FIITJEE Solutions to IIT-JEE-2011

CODE 5

PAPER 2

Time: 3 Hours

Time: 3 Hours Maximum Marks: 240

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. The **question paper CODE** is printed on the right hand top corner of this sheet and on the back page (page No. 36) of this booklet.
2. No additional sheets will be provided for rough work.
3. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets are NOT allowed.
4. Write your name and registration number in the space provided on the back page of this booklet.
5. The answer sheet, a machine-gradable Optical Response Sheet (ORS), is provided separately.
6. DO NOT TAMPER WITH/MULTILATE THE ORS OR THE BOOKLET.
7. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilators.
8. This question Paper contains 36 pages having 69 questions.
9. On breaking the seals, please check that all the questions are legible.

B. Filling the Right Part of the ORS:

10. The ORS also has a CODE printed on its Left and Right parts.
11. Make sure the CODE on the ORS is the same as that on this booklet. **If the codes do not match ask for a change of the booklet.**
12. Write your Name, Registration No. and the name of centre and sign with pen in the boxes provided**. Do not write them anywhere else. Darken the appropriate bubble UNDER** each digit of your Registration No. with a **good quality HB pencil.**

C. Question paper format and Marking scheme:

13. The question paper consists of **3 parts** (Chemistry, Physics and Mathematics). Each part consists of **four sections.**
14. In **Section I** (Total Marks: 24), for each question you will be awarded **3 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** if no bubble is darkened. In all other cases, **minus one (–1)** mark will be awarded.
15. In **Section II** (Total Marks: 16), for each question you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero** marks other wise. There are **no negative marks** in this section.
16. In **Section III** (Total Marks: 24), for each question you will be awarded **4 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** otherwise There are **no negative marks** in this section.
17. In **Section IV** (Total Marks: 16), for each question you will be awarded **2 marks** for each row in which you have darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero marks** otherwise. Thus each question in this section carries **a maximum of 8** Marks. There are **no negative marks** in this section.

PAPER-2 [Code – 5]
IITJEE 2011
PART - I: CHEMISTRY

SECTION – I (Total Marks : 24)
(Single Correct Answer Type)

This Section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. The freezing point (in ^oC) of a solution containing 0.1 g of $K_3[Fe(CN)_6$ (Mol. Wt. 329) in 100 g of water ($K_f = 1.86$ K kg mol^{-1}) is

(A) -2.3×10^{-2} (B) -5.7×10^{-2}
(C) -5.7×10^{-3} (D) -1.2×10^{-2}

Sol. **(A)**

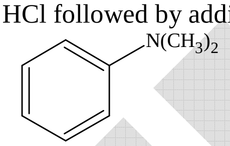
$$K_3\left[Fe(CN)_6\right] \rightarrow 3K^+ + \left[Fe(CN)_6\right]^{3-}$$

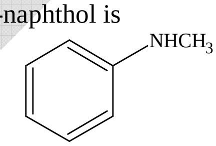
i = 4

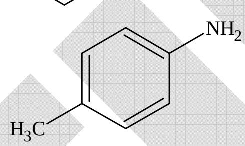
$$\Delta T_f = K_f \times i \times \frac{m}{M} \times \frac{1000}{W} = 1.86 \times 4 \times \frac{0.1}{329} \times \frac{1000}{100} = 2.3 \times 10^{-2}$$

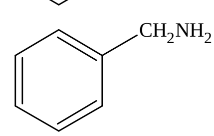
$$T_f^{'} = -2.3 \times 10^{-2}$$

2. Amongst the compounds given, the one that would form a brilliant colored dye on treatment with $NaNO_2$ in dil. HCl followed by addition to an alkaline solution of β-naphthol is

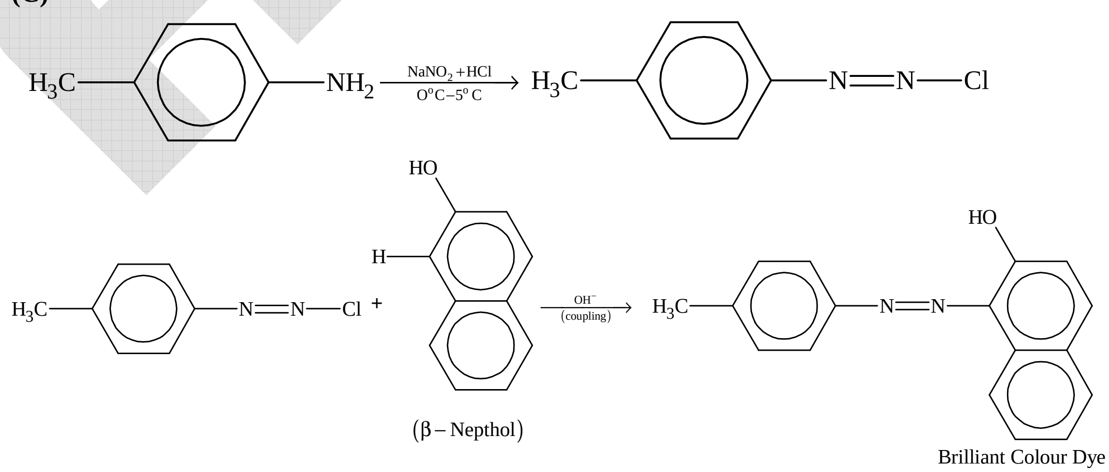
(A) 

(B) 

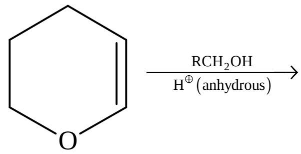
(C) 

(D) 

Sol. **(C)**

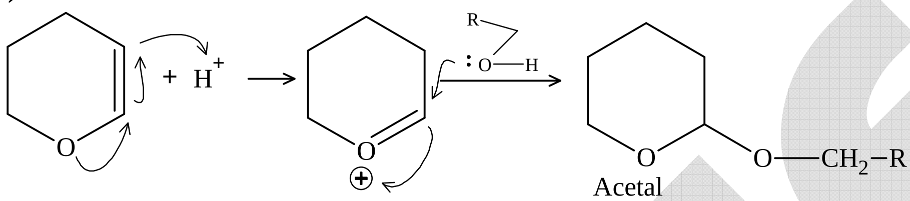


3. The major product of the following reaction is



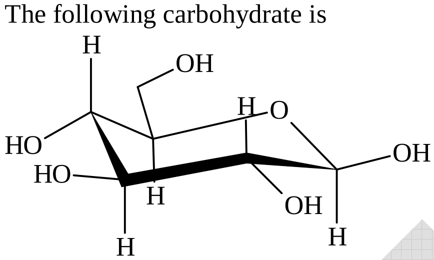
(A) a hemiacetal
(B) an acetal
(C) an ether
(D) an ester

Sol. **(B)**



Acetal

4. The following carbohydrate is



(A) a ketohexose
(B) an aldohexose
(C) an α-furanose
(D) an α-pyranose

Sol. **(B)**

5. Oxidation states of the metal in the minerals haematite and magnetite, respectively, are
(A) II, III in haematite and III in magnetite
(B) II, III in haematite and II in magnetite
(C) II in haematite and II, III in magnetite
(D) III in haematite and II, III in magnetite

Sol. **(D)**
Haematite : Fe_2O_3 : $2x + 3 \times (-2) = 0$
$x = 3$
Magnetite : Fe_3O_4 [an equimolar mixture of FeO and Fe_2O_3]
FeO : $x - 2 = 0 \Rightarrow x = 2$
Fe_2O_3 : $x = 3$

6. Among the following complexes (**K–P**)
$K_3[Fe(CN)_6]$ (**K**), $[Co(NH_3)_6]Cl_3$ (**L**), $Na_3[Co(oxalate)_3]$ (**M**), $[Ni(H_2O)_6]Cl_2$ (**N**), $K_2[Pt(CN)_4]$ (**O**) and $[Zn(H_2O)_6](NO_3)_2$ (**P**)
(A) **K, L, M, N**
(B) **K, M, O, P**
(C) **L, M, O, P**
(D) **L, M, N, O**

6. **(C)**
Following compounds are diamagnetic.
L : $[Co(NH_3)_6]Cl_3$
M : $Na_3[Co(Ox)_3]$
O : $K_2[Pt(CN)_4]$
P : $[Zn(H_2O)_6](NO_3)_2$

7. Passing H_2S gas into a mixture of Mn^{2+}, Ni^{2+}, Cu^{2+} and Hg^{2+} ions in an acidified aqueous solution precipitates
(A) CuS and HgS
(B) MnS and CuS
(C) MnS and NiS
(D) NiS and HgS

Sol. **(A)**
H_2S in presence of aqueous acidified solution precipitates as sulphide of Cu and Hg apart from Pb^{+2}, Bi^{+3}, Cd^{+2}, As^{+3}, Sb^{+3} and Sn^{+2}.

8. Consider the following cell reaction:
$$2Fe_{(s)} + O_{2(g)} + 4H^+_{(aq)} \rightarrow 2Fe^{2+}_{(aq)} + 2H_2O_{(\ell)} \quad E^\circ = 1.67\text{ V}$$
At $[Fe^{2+}] = 10^{-3}$ M, $P(O_2) = 0.1$ atm and pH = 3, the cell potential at 25°C is
(A) 1.47 V
(B) 1.77 V
(C) 1.87 V
(D) 1.57 V

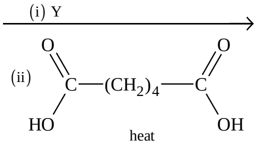
Sol. **(D)**
$$2Fe(s) + O_2(g) + 4H^+(aq) \longrightarrow 2Fe^{+2}(aq) + 2H_2O(\ell)$$
N = 4 (no. of moles of electron involved)
From Nernst's equation,
$$E_{cell} = E^\circ_{cell} - \frac{0.0591}{n}\log Q$$
$$= 1.67 - \frac{0.0591}{4}\log\frac{(10^{-3})^2}{0.1\times(10^{-3})^4} \qquad \{\because [H^+] = 10^{-pH}\}$$
$= 1.67 - 0.106$
$= 1.57$ V

SECTION – II (Total Marks : 16)
(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

9. Reduction of the metal centre in aqueous permanganate ion involves
(A) 3 electrons in neutral medium
(B) 5 electrons in neutral medium
(C) 3 electrons in alkaline medium
(D) 5 electrons in acidic medium

Sol. **(A, D)**
In acidic medium
$$MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$$
In neutral medium
$$MnO_4^- + 2H_2O + 3e^- \longrightarrow MnO_2 + 4OH^-$$
Hence, number of electron loose in acidic and neutral medium 5 and 3 electrons respectively.

10. The correct functional group X and the reagent/reaction conditions Y in the following scheme are
$$X-(CH_2)_4-X \xrightarrow{\text{(i) Y}} \text{condensation polymer}$$


(A) X = $COOCH_3$, Y = H_2/Ni/heat
(B) X = $CONH_2$, Y = H_2/Ni/heat
(C) X = $CONH_2$, Y = Br_2/NaOH
(D) X = CN, Y = H_2/Ni/heat

Sol. **(A, B, C, D)**

Condensation polymers are formed by condensation of a diols or diamine with dicarboxylic acids.

Hence, X may be $-\overset{\overset{\large O}{||}}{C}-OR$ or $-\overset{\overset{\large O}{||}}{C}-NH_2$ or $-C\equiv N$

$$H_3CO-\overset{\overset{O}{||}}{C}-(CH_2)_4-\overset{\overset{O}{||}}{C}-OCH_3 \xrightarrow{H_2/Ni} OH-H_2C(CH_2)_4-CH_2-OH \xrightarrow{HO-\overset{\overset{O}{||}}{C}-(CH_2)_4-\overset{\overset{O}{||}}{C}-OH} \text{Poly}-\text{ester}$$

$$H_2N-\overset{\overset{O}{||}}{C}-(CH_2)_4-\overset{\overset{O}{||}}{C}-NH_2 \xrightarrow{H_2/Ni} H_2N-CH_2(CH_2)_4-CH_2-NH_2 \xrightarrow{HO-\overset{\overset{O}{||}}{C}-(CH_2)_4-\overset{\overset{O}{||}}{C}-OH} \text{Polyamide}$$

$\downarrow$ $Br_2/OH^-, \Delta$

Hoffmann Bromamide Reaction

$$H_2N-(CH_2)_4-NH_2 \xrightarrow{HO-\overset{\overset{O}{||}}{C}-(CH_2)_4-\overset{\overset{O}{||}}{C}-OH} \text{Polyamide}$$

$$N\equiv C-(CH_2)_4-C\equiv N \xrightarrow{H_2/Ni} H_2N-CH_2-(CH_2)-CH_2-NH_2 \xrightarrow{HO-\overset{\overset{O}{||}}{C}-(CH_2)_4-\overset{\overset{O}{||}}{C}-OH} \text{Polyamides}$$

11. For the first order reaction

$2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$

(A) the concentration of the reactant decreases exponentially with time

(B) the half-life of the reaction decreases with increasing temperature

(C) the half-life of the reaction depends on the initial concentration of the reactant

(D) the reaction proceeds to 99.6% completion in eight half-life duration

Sol. **(A, B, D)**

For first order reaction

$[A] = [A]_0 e^{-kt}$

Hence concentration of $[NO_2]$ decreases exponentially.

Also, $t_{1/2} = \dfrac{0.693}{K}$. Which is independent of concentration and $t_{1/2}$ decreases with the increase of temperature.

$$t_{99.6} = \frac{2.303}{K}\log\left(\frac{100}{0.4}\right)$$

$$t_{99.6} = \frac{2.303}{K}(2.4) = 8\times\frac{0.693}{K} = 8\ t_{1/2}$$

12. The equilibrium

$2Cu^{I} \rightleftharpoons Cu^{o} + Cu^{II}$

in aqueous medium at 25°C shifts towards the left in the presence of

(A) NO_3^- (B) Cl^-

(C) SCN^- (D) CN^-

Sol. **(B, C, D)**

Cu^{2+} ions will react with CN^- and SCN^- forming $[Cu(CN)_4]^{3-}$ and $[Cu(SCN)_4]^{3-}$ leading the reaction in the backward direction.

$Cu^{2+} + 2CN^- \rightarrow Cu(CN)_2$

$2Cu(CN)_2 \rightarrow 2CuCN + (CN)_2$

$$CuCN + 3CN^- \rightarrow \left[Cu(CN)_4\right]^{3-}$$

$$Cu^{2+} + 4SCN^- \rightarrow \left[Cu(SCN)_4\right]^{3-}$$

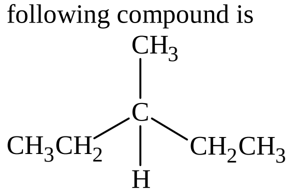
Cu^{2+} also combines with $CuCl_2$ which reacts with Cu to produce CuCl pushing the reaction in the backward direction.

$$CuCl_2 + Cu \rightarrow 2CuCl\downarrow$$

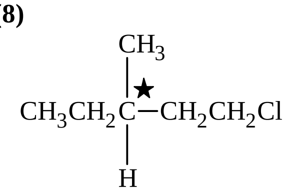
SECTION-III (Total Marks : 24)
(Integer Answer Type)

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the **ORS**.

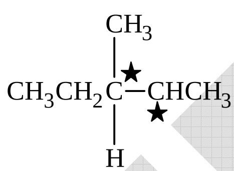
13. The maximum number of isomers (including stereoisomers) that are possible on monochlorination of the following compound is



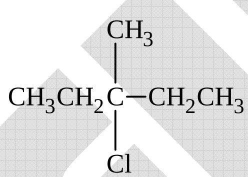
Sol. **(8)**



Enantiomeric pair = 2



Two Enantiomeric pairs = 4



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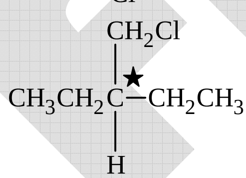
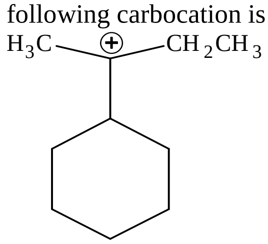


1

Total = 2 + 4 + 1 + 1 = 8

14. The total number of contributing structure showing hyperconjugation (involving C−H bonds) for the following carbocation is



Sol. **(6)**
$6 \times$ H–atoms are there

15. Among the following, the number of compounds than can react with PCl_5 to give $POCl_3$ is
O_2, CO_2, SO_2, H_2O, H_2SO_4, P_4O_{10}

Sol. **(5)**

16. The volume (in mL) of 0.1 M $AgNO_3$ required for complete precipitation of chloride ions present in 30 mL of 0.01 M solution of $[Cr(H_2O)_5Cl]Cl_2$, as silver chloride is close to

Sol. **(6)**
Number of ionisable Cl^- in $[Cr(H_2O)_5Cl]Cl_2$ is 2
$\therefore$ Millimoles of Cl^- = $30 \times 0.01 \times 2 = 0.6$
$\therefore$ Millimoles of Ag^+ required = 0.6
$\therefore$ 0.6 = 0.1 V
V = 6 ml

17. In 1 L saturated solution of AgCl [$K_{sp}(AgCl) = 1.6 \times 10^{-10}$], 0.1 mol of CuCl [$K_{sp}(CuCl) = 1.0 \times 10^{-6}$] is added. The resultant concentration of Ag^+ in the solution is 1.6×10^{-x}. The value of "x" is

Sol. **(7)**
Let the solubility of AgCl is x mollitre^{-1}
and that of CuCl is y mollitre^{-1}

$$\underset{}{AgCl} \rightleftharpoons \underset{x}{Ag^+} + \underset{x}{Cl^-}$$

$$\underset{}{CuCl} \rightleftharpoons \underset{y}{Cu^+} + \underset{y}{Cl^-}$$

$\therefore K_{sp}$ of AgCl = $[Ag^+][Cl^{-1}]$
$1.6 \times 10^{-10} = x(x + y)$ … (i)
Similarly K_{sp} of CuCl = $[Cu^+][Cl^-]$
$1.6 \times 10^{-6} = y(x + y)$ … (ii)
On solving (i) and (ii)
$[Ag^+] = 1.6 \times 10^{-7}$
$\therefore$ x = 7

18. The number of hexagonal faces that are present in a truncated octahedron is

Sol. **(8)**

SECTION-IV (Total Marks : 16)
(Matrix-Match Type)

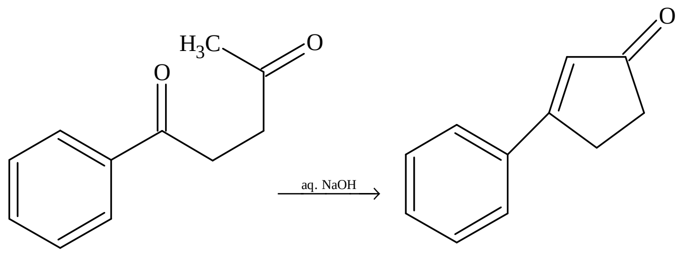
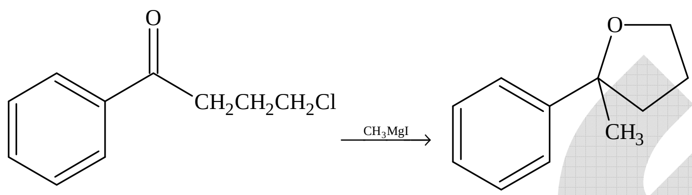
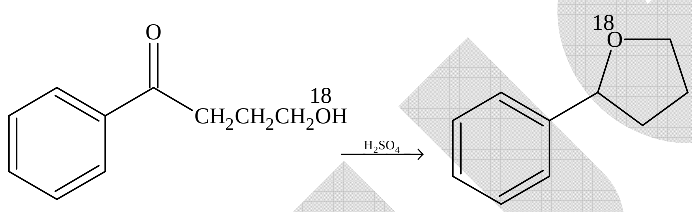
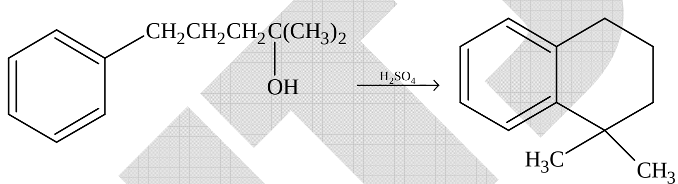
This section contains **2 questions**. Each question has four statements (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

19. Match the transformations in **column I** with appropriate options in **column II**

Column I		Column II	
(A)	$CO_2(s) \rightarrow CO_2(g)$	(p)	phase transition
(B)	$CaCO_3(s) \rightarrow CaO(s) + CO_2(g)$	(q)	allotropic change
(C)	$2H\bullet \rightarrow H_2(g)$	(r)	ΔH is positive
(D)	$P_{(\text{white, solid})} \rightarrow P_{(\text{red, solid})}$	(s)	ΔS is positive
		(t)	ΔS is negative

Sol. **(A) → (p, r, s)** **(B) → (r, s)** **(C) → (t)** **(D) → (p, q, t)**

20. Match the reactions in **column I** with appropriate types of steps/reactive intermediate involved in these reactions as given in **column II**

Column I		Column II
(A)		(p) Nucleophilic substitution
(B)		(q) Electrophilic substitution
(C)		(r) Dehydration
(D)		(s) Nucleophilic addition
		(t) Carbanion

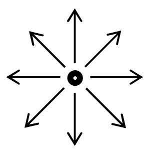
Sol. **(A) → (r, s, t)** **(B) → (p, s)** **(C) → (r, s)** **(D) → (q, r)**

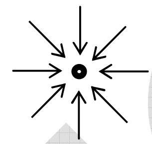
PART - II:

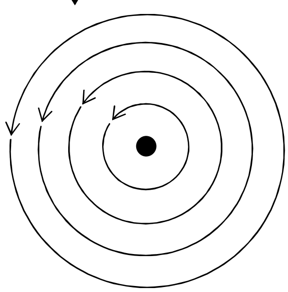
SECTION – I (Total Marks : 24)
(Single Correct Answer Type)

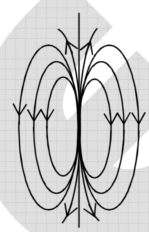
This Section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

21. Which of the field patterns given below is valid for electric field as well as for magnetic field?

(A)


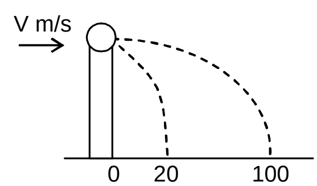
(B)


(C)


(D)


Sol. **(C)**

22. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The velocity V of the bullet is



(A) 250 m/s
(B) $250\sqrt{2}$ m/s
(C) 400 m/s
(D) 500 m/s

Sol. **(D)**

$5 = \frac{1}{2}(10)t^2 \Rightarrow t = 1$ sec

$v_{ball} = 20$ m/s

$v_{bullet} = 100$ m/s

$0.01\text{ V} = 0.01 \times 100 + 0.2 \times 20$

$v = 100 + 400 = 500$ m/s

23. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2 %, the relative percentage error in the density is

(A) 0.9 %
(B) 2.4 %
(C) 3.1 %
(D) 4.2 %

Sol. **(C)**

diameter $= 2.5 + \frac{0.5}{50} \times 20$

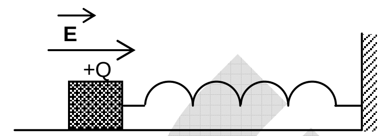
$= 2.70$ mm

$\% \text{ error} = \left(\frac{dm}{m} + 3\frac{dr}{r}\right) \times 100$

$= 2 + 3 \times \frac{0.01}{2.70} \times 100$

$= 3.1\ \%$

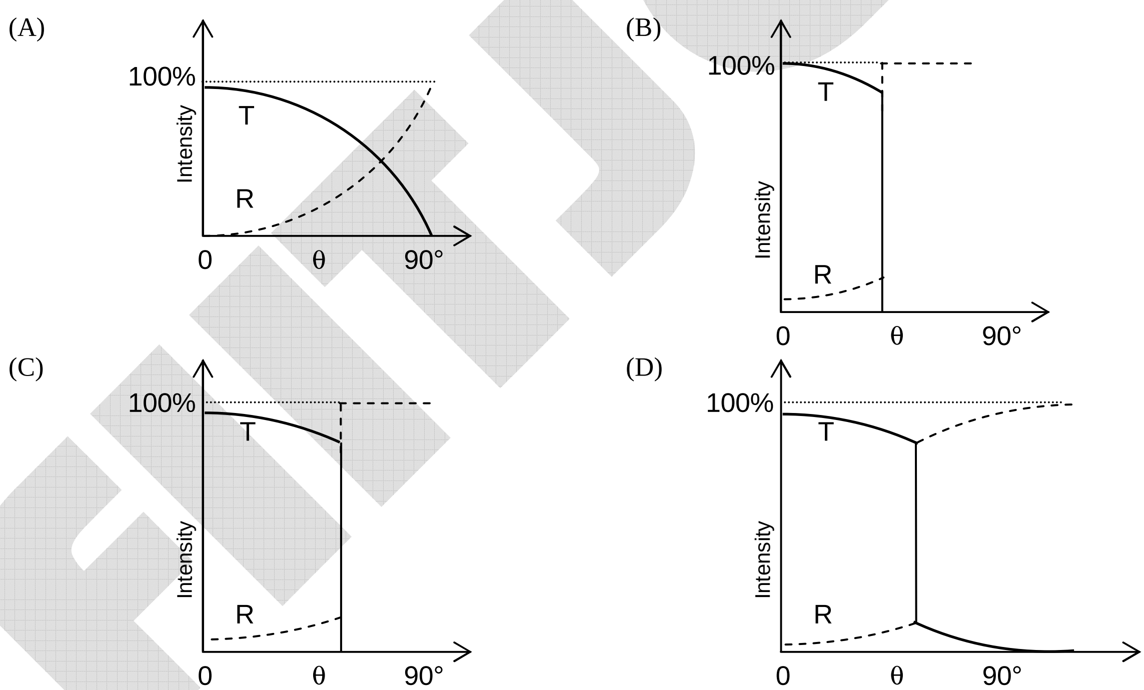
24. A wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge +Q on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be



(A) of the same frequency and with shifted mean position.
(B) of the same frequency and with the same mean position
(C) of changed frequency and with shifted mean position.
(D) of changed frequency and with the same mean position.

Sol. **(A)**

25. A light ray travelling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is



Sol. **(C)**
After total internal reflection, there is no refracted ray.

26. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

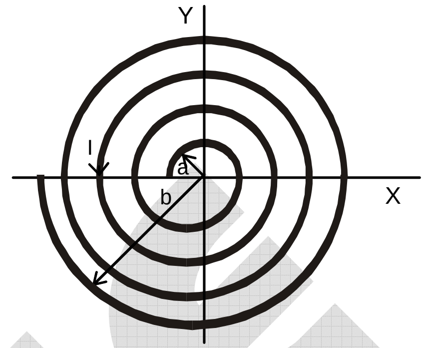
(A) $\frac{1}{2}mV^2$ (B) mV^2

(C) $\frac{3}{2}mV^2$ (D) $2mV^2$

Sol. **(B)**

$$\frac{mV^2}{r} = \frac{GMm}{r^2} \quad \therefore mV^2 = \frac{GMm}{r}$$

27. A long insulated copper wire is closely wound as a spiral of 'N' turns. The spiral has inner radius 'a' and outer radius 'b'. The spiral lies in the XY plane and a steady current 'I' flows through the wire. The Z-component of the magnetic field at the centre of the spiral is



(A) $\frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$

(B) $\frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b+a}{b-a}\right)$

(C) $\frac{\mu_0 NI}{2b}\ln\left(\frac{b}{a}\right)$

(D) $\frac{\mu_0 NI}{2b}\ln\left(\frac{b+a}{b-a}\right)$

Sol. **(A)**

$$\int_a^b \frac{\mu_0 IN}{2r(b-a)}dr$$

$$= \frac{\mu_0 IN}{2(b-a)}\int_a^b \frac{dr}{r} = \frac{\mu_0 IN}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

28. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are

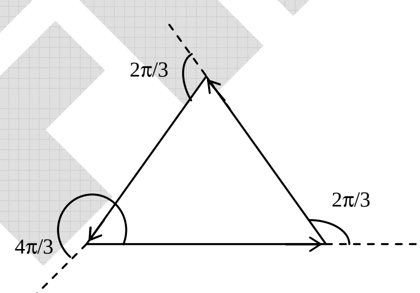
(A) $\sqrt{2}$ A, $\frac{3\pi}{4}$

(B) A, $\frac{4\pi}{3}$

(C) $\sqrt{3}$ A, $\frac{5\pi}{6}$

(D) A, $\frac{\pi}{3}$

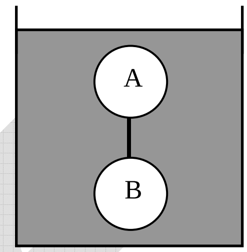
Sol. **(B)**



SECTION – II (Total Marks : 16)
(Multiple Correct Answer(s) Type)

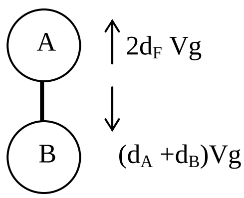
This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

29. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if

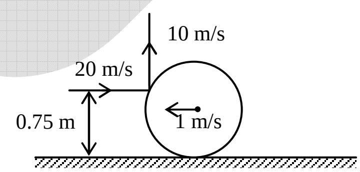


(A) $d_A < d_F$ (B) $d_B > d_F$
(C) $d_A > d_F$ (D) $d_A + d_B = 2d_F$

Sol. **(A, B, D)**



30. A thin ring of mass 2 kg and radius 0.5 m is rolling without on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision



(A) the ring has pure rotation about its stationary CM.
(B) the ring comes to a complete stop.
(C) friction between the ring and the ground is to the left.
(D) there is no friction between the ring and the ground.

Sol. **(C)**
During collision friction is impulsive and immediately after collision the ring will have a clockwise angular velocity hence friction will be towards left.

31. Which of the following statement(s) is/are correct?
(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.
(B) The Gauss law can be used to calculate the field distribution around an electric dipole
(C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.
(D) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$.

Sol. **(C) or (C, D*)**
(D) is correct if we assume it is work done against electrostatic force

32. A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?
(A) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$
(B) $V_C^A > V_C^B$ (D) $V_C^A < V_C^B$

Sol. **(B, C)**

$$I = \frac{V}{Z}$$

$$V^2 = V_R^2 + V_C^2 = (IR)^2 + \left(\frac{I}{\omega C}\right)^2$$

$$z^2 = R^2 + \left(\frac{1}{\omega C}\right)^2$$

SECTION-III (Total Marks : 24)
(Integer Answer Type)

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the **ORS**.

33. A series R-C combination is connected to an AC voltage of angular frequency ω = 500 radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

Sol. **(4)**

$$R\sqrt{1.25} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

RC = 4 ms

34. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^z$ (where $1 < A < 10$). The value of 'Z' is

Sol. **(7)**

Stopping potential $= \frac{hc}{\lambda} - W$

$= 6.2 \text{ eV} - 4.7 \text{ eV}$
$= 1.5 \text{ eV}$

$$V = \frac{Kq}{r} = 1.5$$

$$n = \frac{1.5\times10^{-2}}{9\times10^9\times1.6\times10^{-19}} = 1.05\times10^7$$

Z = 7

35. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is

Sol. **(5)**

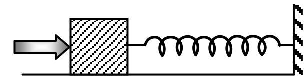
$$0 = 10\frac{\sqrt{3}}{2}t - \frac{1}{2}10t^2 \qquad \ldots\text{(i)}$$

$t = \sqrt{3}$ sec

$$\Rightarrow 1.15 = 5\sqrt{3} - \frac{3}{2}a \qquad \ldots\text{(ii)}$$

$\Rightarrow a \approx 5 \text{ m/s}^2$

36. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V = N/10. Then N is



Sol. **(4)**

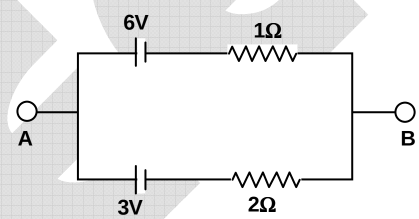
Applying work energy theorem

$$-\frac{1}{2}kx^2 - \mu mgx = -\frac{1}{2}mV^2$$

$$\Rightarrow V = \frac{4}{10}$$

N = 4

37. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is



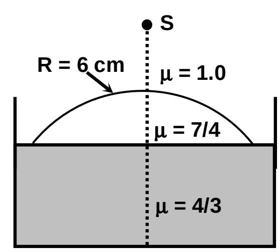
Sol. **(5)**

$$i = \frac{3}{3} = 1 \text{ ampere}$$

$$V_A - 6 + 1 - V_B = 0$$

$$V_A - V_B = 5$$

38. Water (with refractive index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is



Sol. **(2)**

$$\frac{7}{4V_1} - \frac{1}{-24} = \frac{\frac{7}{4}-1}{6} \Rightarrow V_1 = 21 \text{ cm}$$

$$\frac{4/3}{V_2} - \frac{7/4}{21} = 0$$

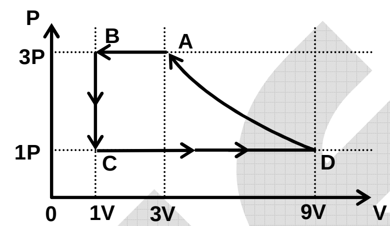
V_2 = 16 cm

x = 18 – 16 = 2 cm

SECTION-IV (Total Marks : 16)
(Matrix-Match Type)

This section contains **2 questions**. Each question has four statements (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

39. One mole of a monatomic gas is taken through a cycle ABCDA as shown in the P-V diagram. Column II give the characteristics involved in the cycle. Match them with each of the processes given in Column I.



	Column I		Column II
(A)	Process A $\rightarrow$ B	(p)	Internal energy decreases
(B)	Process B $\rightarrow$ C	(q)	Internal energy increases.
(C)	Process C $\rightarrow$ D	(r)	Heat is lost
(D)	Process D $\rightarrow$ A	(s)	Heat is gained
		(t)	Work is done on the gas

Sol. **(A) $\rightarrow$ (p, r, t) (B) $\rightarrow$ (p, r) (C) $\rightarrow$ (q, s) (D) (r, t)**

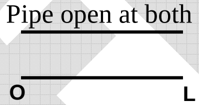
Process A $\rightarrow$ B $\rightarrow$ Isobaric compression
Process B $\rightarrow$ C $\rightarrow$ Isochoric process
Process C $\rightarrow$ D $\rightarrow$ Isobaric expansion
Process D $\rightarrow$ A $\rightarrow$ Polytropic with $T_A = T_D$

40. Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in Column II describing the nature and wavelength of the standing waves.

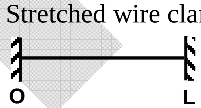
Column I | Column II

(A) Pipe closed at one end — (p) Longitudinal waves

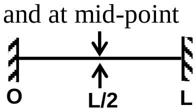
(B) Pipe open at both ends — (q) Transverse waves



(C) Stretched wire clamped at both ends — (r) $\lambda_f = L$



(D) Stretched wire clamped at both ends and at mid-point — (s) $\lambda_f = 2L$



(t) $\lambda_f = 4L$

Sol. **(A) $\rightarrow$ (p, t) (B) $\rightarrow$ (p, s) (C) $\rightarrow$ (q, s) (D) (q, r)**

PART - III:

SECTION – I (Total Marks : 24)
(Single Correct Answer Type)

This Section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

41. If $\lim\limits_{x\to 0}\left[1+x\ln\left(1+b^2\right)\right]^{1/x} = 2b\sin^2\theta$, $b > 0$ and $\theta \in (-\pi, \pi]$, then the value of θ is

(A) $\pm\frac{\pi}{4}$ (B) $\pm\frac{\pi}{3}$

(C) $\pm\frac{\pi}{6}$ (D) $\pm\frac{\pi}{2}$

Sol. **(D)**

$e^{\ln\left(1+b^2\right)} = 2b\sin^2\theta$

$\Rightarrow \sin^2\theta = \frac{1+b^2}{2b}$

$\Rightarrow \sin^2\theta = 1$ as $\frac{1+b^2}{2b} \ge 1$

$\theta = \pm\pi/2$.

42. Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)$ for all $x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} xf(x)dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x-axis. Then

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$

(C) $2R_1 = R_2$ (D) $3R_1 = R_2$

Sol. **(C)**

$R_1 = \int_{-1}^{2} xf\left(x\right)dx = \int_{-1}^{2}\left(2-1-x\right)f\left(2-1-x\right)dx$

$= \int_{-1}^{2}\left(1-x\right)f\left(1-x\right)dx = \int_{-1}^{2}\left(1-x\right)f\left(x\right)dx$

Hence $2R_1 = \int_{-1}^{2} f\left(x\right)dx = R_2$.

43. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in \mathbb{R}$. Then the set of all x satisfying $(f \circ g \circ g \circ f)(x) = (g \circ g \circ f)(x)$, where $(f \circ g)(x) = f(g(x))$, is

(A) $\pm\sqrt{n\pi},\ n \in \{0, 1, 2, ...\}$ (B) $\pm\sqrt{n\pi},\ n \in \{1, 2, ...\}$

(C) $\frac{\pi}{2} + 2n\pi,\ n \in \{..., -2, -1, 0, 1, 2, ...\}$ (D) $2n\pi,\ n \in \{..., -2, -1, 0, 1, 2, ...\}$

Sol. **(A)**

$(fogogof)(x) = \sin^2(\sin x^2)$

$(gogof)(x) = \sin(\sin x^2)$

$\therefore \sin^2(\sin x^2) = \sin(\sin x^2)$

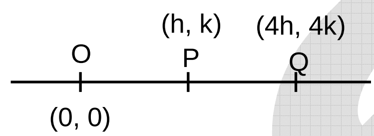
$\Rightarrow \sin(\sin x^2)\,[\sin(\sin x^2) - 1] = 0$

$\Rightarrow \sin(\sin x^2) = 0$ or 1
$\Rightarrow \sin x^2 = n\pi$ or $2m\pi + \pi/2$, where m, $n \in I$
$\Rightarrow \sin x^2 = 0$
$\Rightarrow x^2 = n\pi \Rightarrow x = \pm\sqrt{n\pi}$, $n \in \{0, 1, 2, \ldots\}$.

44. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1 : 3. Then the locus of P is

(A) $x^2 = y$
(B) $y^2 = 2x$
(C) $y^2 = x$
(D) $x^2 = 2y$

Sol. **(C)**



$y^2 = 4x$
and Q will lie on it
$\Rightarrow (4k)^2 = 4 \times 4h$
$\Rightarrow k^2 = h$
$\Rightarrow y^2 = x$ (replacing h by x and k by y)

45. Let P(6, 3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x-axis at (9, 0), then the eccentricity of the hyperbola is

(A) $\sqrt{\frac{5}{2}}$
(B) $\sqrt{\frac{3}{2}}$
(C) $\sqrt{2}$
(D) $\sqrt{3}$

Sol. **(B)**

Equation of normal is $(y - 3) = \frac{-a^2}{2b^2}(x-6) \Rightarrow \frac{a^2}{2b^2} = 1 \Rightarrow e = \sqrt{\frac{3}{2}}$.

46. A value of b for which the equations

$$x^2 + bx - 1 = 0$$
$$x^2 + x + b = 0,$$

have one root in common is

(A) $-\sqrt{2}$
(B) $-i\sqrt{3}$
(C) $i\sqrt{5}$
(D) $\sqrt{2}$

Sol. **(B)**

$x^2 + bx - 1 = 0$
$x^2 + x + b = 0$ … (1)
Common root is
$(b - 1)x - 1 - b = 0$

$\Rightarrow x = \frac{b+1}{b-1}$

This value of x satisfies equation (1)

$\Rightarrow \frac{(b+1)^2}{(b-1)^2} + \frac{b+1}{b-1} + b = 0 \Rightarrow b = \sqrt{3}i,\ -\sqrt{3}i,\ 0$.

47. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a, b, and c is either ω or ω^2. Then the number of distinct matrices in the set S is

(A) 2 (B) 6
(C) 4 (D) 8

Sol. **(A)**
For being non-singular

$$\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix} \neq 0$$

$\Rightarrow ac\omega^2 - (a + c)\omega + 1 \neq 0$
Hence number of possible triplets of (a, b, c) is 2.
i.e. $(\omega, \omega^2, \omega)$ and (ω, ω, ω).

48. The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$
(C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) (–4, 0)

Sol. **(D)**
Circle touching y-axis at (0, 2) is $(x - 0)^2 + (y - 2)^2 + \lambda x = 0$
passes through (– 1, 0)
$\therefore 1 + 4 - \lambda = 0 \Rightarrow \lambda = 5$
$\therefore x^2 + y^2 + 5x - 4y + 4 = 0$
Put $y = 0 \Rightarrow x = -1, -4$
$\therefore$ Circle passes through (– 4, 0)

SECTION – II (Total Marks : 16)
(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

49. If $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \leq -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \leq 0 \\ x - 1, & 0 < x \leq 1 \\ \ln x, & x > 1 \end{cases}$, then

(A) f(x) is continuous at $x = -\pi/2$ (B) f(x) is not differentiable at x = 0
(C) f(x) is differentiable at x = 1 (D) f(x) is differentiable at $x = -3/2$

Sol. **(A, B, C, D)**

$$\lim_{x \to -\frac{\pi}{2}^-} f(x) = 0 = f(-\pi/2)$$

$$\lim_{x \to -\frac{\pi}{2}^+} f(x) = \cos\left(-\frac{\pi}{2}\right) = 0$$

$$f'(x) = \begin{cases} -1, & x \le -\pi/2 \\ \sin x, & -\pi/2 < x \le 0 \\ 1, & 0 < x \le 1 \\ 1/x, & x > 1 \end{cases}$$

Clearly, f (x) is not differentiable at x = 0 as $f'(0^-) = 0$ and $f'(0^+) = 1$.
f (x) is differentiable at x = 1 as $f'(1^-) = f'(1^+) = 1$.

50. Let f : (0, 1) → ℝ be defined by $f(x) = \frac{b-x}{1-bx}$, where be is a constant such that 0 < b < 1. Then

(A) f is not invertible on (0, 1)
(B) $f \ne f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$
(C) $f = f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$
(D) f^{-1} is differentiable on (0, 1)

Sol. **(A)**

$f(x) = \frac{b-x}{1-bx}$

Let $y = \frac{b-x}{1-bx} \Rightarrow x = \frac{b-y}{1-by}$

$0 < x < 1 \Rightarrow 0 < \frac{b-y}{1-by} < 1$

$\frac{b-y}{1-by} > 0 \Rightarrow y < b \text{ or } y > \frac{1}{b}$

$\frac{b-y}{1-by} - 1 < 0 \Rightarrow -1 < y < \frac{1}{b}$

$\therefore -1 < y < b.$

51. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), then L is given by
(A) $y - x + 3 = 0$
(B) $y + 3x - 33 = 0$
(C) $y + x - 15 = 0$
(D) $y - 2x + 12 = 0$

Sol. **(A, B, D)**
$y^2 = 4x$
Equation of normal is $y = mx - 2m - m^3$.
It passes through (9, 6)
$\Rightarrow m^3 - 7m + 6 = 0$
$\Rightarrow m = 1, 2, -3$
$\Rightarrow y - x + 3 = 0,\ y + 3x - 33 = 0,\ y - 2x + 12 = 0.$

52. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then

(A) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$
(B) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$
(C) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$
(D) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

Sol. **(A, D)**

Let P (E) = e and P (F) = f

$P(E \cup F) - P(E \cap F) = \frac{11}{25}$

$\Rightarrow e + f - 2ef = \frac{11}{25}$ … (1)

$P(\bar{E} \cap \bar{F}) = \frac{2}{25}$

$\Rightarrow (1 - e)(1 - f) = \frac{2}{25}$

$\Rightarrow 1 - e - f + ef = \frac{2}{25}$ … (2)

From (1) and (2)

$ef = \frac{12}{25}$ and $e + f = \frac{7}{5}$

Solving, we get

$e = \frac{4}{5},\ f = \frac{3}{5}$ or $e = \frac{3}{5},\ f = \frac{4}{5}$.

SECTION-III (Total Marks : 24)
(Integer Answer Type)

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the **ORS**.

53. Let $\vec{a} = -\hat{i} - \hat{k},\ \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then the value of $\vec{r} \cdot \vec{b}$ is

Sol. **(9)**

$\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$

taking cross with a

$\vec{a} \times (\vec{r} \times \vec{b}) = \vec{a} \times (\vec{c} \times \vec{b})$

$(\vec{a} \cdot \vec{b})\vec{r} - (\vec{a} \cdot \vec{r})\vec{b} = \vec{a} \times (\vec{c} \times \vec{b})$

$\Rightarrow \vec{r} = -3\hat{i} + 6\hat{j} + 3\hat{k}$

$\vec{r} \cdot \vec{b} = 3 + 6 = 9$.

54. The straight line 2x – 3y = 1 divides the circular region $x^2 + y^2 \le 6$ into two parts. If

$$S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\},$$

then the number of point(s) in S lying inside the smaller part is

Sol. **(2)**

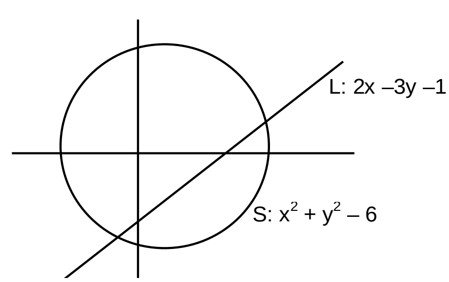
L: 2x – 3y – 1

S: $x^2 + y^2 - 6$

If $L_1 > 0$ and $S_1 < 0$

Then point lies in the smaller part.

$\therefore \left(2, \frac{3}{4}\right)$ and $\left(\frac{1}{4}, -\frac{1}{4}\right)$ lie inside.



55. Let $\omega = e^{i\pi/3}$, and a, b, c, x, y, z be non-zero complex numbers such that

$$a + b + c = x$$
$$a + b\omega + c\omega^2 = y$$
$$a + b\omega^2 + c\omega = z.$$

Then the value of $\dfrac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is

Sol. **(3)**

The expression may not attain integral value for all a, b, c

If we consider a = b = c, then

$x = 3a$

$y = a(1 + \omega + \omega^2) = a\left(1 + i\sqrt{3}\right)$

$z = a(1 + \omega^2 + \omega) = a\left(1 + i\sqrt{3}\right)$

$\therefore |x|^2 + |y|^2 + |z|^2 = 9|a|^2 + 4|a|^2 + 4|a|^2 = 17|a|^2$

$\therefore \dfrac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2} = \dfrac{17}{3}$

Note: However if $\omega = e^{i(2\pi/3)}$, then the value of the expression = 3.

56. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is

Sol. **(2)**

Let $f(x) = x^4 - 4x^3 + 12x^2 + x - 1 = 0$

$f'(x) = 4x^3 - 12x^2 + 24x + 1 = 4(x^3 - 3x^2 + 6x) + 1$

$f''(x) = 12x^2 - 24x + 24 = 12(x^2 - 2x + 2)$

$f''(x)$ has 0 real roots

f (x) has maximum 2 distinct real roots as f (0) = –1.

57. Let $y'(x) + y(x)g'(x) = g(x)g'(x)$, $y(0) = 0$, $x \in \mathbb{R}$, where $f'(x)$ denotes $\dfrac{df(x)}{dx}$ and g(x) is a given non-constant differentiable function on $\mathbb{R}$ with $g(0) = g(2) = 0$. Then the value of y(2) is

Sol. **(0)**

$y'(x) + y(x)\, g'(x) = g(x)\, g'(x)$

$\Rightarrow e^{g(x)}\, y'(x) + e^{g(x)}\, g'(x)\, y(x) = e^{g(x)}\, g(x)\, g'(x)$

$\Rightarrow \dfrac{d}{dx}\left(y(x)e^{g(x)}\right) = e^{g(x)}\, g(x)\, g'(x)$

$\therefore y(x) = e^{g(x)} = \int e^{g(x)} g(x) g'(x)\, dx$

$= \int e^t t\, dt$, where $g(x) = t$

$= (t - 1)\, e^t + c$

$\therefore y(x)\, e^{g(x)} = (g(x) - 1)\, e^{g(x)} + c$

Put $x = 0 \Rightarrow 0 = (0 - 1) . 1 + c \Rightarrow c = 1$

Put $x = 2 \Rightarrow y(2) . 1 = (0 - 1) . (1) + 1$

$y(2) = 0$.

58. Let M be a 3×3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix},\ M\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}, \text{ and } M\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\12\end{bmatrix}.$$

Then the sum of the diagonal entries of M is

Sol. **(9)**

Let $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow b = -1,\ e = 2,\quad h = 3$

$M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \Rightarrow a = 0,\ \ d = 3,\quad g = 2$

$M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix} \Rightarrow g + h + i = 12 \Rightarrow i = 7$

$\therefore$ Sum of diagonal elements = 9.

SECTION-IV (Total Marks : 16)
(Matrix-Match Type)

This section contains **2 questions**. Each question has four statements (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

59. Match the statements given in Column I with the values given in Column II

	Column – I		Column – II
(A)	If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}$, $\vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(p)	$\frac{\pi}{6}$
(B)	If $\int_a^b (f(x) - 3x)\,dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(q)	$\frac{2\pi}{3}$
(C)	The value of $\frac{\pi^2}{\ln 3}\int_{7/6}^{5/6} \sec(\pi x)\,dx$ is	(r)	$\frac{\pi}{3}$
(D)	The maximum value of $\left\lvert \text{Arg}\left(\frac{1}{1-z}\right) \right\rvert$ for $\lvert z \rvert = 1$, $z \neq 1$ is given by	(s)	π
		(t)	$\frac{\pi}{2}$

Sol. **(A) $\rightarrow$ (q)** **(B) $\rightarrow$ (p)** **(C) $\rightarrow$ (s)** **(D) $\rightarrow$ (t)**

(A). $\vec{a} \cdot \vec{b} = -1 + 3 = 2$

$|\vec{a}| = 2,\ |\vec{b}| = 2$

$\cos\theta = \frac{2}{2 \times 2} = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$ but its $\frac{2\pi}{3}$ as its opposite to side of maximum length.

(B). $\int_a^b (f(x) - 3x)\,dx = a^2 - b^2$

$\int_a^b f(x)\,dx = \frac{3}{2}(b^2 - a^2) + a^2 - b^2 = \frac{-a^2 + b^2}{2}$

$\Rightarrow$ f(x) = x.

(C). $\frac{\pi^2}{\ln 3}\left(\frac{\ln|(\sec \pi x + \tan \pi x)|\Big|_{7/6}^{5/6}}{\pi}\right) = \frac{\pi}{\ln 3}\left(\ln\left|\sec\frac{5\pi}{6} + \tan\frac{5\pi}{6}\right| - \ln\left|\sec\frac{7\pi}{6} + \tan\frac{7\pi}{6}\right|\right) = \pi.$

(D). Let $u = \frac{1}{1-z} \Rightarrow z = 1 - \frac{1}{u}$

$|z| = 1 \Rightarrow \left|1 - \frac{1}{u}\right| = 1$

$\Rightarrow |u - 1| = |u|$

$\therefore$ locus of u is perpendicular bisector of line segment joining 0 and 1

$\Rightarrow$ maximum arg u approaches $\frac{\pi}{2}$ but will not attain.

60. Match the statements given in Column I with the intervals/union of intervals given in Column II

	Column – I		Column – II
(A)	The set $\left\{\text{Re}\left(\frac{2iz}{1-z^2}\right) : z \text{ is a complex number}, \|z\| = 1, z \neq \pm 1\right\}$ is	(p)	$(-\infty, -1) \cup (1, \infty)$
(B)	The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	(q)	$(-\infty, 0) \cup (0, \infty)$
(C)	If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\left\{f(\theta) : 0 \le \theta < \frac{\pi}{2}\right\}$ is	(r)	$[2, \infty)$
(D)	If $f(x) = x^{3/2}(3x - 10)$, $x \ge 0$, then f(x) is increasing in	(s)	$(-\infty, -1] \cup [1, \infty)$
		(t)	$(-\infty, 0] \cup [2, \infty)$

Sol. **(A) → (s)** **(B) → (t)** **(C) → (r)** **(D) → (r)**

(A). $z = \frac{2i(x+iy)}{1-(x+iy)^2} = \frac{2i(x+iy)}{1-(x^2-y^2+2ixy)}$

Using $1 - x^2 = y^2$

$Z = \frac{2ix - 2y}{2y^2 - 2ixy} = -\frac{1}{y}.$

$\because\ -1 \le y \le 1 \Rightarrow -\dfrac{1}{y} \le -1$ or $-\dfrac{1}{y} \ge 1$.

(B). For domain

$-1 \le \dfrac{8\cdot 3^{x-2}}{1-3^{2(x-1)}} \le 1$

$\Rightarrow -1 \le \dfrac{3^x - 3^{x-2}}{1-3^{2x-2}} \le 1$.

Case –I: $\dfrac{3^x - 3^{x-2}}{1-3^{2x-2}} - 1 \le 0$

$\Rightarrow \dfrac{(3^x-1)(3^{x-2}-1)}{(3^{2x-2}-1)} \ge 0$

$\Rightarrow x \in (-\infty, 0] \cup (1, \infty)$.

Case –II: $\dfrac{3^x - 3^{x-2}}{1-3^{2x}-2} + 1 \ge 0$

$\Rightarrow \dfrac{(3^{x-2}-1)(3^x+1)}{(3^x\cdot 3^{x-2}-1)} \ge 0$

$\Rightarrow x \in (-\infty, 1) \cup [2, \infty)$

So, $x \in (-\infty, 0] \cup [2, \infty)$.

(C). $R_1 \to R_1 + R_3$

$f(\theta) = \begin{vmatrix} 0 & 0 & 2 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$

$= 2(\tan^2\theta + 1) = 2\sec^2\theta$.

(D). $f'(x) = \dfrac{3}{2}(x)^{1/2}(3x-10) + (x)^{3/2} \times 3 = \dfrac{15}{2}(x)^{1/2}(x-2)$

Increasing, when $x \ge 2$.