MATHEMATICS
PART – A

1. ABC is a triangle, right angled at A. The resultant of the forces acting along $\overrightarrow{AB}, \overrightarrow{AC}$ with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively is the force along $\overrightarrow{AD}$, where D is the foot of the perpendicular from A onto BC. The magnitude of the resultant is

(1) $\frac{AB^2 + AC^2}{(AB)^2(AC)^2}$ (2) $\frac{(AB)(AC)}{AB + AC}$

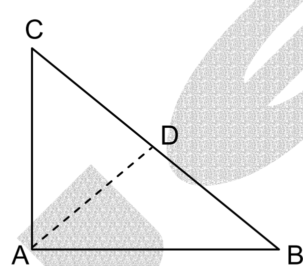
(3) $\frac{1}{AB} + \frac{1}{AC}$ (4) $\frac{1}{AD}$

Ans. (4)

Sol: Magnitude of resultant

$= \sqrt{\left(\frac{1}{AB}\right)^2 + \left(\frac{1}{AC}\right)^2} = \frac{\sqrt{AB^2 + AC^2}}{AB \cdot AC}$

$= \frac{BC}{AB \cdot AC} = \frac{BC}{AD \cdot BC} = \frac{1}{AD}$



2. Suppose a population A has 100 observations 101, 102, … , 200, and another population B has 100 observations 151, 152, … , 250. If V_A and V_B represent the variances of the two populations, respectively, then $\frac{V_A}{V_B}$ is

(1) 1 (2) 9/4
(3) 4/9 (4) 2/3

Ans. (1)

Sol: $\sigma_x^2 = \frac{\sum d_i^2}{n}$. (Here deviations are taken from the mean)

Since A and B both has 100 consecutive integers, therefore both have same standard deviation and hence the variance.

$\therefore \frac{V_A}{V_B} = 1 \left(\text{As } \sum d_i^2 \text{ is same in both the cases}\right)$.

3. If the roots of the quadratic equation $x^2 + px + q = 0$ are tan30° and tan15°, respectively then the value of 2 + q – p is

(3) 2 (2) 3
(3) 0 (4) 1

Ans. (2)

Sol: $x^2 + px + q = 0$

$\tan 30° + \tan 15° = -p$

$\tan 30° \cdot \tan 15° = q$

$\tan 45° = \dfrac{\tan 30° + \tan 15°}{1 - \tan 30° \tan 15°} = \dfrac{-p}{1-q} = 1$

$\Rightarrow -p = 1 - q$

$\Rightarrow q - p = 1 \quad \therefore\ 2 + q - p = 3.$

4. The value of the integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is

(1) 1/2 (2) 3/2
(3) 2 (4) 1

Ans. (2)

Sol: $I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$

$I = \int_3^6 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}}dx$

$2I = \int_3^6 dx = 3 \Rightarrow I = \frac{3}{2}.$

5. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is

(1) 4 (2) 6
(3) 1 (4) 2

Ans. (1)

Sol: $2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow (\sin x + 3)(2\sin x - 1) = 0$

$\Rightarrow \sin x = \frac{1}{2} \quad \therefore$ In $(0, 3\pi)$, x has 4 values

6. If $\left(\bar{a}\times\bar{b}\right)\times\bar{c} = \bar{a}\times\left(\bar{b}\times\bar{c}\right)$, where $\bar{a}, \bar{b}$ and $\bar{c}$ are any three vectors such that $\bar{a}\cdot\bar{b}\neq 0$, $\bar{b}\cdot\bar{c}\neq 0$, then $\bar{a}$ and $\bar{c}$ are

(1) inclined at an angle of $\pi/3$ between them
(2) inclined at an angle of $\pi/6$ between them
(3) perpendicular
(4) parallel

Ans. (4)

Sol: $\left(\bar{a}\times\bar{b}\right)\times\bar{c} = \bar{a}\times\left(\bar{b}\times\bar{c}\right)$, $\bar{a}\cdot\bar{b}\neq 0$, $\bar{b}\cdot\bar{c}\neq 0$

$\Rightarrow (\bar{a}\cdot\bar{c})\bar{b} - (\bar{b}\cdot\bar{c})\bar{a} = (\bar{a}\cdot\bar{c})\bar{b} - (\bar{a}\cdot\bar{b})\bar{c}$

$(\bar{a}\cdot\bar{b})\bar{c} = (\bar{b}\cdot\bar{c})\bar{a}$

$\bar{a}\parallel\bar{c}$

7. Let W denote the words in the English dictionary. Define the relation R by :

R = {(x, y) $\in$ W × W | the words x and y have at least one letter in common}. Then R is
(1) not reflexive, symmetric and transitive
(2) reflexive, symmetric and not transitive
(3) reflexive, symmetric and transitive
(4) reflexive, not symmetric and transitive

Ans. (2)
Sol: Clearly (x, x) $\in$ R $\forall$ x $\in$ W. So, R is reflexive.
Let (x, y) $\in$ R, then (y, x) $\in$ R as x and y have at least one letter in common. So, R is symmetric.
But R is not transitive for example
Let x = DELHI, y = DWARKA and z = PARK
then (x, y) $\in$ R and (y, z) $\in$ R but (x, z) $\notin$ R.

8. If A and B are square matrices of size n × n such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true ?
(1) A = B
(2) AB = BA
(3) either of A or B is a zero matrix
(4) either of A or B is an identity matrix

Ans. (2)
Sol: $A^2 - B^2 = (A - B)(A + B)$
$A^2 - B^2 = A^2 + AB - BA - B^2$
$\Rightarrow AB = BA$.

9. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$ is
(1) i (2) 1
(3) –1 (4) –i

Ans. (4)
Sol: $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right) = \sum_{k=1}^{10}\sin\frac{2k\pi}{11} + i\sum_{k=1}^{10}\cos\frac{2k\pi}{11}$
$= 0 + i(-1) = -i$.

10. All the values of m for which both roots of the equations $x^2 - 2mx + m^2 - 1 = 0$ are greater than –2 but less than 4, lie in the interval
(1) $-2 < m < 0$ (2) $m > 3$
(3) $-1 < m < 3$ (4) $1 < m < 4$

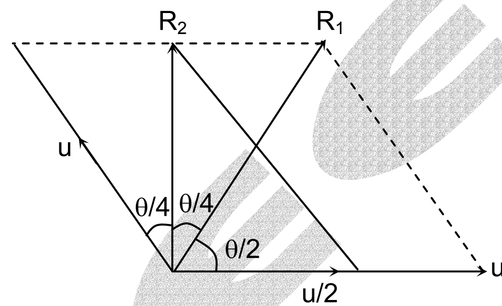
Ans. (3)
Sol: Equation $x^2 - 2mx + m^2 - 1 = 0$
$(x - m)^2 - 1 = 0$
$(x - m + 1)(x - m - 1) = 0$
$x = m - 1, m + 1$
$-2 < m - 1$ and $m + 1 < 4$

$m > -1$ and $m < 3$
$-1 < m < 3$.

11. A particle has two velocities of equal magnitude inclined to each other at an angle θ. If one of them is halved, the angle between the other and the original resultant velocity is bisected by the new resultant. Then θ is

(1) 90° (2) 120°
(3) 45° (4) 60°

Ans. (2)



Sol: $\tan\frac{\theta}{4} = \dfrac{\frac{u}{2}\sin\theta}{u + \frac{u}{2}\cos\theta}$

$\Rightarrow \sin\frac{\theta}{4} + \frac{1}{2}\sin\frac{\theta}{4}\cos\theta = \frac{1}{2}\sin\theta\cos\frac{\theta}{4}$

$\therefore\ 2\sin\frac{\theta}{4} = \sin\frac{3\theta}{4} = 3\sin\frac{\theta}{4} - 4\sin^3\frac{\theta}{4}$

$\therefore\ \sin^2\frac{\theta}{4} = \frac{1}{4} \quad \Rightarrow \frac{\theta}{4} = 30°$ or $\theta = 120°$.

12. At a telephone enquiry system the number of phone cells regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10-minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is

(1) $\frac{6}{5^e}$ (2) $\frac{5}{6}$
(3) $\frac{6}{55}$ (4) $\frac{6}{e^5}$

Ans. (4)

Sol: $P(X = r) = \dfrac{e^{-m}m^r}{r!}$

$P(X \le 1) = P(X = 0) + P(X = 1)$

$= e^{-5} + 5 \times e^{-5} = \dfrac{6}{e^5}$.

13. A body falling from rest under gravity passes a certain point P. It was at a distance of 400 m from P, 4s prior to passing through P. If $g = 10\ m/s^2$, then the height above the point P from where the body began to fall is

(1) 720 m (2) 900 m
(3) 320 m (4) 680 m

Ans. (1)

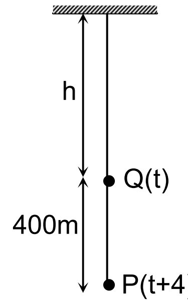
Sol: We have $h = \frac{1}{2}gt^2$ and h + 400 = $\frac{1}{2}g(t+4)^2$.

Subtracting we get 400 = 8g + 4gt

$\Rightarrow$ t = 8 sec

$\therefore\ h = \frac{1}{2} \times 10 \times 64 = 320m$

$\therefore$ Desired height = 320 + 400 = 720 m.



14. $\int_0^{\pi} xf(\sin x)dx$ is equal to

(1) $\pi\int_0^{\pi} f(\cos x)dx$ (2) $\pi\int_0^{\pi} f(\sin x)dx$

(3) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$ (4) $\pi\int_0^{\pi/2} f(\cos x)dx$

Ans. (4)

Sol: I = $\int_0^{\pi} xf(\sin x)\,dx = \int_0^{\pi} (\pi - x)f(\sin x)\,dx$

= $\pi\int_0^{\pi} f(\sin x)\,dx - I$

2I = $\pi\int_0^{\pi} f(\sin x)\,dx$

I = $\frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx = \pi\int_0^{\pi/2} f(\sin x)\,dx$

= $\pi\int_0^{\pi/2} f(\cos x)dx$.

15. A straight line through the point A(3, 4) is such that its intercept between the axes is bisected at A. Its equation is

(1) x + y = 7 (2) 3x – 4y + 7 = 0

(3) 4x + 3y = 24 (4) 3x + 4y = 25

Ans. (3)

Sol: The equation of axes is xy = 0

$\Rightarrow$ the equation of the line is

$\frac{x \cdot 4 + y \cdot 3}{2} = 12 \Rightarrow$ 4x + 3y = 24.

16. The two lines x = ay + b, z = cy + d; and x = a′y + b′, z = c′y + d′ are perpendicular to each other if

(1) aa′ + cc′ = –1 (2) aa′ + cc′ = 1

(3) $\frac{a}{a'} + \frac{c}{c'} = -1$ (4) $\frac{a}{a'} + \frac{c}{c'} = 1$

Ans. (1)

Sol: Equation of lines $\frac{x-b}{a} = y = \frac{z-d}{c}$

$\frac{x-b'}{a'} = y = \frac{z-d'}{c'}$

Lines are perpendicular $\Rightarrow aa' + 1 + cc' = 0$.

17. The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is

(!) $xy = \frac{105}{64}$

(2) $xy = \frac{3}{4}$

(3) $xy = \frac{35}{16}$

(4) $xy = \frac{64}{105}$

Ans. (1)

Sol: Parabola: $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$

Vertex: (α, β)

$$\alpha = \frac{-a^2/2}{2a^3/3} = -\frac{3}{4a},\ \beta = \frac{-\left(\frac{a^4}{4} + 4\cdot\frac{a^3}{3}\cdot 2a\right)}{4\frac{a^3}{3}} = -\frac{-\left(\frac{1}{4}+\frac{8}{3}\right)a^4}{\frac{4}{3}a^3}$$

$$= -\frac{35}{12}\frac{a}{4}\times 3 = -\frac{35}{16}a$$

$$\alpha\beta = -\frac{3}{4a}\left(-\frac{35}{16}\right)a = \frac{105}{64}.$$

18. The values of a, for which the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a right-angled triangle with $C = \frac{\pi}{2}$ are

(1) 2 and 1

(2) –2 and –1

(3) –2 and 1

(4) 2 and –1

Ans. (1)

Sol: $\Rightarrow \overrightarrow{BA} = \hat{i} - 2\hat{j} + 6\hat{k}$

$\overrightarrow{CA} = (2-a)\hat{i} + 2\hat{j}$

$\overrightarrow{CB} = (1-a)\hat{i} - 6\hat{k}$

$\overrightarrow{CA}\cdot\overrightarrow{CB} = 0 \Rightarrow (2-a)(1-a) = 0$

$\Rightarrow a = 2, 1.$

19. $\int_{-3\pi/2}^{-\pi/2}\left[(x+\pi)^3+\cos^2(x+3\pi)\right]dx$ is equal to

(1) $\frac{\pi^4}{32}$
(2) $\frac{\pi^4}{32}+\frac{\pi}{2}$
(3) $\frac{\pi}{2}$
(4) $\frac{\pi}{4}-1$

Ans. (3)

Sol: $I = \int_{-3\pi/2}^{-\pi/2}\left[(x+\pi)^3+\cos^2(x+3\pi)\right]dx$

Put x + π = t

$I = \int_{-\pi/2}^{\pi/2}\left[t^3+\cos^2 t\right]dt = 2\int_{0}^{\pi/2}\cos^2 t\,dt$

$= \int_{0}^{\pi/2}(1+\cos 2t)\,dt = \frac{\pi}{2}+0$.

20. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is

(1) 1/4
(2) 41
(3) 1
(4) 17/7

Ans. (2)

Sol: $y = \frac{3x^2+9x+17}{3x^2+9x+7}$

$3x^2(y-1) + 9x(y-1) + 7y - 17 = 0$

$D \geq 0 \quad \because$ x is real

$81(y-1)^2 - 4x3(y-1)(7y-17) \geq 0$

$\Rightarrow (y-1)(y-41) \leq 0 \Rightarrow 1 \leq y \leq 41$.

21. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is

(1) $\frac{3}{5}$
(B) $\frac{1}{2}$
(C) $\frac{4}{5}$
(D) $\frac{1}{\sqrt{5}}$

Ans. (1)

Sol: $2ae = 6 \Rightarrow ae = 3$

$2b = 8 \Rightarrow b = 4$

$b^2 = a^2(1-e^2)$

$16 = a^2 - a^2e^2$

$a^2 = 16 + 9 = 25$

$a = 5$

$\therefore e = \frac{3}{a} = \frac{3}{5}$

22. Let A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and B = $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, a , b $\in$ N. Then
(1) there cannot exist any B such that AB = BA
(2) there exist more than one but finite number of B's such that AB = BA
(3) there exists exactly one B such that AB = BA
(4) there exist infinitely many B's such that AB = BA

Ans. (4)

Sol: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$AB = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$

$BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$

AB = BA only when a = b

23. The function f(x) = $\frac{x}{2} + \frac{2}{x}$ has a local minimum at
(1) x = 2
(2) x = –2
(3) x = 0
(4) x = 1

Ans. (1)

Sol: $\frac{x}{2} + \frac{2}{x}$ is of the form $x + \frac{1}{x} \geq 2$ & equality holds for x = 1

24. Angle between the tangents to the curve y = x^2 – 5x + 6 at the points (2, 0) and (3, 0) is
(1) $\frac{\pi}{2}$
(2) $\frac{\pi}{2}$
(3) $\frac{\pi}{6}$
(4) $\frac{\pi}{4}$

Ans. (2)

Sol: $\frac{dy}{dx} = 2x - 5$

$\therefore m_1 = (2x - 5)_{(2,\,0)} = -1$, $m_2 = (2x - 5)_{(3,\,0)} = 1$

$\Rightarrow m_1 m_2 = -1$

25. Let $a_1, a_2, a_3, \ldots$ be terms of an A.P. If $\frac{a_1 + a_2 + \cdots a_p}{a_1 + a_2 + \cdots + a_q} = \frac{p^2}{q^2}, p \neq q$, then $\frac{a_6}{a_{21}}$ equals
(1) $\frac{41}{11}$
(2) $\frac{7}{2}$
(3) $\frac{2}{7}$
(4) $\frac{11}{41}$

Ans. (4)

Sol: $\dfrac{\frac{p}{2}\left[2a_1+(p-1)d\right]}{\frac{q}{2}\left[2a_1+(q-1)d\right]}=\dfrac{p^2}{q^2} \Rightarrow \dfrac{2a_1+(p-1)d}{2a_1+(q-1)d}=\dfrac{p}{q}$

$$\frac{a_1+\left(\frac{p-1}{2}\right)d}{a_1+\left(\frac{q-1}{2}\right)d}=\frac{p}{q}$$

For $\dfrac{a_6}{a_{21}}$, $p=11,\ q=41 \ \rightarrow \dfrac{a_6}{a_{21}}=\dfrac{11}{41}$

26. The set of points where $f(x)=\dfrac{x}{1+|x|}$ is differentiable is

(1) $(-\infty, 0) \cup (0, \infty)$
(2) $(-\infty, -1) \cup (-1, \infty)$
(3) $(-\infty, \infty)$
(4) $(0, \infty)$

Ans. (3)

Sol: $f(x)=\begin{cases}\dfrac{x}{1-x}, & x<0\\ \dfrac{x}{1+x}, & x\ge 0\end{cases} \Rightarrow f'(x)=\begin{cases}\dfrac{1}{(1-x)^2}, & x<0\\ \dfrac{1}{(1+x)^2}, & x\ge 0\end{cases}$

$\therefore$ $f'(x)$ exist at everywhere.

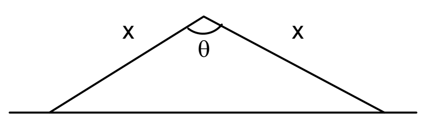
27. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is

(1) $\dfrac{3}{2}x^2$
(2) $\sqrt{\dfrac{x^3}{8}}$
(3) $\dfrac{1}{2}x^2$
(4) πx^2

Ans. (3)

Sol: Area = $\dfrac{1}{2}x^2 \sin\theta$

$A_{max}=\dfrac{1}{2}x^2\left(\text{at } \sin\theta=1,\quad \theta=\dfrac{\pi}{2}\right)$



28. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are of be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote is
(1) 5040
(2) 6210
(3) 385
(4) 1110

Ans. (3)
Sol: ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$
= 10 + 45 + 120 + 210 = 385

29. If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$, then a_n is

(1) $\frac{b^n - a^n}{b-a}$
(2) $\frac{a^n - b^n}{b-a}$
(3) $\frac{a^{n+1} - b^{n+1}}{b-a}$
(4) $\frac{b^{n+1} - a^{n+1}}{b-a}$

Ans. (4)

Sol: $(1-ax)^{-1}(1-bx)^{-1} = \left(1+ax+a^2x^2+\ldots\ldots\right)\left(1+bx+b^2x^2+\ldots.\right)$

$\therefore$ coefficient of $x^n = b^n + ab^{n-1} + a^2b^{n-2} + \ldots. + a^{n-1}b + a^n = \frac{b^{n+1}-a^{n+1}}{b-a}$

$\therefore\ a_n = \frac{b^{n+1}-a^{n+1}}{b-a}$

30. For natural numbers m, n if $(1 - y)^m (1 + y)^n = 1 + a_1y + a_2y^2 + \ldots$, and $a_1 = a_2 = 10$, then (m, n) is

(1) (20, 45)
(2) (35, 20)
(3) (45, 35)
(4) (35, 45)

Ans. (4)

Sol: $(1-y)^m(1+y)^n = \left[1-{}^mC_1y+{}^mC_2y^2-\ldots.\right]\left[1+{}^nC_1y+{}^nC_2y^2+\ldots\right]$

$= 1+(n-m)+\left\{\frac{m(m-1)}{2}+\frac{n(n-1)}{2}-mn\right\}y^2+\ldots..$

$\therefore a_1 = n-m = 10$ and $a_2 = \frac{m^2+n^2-m-n-2mn}{2} = 10$

So, $n - m = 10$ and $(m - n)^2 - (m + n) = 20 \quad \Rightarrow m + n = 80$

$\therefore\ m = 35,\quad n = 45$

31. The value of $\int_1^a [x]f'(x)dx$, a > 1, where [x] denotes the greatest integer not exceeding x is

(1) af(a) – {f(1) + f(2) + … + f([a])}
(2) [a] f(a) – {f(1) + f(2) + … + f([a])}
(3) [a] f([a]) – {f(1) + f(2) + … + f(a)}
(4) af([a]) – {f(1) + f(2) + … + f(a)}

Ans. (2)

Sol: Let a = k + h, where [a] = k and $0 \le h < 1$

$\therefore\ \int_1^a [x]f'(x)dx = \int_1^2 1f'(x)dx + \int_2^3 2f'(x)dx + \ldots\ldots\ldots \int_{k-1}^{k}(k-1)dx + \int_k^{k+h} kf'(x)dx$

{f(2) – f(1)} + 2{f(3) – f(2)} + 3{f(4) – f(3)}+…….+ (k–1) – {f(k) – f(k – 1)} + k{f(k + h) – f(k)}

= – f(1) – f(2) – f(3)……. – f(k) + k f(k + h)

= [a] f(a) – {f(1) + f(2) + f(3) + …. + f([a])}

32. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is
(1) $x^2 + y^2 + 2x - 2y - 47 = 0$
(2) $x^2 + y^2 + 2x - 2y - 62 = 0$
(3) $x^2 + y^2 - 2x + 2y - 62 = 0$
(4) $x^2 + y^2 - 2x + 2y - 47 = 0$

Ans. (4)
Sol: Point of intersection of $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ is $(1, -1)$, which is the centre of the circle and radius = 7.
$\therefore$ Equation is $(x - 1)^2 + (y + 1)^2 = 49 \Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0$.

33. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of
(1) second order and second degree
(2) first order and second degree
(3) first order and first degree
(4) second order and first degree

Ans. (4)
Sol: $Ax^2 + By^2 = 1$ … (1)

$Ax + By\frac{dy}{dx} = 0$ … (2)

$A + By\frac{d^2y}{dx^2} + B\left(\frac{dy}{dx}\right)^2 = 0$ … (3)

From (2) and (3)

$$x\left\{-By\frac{d^2y}{dx^2} - B\left(\frac{dy}{dx}\right)^2\right\} + By\frac{dy}{dx} = 0$$

$$\Rightarrow xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

34. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre is
(1) $x^2 + y^2 = \frac{3}{2}$
(B) $x^2 + y^2 = 1$
(3) $x^2 + y^2 = \frac{27}{4}$
(D) $x^2 + y^2 = \frac{9}{4}$

Ans. (4)
Sol: $\cos\frac{\pi}{3} = \frac{\sqrt{h^2 + k^2}}{3} \Rightarrow h^2 + k^2 = \frac{9}{4}$

35. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}, x > 0$ and $y = 3x$, $x > 0$, then a belongs to
(1) $\left(0, \frac{1}{2}\right)$
(2) $(3, \infty)$
(3) $\left(\frac{1}{2}, 3\right)$
(4) $\left(-3, -\frac{1}{2}\right)$

Ans. (3)

Sol: $a^2 - 3a < 0$ and $a^2 - \frac{a}{2} > 0 \Rightarrow \frac{1}{2} < a < 3$

36. The image of the point (–1, 3, 4) in the plane x – 2y = 0 is

(1) $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$ (2) (15, 11, 4)

(3) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$ (4) (8, 4, 4)

Sol: If (α, β, γ) be the image then $\frac{\alpha-1}{2} - 2\left(\frac{\beta+3}{2}\right) = 0$

$\therefore \alpha - 1 - 2\beta - 6 \Rightarrow \alpha - 2\beta = 7$ … (1)

and $\frac{\alpha+1}{1} = \frac{\beta-3}{-2} = \frac{\gamma-4}{0}$ … (2)

From (1) and (2)

$\alpha = \frac{9}{5}, \ \beta = -\frac{13}{5}, \ \gamma = 4$

No option matches.

37. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of

$\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \cdots + \left(z^6 + \frac{1}{z^6}\right)^2$ is

(1) 18 (2) 54

(3) 6 (4) 12

Ans. (4)

Sol: $z^2 + z + 1 = 0 \Rightarrow z = \omega$ or ω^2

so, $z + \frac{1}{z} = \omega + \omega^2 = -1, \ z^2 + \frac{1}{z^2} = \omega^2 + \omega = -1, \ z^3 + \frac{1}{z^3} = \omega^3 + \omega^3 = 2$

$z^4 + \frac{1}{z^4} = -1, \ z^5 + \frac{1}{z^5} = -1$ and $z^6 + \frac{1}{z^6} = 2$

$\therefore$ The given sum = 1 + 1 + 4 + 1 + 1 + 4 = 12

38. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then tanx is

(1) $\frac{(1-\sqrt{7})}{4}$ (B) $\frac{(4-\sqrt{7})}{3}$

(3) $-\frac{(4+\sqrt{7})}{3}$ (4) $\frac{(1+\sqrt{7})}{4}$

Ans. (3)

Sol: $\cos x + \sin x = \frac{1}{2} \Rightarrow 1 + \sin 2x = \frac{1}{4} \Rightarrow \sin 2x = -\frac{3}{4}$, so x is obtuse

and $\frac{2\tan x}{1+\tan^2 x} = -\frac{3}{4} \Rightarrow 3\tan^2 x + 8\tan x + 3 = 0$

$\therefore \tan x = \dfrac{-8 \pm \sqrt{64-36}}{6} = \dfrac{-4 \pm \sqrt{7}}{3}$

$\because \tan x < 0 \qquad \therefore \tan x = \dfrac{-4-\sqrt{7}}{3}$

39. If $a_1, a_2, \ldots, a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + \ldots + a_{n-1}a_n$ is equal to
(1) $n(a_1 - a_n)$
(2) $(n - 1)(a_1 - a_n)$
(3) na_1a_n
(4) $(n - 1)a_1a_n$

Ans. (4)

Sol: $\dfrac{1}{a_2} - \dfrac{1}{a_1} = \dfrac{1}{a_3} - \dfrac{1}{a_2} = \ldots.. = \dfrac{1}{a_n} - \dfrac{1}{a_{n-1}} = d$ (say)

Then $a_1a_2 = \dfrac{a_1-a_2}{d}$, $a_2a_3 = \dfrac{a_2-a_3}{d}$,......., $a_{n-1}a_n = \dfrac{a_{n-1}-a_n}{d}$

$\therefore a_1a_2 + a_2a_3 + \ldots\ldots. + a_{n-1}a_n = \dfrac{a_1-a_n}{d}$ Also, $\dfrac{1}{a_n} = \dfrac{1}{a_1} + (n-1)d$

$\Rightarrow \dfrac{a_1-a_n}{d} = (n-1)a_1a_n$

40. If $x^m \cdot y^n = (x+y)^{m+n}$, then $\dfrac{dy}{dx}$ is
(1) $\dfrac{y}{x}$
(2) $\dfrac{x+y}{xy}$
(3) xy
(4) $\dfrac{x}{y}$

Ans. (1)

Sol: $x^m.y^n = (x+y)^{m+n} \Rightarrow m\ln x + n\ln y = (m+n)\ln(x+y)$

$\therefore \dfrac{m}{x} + \dfrac{n}{y}\dfrac{dy}{dx} = \dfrac{m+n}{x+y}\left(1+\dfrac{dy}{dx}\right) \Rightarrow \left(\dfrac{m}{x} - \dfrac{m+n}{x+y}\right) = \left(\dfrac{m+n}{x+y} - \dfrac{n}{y}\right)\dfrac{dy}{dx}$

$\Rightarrow \dfrac{my-nx}{x(x+y)} = \left(\dfrac{my-nx}{y(x+y)}\right)\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x}$