 200 nm

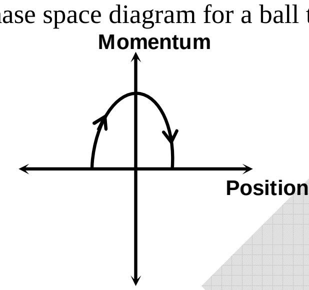
Sol. **(B)**
$\omega = 2\pi c / \lambda$

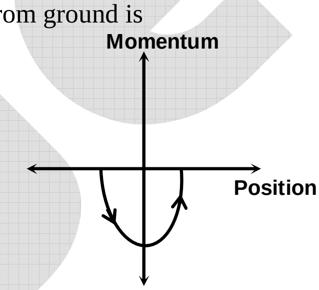
Paragraph for Question Nos. 37 to 39

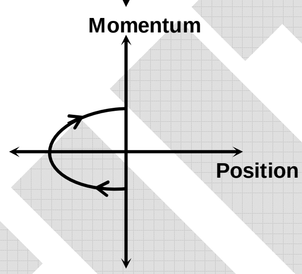
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momenum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which positon or momentum upwards (or to right) is positive and downwards (or to left) is negative.

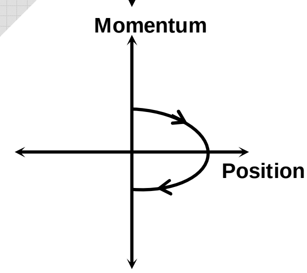


37. The phase space diagram for a ball thrown vertically up from ground is

(A)


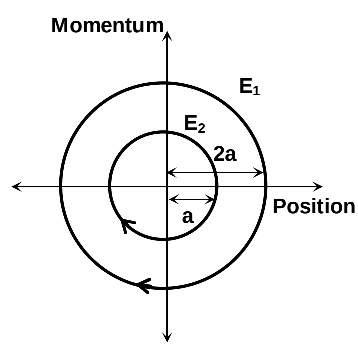
(B)


(C)


(D)


Sol. **(D)**

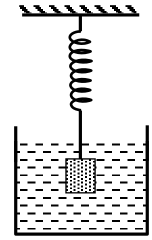
38. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then



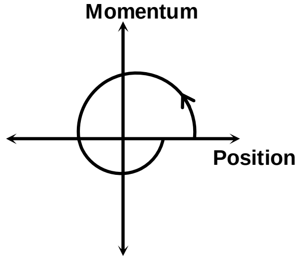
(A) $E_1 = \sqrt{2}E_2$ (B) $E_1 = 2E_2$
(C) $E_1 = 4E_2$ (D) $E_1 = 16E_2$

Sol. **(C)**
$E \propto A^2$

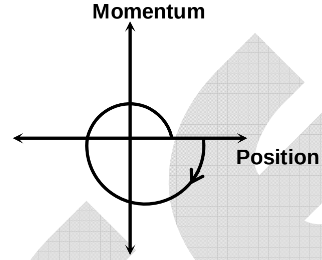
39. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



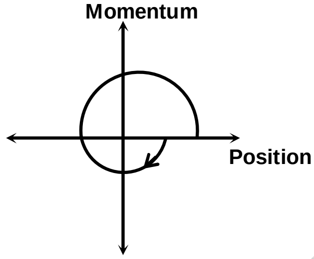
(A)



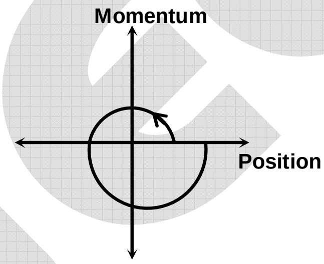
(B)



(C)



(D)

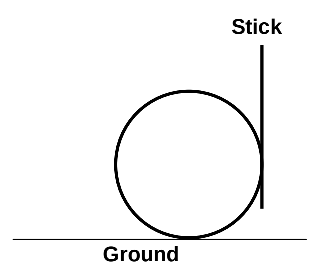


Sol. **(B)**

SECTION-IV (Total Marks : 28)
(Integer Answer Type)

This section contains **7** questions. The answer to each of the questions is a **single digit integer,** ranging from 0 to 9. The bubble corresponding to the correct is to be darkened in the ORS.

40. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficeint of friction between the stick and the ring is (P/10). The value of P is



Sol. **(4)**

Now

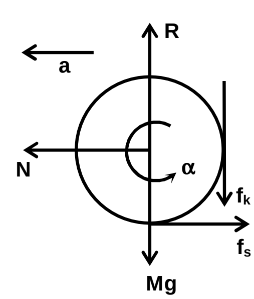
$N - f_s = ma$ $\therefore f_s = 1.4$ N

and $(f_s - f_K)R = I\alpha$, $a = R\alpha$

$\therefore f_K = 0.8$ N

So, $\mu = \frac{P}{10} = 0.4$

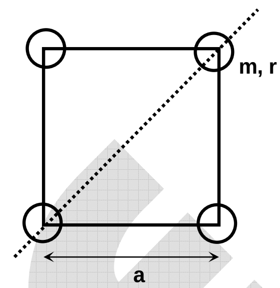
P = 4



41. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kg-m^2, then N is

Sol. **(9)**

$$I = 2\left(\frac{2}{5}\times mr^2\right) + 2\left[\frac{2}{5}mr^2 + m\left(\frac{a}{\sqrt{2}}\right)^2\right]$$



$I = 9 \times 10^{-4}$ kg-m^2
$\therefore$ N = 9

42. Steel wire of lenght 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is 10^{-5}/°C, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that L $\gg$ diameter of the wire. Then the value of 'm' in kg is nearly

Sol. **(3)**

Change in length $\Delta L = \frac{MgL}{YA} = L\alpha\Delta T$

$\therefore$ m $\approx$ 3 kg

43. Four point charges, each of +q, are rigidly fixed at the four corners of a square planar soap film of side 'a'. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, where 'k' is a constant. Then N is

Sol. **(3)**

Since $F_{electric} \propto \frac{q^2}{a^2} \propto \gamma a$

$\therefore a = k\left(\frac{q^2}{\gamma}\right)^{1/3}$

$\therefore$ N = 3

44. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define N = 10 μ, then N is

Sol. **(5)**
mg(sin θ + μcosθ) = 3mg(sinθ – μcosθ)
$\therefore$ μ = 0.5
$\therefore$ N = 5

45. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is

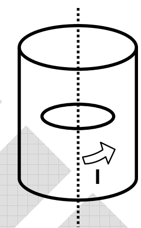
Sol. **(1)**

$$\frac{dN}{dt} = \lambda N$$

$$\therefore\ N = \frac{\left(\frac{dN}{dt}\right)}{\lambda} = 10^{10} \times 10^{9} = 10^{19} \text{ atoms}$$

$m_{sample} = 10^{-25} \times 10^{19}$ kg = 1 mg

46. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0\cos(300\ t)$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin(300\ t)$, then 'N' is



Sol. **(6)**

$$B_{inside} = \mu_0 ni = \frac{\mu_o I}{L}$$

$$\therefore\ E_{ind} = -\frac{d\phi}{dt} = -\frac{\mu_0(\pi r^2)}{L}\frac{dI}{dt}$$

So magnetic moment = $\left(\frac{E_{ind}}{R}\right)\pi r^2$

$= 6\mu_0 I_0 \sin(300t)$

Therefore, n = 6

PART - III:

SECTION – I (Total Marks : 21)
(Single Correct Answer Type)

This section contains **7 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

47. Let (x_0, y_0) be solution of the following equations

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$
$$3^{\ln x} = 2^{\ln y}$$

Then x_0 is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) 6

Sol. **(C)**

$(2x)^{\ln 2} = (3y)^{\ln 3}$ …(i)

$3^{\ln x} = 2^{\ln y}$ …(ii)

$\Rightarrow$ (logx) (log3) = (logy)log2

$\Rightarrow \log y = \dfrac{(\log x)(\log 3)}{\log 2}$ …(iii)

In (i) taking log both sides

(log2) {log2 + logx} = log3{log3 + logy}

$(\log 2)^2 + (\log 2)(\log x) = (\log 3)^2 + \dfrac{(\log 3)^2(\log x)}{\log 2}$ from (iii)

$\Rightarrow (\log 2)^2 - (\log 3)^2 = \dfrac{(\log 3)^2 - (\log 2)^2}{\log 2}(\log x) \Rightarrow -\log 2 = \log x$

$\Rightarrow x = \dfrac{1}{2} \Rightarrow x_0 = \dfrac{1}{2}$.

48. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then

(A) $P \subset Q$ and $Q - P \neq \varnothing$ (B) $Q \not\subset P$

(C) $P \not\subset Q$ (D) $P = Q$

Sol. **(D)**

In set P, $\sin\theta = \left(\sqrt{2}+1\right)\cos\theta \Rightarrow \tan\theta = \sqrt{2}+1$

In set Q, $\left(\sqrt{2}-1\right)\sin\theta = \cos\theta \Rightarrow \tan\theta = \dfrac{1}{\sqrt{2}-1} = \sqrt{2}+1 \Rightarrow P = Q$.

49. Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}, \vec{b} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{c} = \hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\dfrac{1}{\sqrt{3}}$, is given by

(A) $\hat{i}-3\hat{j}+3\hat{k}$ (B) $-3\hat{i}-3\hat{j}+\hat{k}$

(C) $3\hat{i}-\hat{j}+3\hat{k}$ (D) $\hat{i}+3\hat{j}-3\hat{k}$

Sol. **(C)**

$\vec{v} = \lambda\bar{a} + \mu\bar{b}$

$= \lambda\left(\hat{i}+\hat{j}+\hat{k}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)$

Projection of $\vec{v}$ on $\bar{c}$

$\frac{\bar{v}\cdot\bar{c}}{|\bar{c}|} = \frac{1}{\sqrt{3}} \Rightarrow \frac{\left[(\lambda+\mu)\hat{i}+(\lambda-\mu)\hat{j}+(\lambda+\mu)\hat{k}\right]\cdot\left(\hat{i}-\hat{j}-\hat{k}\right)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \lambda + \mu - \lambda + \mu - \lambda - \mu = 1 \Rightarrow \mu - \lambda = 1 \Rightarrow \lambda = \mu - 1$

$\bar{v} = (\mu-1)\left(\hat{i}+\hat{j}+\hat{k}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right) \;\; = \mu\left(2\hat{i}+2\hat{k}\right)-\hat{i}-\hat{j}-\hat{k}$

$\bar{v} = (2\mu-1)\hat{i}-\hat{j}+(2\mu-1)\hat{k}$

At $\mu = 2$, $\bar{v} = 3\hat{i}-\hat{j}+3\hat{k}$.

50. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x\sin x^2}{\sin x^2+\sin\left(\ln 6 - x^2\right)}dx$ is

(A) $\frac{1}{4}\ln\frac{3}{2}$
(B) $\frac{1}{2}\ln\frac{3}{2}$
(C) $\ln\frac{3}{2}$
(D) $\frac{1}{6}\ln\frac{3}{2}$

Sol. **(A)**

$x^2 = t \Rightarrow 2x\,dx = dt$

$I = \frac{1}{2}\int_{\ln 2}^{\ln 3}\frac{\sin t}{\sin t+\sin(\ln 6 - t)}dt$ and $I = \frac{1}{2}\int_{\ln 2}^{\ln 3}\frac{\sin(\ln 6 - t)}{\sin(\ln 6 - t)+\sin t}dt$

$2I = \frac{1}{2}\int_{\ln 2}^{\ln 3}1dt \Rightarrow I = \frac{1}{4}\ln\frac{3}{2}$.

51. A straight line L through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x+y=1$. If L also intersects the x-axis, then the equation of L is

(A) $y+\sqrt{3}x+2-3\sqrt{3}=0$
(B) $y-\sqrt{3}x+2+3\sqrt{3}=0$
(C) $\sqrt{3}y-x+3+2\sqrt{3}=0$
(D) $\sqrt{3}y+x-3+2\sqrt{3}=0$

Sol. **(B)**

$\left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$

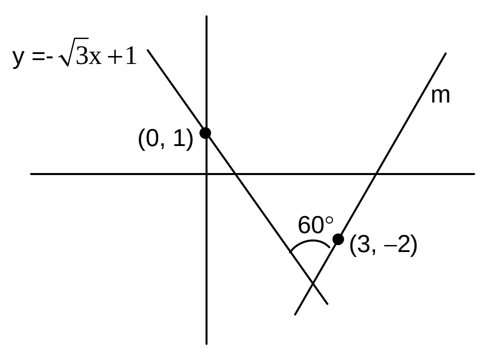
$\Rightarrow m+\sqrt{3} = \pm\left(\sqrt{3}-3m\right)$

$\Rightarrow 4m = 0 \Rightarrow m = 0$

or $2m = 2\sqrt{3} \Rightarrow m = \sqrt{3}$

$\therefore$ Equation is $y + 2 = \sqrt{3}(x-3)$

$\Rightarrow \sqrt{3}x - y - \left(2+3\sqrt{3}\right) = 0$



52. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is

(A) 1
(B) 2
(C) 3
(D) 4

Sol. **(C)**

$a_n = \alpha^n - \beta^n$

$\alpha^2 - 6\alpha - 2 = 0$

Multiply with α^8 on both sides

$\Rightarrow \alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$ …(i)

similarly $\beta^{10} - 6\beta^9 - 2\beta^8 = 0$ …(ii)

(i) and (ii)

$\Rightarrow \alpha^{10} - \beta^{10} - 6(\alpha^9 - \beta^9) = 2(\alpha^8 - \beta^8)$

$\Rightarrow a_{10} - 6a_9 = 2a_8 \Rightarrow \dfrac{a_{10} - 2a_8}{2a_9} = 3$.

53. Let the straight line x = b divides the area enclosed by $y = (1 - x)^2$, y = 0 and x = 0 into two parts $R_1(0 \le x \le b)$ and $R_2(b \le x \le 1)$ such that $R_1 - R_2 = \dfrac{1}{4}$. Then b equals

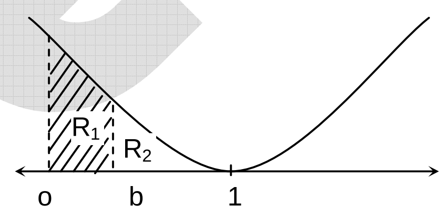
(A) $\dfrac{3}{4}$

(B) $\dfrac{1}{2}$

(C) $\dfrac{1}{3}$

(D) $\dfrac{1}{4}$

Sol. **(B)**



$\because \displaystyle\int_0^b (1-x)^2\,dx - \int_b^1 (1-x)^2\,dx = \frac{1}{4}$

$\Rightarrow \left.\dfrac{(x-1)^3}{3}\right|_0^b - \left.\dfrac{(x-1)^3}{3}\right|_b^1 = \dfrac{1}{4}$

$\Rightarrow \dfrac{(b-1)^3}{3} + \dfrac{1}{3} - \left(0 - \dfrac{(b-1)^3}{3}\right) = \dfrac{1}{4}$

$\Rightarrow \dfrac{2(b-1)^3}{3} = -\dfrac{1}{12} \Rightarrow (b - 1)^3 = -\dfrac{1}{8} \Rightarrow b = \dfrac{1}{2}$.

SECTION – II (Total Marks : 16)
(Multiple Correct Answers Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

54. Let f: R → R be a function such that f(x + y) = f(x) + f(y), ∀ x, y ∈ R. If f(x) is differentiable at x = 0, then

(A) f(x) is differentiable only in a finite interval containing zero

(B) f(x) is continuous ∀ x ∈ R

(C) f′(x) is constant ∀ x ∈ R

(D) f(x) is differentiable except at finitely many points

Sol. **(B, C)**

$\because$ f(0) = 0

and $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$= = \lim\limits_{h\to 0}\dfrac{f(h)}{h} = f'(0) = k$(say)

$\Rightarrow f(x) = kx + c \Rightarrow f(x) = kx$ ($\because$ f(0) = 0).

55. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are

(A) $\hat{j}-\hat{k}$
(B) $-\hat{i}+\hat{j}$
(C) $\hat{i}-\hat{j}$
(D) $-\hat{j}+\hat{k}$

Sol. **(A, D)**

Any vector in the plane of $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ is $\vec{r}=(\lambda+\mu)\hat{i}+(\lambda+2\mu)\hat{j}+(2\lambda+\mu)\hat{k}$

Also, $\vec{r}\cdot\vec{c}=0 \Rightarrow \lambda+\mu=0$

$\Rightarrow \left[\vec{r}\ \vec{a}\ \vec{b}\right]=0$.

56. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then

(A) the equation of the hyperbola is $\frac{x^2}{3}-\frac{y^2}{2}=1$
(B) a focus of the hyperbola is (2, 0)
(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$
(D) the equation of the hyperbola is $x^2 - 3y^2 = 3$

Sol. **(B, D)**

Ellipse is $\frac{x^2}{2^2}+\frac{y^2}{1^2}=1$

$1^2 = 2^2(1-e^2) \Rightarrow e = \frac{\sqrt{3}}{2}$

$\therefore$ eccentricity of the hyperbola is $\frac{2}{\sqrt{3}} \Rightarrow b^2 = a^2\left(\frac{4}{3}-1\right) \Rightarrow 3b^2 = a^2$

Foci of the ellipse are $\left(\sqrt{3},\ 0\right)$ and $\left(-\sqrt{3},\ 0\right)$.

Hyperbola passes through $\left(\sqrt{3},\ 0\right)$

$\frac{3}{a^2}=1 \Rightarrow a^2 = 3$ and $b^2 = 1$

$\therefore$ Equation of hyperbola is $x^2 - 3y^2 = 3$

Focus of hyperbola is $(ae, 0) \equiv \left(\sqrt{3}\times\frac{2}{\sqrt{3}},\ 0\right) \equiv (2, 0)$.

57. Let M and N be two 3 × 3 non-singular skew symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to

(A) M^2
(B) $-N^2$
(C) $-M^2$
(D) MN

Sol. **(C)**

MN = NM

$M^2N^2(M^TN)^{-1}(MN^{-1})^T$

$M^2N^2N^{-1}(M^T)^{-1}(N^{-1})^T.M^T$

$= M^2N.(M^T)^{-1}(N^{-1})^TM^T = -M^2.N(M)^{-1}(N^T)^{-1}M^T$

$= +M^2NM^{-1}N^{-1}M^T = -M.NMM^{-1}N^{-1}M = -MNN^{-1}M = -M^2$.

Note: A skew symmetric matrix of order 3 cannot be non-singular hence the question is wrong.

SECTION-III (Total Marls : 15)
(Paragraph Type)

This section contains **2 paragraphs.** Based upon one of paragraphs **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of **which ONLY ONE** is correct.

Paragraph for Question Nos. 58 to 59

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

58. The probability of the drawn ball from U_2 being white is

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$

(C) $\frac{19}{30}$ (D) $\frac{11}{30}$

Sol. **(B)**

$H \rightarrow$ 1 ball from U_1 to U_2

$T \rightarrow$ 2 ball from U_1 to U_2

E : 1 ball drawn from U_2

$$\text{P/W from } U_2 = \frac{1}{2}\times\left(\frac{3}{5}\times 1\right)+\frac{1}{2}\times\left(\frac{2}{5}\times\frac{1}{2}\right)+\frac{1}{2}\times\left(\frac{{}^3C_2}{{}^5C_2}\times 1\right)+\frac{1}{2}\times\left(\frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3}\right)+\frac{1}{2}\times\left(\frac{{}^3C_1\cdot{}^2C_1}{{}^5C_2}\times\frac{2}{3}\right) = \frac{23}{30}.$$

59. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$

(C) $\frac{15}{23}$ (D) $\frac{12}{23}$

Sol. **(D)**

$$P\left(\frac{H}{W}\right)=\frac{P(W/H)\times P(H)}{P(W/T)\cdot P(T)+(W/H)\cdot P(H)} = \frac{\frac{1}{2}\left(\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right)}{23/30}=\frac{12}{23}.$$

Paragraph for Question Nos. 60 to 62

Let a, b and c be three real numbers satisfying $\begin{bmatrix} a & b & c\end{bmatrix}\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7\end{bmatrix}=\begin{bmatrix} 0 & 0 & 0\end{bmatrix}$ ……(E)

60. If the point P(a, b, c), with reference to (E), lies on the plane 2x + y + z = 1, then the value of 7a + b + c is

(A) 0 (B) 12

(C) 7 (D) 6

Sol. **(D)**

a + 8b + 7c = 0

9a + 2b + 3c = 0

a + b + c = 0
Solving these we get
b = 6a $\Rightarrow$ c = – 7a
now 2x + y + z = 0
$\Rightarrow$ 2a + 6a + (–7a) = 1 $\Rightarrow$ a = 1, b = 6, c = – 7.

61. Let ω be a solution of $x^3 - 1 = 0$ with Im(ω) > 0. If a = 2 with b and c satisfying (E), then the value of $\frac{3}{\omega^a}+\frac{1}{\omega^b}+\frac{3}{\omega^c}$ is equal to

(A) –2
(B) 2
(C) 3
(D) – 3

Sol. **(A)**
a = 2, b and c satisfies (E)
b = 12, c = – 14

$$\frac{3}{\omega^a}+\frac{1}{\omega^b}+\frac{3}{\omega^c}=\frac{3}{\omega^2}+\frac{1}{\omega^{12}}+\frac{3}{\omega^{-14}} = -2.$$

62. Let b = 6, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n$ is

(A) 6
(B) 7
(C) $\frac{6}{7}$
(D) ∞

Sol. **(B)**
$ax^2 + bx + c = 0 \Rightarrow x^2 + 6x - 7 = 0$
$\Rightarrow \alpha = 1, \beta = -7$

$$\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n = \sum_{n=0}^{\infty}\left(\frac{1}{1}-\frac{1}{7}\right)^n = 7.$$

SECTION-IV (Total Marks : 28)
(Integer Answer Type)

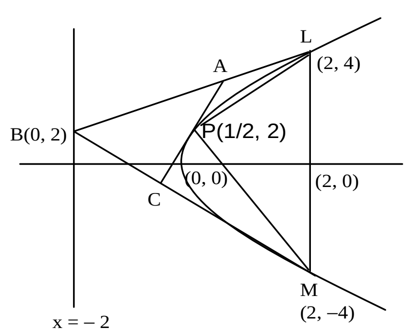
This section contains **7** questions. The answer to each of the questions is a **single digit integer,** ranging from 0 to 9. The bubble corresponding to the correct is to be darkened in the ORS.

63. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is

Sol. **(2)**
$y^2 = 8x = 4.2.x$

$$\frac{\Delta LPM}{\Delta ABC} = 2$$

$$\frac{\Delta_1}{\Delta_2} = 2$$



64. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

Sol. **(1)**

$\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $\theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{2\cos^2\theta - 1}}\right)\right)$

$= \sin\left(\sin^{-1}(\tan\theta)\right) = \tan\theta$.

$\frac{d(\tan\theta)}{d(\tan\theta)} = 1$.

65. Let f: $[1, \infty) \to [2, \infty)$ be a differentiable function such that f(1) = 2. If $6\int_1^x f(t)dt = 3xf(x) - x^3$ for all $x \geq 1$, then the value of f(2) is

Sol. **(6)**

$6\int_1^x f(t)dt = 3x\,f(x) - x^3 \Rightarrow 6f(x) = 3f(x) + 3xf'(x) - 3x^2$

$\Rightarrow 3f(x) = 3xf'(x) - 3x^2 \Rightarrow xf'(x) - f(x) = x^2$

$\Rightarrow x\frac{dy}{dx} - y = x^2 \Rightarrow \frac{dy}{dx} - \frac{1}{x}y = x \quad \ldots(i)$

I.F. $= e^{\int -\frac{1}{x}dx} = e^{-\log_e x}$

Multiplying (i) both sides by $\frac{1}{x}$

$\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = 1 \Rightarrow \frac{d}{dx}\left(y.\frac{1}{x}\right) = 1$

integrating

$\frac{y}{x} = x + c$

Put x = 1, y = 2

$\Rightarrow 2 = 1 + c \Rightarrow c = 1 \Rightarrow y = x^2 + x$

$\Rightarrow f(x) = x^2 + x \Rightarrow f(2) = 6$.

Note: If we put x = 1 in the given equation we get f(1) = 1/3.

66. The positive integer value of n > 3 satisfying the equation $\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

Sol. **(7)**

$$\frac{1}{\sin\frac{\pi}{n}} - \frac{1}{\sin\frac{3\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}} \Rightarrow \frac{\sin\frac{3\pi}{n} - \sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}} \quad \frac{\left(2\sin\frac{\pi}{n}\cos\frac{2\pi}{n}\right)\sin\frac{2\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}} = 1$$

$\Rightarrow \sin\frac{4\pi}{n} = \sin\frac{3\pi}{n} \Rightarrow \frac{4\pi}{n} + \frac{3\pi}{n} = \pi \Rightarrow n = 7.$

67. Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i$, $1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is

Sol. **(9)**

$a_1, a_2, a_3, \ldots a_{100}$ is an A.P.

$a_1 = 3$, $S_p = \sum_{i=1}^{p} a_i$, $1 \le p \le 1\,00$

$$\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}\left(6+(5n-1)d\right)}{\frac{n}{2}\left(6-d+nd\right)}$$

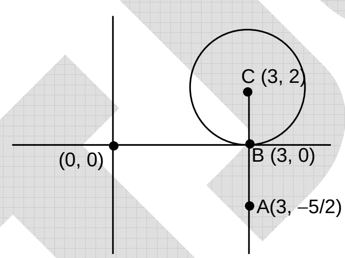
$\frac{S_m}{S_n}$ is independent of n of $6 - d = 0 \Rightarrow d = 6$.

68. If z is any complex number satisfying $|z - 3 - 2i| \le 2$, then the minimum value of $|2z - 6 + 5i|$ is

Sol. **(5)**

Length AB = $\frac{5}{2}$

$\Rightarrow$ Minimum value = 5.



69. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} with $a > 0$ is

Sol. **(8)**

$$\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + a^{8} + a^{10} + 1}{8} \ge 1$$

minimum value = 8.