AIEEE–2011 (Set – Q)

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of Pencil is strictly prohibited.

2. The Answer Sheet is kept inside the Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.

3. The test is of 3 **hours** duration.

4. The Test Booklet consists of 90 questions. The maximum marks are 360.

5. There are **three** parts in the question paper A, B, C consisting of Physics, Mathematics, Chemistry having 30 questions in each part of equal weight age. Each question is allotted 4(four) marks for each correct response.
6. Candidates will be awarded marks as stated above in instruction No. 5 for correct response of each question ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.

8. Use Blue/Black Ball Point Pen only for writing particulars/marking responses on Side-1 and Side-2 of the Answer Sheet. Use of pencil is strictly prohibited.

9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.

10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in 3 pages (Pages 21 – 23) at the end of the booklet.

11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them.**

12. The CODE for this Booklet is **Q**. Make sure that the CODE printed on **Side-2** of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.

13. **Do not fold or make any stray marks on the Answer Sheet.**

PART A : PHYSICS

1. The transverse displacement $y(x,t)$ of a wave on a string is given by $y(x,t) = e^{-\left(ax^2+bt^2+2\sqrt{ab}\,xt\right)}$. This represents a
(1) wave moving in – x direction with speed $\sqrt{\frac{b}{a}}$ (2) standing wave of frequency $\sqrt{b}$
(3) standing wave of frequency $\frac{1}{\sqrt{b}}$ (4) wave moving in + x direction with $\sqrt{\frac{a}{b}}$

2. A screw gauge gives the following reading when used to measure the diameter of a wire.
Main scale reading : 0 mm
Circular scale reading : 52 divisions
Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.
The diameter of wire from the above date is :
(1) 0.052 cm (2) 0.026 cm (3) 0.005 cm (4) 0.52 cm

3. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is
(1) g (2) $\frac{2}{3}g$ (3) $\frac{g}{3}$ (4) $\frac{3}{2}g$

4. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = 0.03 Nm^{-1}):
(1) 0.2π mJ (2) 2π mJ (3) 0.4 π mJ (4) 4π mJ

5. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc:
(1) continuously decreases (2) continuously increases
(3) first increases and then decreases (4) remains unchanged

6. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is :
(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{2}$

7. Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is:
(1) $-\frac{4Gm}{r}$ (2) $-\frac{6Gm}{r}$ (3) $-\frac{9Gm}{r}$ (4) zero

8. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d << 1)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them,
(1) $v \propto x^{-1}$ (2) $v \propto x^{1/2}$ (3) $v \propto x$ (4) $v \propto x^{-1/2}$

9. A boat is moving due east in a region where the earth's magnetic field is $5.0 \times 10^{-5} NA^{-1}m^{-1}$ due north and horizontal. The boat carries a vertical aerial 2m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is :
(1) 0.75 mV (2) 0.50 mV (3) 0.15 mV (4) 1 mV

10. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by :
$$\frac{dv}{dt} = -2.5\sqrt{v}$$
where v is the instantaneous speed. The time taken by the object, to come to rest, would be :
(1) 2 s (2) 4 s (3) 8 s (4) 1 s

11. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic field is :
(1) $\frac{\pi}{4}\sqrt{LC}$ (2) $2\pi\sqrt{LC}$ (3) $\sqrt{LC}$ (4) $\pi\sqrt{LC}$

12. Let the x – z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with z < 0 has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is
(1) 45^0 (2) 60^0 (3) 75^0 (4) 30^0

13. A current I flows in an infinitely long wire with cross section in the form of a semicircular ring of radius R. The magnitude of the magnetic induction along its axis is
(1) $\frac{\mu_0 I}{2\pi^2 R}$ (2) $\frac{\mu_0 I}{2\pi R}$ (3) $\frac{\mu_0 I}{4\pi^2 R}$ (4) $\frac{\mu_0 I}{\pi^2 R}$

14. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed υ and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by :
(1) $\frac{(\gamma-1)}{2\gamma R}M\upsilon^2 K$ (2) $\frac{\gamma M\upsilon^2}{2R}K$ (3) $\frac{(\gamma-1)}{2R}M\upsilon^2 K$ (4) $\frac{(\gamma-1)}{2(\gamma+1)R}M\upsilon^2 K$

15. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is :
(1) $\frac{M+m}{M}$ (2) $\left(\frac{M}{M+m}\right)^{1/2}$ (3) $\left(\frac{M+m}{M}\right)^{1/2}$ (4) $\frac{M}{M+m}$

16. Water is flowing continuously from a tap having an internal diameter 8×10^{-3}m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1}m below the lap is close to :
(1) 7.5×10^{-3}m (2) 9.6×10^{-3}m (3) 3.6×10^{-3}m (4) 5.0×10^{-3}m

17. This question has Statement – 1 and Statement – 2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement-1 : Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.
Statement-2 : The state of ionosphere varies from hour to hour, day to day and season to season.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

18. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is :
(1) $\frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$ (2) $\frac{n_1T_1 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$ (3) $\frac{n_1^2T_1^2 + n_2^2T_2^2 + n_3^2T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$ (4) $\frac{(T_1 + T_2 + T_3)}{3}$

19. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ Newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation made by the pulley before its direction of motion if reversed, is :
(1) more than 3 but less than 6 (2) more than 6 but less than 9
(3) more than 9 (4) less than 3

20. A resistor 'R' and 2μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. $(\log_{10} 2.5 = 0.4)$
(1) $1.7 \times 10^5 \Omega$ (2) $2.7 \times 10^6 \Omega$ (3) $3.3 \times 10^7 \Omega$ (4) $1.3 \times 10^4 \Omega$

21. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively :
(1) 372 K and 330 K (2) 330 K and 268 K (3) 310 K and 248 K (4) 372 K and 310 K

22. If a wire is stretched to make it 0.1% longer, its resistance will :
(1) increase by 0.2% (2) decrease by 0.2% (3) decrease by 0.05% (4) increases by 0.05%

23. **Direction:**
The question has a paragraph followed by two statements, Statement – 1 and statement – 2. Of the given four alternatives after the statements, choose the one that describes the statements.
A thin air film is formed by putting the convex surface of a plane – convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.
Statement-1 : When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.
Statement-2 : The centre of the interference pattern is dark.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(3) Statement-1 is false, Statement-2 is true. (4) Statement-1 is true, Statement-2 is false.

24. A car is fitted with a convex side-view mirror of focal length 20cm. A second car 2.8 m behind the first car is overtaking the first car at relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is :
(1) $\frac{1}{15}$ m/s (2) 10m/s (3) 15m/s (4) $\frac{1}{10}$ m/s

25. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is :
(1) 36.3 eV (2) 108.8 eV (3) 122.4 eV (4) 12.1 eV

26. The electrostatic potential inside a charged spherical ball is given by $\phi = \alpha\rho^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside ball is
(1) $-6a\varepsilon_0 r$ (2) $-24\pi a\varepsilon_0 r$ (3) $-6a\varepsilon_0$ (4) $-24\pi a\varepsilon_0 r$

27. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is :
(1) $\pi\frac{v^4}{g^2}$ (2) $\frac{\pi}{2}\frac{v^4}{g^2}$ (3) $\pi\frac{v^2}{g^2}$ (4) $\pi\frac{v^4}{g}$

28. 100g of water is heated from 30^0C to 50^0C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4148 J/kg/K):
(1) 8.4 kJ (2) 84 kJ (3) 2.1 kJ (4) 4.2 kJ

29. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 and $\frac{1}{3}$ of it had decayed is :
(1) 14 min (2) 20 min (3) 28 min (4) 7 min

30. This question has Statement – 1 and Statement – 2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement-1 : A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface doubled, both the K_{max} and V_0 are also doubled.
Statement-2 : The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

PART B: MATHEMATICS

31. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersect L_3 at R.

Statement – 1 : The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$.

Statement – 2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(1) Statement – 1 is true, Statement – 2 is true; Statement – 2 is not a correct explanation for Statement – 1

(2) Statement – 1 is true, Statement– 2 is false.

(3) Statement – 1 is false, Statement– 2 is true.

(4) Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1

32. If $A = \sin^2 x + \cos^4 x$, then for all real x

(1) $\frac{13}{16} \le A \le 1$ (2) $1 \le A \le 2$ (3) $\frac{3}{4} \le A \le \frac{13}{16}$ (4) $\frac{3}{4} \le A \le 1$

33. The coefficient of x^7 in the expansion of $\left(1 - x - x^2 + x^3\right)^6$ is

(1) -132 (2) -144 (3)132 (4) 144

34. $\lim\limits_{x \to 2} \left(\dfrac{\sqrt{1 - \cos\{2(x-2)\}}}{x-2} \right)$

(1) equals $\sqrt{2}$ (2) equals $-\sqrt{2}$ (3) equals $\frac{1}{\sqrt{2}}$ (4) does not exist

35. Statement – 1 : The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$

Statement – 2 : The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(1) Statement – 1 is true, Statement – 2 is true; Statement – 2 is not a correct explanation for Statement – 1

(2) Statement – 1 is true, Statement– 2 is false.

(3) Statement – 1 is false, Statement– 2 is true.

(4) Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1

36. $\dfrac{d^2x}{dy^2}$ equals

(1) $-\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$ (2) $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-2}$ (3) $-\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$ (4) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$

37. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to

(1) 5 (2) 13 (3) -2 (4) 7

38. Let R be the set of real numbers

Statement – 1 : $A = \{(x, y) \in R \times R : y - x \text{ is an integer}\}$ is an equivalence relation on R.

Statement – 2 : $B = \{(x, y) \in R \times R : x = \alpha y \text{ for some rational number } \alpha\}$ is an equivalence relation on R.

(1) Statement – 1 is true, Statement – 2 is true; Statement – 2 is not a correct explanation for Statement – 1

(2) Statement – 1 is true, Statement– 2 is false.

(3) Statement – 1 is false, Statement– 2 is true.

(4) Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1

39. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2} dx$ is

(1) $\frac{\pi}{8}\log 2$ (2) $\frac{\pi}{2}\log 2$ (3) $\log 2$ (4) $\pi \log 2$

40. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line $\text{Re}\, z = 1$, then it is necessary that

(1) $\beta \in (-1, 0)$ (2) $|\beta| = 1$ (3) $\beta \in (1, \infty)$ (4) $\beta \in (0, 1)$

41. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval

(1) $\left(\frac{3}{4}, \frac{11}{12}\right]$ (2) $\left[0, \frac{1}{2}\right]$ (3) $\left(\frac{11}{12}, 1\right]$ (4) $\left(\frac{1}{2}, \frac{3}{4}\right]$

42. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after

(1) 19 months (2) 20 months (3) 21 months (4) 18 months

43. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is

(1) $(0, \infty)$ (2) $(-\infty, 0)$ (3) $(-\infty, \infty) - \{0\}$ (4) $(-\infty, \infty)$

44. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals

(1) $\frac{3}{2}$ (2) $\frac{2}{5}$ (3) $\frac{5}{3}$ (4) $\frac{2}{3}$

45. If $\vec{a} = \frac{1}{\sqrt{10}}\left(3\hat{i} + \hat{k}\right)$ and $\vec{b} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$, then the value of $\left(2\vec{a} - \vec{b}\right).\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} + 2\vec{b}\right)\right]$ is

(1) -3 (2) 5 (3) 3 (4) -5

46. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point $(-3, 1)$ and has eccentricity $\sqrt{\frac{2}{5}}$ is

(1) $5x^2 + 3y^2 - 48 = 0$ (2) $3x^2 + 5y^2 - 15 = 0$ (3) $5x^2 + 3y^2 - 32 = 0$ (4) $3x^2 + 5y^2 - 32 = 0$

47. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T - t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is

(1) $I - \frac{kT^2}{2}$ (2) $I - \frac{k(T-t)^2}{2}$ (3) e^{-kT} (4) $T^2 - \frac{I}{k}$

48. The vector $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying: $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a}.\vec{d} = 0$. Then the vector $\vec{d}$ is equal to

(1) $\vec{c} + \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (2) $\vec{b} + \left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ (3) $\vec{c} - \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (4) $\vec{b} - \left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

49. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 \ (c > 0)$ touch each other if

(1) $|a| = c$ (2) $a = 2c$ (3) $|a| = 2c$ (4) $2|a| = c$

50. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is

(1) $P(C|D) \geq P(C)$ (2) $P(C|D) < P(C)$ (3) $P(C|D) = \frac{P(D)}{P(C)}$ (4) $P(C|D) = P(C)$

51. The number of values of k for which the linear equations
$4x + ky + 2z = 0$; $kx + 4y + z = 0$; $2x + 2y + z = 0$ possess a non-zero solution is

(1) 2 (2) 1 (3) zero (4) 3

52. Consider the following statements
P : Suman is brilliant
Q : Suman is rich
R : Suman is honest
The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as

(1) $\sim\left(Q \leftrightarrow (P \wedge \sim R)\right)$ (2) $\sim Q \leftrightarrow \sim P \wedge R$ (3) $\sim(P \wedge \sim R) \leftrightarrow Q$ (4) $\sim P \wedge (Q \leftrightarrow \sim R)$

53. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is

(1) $\frac{3\sqrt{2}}{8}$ (2) $\frac{8}{3\sqrt{2}}$ (3) $\frac{4}{\sqrt{3}}$ (4) $\frac{\sqrt{3}}{4}$

54. If the mean deviation about the median of the numbers a, 2a, …, 50a is 50, then $|a|$ equals

(1) 3 (2) 4 (3) 5 (4) 2

55. Statement – 1 : The point $A(1, 0, 7)$ is the mirror image of the point $B(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

Statement – 2 : The line: $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$.

(1) Statement – 1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement – 1
(2) Statement – 1 is true, Statement– 2 is false.
(3) Statement – 1 is false, Statement– 2 is true.
(4) Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1

56. Let A and B be two symmetric matrices of order 3.

Statement – 1 : $A(BA)$ and $(AB)A$ are symmetric matrices.

Statement – 2 : AB is symmetric matrix if matrix multiplication of A and B is commutative.

(1) Statement – 1 is true, Statement – 2 is true; Statement – 2 is not a correct explanation for Statement – 1
(2) Statement – 1 is true, Statement– 2 is false.
(3) Statement – 1 is false, Statement– 2 is true.
(4) Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1

57. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals

(1) (1, 1) (2) (1, 0) (3) (-1, 1) (4) (0, 1)

58. The value of p and q for which the function $f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x} & , \ x < 0 \\ q & , \ x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}} & , \ x > 0 \end{cases}$

is continuous for all x in R, is

(1) $p = \frac{5}{2}, q = \frac{1}{2}$ (2) $p = -\frac{3}{2}, q = \frac{1}{2}$ (3) $p = \frac{1}{2}, q = \frac{3}{2}$ (4) $p = \frac{1}{2}, q = -\frac{3}{2}$

59. The area of the region enclosed by the curves $y = x, x = e, y = \frac{1}{x}$ and the positive x-axis is

(1) 1 square units (2) $\frac{3}{2}$ square units (3) $\frac{5}{2}$ square units (4) $\frac{1}{2}$ square units

60. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t}\sin t \, dt$. Then f has

(1) local minimum at π and 2π
(2) local minimum at π and local maximum at 2π
(3) local maximum at π and local minimum at 2π
(4) local maximum at π and 2π

PART C: CHEMISTRY

61. Among the following the maximum covalent character is shown by the compound :
(1) $SnCl_2$ (2) $AlCl_3$ (3) $MgCl_2$ (4) $FeCl_2$

62. The presence or absence of hydroxyl group on which carbon atom of sugar differentiates RNA and DNA ?
(1) 2[nd] (2) 3[rd] (3) 4[th] (4) 1[st]

63. Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate and another compound. The other compound is :
(1) Trichloromethanol (2) 2, 2, 2-Trichloropropanol
(3) Chloroform (4) 2, 2, 2-Trichloroethanol

64. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in the above reaction is :
(1) 2-Butanone (2) Ethyl chloride (3) Ethyl ethanoate (4) Diethyl ether

65. The reduction potential of hydrogen half cell will be negative if :
(1) $p(H_2) = 1$ atm and $[H^+] = 1.0$ M (2) $p(H_2) = 2$ atm and $[H^+] = 1.0$ M
(3) $p(H_2) = 2$ atm and $[H^+] = 2.0$ M (4) $p(H_2) = 1$ atm and $[H^+] = 2.0$ M

66. The strongest acid amongst the following compounds is :
(1) HCOOH (2) $CH_3CH_2CH(Cl)CO_2H$
(3) $ClCH_2CH_2CH_2COOH$ (4) CH_3COOH

67. The degree of dissociation (α) of a weak electrolyte, A_xB_y is related to van't Hoff factor (i) by the expression :
(1) $\alpha = \frac{i-1}{x+y+1}$ (2) $\alpha = \frac{x+y-1}{i-1}$ (3) $\alpha = \frac{x+y+1}{i-1}$ (4) $\alpha = \frac{i-1}{(x+y-1)}$

68. `a' and `b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because
(1) a and b for $Cl_2 <$ a and b for C_2H_6
(2) a for $Cl_2 <$ a for C_2H_6 but b for $Cl_2 >$ b for C_2H_6
(3) a for $Cl_2 >$ a for C_2H_6 but b for $Cl_2 <$ b for C_2H_6
(4) a and b for $Cl_2 >$ a and b for C_2H_6

69. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm, the value of K is
(1) 3 atm (2) 0.3 atm (3) 0.18 atm (4) 1.8 atm

70. Boron cannot form which one of the following anions ?
(1) BH_4^- (2) $B(OH)_4^-$ (3) BO_2^- (4) BF_6^{3-}

71. Which of the following facts about the complex $[Cr(NH_3)_6]Cl_3$ s wrong ?
(1) The complex is paramagnetic (2) The complex is an outer orbital complex
(3) The complex gives white precipitate with silver nitrate solution
(4) The complex involves d^2sp^3 hybridization and is octahedral in shape.

72. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it from freezing at $-6°C$ will be :
[K_f for water = 1.86 K kg mol^{-1}, and molar mass of ethylene glycol = 62g mol^{-1})
(1) 204.30g (2) 400.00 g (3) 304.60 g (4) 804.32g

73. Which one of the following order represents the correct sequence of the increasing basic nature of the given oxides ?
(1) $MgO < K_2O < Al_2O_3 < Na_2O$
(2) $Na_2O < K_2O < MgO < Al_2O_3$
(3) $K_2O < Na_2O < Al_2O_3 < MgO$
(4) $Al_2O_3 < MgO < Na_2O < K_2O$

74. The rate of a chemical reaction doubles for every $10°C$ rise of temperature. If the temperature is raised by $50°C$, the rate of the reaction increases by about :
(1) 24 times (2) 32 times (3) 64 times (4) 10 times

75. The magnetic moment (spin only) of $[NiCl_4]^{2-}$ is
(1) 5.46 BM (2) 2.83 BM (3) 1.41 BM (4) 1.82 BM

76. The hybridization of orbitals of N atom in NO_3^-, NO_2^+ and NH_4^+ are respectively :
(1) sp^2, sp, sp^3 (2) sp, sp^3, sp^2 (3) sp^2, sp^3, sp (4) sp, sp^2, sp^3

77. In context of the lanthanoids, which of the following statements is not correct ?
(1) All the members exhibit +3 oxidation state
(2) Because of similar properties the separation of lanthanoids is not easy.
(3) Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series.
(4) There is a gradual decrease in the radii of the members with increasing atomic number in the series.

78. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH, is supplied. What is the mole fraction of methyl alcohol in the solution ?
(1) 0.190 (2) 0.086 (3) 0.050 (4) 0.100

79. Which of the following statement is wrong ?
(1) Nitrogen cannot form $d\pi$ - $p\pi$ bond.
(2) Single N- N bond is weaker than the single P – P bond,
(3) N_2O_4 has two resonance structures
(4) The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table

80. The outer electron configuration of Gd (Atomic No : 64 is :
(1) $4f^8\ 5d^0\ 6s^2$ (2) $4f^4\ 5d^4\ 6s^2$ (3) $4f^7\ 5d^1\ 6s^2$ (4) $4f^3\ 4d^5\ 6s^2$

81. Which of the following statements regarding sulphur is incorrect ?
(1) The vapour at $200°C$ consists mostly of S_8 rings
(2) At $600°C$ the gas mainly consists of S_2 molecules
(3) The oxidation state of sulphur is never less than +4 in its compounds
(4) S_2 molecule is paramagnetic.

82. The structure of IF_7 is :
(1) trigonal bipyramid (2) octahedral (3) pentagonal bipyramid (4) square pyramid

83. Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of :
(1) a vinyl group (2) an isopropyl group
(3) an acetylenic triple bond (4) two ethylenic double bonds

84. A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emissions is at 680 nm, the other is at :
(1) 325 nm (2) 743 nm (3) 518 nm (4) 1035 nm

85. Silver Mirror test is given by which one of the following compounds ?
(1) Acetone (2) Formaldehyde (3) Benzophenone (4) Acetaldehyde

86. Which of the following reagents may be used to distinguish between phenol and benzoic acid ?
(1) Tollen's reagent (2) Molisch reagent (3) Neutral $FeCl_3$ (4) Aqueous NaOH

87. Phenol is heated with a solution of mixture of KBr and $KBrO_3$. The major product obtained in the above reaction is
(1) 3-Bromophenol (2) 4-Bromophenol (3) 2, 4, 6- Tribromophenol (4) 2-Bromophenol

88. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing from one of the face centred points, the formula of the compound is :
(1) AB_2 (2) A_2B_3 (3) A_2B_5 (4) A_2B

89. The entropy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of 10 dm^3 to a volume of 100 dm^3 at 27°C is :
(1) 35.8J $mol^{-1}K^{-1}$ (2) 32.3J $mol^{-1}K^{-1}$ (3) 42.3J $mol^{-1}K^{-1}$ (4) 38.3J $mol^{-1}K^{-1}$

90. Identify the compound that exhibits tautomerism.
(1) Lactic acid (2) 2-Pentanone (3) Phenol (4) 2- Butene

READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. The candidates should fill in the required particulars on the Test Booklet and Answer Sheet **(Side–1)** with **Blue/Black Ball Point Pen.**

2. For writing/marking particulars on **Side-2** of the Answer Sheet, use **Blue/Black Ball Point Pen only.**

3. The candidates should not write their Roll Numbers anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.

4. Out of the four options given for each question, only one option is the correct answer.

5. For each **incorrect response, one-fourth** (1/4) of the total marks allotted to the question would be deducted from the total score. **No deduction** from the total score, however, will be made **if no response** is indicated for an item in the Answer Sheet.

6. Handle the Test Booklet and Answer Sheet with care, as under no circumstances (except for discrepancy in Test Booklet Code and Answer Sheet Code), will another set be provided.

7. The candidates are not allowed to do any rough work or writing work on the Answer Sheet. All calculations/writing work are to be done in the space provided for this purpose in the Test Booklet itself, marked 'Space for Rough Work'. This space is given at the bottom of each page and in 4 pages (Pages 20 – 23) at the end of the booklet.

8. On completion of the test, the candidates must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them.**

9. Each candidate must show on demand his/her Admit Card to the Invigilator.

10. No candidate, without special permission of the Superintendent or Invigilator, should leave his/her seat.

11. The candidates should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and sign the Attendance Sheet again. Cases where a candidate has not signed the Attendance Sheet a second time will be deemed not to have handed over the Answer Sheet and dealt with as an unfair means case. **The candidates are also required to put their left hand THUMB impression in the space provided in the Attendance Sheet.**

12. Use of Electronic/Manual Calculator and any Electronic Item like mobile phone, pager etc. is prohibited.

13. The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Board.

14. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.

15. **Candidates are not allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, electronic device or any other material except the Admit Card inside the examination hall/room.**

SOLUTIONS

PART A
PHYSICS

1. 1

Sol. $y_{(x,t)} = e^{-\left(\sqrt{a}\,x+\sqrt{b}\,t\right)^2}$ $V=\sqrt{\frac{b}{a}}$

Wave moving in $-$ve x –direction.

2. 1

Sol. Diameter of wire $= \frac{1}{100}\times 52 = 0.52\text{mm} = 0.052\text{cm}$

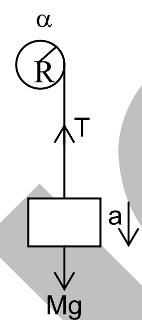
3. 2

Sol. Mg –T = Ma ….. (1)

$T\times R = I\alpha = \frac{1}{2}MR^2\alpha$

$T=\frac{1}{2}Ma \quad (a=\alpha R)$ ….. (2)

From (1) and (2) $a=\frac{2g}{3}$



4. 3

Sol. $W = T\times\Delta A = T\times 8\pi\left(r_2^{\,2}-r_1^{\,2}\right) = 0.4\pi\text{mJ}$

5. 3

Sol. $\tau = 0$

Angular momentum is conserve

$I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \frac{I_1\omega_1}{I_2}$

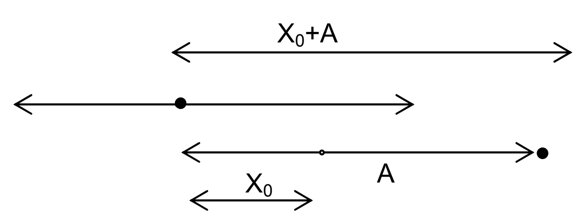
I_2 first decreases and then increases

$\therefore \omega$ first increases and then decreases.

6. 4

Sol. $\phi_1 = 0$

$\phi_2 = \frac{\pi}{2}$



7. 3

Sol. Position of the null point from mass m, $x = \frac{r}{1+\sqrt{\frac{4m}{m}}} = \frac{r}{3}$

$V = -Gm\left(\frac{3}{r}+\frac{12}{2r}\right) = -9\frac{Gm}{r}$

8. 4

Sol. At any instant of separation between charges is x.

equilibrium condition $= K\frac{Q^2}{x^2} = \omega\frac{x}{2\ell}$

$\Rightarrow Q^2 = Cx^3$

$\Rightarrow 2Q\frac{dQ}{dt} = C3x^2\frac{dx}{dt}$

$\Rightarrow \frac{dx}{dt} \propto \frac{x^{3/2}}{x^2} \propto x^{-1/2}$

9. 3

Sol. $E = B_H \ell V = 0.15 mV$

10. 1

Sol. $\frac{dv}{dt} = -2.5\sqrt{v}$

Integrating the above equation.

$\Rightarrow 2\sqrt{v} = -2.5t + C$

at $t = 0, v = 6.25 \Rightarrow C = 5$

at $v = 0 \Rightarrow t = \frac{5}{2.5} = 2s$

11. 1

Sol. Charge oscillates simple harmonic motion $q = q_0 \sin \omega t$, $U = \frac{1}{2}\frac{q^2}{C}$

$q = \frac{q_0}{\sqrt{2}} \Rightarrow \omega t = \frac{\pi}{4}$

$\Rightarrow t = \frac{T}{8} = \frac{2\pi}{8}\sqrt{LC} = \frac{\pi}{4}\sqrt{LC}$

12.

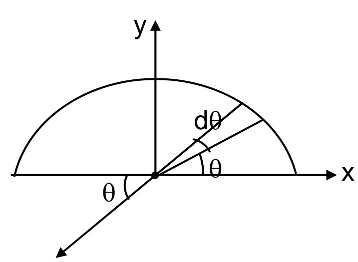
Sol. Normal to the plane is z –axis

$\cos\theta_1 = \frac{A_z}{A} = \frac{10}{20} = \frac{1}{2}, \theta_1 = 60$

$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 \Rightarrow \sqrt{2}\times\frac{\sqrt{3}}{2} = \sqrt{3}\sin\theta_2 \Rightarrow \theta_2 = 45^0$

13. 4

Sol. $d\vec{B} = \frac{\mu_0 di}{2\pi R}\left[-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right]$



$di = \frac{T}{\pi R}Rd\theta$

$= \frac{I}{\pi}d\theta$

$d\vec{B} = \frac{\mu_0 I}{2\pi^2 R}\left(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right)$

$\vec{B} = -\frac{\mu_0 I}{\pi^2 R}\hat{j}$

14. 3

Sol. $W = \Delta U$

$\frac{1}{2}mv^2 = nC_v dT$

$= \frac{m}{M}\frac{R}{\gamma - 1}dT$

$dT = \frac{M(\gamma - 1)v^2}{2R}K$

15. 3

Sol. Energy of simple harmonic oscillator is constant.

$\Rightarrow \frac{1}{2}M\omega^2 A_1^2 = \frac{1}{2}(m+M)\omega^2 A_2^2$

$\frac{A_1^2}{A_2^2} = \frac{M+m}{M}$

$\therefore \frac{A_1}{A_2} = \sqrt{\frac{M+m}{M}}$

16. 3

Sol. Equation of continuity

$\Rightarrow (a \times v)\text{top} = (a \times v)\text{bottom}$

$v_b^2 - (0.4)^2 = 2 \times 9.8 \times 0.2\left[v^2 - u^2 = 2gh \text{ is used}\right]$

v_b = 2m/s (nearly)

$\pi\left[8 \times 10^{-3}\right] \times 0.4 = \pi d^2 \times 4$

$d \approx 3.6 \times 10^{-3}\text{m}$

17. 1

Sol. Since ionospheric properties change with time, these signals are in general less stable than ground wave signals.

18. 1

Sol. Data $\Rightarrow n_1 k_1 t_1 + n_2 kT_2 + n_3 kT_3 = (n_1 + n_2 + n_3)kT$

$\therefore T = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

19. 1

Sol. r × F = I × α

$2(20t - 5t^2) = 10\alpha \Rightarrow \alpha = 4t - t^2$

$\frac{d\omega}{dt} = 4t - t^2$

$d\omega = (4t^2 - t^2)dt$

$\omega = 2t^2 - \frac{t^3}{3}$ (on integration)

$\omega = 0 \Rightarrow t = 6s$

$$\omega = \frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$$

$$d\theta = \left(2t^2 - \frac{t^3}{3}\right)dt$$

$$\Rightarrow \theta = \frac{2t^3}{3} - \frac{t^4}{12} \text{ (on integration)}$$

$$\theta(\text{in } 6s) = 36\,\text{rad}$$

$$\Rightarrow 2\pi n = 36$$

$$n = \frac{36}{2\pi} =< 6$$

20. 2

Sol. $V_c = E\left(1 - e^{-t/Rc}\right)$

$$1 - e^{-t/Rc} = \frac{120}{200} = \frac{3}{5}$$

$$\Rightarrow R = \frac{5}{1.84 \times 10^{-6}} = 2.7 \times 10^6\,\Omega$$

21. 4

Sol. $\eta_1 = \frac{T_1 - T_2}{T_1} = \frac{1}{6}$

$$\eta_2 = \frac{T_1 - (T_2 - 62)}{T_1} = \frac{1}{3}$$

$$\Rightarrow \frac{T_1 - T_2}{T_1} + \frac{62}{T_1} = \frac{1}{3}$$

$$\frac{1}{6} + \frac{62}{T_1} = \frac{1}{3}$$

$$\frac{62}{T_1} = \frac{1}{6}$$

$$\therefore T_1 = 62 \times 6 = 372K$$

$$\frac{T_1 - T_2}{T_1} = \frac{1}{6}$$

$$1 - \frac{T_2}{T_1} = \frac{1}{6}$$

$$\frac{T_2}{372} = \frac{5}{6}$$

$$\Rightarrow T_2 = 310K$$

22. 1

Sol. $R \propto \ell^2$ (for a given volume)

$$\Rightarrow \frac{\Delta R}{R}\% = \frac{2\Delta\ell}{\ell}\%$$

Thus when wire is stretched by 0.1% resistance increases by 0.2%

23. 1

Sol. As light enters from air to glass it suffers a phase change on π and therefore at centre there will be destructive interference.

24. 1

Sol. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$-\frac{1}{v^2}\frac{dv}{dt}-\frac{1}{u^2}\frac{du}{dt}=0$

$\frac{dv}{dt}=-\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$

f = 20 cm

$\frac{1}{u}+\frac{1}{-280}=\frac{1}{20}$

$\Rightarrow v=\frac{280}{15}\text{cm}$

$v_I=-\left(\frac{280}{15\times 280}\right)^2\times 15$

$=\frac{1}{15}\text{m/s}$

25. 2

Sol. $E_n=-13.6\frac{Z^2}{n^2}$

$E_{Li^{++}}=-13.6\times\frac{9}{1}=-122.4\text{eV}$

$E_{Li^{+++}}=-13.6\times\frac{9}{9}=-13.6\text{eV}$

$\Delta E=-13.6-(-122.4)$

= 108.8 eV

26. 3

Sol. Potential inside (ϕ) = $ar^2 + b$

$\therefore E_r=-\frac{\delta v}{\delta r}=-2ar$

Electric field inside uniformly charged solid volume varies with ‘r’. So charge density is constant

$\phi_{net}=(-2ar)4\pi r^2=-8\pi ar^3$

$-8\pi ar^3=\frac{\sigma\times\frac{4}{3}\pi r^3}{\varepsilon_0}$

$\therefore \sigma=-6a\varepsilon_0$

27. 1

Sol. Max. range $=\frac{u^2}{g}$ i.e., $\frac{v^2}{g}$ (radius of circle)

Area occupied $=\pi\left(\frac{v^2}{g}\right)^2=\frac{\pi v^4}{g^2}$

28. 1

Sol. $\Delta Q = \Delta U + \Delta W$ (ignoring expansion)

$\Delta U = ms\Delta T = 0.1 \times 4.184 \times 20 = 8.368\,kJ$

29. 2

Sol. $t_{\frac{1}{2}} = 20$ minutes

$N = N_0 e^{-\lambda t_2} \quad \lambda t_1 = \ln 3$

$\frac{2}{3}N_0 = N_0 e^{-\lambda t_2}\; t_1 = \frac{1}{\lambda}\ln 3$

$\frac{2}{3}N_0 = N_0 e^{-\lambda t_2}$

$t_2 = \frac{1}{\lambda}\ln\frac{3}{2}$

$t_2 - t_1 = \frac{1}{\lambda}\left[\ln\frac{3}{2} - \ln 3\right]$

$= \frac{1}{\lambda}\ln\left[\frac{1}{2}\right] = \frac{0.693}{\lambda}$

= 20 min

30. 3

Sol. $KE_{max} = h\upsilon - h\upsilon_0$

$h\upsilon - h\upsilon_0 = e \times \Delta v$

$V_0 = \frac{h\upsilon}{e} - \frac{h\upsilon_0}{e}$

'υ' is doubled

$KE_{max} = 2h\upsilon - h\upsilon_0$

$V_0' = (\Delta V)' = \frac{2h\upsilon}{e} - \frac{h\upsilon_0}{e}$

$\frac{KE_{max}}{KE_{max}}$ may not be equal to 2

$\Rightarrow \frac{V_0'}{V_0}$ may not equal to 2

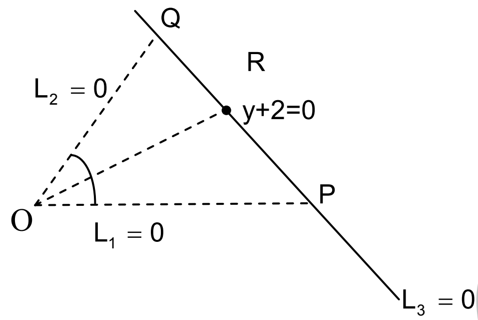
KE max = $h\upsilon - h\upsilon_0$

$V = \frac{h\upsilon}{e} - \frac{h\upsilon_0}{e}$

PART B: MATHEMATICS

31. **2**

Sol:



$P(-2, -2); Q = (1, -2)$

Equation of angular bisector $\overline{OR}$ is $\left(\sqrt{5}+2\sqrt{2}\right)x = \left(\sqrt{5}-\sqrt{2}\right)y$

$\therefore PR : RQ = 2\sqrt{2} : \sqrt{5}$

32. **4**

Sol: $A = \sin^2 x + \cos^4 x = \dfrac{7+\cos 4x}{8} \Rightarrow \dfrac{3}{4} \le A \le 1$

33. **2**

Sol: $\left[1 - x - x^2(1-x)\right]^6 = (1-x)^6\left(1-x^2\right)^6$

$= \left[{}^6C_0 - {}^6C_1x + {}^6C_2x^2 - {}^6C_3x^3 + {}^6C_4x^4 - {}^6C_5x^5 + {}^6C_6x^6\right] \times \left[{}^6C_0 - {}^6C_1x^2 + {}^6C_2x^4 - {}^6C_3x^6 + ...\right]$

Coefficient of $x^7 = {}^6C_1\,{}^6C_3 - {}^6C_3\,{}^6C_2 + {}^6C_5\,{}^6C_1 = 120 - 300 + 36 = -144$

34. **4**

Sol: $\lim\limits_{x\to 2} \dfrac{\sqrt{2\sin^2(x-2)}}{x-2}$

$\lim\limits_{x\to 2} \dfrac{\sqrt{2}\left|\sin(x-2)\right|}{x-2}$

R.H.L. $= \sqrt{2}$, L.H.L. $= -\sqrt{2}$

Limit does not exist.

35. **4**

Sol: ${}^{(n-1)}C_{(r-1)} = {}^{(10-1)}C_{(4-1)} = {}^9C_3$

Statement 1 is correct

Statement 2 is also correct

From 9 we can select 3 in ${}^9C_3$ ways. It is correct explanation.

36. **3**

Sol: $\frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{1}{\left(\frac{dy}{dx}\right)}\right) = -\frac{1}{\left(\frac{dy}{dx}\right)^2}\frac{d}{dy}\left(\frac{dy}{dx}\right)$

$= -\left(\frac{dy}{dx}\right)^{-2}\frac{1}{\left(\frac{dy}{dx}\right)}\frac{d}{dx}\left(\frac{dy}{dx}\right) = -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

37. **4**

Sol: $\frac{dy}{dx} = y+3 \Rightarrow \frac{dy}{y+3} = dx$

$\ln(y+3) = x+c$

$x = 0 \Rightarrow y = 2$

$\Rightarrow \ln 5 = 0 + c$

$c = \ln 5$

$\ln(y+3) = x + \ln 5$

$y+3 = e^{x+\ln 5} \Rightarrow y+3 = e^{\ln 2+\ln 5}$

$y+3 = 10 \Rightarrow y = 7$

38. **2**

Sol: $x-y$ is an integer

$x-x=0$ is an integer $\Rightarrow A$ is Reflexive

$x-y$ is an integer $\Rightarrow y-x$ is an integer $\Rightarrow A$ is symmetric

$x-y,\ y-z$ are integers

As sum of two integers is an integer.

$\Rightarrow (x-y)+(y-z) = x-z$ is an integer

$\Rightarrow A$ is transitive. Hence statement – 1 is true.

Also $\frac{x}{x} = 1$ is a rational number $\Rightarrow B$ is reflexive

$\frac{x}{y} = \alpha$ is rational $\Rightarrow \frac{y}{x}$ need not be rational

i.e., $\frac{0}{1}$ is rational $\Rightarrow \frac{1}{0}$ is not rational

Hence B is not symmetric

$\Rightarrow$ B is not an equivalence relation.

39. **4**

Sol: $I = 8\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$

$= 8\int_0^{\frac{\pi}{4}} \frac{\log(1+\tan\theta)}{1+\tan^2\theta}\sec^2\theta\, d\theta \;(\text{let } x = \tan\theta)$

$= 8\int_0^{\frac{\pi}{4}} \log\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right) d\theta \; = 8\int_0^{\frac{\pi}{4}} \log\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right) d\theta \; = 8\int_0^{\frac{\pi}{4}} \log 2\, d\theta - 8\int_0^{\frac{\pi}{4}} \log(1+\tan\theta)\, d\theta$

$= 8\log 2\frac{\pi}{4} - I$

$2I = 2\pi\log 2$

$I = \pi\log 2$

40. **3**

Sol: Suppose roots are $1+pi,\ 1+qi$

Sum of roots $1+pi+1+qi = -\alpha$ which is real

$\Rightarrow$ roots of $1+pi,\ 1-pi$

Product of roots $= \beta = 1+p^2 \in (1, \infty)$

$p \neq 0$ since roots are distinct.

41. **2**

Sol: $n = 5$

Success = p

Failure = q

P (at least one failure) $\geq \frac{31}{32}$

1 – P (no failure) $\geq \frac{31}{32}$

$1 - P(x = 5) \geq \frac{31}{32}$

$1 - {}^5C_5 p^5 \geq \frac{31}{32}$

$-p^5 \geq -\frac{1}{32}$

$p^5 \leq \frac{1}{32}$

$p \leq \frac{1}{2}$

$p \in \left[0, \frac{1}{2}\right]$

42. **3**

Sol:

1	2	3	4	5	6	……
200	200	200	240	280	…….	……..

Sum = 11040

120 + 80 + 160 + 40 + 200 + 240 + … = 11040

$$\frac{n}{2}\left[2a+(n-1)d\right]+80+40=11040$$

$$\frac{n}{2}\left[240+(n-1)40\right]=10920$$

$$n\left[6+n-1\right]=546$$

$$n(n+5)=546$$

$$n=21$$

43. **2**

Sol: $\frac{1}{\sqrt{|x|-x}} \Rightarrow |x|-x>0 \Rightarrow |x|>x \Rightarrow x$ is negative

$x \in (-\infty, 0)$

44. **4**

Sol: $\cos\theta=\sqrt{\frac{5}{14}}$

$$\sin\theta=\frac{3}{\sqrt{14}}$$

$$\sin\theta=\frac{1+4+3\lambda}{\sqrt{1+4+\lambda^2}\sqrt{1+4+9}}$$

$$\frac{3}{\sqrt{14}}=\frac{5+3\lambda}{\sqrt{5+\lambda^2}\sqrt{14}} \Rightarrow \lambda=\frac{2}{3}$$

45. **4**

Sol: $(2\bar{a}-\bar{b}).\left\{(\bar{a}\times\bar{b})\times(\bar{a}+2\bar{b})\right\}=(2\bar{a}-\bar{b}).\left\{\left[\bar{a}.(\bar{a}+2\bar{b})\right]\bar{b}-\left[\bar{b}.(\bar{a}+2\bar{b})\bar{a}\right]\right\}$

$$=-5(\bar{a})^2(\bar{b})^2+5(\bar{a}.\bar{b})^2=-5$$

46. **4**

Sol: $b^2=a^2(1-e^2)=a^2\left(1-\frac{2}{5}\right)=a^2\frac{3}{5}=\frac{3a^2}{5}$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow \frac{9}{a^2}+\frac{5}{3a^2}=1$$

$$a^2=\frac{32}{3}$$

$$b^2=\frac{32}{5}$$

$\therefore$ Required equation of ellipse $3x^2+5y^2-32=0$

47. **1**

Sol: $\frac{dV}{dt} = -k(T-t) \Rightarrow dV = -k(T-t)dt$

Integrate

$V = \frac{-k(T-t)^2}{(-2)} + c \Rightarrow V = \frac{k(T-t)^2}{2} + c$

at $t = 0 \Rightarrow V = I$

$I = \frac{kT^2}{2} + c \Rightarrow c = I - \frac{kT^2}{2} \Rightarrow c = V(T) = I - \frac{kT^2}{2}$

48. **3**

Sol: $\bar{b} \times \bar{c} = \bar{b} \times \bar{d}$

$\Rightarrow \bar{a} \times (\bar{b} \times \bar{c}) = \bar{a} \times (\bar{b} \times \bar{d})$

$\Rightarrow (\bar{a}.\bar{c})\bar{b} - (\bar{a}.\bar{b})\bar{c} = (\bar{a}.\bar{d})\bar{b} - (\bar{a}.\bar{b})\bar{d}$

$\Rightarrow (\bar{a}.\bar{c})\bar{b} - (\bar{a}.\bar{b})\bar{c} = -(\bar{a}.\bar{b})\bar{d}$

$\therefore \bar{d} = \bar{c} - \left(\frac{\bar{a}.\bar{c}}{\bar{a}.\bar{b}}\right)\bar{b}$

49. **1**

Sol: $c_1 = \left(\frac{a}{2}, 0\right); c_2 = (0, 0)$

$r_1 = \frac{a}{2}; r_2 = c$

$c_1c_2 = r_1 - r_2 \Rightarrow \frac{a}{2} = c - \frac{a}{2} \Rightarrow c = a$

50. **1**

Sol: $C \cap D = C \Rightarrow P(C \cap D) = P(C) \Rightarrow P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} \geq P(C)$

51. **1**

Sol: $\begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0 \Rightarrow k^2 - 6k + 8 = 0 \Rightarrow k = 4, 2$

52. **1**

Sol: $\sim\{(P \wedge \sim R) \leftrightarrow Q\} = \sim\{Q \leftrightarrow (P \wedge \sim R)\}$

53. **1**

Sol: $P = (y^2, y)$

Perpendicular distance from P to $x - y + 1 = 0$ is $\dfrac{|y^2 - y + 1|}{\sqrt{2}}$

$y^2 - y + 1 > 0\ \forall y \in R$

$\therefore$ Coefficient $y^2 > 0$

$\therefore$ Min value $= \dfrac{1}{\sqrt{2}}\left(\dfrac{4ac - b^2}{4a}\right) = \dfrac{3}{4\sqrt{2}}$

54. **2**

Sol: $\dfrac{1}{n}\sum |x_i - A|$

A = Median = $\dfrac{25a + 26a}{2} = 25.5a$

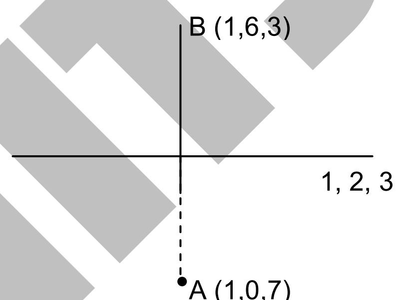
Mean deviation $= \dfrac{1}{50}\{|a - 25.5a| + |2a - 25.5a|\} = \dfrac{2}{50}\{(24.5a + 23.5a) + ...(0.5a)\}$

$= \dfrac{2}{50}\{312.5a\} = 50$ (Given)

$\Rightarrow 625a = 2500 \Rightarrow a = 4$

55. **1**

Sol:



Statement – 1 : AB is perpendicular to given line and mid point of AB lies on line
Statement – 2 is true but it is not correct explanation as it is bisector only.
If it is perpendicular bisector then only statement – 2 is correct explanation.

56. **1**

Sol: $A^T = A, B^T = B$

$(A(BA))^T = (BA)^T A^T = (A^T B^T)A = (AB)A = A(BA)$

$((AB)A)^T = A^T (AB)^T = A(B^T A^T) = A(BA) = (AB)A$

$\therefore$ Statement – 1 is correct

Statement – 2

$(AB)^T = B^T A^T = BA = AB$ ($\because$ AB is commutative)

Statement – 2 is also correct but it is not correct explanation of Statement – 1

57. **1**

Sol: $1+\omega=-\omega^2$

$(1+\omega)^7=\left(-\omega^2\right)^7=-\omega^{14}=-\omega^2=1+\omega=A+B\omega \Rightarrow (A, B)=(1, 1)$

58. **2**

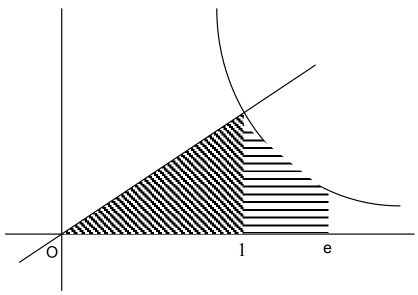
$$\lim_{x\to 0}\frac{\sin(p+1)+\sin x}{x}=q=\lim_{x\to 0}\frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}$$

$$\lim_{x\to 0}(p+1)\cos(p+1)x+\cos x=q=\frac{1}{2}$$

$$\Rightarrow p+1+1=\frac{1}{2}\Rightarrow p=-\frac{3}{2};\ q=\frac{1}{2}$$

59. **2**

Sol:



$$\text{Area}=\int_0^1 x\,dx+\int_1^e \frac{1}{x}dx=\frac{1}{2}+1=\frac{3}{2}$$

60. **3**

Sol: $f'(x)=\sqrt{x}\sin x$

Given $x\in\left(0, \frac{5\pi}{2}\right)$

$f'(x)$ changes sign from +ve to –ve at π

$f'(x)$ changes sign from -ve to +ve at 2π

f has local max at π, local min at 2π

PART C: CHEMISTRY

61. (2)

Sol : Greater charge and small size of cation cause more polarization and more covalent is that compound

62. (1)

Sol : In RNA, the sugar is $\beta-D-$Ribose, where as in DNA the Sugar is β-D-2-deoxy Ribose

63. (4)

Sol : $2CCl_3CHO \xrightarrow{OH^{(-)}} CCl_3COONa + CCl_3CH_2OH$

Cannizaro reaction is a disproportionation reaction

One aldehyde molecule is oxidized to salt of the carboxylic Acid, other one is reduced to Alcohol. So the compound is CCl_3CH_2OH

IUPAC Name is 2, 2, 2, - Trichloro ethanol

64. (3)

Sol : $C_2H_5\overset{(-)}{O}\overset{\oplus}{N}a + CH_3-\underset{\underset{O}{\|}}{C}-Cl \rightarrow CH_3-\underset{\underset{O}{\|}}{C}-O-C_2H_5$ Ethyl ethanoate

65. (2)

Sol : $2H^+ + 2e^- \rightarrow H_2(g)$

$E = E^\circ - 0.059 \ \log\left(\frac{P_{H_2}}{[H^+]^2}\right)$ (here E is –ve when $P_{H_2} > [H^+]^2$)

$= \frac{-0.0591}{2}\log_{10}\left(\frac{2}{1}\right) = \frac{-.0591}{2} \times .3010$ = negative value

66. (2)

Sol : Electron releasing groups (Alkyl groups) de stabilizes conjugate base.

The +I effect of C_3H_7 is less than - I effect of Cl

K_a of HCOOH is 17.9×10^{-5}

K_a of $CH_3CH_2\underset{\underset{Cl}{|}}{CH}-COOH$ is 139×10^{-5}

67. (4)

Sol : $i = 1-\alpha + n\alpha = 1 + \alpha(n-1)$

$\frac{i-1}{n-1} = \alpha$

$A_xB_y \rightarrow xA^{+y} + yB^{-x}$

n = x+y

So $\alpha = \frac{i-1}{x+y-1}$

68. (3)

Sol : ease of liquefaction $\propto \frac{a}{b}$

for ethane a = 5.49, b=0.0638

for Cl_2 a = 6.49, b = 0.0562

69. (4)

Sol : $CO_2(g) + C \rightleftharpoons 2CO(g)$

Initial moles	p	O
Equilibriumm moles	p-x	2x

Total pressure at equilibrium = 0.8 atm ; Total no.of moles = p + x.

Therefore $p \propto n$; $\frac{0.5}{0.8} = \frac{p}{p+x} \Rightarrow x = 0.3$

$K_p = \frac{P_{CO}^2}{P_{CO_2}} = \frac{0.6 \times 0.6}{0.2} = 1.8\text{ atm}$

70. (4)

Sol : As Boron has only four orbitals in the valence shell (i.e. 2s, $2p_x$, $2p_y$ & $2p_z$) it can show a maximum valency of four only.

Therefore $[BF_6]^{3-}$ is not possible

71. (2)

Sol : $[Cr(NH_3)_6]Cl_3$ involves d^2sp^3 hybridization and it is an inner orbital complex.

72. (4)

Sol : $\Delta T_f = K_f \times m = K_f \times \frac{w_2 \times 1000}{w_1 \times m_2}$

w_1 & w_2 = wt of solvent & solute respecting

m_2 = mw of solute

$\Delta T_f = 0^o - (-6^0) = 6 = 1.86 \times \frac{w_2 \times 1000}{4000 \times 62}$

Therefore $w_2 = 800g$

73. (4)

Sol : Across a period metallic strength decreases & down the group it increases

74. (2)

Sol : Temperature coefficient μ =2;

$\mu^{\frac{\Delta T}{10}} = \frac{k_2}{k_1}$;

$2^{\frac{50}{10}} = 2^5 = 32 = \frac{k_2}{k_1}$

Therefore $32k_1 = k_2$

75. (2)

Sol : In $[NiCl_4]^{2-}$, n = 2

$\mu = \sqrt{n(n+2)}$ BM

$= \sqrt{2(2+2)} = 2.82 BM$

76. (1)

Sol :

77. (3)

Sol : The general o.s of lanthanides is +3, only few elements exhibit +4 o.s.

78. (2)

Sol : Molefraction of solute (X_2) in aqueous solution = $\dfrac{m}{m + \dfrac{1000}{18}}$

$= \dfrac{5.2}{5.2 + \dfrac{1000}{18}} = 0.09$

79. (4)

Sol : Stability of hydrides decreases down the group from NH_3 to BiH_3 as M-H bond energy decreases.

80. (3)

81. (3)

Sol : `S' can exhibit a minimum oxidation state of -2

(Ex. H_2S)

82. (3)

Sol : In IF_7, I undergoes sp^3d^3 hybridisation

83. (1)

Sol : Vinyl group

$\left(CH_2 = CH-\right)$

on ozonolosys give formaldehyde

84. (2)

Sol : $\dfrac{1}{\lambda_{absorbed}} = \dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2}$

$\Rightarrow \dfrac{1}{355} = \dfrac{1}{680} + \dfrac{1}{\lambda_2}$

$\Rightarrow \lambda_2 = 742.8 \cong 743$ nm

85. (2 , 4)

Sol : Formaldehyde and Acetaldehyde can be oxidized by tollen's reagent to give silver mirror.

86. (3)

Sol : Phenol gives violet coloured comlex compound with neutral $FeCl_3$, benzoic acid gives pale dull yellow ppt. with neutral $FeCl_3$

87. (3)

Sol : In acidic medium, KBr + $KBrO_3$ in turn produces Br_2. Phenol reacts with Br_2 (aq) to give 2, 4, 6-trinitrophenol

88. (3)

Sol : Effective no.of A atoms = $\frac{1}{8} \times 8 = 1$

Effective no.of B atoms = $\frac{1}{2} \times 5$ (One is missing) = $\frac{5}{2}$

Therefore formula is $A_1B_{\frac{5}{2}} = A_2B_5$

89. (4)

Sol : For an ideal gas, for isothermal reversible process,

$\Delta S = 2.303 \text{ nR log } \left(\frac{V_2}{V_1}\right)$

= $2.303 \times 2 \times 8.314 \times \log\left(\frac{100}{10}\right)$

= $38.3 \text{ J mol}^{-1}.\text{k}^{-1}$

90. 2, (2, 3)

Sol : both 2-pentanone, phenol can exhibit tautomerism