IIT JEE 2013 : Solution to Full Length Test 8 : PAPER - I & II

PAPER – I

PHYSICS

1. (B)

$\Delta x_m = (X - L)$

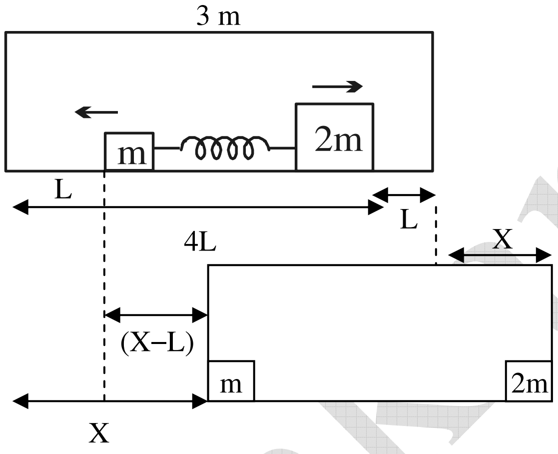
$\Delta x_{2m} = (X + L)$

$\Delta x_{3m} = X$

$\Rightarrow$ m (X − L) + 2m (X + L) + 3mx = 0

$\Rightarrow$ 6mx − mL = 0

$\Rightarrow x = \dfrac{L}{6}$



2. (A)

$q_2 = q_0\, C_2 / (C_1 + C_2)\,(1 - e^{-t/\tau})$, where τ is $\dfrac{RC_1C_2}{C_1 + C_2}$

3. (C)

Just after closing switch no current flows through R_2 so I_1 = 3mA

Long time after closing switch no current flows through C so I_2 = 2mA

Directly after re-opening the switch no current flows through R_1 and the capacitor will discharge through R_2 so I_3 = 2mA]

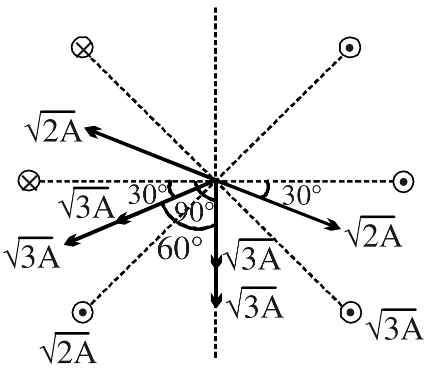
4. (C)

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{\sqrt{3}\times 2}{3\times 10^{-2}} = 10^{-5}\times\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}\times 10^{-5}$$

$$= B = \sqrt{(2B_1)^2 + (2B_2)^2 + 2(2B_1)\times(2B_2)\cos 60^\circ}$$

$$= B\sqrt{4+4+4}$$

$= 4 \times 10^{-5} = 40 \times 10^{-6}$ along 9



5. (C)

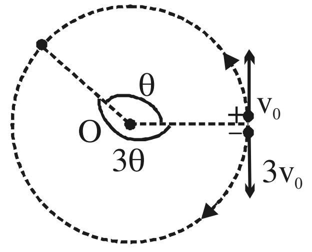
$r = \dfrac{p}{qB}$ = same, $T_+ = \dfrac{2\pi m_+}{qB} = \dfrac{6\pi m}{qB}$, $T_- = \dfrac{2\pi m}{qB}$

as $T_+ = 3T_-$, They will meet at $\theta = \pi/2$

q = 1 μC, B = 2πμT , m = 10^{-15} kg

The time is $= \dfrac{T_+}{4} = \dfrac{6\pi m}{4qB} = \dfrac{6\times\pi\times 10^{-15}}{4\times 1\times 10^{-6}\times 2\pi\times 10^{-6}}$

$= 0.75 \times 10^{-3}$ S = 750 μS



6. (C)

for 1 loop $\oint_0^{\ell_1} B\cdot\vec{d}s = \mu_0 I$

$\Rightarrow$ for N loop $\oint_0^{\ell} B\cdot\vec{d}s = N\oint_0^{\ell_1}\vec{B}\cdot\vec{d}s = \mu_0 NI$

7. (A)

$R = \dfrac{20^2\times\sin 120°}{g} = 20\sqrt{3} = \dfrac{\Delta R}{R} = \dfrac{2\Delta U}{U} \Rightarrow \Delta R = \dfrac{2\times 5}{100}\times 20\sqrt{3} = 2\sqrt{3}$

$20\sqrt{3} - 2\sqrt{3} < R < 20\sqrt{3} + 2\sqrt{3} \quad \Rightarrow\ 31.1\text{m} < R < 38.1\ \text{m}$

8. (C)

TV^{r-1} = const.

9. (C)

$\Delta\phi = 2n\pi$

$\Rightarrow \dfrac{\pi}{2} + \dfrac{2\pi}{\lambda} d\sin\theta = 2n\pi$

$\dfrac{2\pi}{\lambda} d\sin\theta = \left(2n - \dfrac{1}{2}\right)\pi$

$\sin\theta = \left(2n - \dfrac{1}{2}\right)\dfrac{\lambda}{2d} = \dfrac{1}{2}\times\dfrac{\lambda}{2\times 3\lambda} = \dfrac{1}{12}$

$\Rightarrow = \dfrac{y}{\sqrt{(100\lambda)^2}} = \dfrac{1}{12}$

$144y^2 = (100\lambda)^2$

$y \approx \dfrac{100\lambda}{12} = \dfrac{25\lambda}{3}$

10. (A)

Radius $R_1 = \dfrac{mv}{qB_1}$

In a given fields radius can be same for every entry, if magnitude of B_1 and B_2 are equal.

11. (A), (C), (D)

12. (B), (D)

The flux through the differential cube is

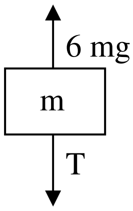
$d\phi = \dfrac{\partial E}{\partial x}(dx\,dy\,dz) + \dfrac{\partial E}{\partial y}(dx\,dy\,dz) + \dfrac{\partial E}{\partial z}(dx\,dy\,dz)$

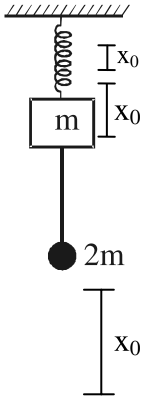
$= (3 + 4 + 5)\ dx\ dy\ dz$

$d\phi = 12\ dx\ dy\ dz = 12\ dV$

$dq = 12\ \varepsilon_0\ dV$

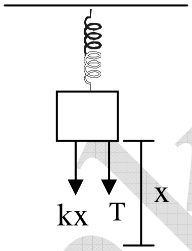
13. (A), (D)

(A) 



$2(6\text{ mg} - T - \text{mg} = \text{ma})$
$-(T - 2\text{ mg} = 2\text{ ma})$
$12\text{ mg} - 3\text{ T} = 0$
$T = 4\text{ mg}$

(B) $Kx + T + mg = ma$
$2mg - T = 2ma$
for $T = 0$, $a = g$
$\Rightarrow\ x = 0$
as particle is released x_0 below equilibrium
so it will go x_0 above equilibrium.
i.e. at $x = 0 \Rightarrow T_{min} = 0$



(C) & (D) for $x_0 > \dfrac{3mg}{k}$ it will be no longer SHM as string will block.

14. (A), (D)

(A) $\dfrac{\mu_0 K}{2}$. Hence, $K = \sigma V$

(D) $F = q\vec{u}\times\vec{B}$ hence upwards.

15. (A), (B)

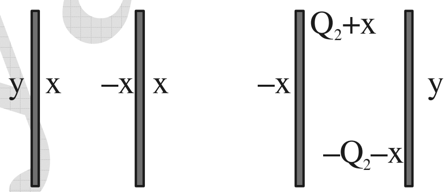
$xa + xb + (Q_2 + x)\,c = 0$

$x = \dfrac{-Q_2 c}{a+b+c}$

$Q_1 = y + x + y - Q_2 - x$

$y = \dfrac{Q_1 + Q_2}{2}$

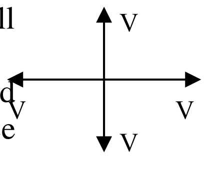
$V = \dfrac{Q_2 ca}{(a+b+c)S\varepsilon_0}$]



16. [2]

Initially $V_{CM} = 0$ and particles are spreading symmetrically in all possible direction.

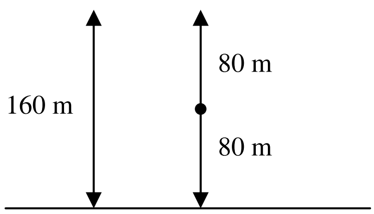
Finally if topmost particle come back to initial point, its speed must be v and directed downward at this moment distance travelled by CM is 80 m



(as acc. of CM is $t = \sqrt{\dfrac{2h}{g}} = 4$ sec.)

$\Rightarrow\ 4 = \dfrac{2V}{g}$

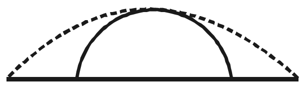
$\Rightarrow\ 4 = \dfrac{2V}{g} \Rightarrow v - 2g \qquad \therefore\ v = 20$ m/s



17. [2]

For minimum condition it should just touch topmost

This imply three facts



$$D = \frac{u^2 \sin\theta\cos\theta}{g} \quad \text{............ (I)}$$

$$h_{max} = \frac{u^2 \sin^2\theta}{2g} \quad \text{............ (II)}$$

r = radius of curvature at topmost point

$$= \frac{u^2 \cos^2\theta}{g} \quad \text{............... (III)}$$

from (II) & (III)

$$\frac{u^2\cos^2\theta}{g} = \frac{u^2\sin^2\theta}{2g}$$

$\Rightarrow$ $\tan\theta = \sqrt{2}$

from (I) & (II)

$$\frac{D}{r} = \tan\theta$$

$D = r\tan\theta \quad \therefore \quad D = 2$ m

18. [1]

as object is moving in x − y plane with centre at origin

$\Rightarrow$ z = 0

$$\vec{F} = \left[(2x - y + 3z)\hat{i} + (x + y - z)\,\hat{j} + (5x - 2y - z)\,\hat{k}\right]$$

$$\vec{F} = \left[(2x - y)\hat{i} + (x + y)\,\hat{j} + (5x - 2y)\,\hat{k}\right]$$

$y = R\sin\theta \Rightarrow dy = R\cos\theta\, d\theta$

$x = R\cos\theta \Rightarrow dx = -R\sin\theta\, d\theta$

$$\overrightarrow{dS} = dx\,\hat{i} + dy\,\hat{j}$$

$$= -R\sin\theta\, d\theta\,\hat{i} + R\cos\theta\, d\theta\,\hat{j}$$

$$\Rightarrow d\omega = \vec{F}\cdot\overrightarrow{dS}$$

$$= \int(2x - y)(-y\,d\theta) + \int(x + y)x\,d\theta$$

$$= \int\left(y^2 - 2xy\right)d\theta + \int\left(x^2 + xy\right)d\theta$$

$$= R^2\left[\int_0^{2\pi}\left(\sin^2\theta - 2\sin\theta\cos\theta\right)d\theta + \int_0^{2\pi}\left(\cos^2\theta + \sin\theta - \cos\theta\right)d\theta\right.$$

$$= R^2\left[\int_0^{2\pi}\left(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta\right)d\theta\right.$$

$$= R^2\left[\int_0^{2\pi}\left(1 - \frac{\sin 2\theta}{2}\right)d\theta\right.$$

$$= R^2(2\pi) = 32\pi \simeq 100 \text{ joule}$$

19. [2]

$13(2x) + R = 200$

$13(2\ (10 - x) + R = 100$

$260 + 2R = 300$

$R = 20\ \Omega \quad = 2$ deca ohm

20. [5]

By Energy Conservation

$$= mg\frac{R}{\sqrt{2}} = \frac{1}{2}\frac{m(\sqrt{2}R)^2\omega^2}{3} \quad \Rightarrow \omega^2 = \frac{3g}{\sqrt{2}R}$$

$$\text{Now, } 2N\cos 45° - mg = m\times\frac{3g}{\sqrt{2}R}\times\frac{R}{\sqrt{2}} \Rightarrow N = \frac{5mg}{2\sqrt{2}} = 50$$

CHEMISTRY

21. (A)

3 mole atoms of oxygen are present in 1 mole of $BaCO_3$

So, 1.5 mole atoms of oxygen will be present in

$= \frac{1}{3} \times 1.5 = 0.5$ mole of $BaCO_3$

22. (D)

H_2O is polar, hence it has higher critical temperature.

23. (B)

Let the wavelength of particle = x

So, $\quad$ velocity $= \frac{x}{100}$

$$\lambda = \frac{h}{mv}; \qquad x = \frac{h\times 100}{m\times x}$$

$$x^2 = \frac{100h}{m}; \qquad x = 10\sqrt{\frac{h}{m}}$$

24. (B)

25. (C)

$K_{sp} = S^2$

$S = \sqrt{10^{-8}} = 10^{-4}$ mol L^{-1}

$= 10^{-4} \times 283$ g L^{-1}

$= 2.83 \times 10^{-2}$ g L^{-1}

1000 mL of the solution contains $AgIO_3 = 2.83 \times 10^{-2}$ g

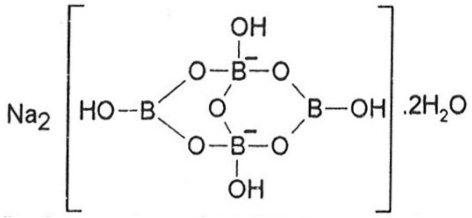
100 mL of the solution contains $AgIO_3 = 2.83 \times 10^{-3}$ g

26. (A)

$$\Delta S_f = \frac{\Delta H_f}{T_f} = \frac{2930}{300} = 9.77\ J\ K^{-1}\ mol^{-1}$$

27. (A)

28. (A)



The boron atoms bearing negative charges are sp^3 hybridized, while other two boron atoms are sp^2 hybridized.

29. (B)

30. (A)
In presence of light (a free radical producing agent), toluene undergoes free radical substitution in the side chain to form benzyl chloride.

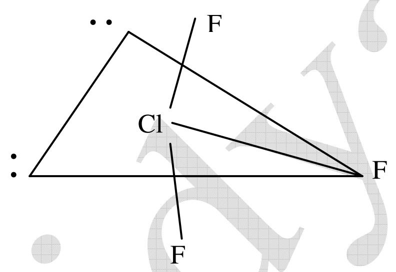
31. (B), (C)

32. (A), (B), (C), (D)
Fact

33. (A), (B), (D)

34. (A), (C), (D)
$Mg(NO_3)_2$ is more covalent and more readily decomposes.
$Mg(NO_3)_2(s) \rightarrow MgO\ (s) + 2NO_2(g) + 1/2O_2(g)$

35. (A), (B), (C)
$BrF_5 + 3H_2O \rightarrow 5HF + HBrO_3$
In ClF_3, Cl is sp^3d hybridized. It is arrow−shaped (also mentioned as T−shaped)



Equatorial Cl−F bond distance is shorter than the other two.

36. [5]

$CH_3CH_2CH_2CH_2CH_2OH$ $\qquad$ $CH_3\overset{\overset{CH_3}{|}}{C}HCH_2CH_2OH$

$C_2H_5\overset{\overset{CH_3}{|}}{C}HCH_2OH$ (dl) $\qquad$ $(CH_3)_3CCH_2OH$

37. [4] **38.** [3] **39.** [6]

40. [2]
$\overset{sp^3}{CH_3}-\overset{sp^3}{CH_2}-\overset{sp}{C}\equiv\overset{sp}{CH}$

MATHEMATICS

41. (A)

Assuming arg $z_1 = \theta_1$ and arg $z_2 = \theta_1 + \theta_2$

$$\frac{\alpha\beta z_1}{\gamma\delta z_2} + \frac{\gamma\delta z_2}{\alpha\beta z_1} = \frac{\alpha\beta|z_1|e^{i\theta_1}}{\gamma\delta|z_2|e^{i(\theta_1+\theta_2)}} + \frac{\gamma\delta|z_2|e^{i(\theta_1+\theta_2)}}{\alpha\beta|z_1|e^{i\theta_1}}$$

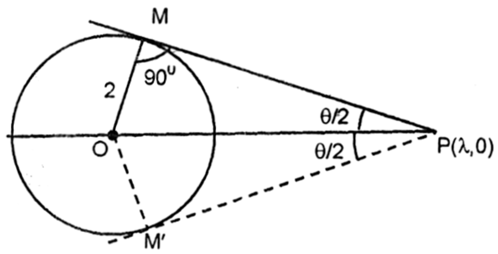
$= e^{-i\theta_2} + e^{i\theta_2} = 2\cos\theta_2$

which lies in [−2, 2]

42. (D)

We have $\frac{\pi}{2} < \theta < \frac{2\pi}{3}$,

i.e. $\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{3}$



$\Rightarrow \frac{1}{\sqrt{2}} < \sin\frac{\theta}{2} < \frac{\sqrt{3}}{2}$

But $\sin\frac{\theta}{2} = \frac{2}{|\lambda|} \Rightarrow \frac{1}{\sqrt{2}} < \frac{2}{|\lambda|} < \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{4}{\sqrt{3}} < |\lambda| < 2\sqrt{2}$

$\Rightarrow \frac{4}{\sqrt{3}} < \lambda < 2\sqrt{2}$ or $-2\sqrt{2} < \lambda < -\frac{4}{\sqrt{3}}$

43. (B)

f (x) is piecewise continuous and decreasing function and the graph of function will cut x−axis at 2008 points. Hence answer is (B).

44. (D)

For real roots $b^2 \ge 4ac$ or $\left(\frac{b}{2}\right)^2 \ge ac$

$\therefore$ Required probability = $\frac{4}{27}$

45. (D)

Case – I : We choose first square from corner square. In this total number of ways of choosing 2 squares = 4×2.

Case – II : We choose first square from first or last row or column except corner one. Total number of ways = 24×3

Case – III : Any square except from boundary one

Total ways = 36×4

Total ways = 8 + 72 + 144 = 224

Now any square can be chosen as first or second

$\therefore$ Required probability = $\frac{112}{^{64}C_2} = \frac{1}{18}$

46. (C)

$\cos^{-1}(\log_3(x^2 + 17x + 75))$ is defined if $-1 \le \log_3(x^2 + 17x + 75) \le 1$.

$\Rightarrow \frac{1}{3} \le x^2 + 17x + 75 \le 3$

Now, $x^2 + 17x + 75 \le 3$

$\Rightarrow x^2 + 17x + 72 \le 0$

$\Rightarrow (x + 8)(x + 9) \le 0$

$\Rightarrow x \in [-9, -8]$

and $x^2 + 17x + 75 \ge \frac{1}{3}$

$\Rightarrow 3x^2 + 51x + 225 - 1 \ge 0$

$\Rightarrow 3x^2 + 51x + 224 \ge 0$

Here $D < 0$

$\Rightarrow 3x^2 + 51x + 224 > 0\ \forall\ x \in R$

Hence $x \in [-9, -8]$.

47. (B)

Let d be the common difference of the A.P. Then $a_{2r} = a_{2r-1} + d$

$$\Rightarrow \sum_{r=1}^{10^{99}} a_{2r} = \sum_{r=1}^{10^{99}} \left(a_{2r-1} + d\right) \Rightarrow \sum_{r=1}^{10^{99}} a_{2r} = \sum_{r=1}^{10^{99}} a_{2r-1} + 10^{99} d$$

$\Rightarrow 10^{100} = 10^{99} + 10^{99} d$

$$\Rightarrow d = \frac{10^{100} - 10^{99}}{10^{99}} = \frac{10^{99}(10-1)}{10^{99}} = 9$$

48. (C)

$\log_{0.09}(x^2 + 2x)$ is defined when $x^2 + 2x > 0$

$\Rightarrow (x)(x + 2) > 0 \Rightarrow x \in (-\infty, -2) \cup (0, \infty)$ ………… (1)

and $\log_{0.3}\sqrt{x+2}$ is defined when $x + 2 > 0$

$\Rightarrow x \in (-2, \infty)$ ………… (2)

Also $\log_{0.09}(x^2 + 2x) \ge \log_{0.3}\sqrt{x+2}$

$\Rightarrow \frac{1}{2} \log_{0.3}(x^2 + 2x) \ge \log_{0.3}\sqrt{x+2}$

$\Rightarrow \log_{0.3}(x^2 + 2x) \ge \log_{0.3}(x + 2)$

$\Rightarrow x^2 + 2x \le x + 2 \Rightarrow x^2 + x - 2 \le 0$

$\Rightarrow x \in [-2, 1]$ ………….. (3)

From (1), (2) and (3) the solution is $x \in (0, 1]$

49. (C)

Given $\alpha + i\beta = \left(\frac{-1+i\sqrt{3}}{2}\right)^{3n_1/4} (1-i)^{-2n_2}$

Here $\frac{-1+i\sqrt{3}}{2} = e^{\frac{i2\pi}{3}}$ and $1 - i = \sqrt{2}e^{\frac{-i\pi}{4}}$

Now, $\alpha + i\beta = e^{\frac{i\pi n_1}{2}} \cdot \left(\sqrt{2}\right)^{-2n_2} e^{\frac{i\pi}{2}n_2}$

$\Rightarrow \alpha + i\beta = 2^{-n_2} e^{i(n_1+n_2)\pi/2}$

$\Rightarrow\ \alpha = 2^{-2n_2}\cos\left(n_1+n_2\right)\dfrac{\pi}{2} = 0$, when $n_1 + n_2$ is odd

$\beta = 2^{-n_2}\sin\left(n_1+n_2\right)\dfrac{\pi}{2} = 0$, when $n_1 + n_2$ is even

Thus (C) is false.

50. (B)

The form being 0/0, the required limit by L' Hospital's rule is

$$\lim_{x\to 0}\frac{2x}{2xf'\left(x^2\right)-360xf'\left(9x^2\right)+396xf'\left(99x^2\right)} = \lim_{x\to 0}\frac{1}{f'\left(x^2\right)-180f'\left(9x^2\right)+198f'\left(99x^2\right)}$$

for f (x) = 0; the slope of tangent is $f'(x)$ and slope of normal at x = 0 is $-\dfrac{1}{f'(0)} = 1$ (given)

$\Rightarrow\ f'(0) = -1$

Limit = $\dfrac{1}{(1-180+198).f'(0)} = -\dfrac{1}{19}$

51. (A), (B)

$f(x + y) + f(x - y) = 2f(x)\cdot f(y)$

Put x = 0, $\Rightarrow f(y) + f(-y) = 2f(0)\cdot f(y)$

Put x = y = 0

$\Rightarrow\ f(0) = 1\ (\because\ f(0) \neq 0))$

$\Rightarrow\ f(-y) = f(y)$

f is even $\Rightarrow f(-2) = f(2) = a$

52. (A), (C), (D)

We have

${}^{100}C_{50} = \dfrac{(100)!}{(50)!(50)!} = 2^{\alpha}, 3^{\beta}, 5^{\gamma}, 7^{\delta}$,……. where $\alpha, \beta, \gamma, \delta$……. are non−negative integers.

Exponent of 2 in (100)! is = $\left[\dfrac{100}{2}\right]+\left[\dfrac{100}{2^2}\right]+\left[\dfrac{100}{2^3}\right]+\left[\dfrac{100}{2^4}\right]+\left[\dfrac{100}{2^5}\right]+\left[\dfrac{100}{2^6}\right]$

= 50 + 25 + 12 + 6 + 3 + 1 = 97

Exponent of 2 in (50)! is = $\left[\dfrac{50}{2}\right]+\left[\dfrac{50}{2^2}\right]+\left[\dfrac{50}{2^3}\right]+\left[\dfrac{50}{2^4}\right]+\left[\dfrac{50}{2^5}\right]$

= 25 + 12 + 6 + 3 + 1 = 47

Exponent of 2 in ${}^{100}C_{50}$ is = 3 $\left\{\because\ {}^{100}C_{50} = \dfrac{2^{97}}{2^{47}\cdot 2^{47}}\ I = 2^3\cdot I\right\}$

In the similar way exponent of 3, 5 and 7 in ${}^{100}C_{50}$ are 4, 0 and 0 respectively.

$\therefore\ {}^{100}C_{50} = 2^3\cdot 3^4\cdot 5^0\cdot 7^0$ ……………

$\Rightarrow\ \alpha = 3, \beta = 4, \gamma = 0, \delta = 0$

53. (B), (C), (D)

Let $\dfrac{3x}{\pi} = t$, $\quad f(t) = [2t]\cos[t]$

$x \in \left[\frac{\pi}{6}, \pi\right] \Rightarrow t \in \left[\frac{1}{2}, 3\right]$

$\therefore\ t = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3$

$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$

54. (A), (B), (C)
$a^{1/3} + b^{1/3} = (a + b)^{1/3}$ is true
when either a = 0 or b = 0 or a + b = 0
$\Rightarrow\ x = 0, \frac{3}{2}, 1.$

55. (B), (C)
Let O be the circumcentre. Then OP + OR ≥ PR ≥ AD = 1, so the radius is at least 1/2. P, Q, R always lie inside or on the circle through A, B, C, D which has radius $1/\sqrt{2}$, so the radius is at most $1/\sqrt{2}$.

56. [9]
$\log_{1/3}(\log_{1/2}y) = \log_3(\log_{1/2}y) \times \log_{1/3}3$ also $\log_{1/2}y = \log_2 y \times \log_{1/2}2$

$\Rightarrow\ \log_3(\log_2 x) - \log_3(-\log_2 y) = 1 \Rightarrow \log_3\left(-\frac{\log_2 x}{\log_2 y}\right) = 1$

$-\frac{\log_2 x}{\log_2 y} = 3 \Rightarrow xy^3 = 1$ also, $xy^2 = 9 \Rightarrow y = \frac{1}{9}\ \therefore\ x = 729.$

57. [5]
Suppose the axes are rotated in the anti−clockwise direction through an angle α.

The equation of the line L with respect to the old axes is given by $\frac{x}{1} + \frac{y}{1/2} = 1.$

To find the equation of L with respect to the new axes, replace x by (x cos α − y sin α) and y by (x sin α + y cos α).

$\therefore$ equation is $(x\cos\alpha - y\sin\alpha) + \frac{1}{1/2}(x\sin\alpha + y\cos\alpha) = 1$

Since p, q are the intercepts made by this line on co−ordinate axes, we have on putting (p, 0) and then (0, q) $\Rightarrow \frac{1}{p} = 1\ \cos\alpha + \frac{1}{1/2}\sin\alpha$ and $\frac{1}{q} = -(1)\sin\alpha + \frac{1}{1/2}\cos\alpha.$

Eliminate α.

Squaring and adding, we get $\frac{1}{p^2} + \frac{1}{q^2} = 1 + \frac{1}{(1/2)^2} = 5$

58. [0]
$(1 + \cot^2 A)\cot^2 A - (1 + \tan^2 A)\tan^2 A - (\cot^2 A - \tan^2 A)[(1 + \tan^2 A)(1 + \cot^2 A) - 1]$
$= \cot^2 A + \cot^4 A - \tan^2 A - \tan^4 A - (\cot^2 A - \tan^2 A)(\cot^2 A + \tan^2 A + 1)$
$= \cot^2 A + \cot^4 A - \tan^2 A - \tan^4 A - (\cot^2 A - \tan^2 A) - (\cot^4 A - \tan^4 A) = 0$

59. [2]

$$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} 0 \Rightarrow \begin{vmatrix} p & b & c \\ a-p & q-b & 0 \\ 0 & b-q & r-c \end{vmatrix} = 0$$

$R_3 \to R_3 - R_2$

$R_2 \to R_2 - R_1$

$$\begin{vmatrix} \frac{p}{p-a} & \frac{b}{q-b} & \frac{c}{r-c} \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix} = 0 \Rightarrow \frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c} = 0$$

$$\Rightarrow \frac{p}{p-a} + \left(\frac{q}{q-b} - 1\right) + \left(\frac{r}{r-c} - 1\right) = 0 \Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

60. [4]

Using Lagrange's mean value theorem,

for f in [1, 2], $\forall \subset \in (1, 2)$

$$\frac{f(2) - f(1)}{2-1} = f'(c) \le 2$$

$f(2) - 2 \le 2 \Rightarrow f(2) \le 4$ …………….. (1)

Again using Lagrange's mean value theorem,

for f in [2, 4], $\forall$ d $\in$ (2, 4)

$$\frac{f(4) - f(2)}{4-2} = f'(d) \le 2$$

$f(4) - f(2) \le 4$

$4 \le f(2)$ …………….. (2)

$\therefore$ from (1) and (2)

$f(2) = 4$

PAPER – II

PHYSICS

1. (A)

Let m_1 and m_2 be the masses of particles.

By principle of conservation of momentum

$m_1 u_1 + 0 = m_2 v - m_1 v$

By Newton's law

$v + v = -e(0 - u_1)$

$e = 1$

$2v = u_1$

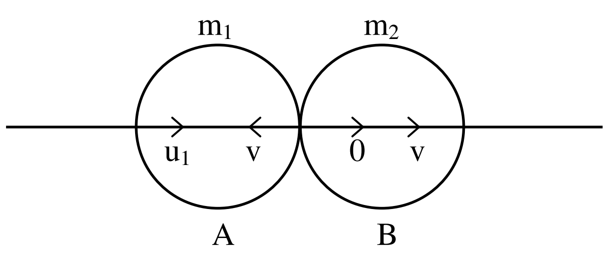
Substituting this in (1)

$2m_1 v = m_2 v - m_1 v$

$2m_1 = m_2 - m_1$

$3m_1 = m_2$

$$\frac{m_1}{m_2} = \frac{1}{3}$$



2. (B)

Frequency = 200 Hz

Velocity = 50 m/sec.

$\therefore$ Wavelength = $\dfrac{50}{200}$ = 0.25 m

The equation for stationary wave.

$y = 2\,A\cos\dfrac{2\pi x}{\lambda}\sin\dfrac{2\pi t}{T}$

$\therefore$ y = 10 cos 8πx sin 400 πt

and y = 5 sin 2π (200 t – 4x)

3. (A)

Applying Newton's second law to the circular orbit.

we have $\dfrac{mv^2}{r} = \dfrac{GMm}{r^2}$ But $v = \dfrac{2\pi r}{T}$

$$m\frac{4\pi^2 r^2}{rT^2} = \frac{GMm}{r^2}; T^2 = \frac{4\pi^2 r^3}{GM}; T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

But $M = \dfrac{4}{3}\pi r^3\rho$ and $r = r_p$

$$\therefore T = \frac{2\pi r_p^{3/2}}{\sqrt{G\cdot\frac{4}{3}\pi r_p^3\rho}} = 2\pi\sqrt{\frac{3}{4\pi Gp}}$$

$$= \sqrt{\frac{3\pi}{G\rho}}$$

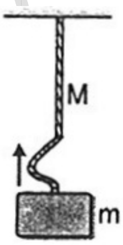
4. (D)

$f = \left(\dfrac{v}{\lambda}\right)$ at lowest point

$f = \dfrac{1}{\lambda}\sqrt{\dfrac{mg}{\mu}}$

$f^1 = \dfrac{v^1}{\lambda^1}$ at highest point

$f^1 = \dfrac{1}{\lambda^1}\sqrt{\dfrac{(M+m)g}{\mu}}$

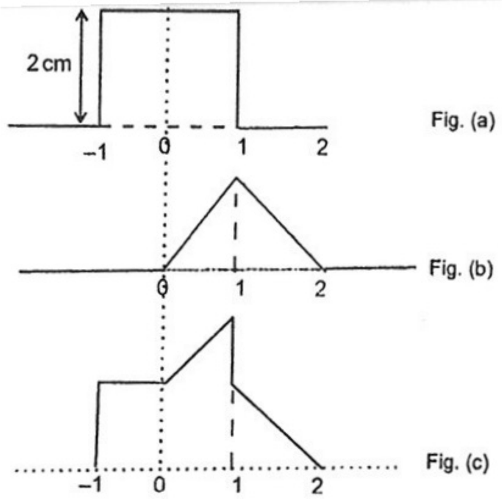


$f^1 = f$: no any change in source freq.

$$\frac{1}{\lambda}\sqrt{\frac{mg}{\mu}} = \frac{1}{\lambda'}\sqrt{\frac{(M+m)g}{\mu}} \Rightarrow \left[\lambda^1 = \lambda\sqrt{\frac{M+m}{m}}\right]$$

5. (D)

At t = 2 second, the position of both pulses are separately given by figure (a) and figure (b); the superposition of both pulses is given by figure (c).



6. (D)

Before heating let the pressure of gas be P_1, from the equilibrium piston,

$PA = kx_1$

$\therefore\ x_1 = \dfrac{PA}{K} = \left(\dfrac{nRT}{V}\right)\dfrac{A}{K} = \dfrac{1\times 8.3\times 100\times 10^{-2}}{0.83\times 100}$

$= 10^{-1} = 0.1$ m

Since during heating process,

The spring is compressed further by 0.1 m

$\therefore\ x_2 = 0.2$ m

work done by gas = $\dfrac{1}{2}.100\left(0.2^2 - 0.1^2\right)$

$= \dfrac{1}{2}$. 100 (0.1) (0.3) = 1.50 = 1.5 J

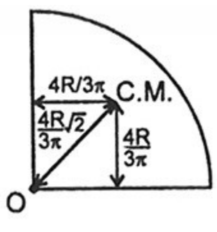
7. (B)

M. I. about 'O' is $\dfrac{MR^2}{2}$

By parallel−axis theorem :

$$\frac{MR^2}{2} = I_{cm} + M\left(\frac{4R}{3\pi}\cdot\sqrt{2}\right)^2$$

$$\Rightarrow\ I_{cm} = \frac{MR^2}{2} - M\left(\sqrt{2}\,\frac{4R}{3\pi}\right)^2$$



8. (A)

$S_1P - S_2P = 0$

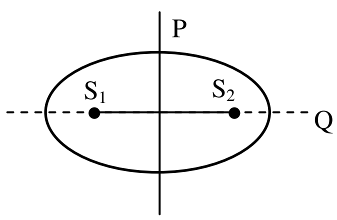
$S_1Q - S_2Q = 4\lambda$

$\therefore$ 3 maximum lie on are between P & Q.

Counting all quadrants; the total number is 12.

P & Q symmetrical points are also maxima.

$\therefore$ Total number is 16



9. (A)

$\dfrac{Q}{A} = 0.06 \times \dfrac{30-25}{1.5} = 0.2$ [From first slab]

$t_1 = 30°C$, $t_2 = 25°C$, t_3, t_4, $t_5 = -10°C$ be temperatures at interfaces.

$\therefore \dfrac{Q}{A} = 0.2 = 0.1 \times \dfrac{t_4+10}{3.5} \Rightarrow t_4 = -3°C$

$\therefore \dfrac{Q}{A} = 0.2 = 0.04 \times \dfrac{t_3+3}{2.8} \Rightarrow t_3 = 11°C$

10. (A)

From problem (9) :

$\dfrac{Q}{A} = 0.2 = K_2 \;\; \dfrac{14}{1.4} \Rightarrow k_2 = 0.02$

11. (C)

For a wire of radius R carrying a current I, the magnetic field at distance r is given by :

$$B = \begin{cases} \dfrac{\mu_0 I}{2\pi R}\cdot\left(\dfrac{r}{R}\right) & 0 \le r \le R \\ \dfrac{\mu_0 I}{2\pi r} & r \ge R \end{cases}$$

Clearly $\left.\begin{array}{l}\text{a \& c carry same currents} \\ \text{b \& d carry same currents}\end{array}\right\}$ (i)

$\left.\begin{array}{l}\text{a \& b have same current densities} \\ \text{c \& d have same current densities}\end{array}\right\}$ (ii)

From condition (ii); $R_a > R_b$ & $R_c > R_d$

From condition (i); $R_c > R_a$

$\therefore$ (C) has largest radius.

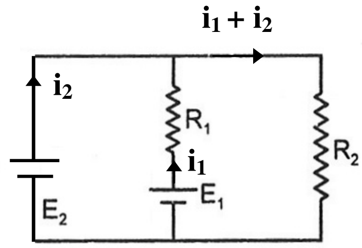
12. (A)

a & c carry same current in graph but radius of 'a' is smaller than radius of c.

$\therefore$ Current density in 'a' is larger.

13. (B) **14.** (D)

Soln. for Q. No. 13 & 14



$i_1 + i_2 = \dfrac{E_2}{R_2}$ ………. (1)

$E_2 = E_1 - i_1 R_1$

$\Rightarrow i_1 = \left(\dfrac{E_1 - E_2}{R_1}\right)$ ………… (2) [Negative slope]

$i_2 = \dfrac{E_2}{R_2} - i_1 = E_2\left[\dfrac{1}{R_1} + \dfrac{1}{R_2}\right] - \dfrac{E_1}{R_1}$ [Positive slope]

$i_1 = 0 \Rightarrow E_1 = E_2$ from Eq. (1) according to graph; $i_1 = 0$ at $E_2 = 6V \;\therefore\; E_1 = 6V$

Slope of i_1 vs E_2 graph $= -\dfrac{1}{R_1} = -\dfrac{0.1}{2} \Rightarrow R_1 = 20\ \Omega$

Slope of i_2 *vs* E_2 graph = $\frac{1}{R_1}+\frac{1}{R_2}=\frac{0.3}{4}=\frac{3}{40} \Rightarrow \frac{1}{R_3}=\frac{3}{40}-\frac{1}{R_1}=\frac{3}{40}-\frac{1}{20}$

$\Rightarrow$ $R_2 = 40\ \Omega$

15. (A), (C)

We know the acceleration of particle executing S.H.M. = $-\omega^2 x$.

Velocity $v^2 = \omega^2 (a^2 - x^2)$

$\therefore u^2 = \omega^2 \left(a^2 - x_1^2\right)$

$v^2 = \omega^2 \left(a^2 - x_2^2\right)$

$\alpha = -\omega^2 x_1$

$\beta = -\omega^2 x_2$

(1) − (2) $\quad u^2 - v^2 = \omega^2 \left(x_2^2 - x_1^2\right)$

(3) + (4) $\quad \alpha + \beta = -\omega^2 (x_1 + x_2)$

Dividing (5) by (6)

$$\frac{u^2 - v^2}{\alpha+\beta} = -\left(x_2 - x_1\right) \qquad \text{so } d = \left|\frac{u^2 - v^2}{\alpha+\beta}\right|$$

Hence when the accelerations are α and β the distance between the particles

$= x_2 - x_1 = \frac{u^2 - v^2}{\alpha+\beta}$

From (3) and (4) we get

$\beta - \alpha = \omega^2 (x_2 - x_1)$

$$\omega^2 = \frac{\beta-\alpha}{\left(x_2 - x_1\right)}$$

$$T = \frac{2\pi}{\omega} = 2\pi \Big/ \sqrt{\frac{\beta-\alpha}{\left(x_2 - x_1\right)}} = 2\pi\sqrt{\frac{\left(x_2 - x_1\right)}{\beta-\alpha}}$$

But $x_2 - x_1 = \frac{u^2 - v^2}{\alpha+\beta}$

$$\therefore T = 2\pi \sqrt{\frac{u^2 - v^2}{(\alpha+\beta)(\beta-\alpha)}} = 2\pi\sqrt{\frac{u^2 - v^2}{\beta^2 - \alpha^2}}$$

16. (A), (C), (D)

For any curve slope of the tangent at the point = $\frac{dy}{dx}$

Here slope $\frac{ds}{dt}$

But $\frac{ds}{dt}$ is the velocity at the point.

$\therefore$ The velocity of particle at any point is given by the slope of the tangent at that point.

i) Between O and A the slope is positive. Hence velocity is positive. As we move from O to A the slope decreases. So the velocity decreases and hence acceleration is negative.

ii) Between A and B slope of tangent is zero at every point. Velocity and acceleration are both zero.

iii) Between C and D the slope of tangent is negative but constant. The velocity is negative and the acceleration is zero.

iv) Between D and E the slope of the tangent is positive and increasing. Hence velocity is positive and increasing. The acceleration is positive.

17. (A), (C)

$f \propto (T)^{1/2} \Rightarrow \dfrac{\Delta f}{f} = \dfrac{1}{2}\dfrac{\Delta T}{T}$

$\therefore\ \dfrac{15\text{Hz}}{f} = \dfrac{1}{2}\times\dfrac{21}{100} \Rightarrow f = \dfrac{30\times100}{21} \Rightarrow f = \dfrac{1000}{7}\text{Hz}$

$\therefore\ f = 143$ Hz

and $v \propto \sqrt{T}$ So $\dfrac{\Delta v}{v} = \dfrac{1}{2}\dfrac{\Delta T}{T} = \dfrac{21}{2} = 10.5\%$

18. (A), (C)

For O_1; observer stationary, source is moving;

$f_1 = \dfrac{V}{V - \dfrac{V}{5}}\cdot f = \dfrac{5f}{4} \qquad \therefore\ \lambda_1 = \dfrac{V}{f_1} = \dfrac{4V}{5f}$

Frequency passed in water is $\dfrac{5f}{4}$, now for observer O_2

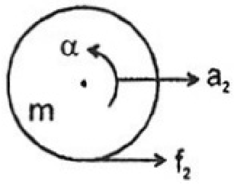
$\therefore\ f_2 = \dfrac{4V + \dfrac{V}{5}}{4V}\cdot\dfrac{5f}{4} = \dfrac{21f}{16}$

$\lambda_2 = \dfrac{4V}{f_2} = \dfrac{4V}{21f}\times16$

19. (A), (B), (C)

20. (B), (C)

Friction on plate due to ground



$f_1 = 7.5 \times 0.2 \times 10 = 15$

$25 - 15 - f_2 = 1.5\, a_1$

$f_2 = 6a_2$

$10 = 1.5\, a_1 + 6a_2$ ………….. (i)

$f_2 \cdot r = mr \cdot \alpha$

$\Rightarrow\ f_2 = ma_2$ ………… (ii)

$f_2 = ma_1 - ma_2$

$a_2 + r\alpha = a_1 \quad \Rightarrow \quad a_1 - a_2 = a_2$

$\Rightarrow\ a_2 = a_1 - a_2 \quad \Rightarrow \quad a_1 = 2a_2$

$10 - 3a_1 = 1.5\, a_1$

$\Rightarrow\ a_1 = \dfrac{100}{45} = \dfrac{20}{9}$

$a_2 = \dfrac{a_1}{2} = \dfrac{20}{18}$

$v_1 = a_1 t = \dfrac{20}{9}\times\dfrac{3}{4} = \dfrac{5}{3}$ (Plate)

$v_2 = a_2 t = \dfrac{20}{18}\times\dfrac{3}{4} = \dfrac{5}{6}$ (ring)

CHEMISTRY

21. (B)
dsp^2 hybridisation gives square planar structure with s, p_x, p_y and $d_{x^2-y^2}$ orbitals involved forming angles of 90°.

22. (B)

$P \times V = \frac{4}{M} \times R \times T$ Case I

$P \times V = \frac{4-0.8}{M} \times R \times (T+50)$ Case II

$\therefore \frac{4}{3.2} \times \frac{T}{(T+50)} = 1$ or $4T = 3.2\,T + 160$

$\therefore 0.8\,T = 160$

or $T = \frac{160}{0.8} = 200$ K

23. (B)
As conversion of X to AB is fast, it means the process has a very low activation energy.

24. (B)

25. (C)

Number of atoms in 1 g ${}^{235}_{92}U = \frac{6.023 \times 10^{23}}{235}$

Energy obtained by fission of one atom = 3.20×10^{-11} J

$\therefore$ Energy obtained by fission of $\frac{6.023 \times 10^{23}}{235}$ atoms

$= \frac{3.20 \times 10^{-11} \times 6.023 \times 10^{23}}{235} = 8.20 \times 10^{10}$ J $= 8.20 \times 10^{7}$ kJ

26. (C)

Energy available for muscular work = $\frac{2880 \times 25}{100} = 720$ kJ mol^{-1}

$\therefore$ Energy available for muscular work for 120 g of glucose

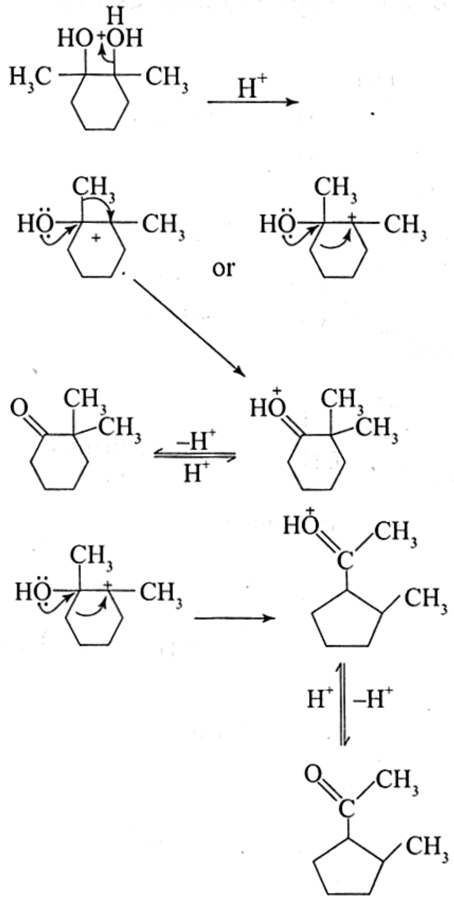
$= 720 \times \frac{120}{180} = 480$ kJ (molecular mass of glucose = 180)

$\because$ 100 kJ of energy is used for walking = 1 km

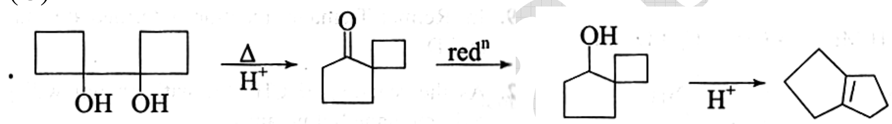
$\therefore$ 480 kJ of energy is used for walking = $1 \times \frac{480}{100} = 4.8$ km

27. (A) **28.** (A) **29.** (C) **30.** (D)

31. (D)



32. (C)



33. (B) **34.** (D) **35.** (B), (C)

36. (A), (B), (C)

Particle size in crystalloids are smaller than that in colloids, crystalloid particles passed through a membrane whereas colloidal particles do not. Crystalloids do not exhibit Tyndall effect whereas colloids exhibit Tyndall effect.

37. (A), (B), (D) **38.** (A), (C)

39. (A), (C), (D)

(A) For exothermic reaction ΔH is −ve.

As $\Delta H = H_p - H_r$

For ΔH to be −ve

$H_r > H_p$

(B) Enthalpy of combustion is always negative

(C) As $\Delta G = \Delta H - T\Delta S$

for a reaction to be feasible at all temperatures

ΔG = −ve which is possible if

ΔH = −ve and ΔS = +ve

(D) for the reaction

$$C_{(s)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{(g)}$$

$$\Delta n = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore \ \Delta H = \Delta E + \frac{1}{2}RT$$

$$\therefore \ \Delta H > \Delta E$$

40. (A), (B)

When the dispersion medium is gas, the colloidal system is called aerosol, smoke, dust are examples of aerosols of solids whereas fog, clouds are examples of aerosol of liquids.

MATHEMATICS

41. (C)

$\alpha - p > 0$, $\beta - p > 0$ and $D \ge 0$

$\Rightarrow \ \alpha + \beta - 2p > 0$, $\alpha\beta - p(\alpha + \beta) + p^2 > 0$ and $p^2 + 4p^2 \ge 0$

$\therefore \ p - 2p > 0$, $-p^2 - p \cdot p + p^2 > 0 \qquad \Rightarrow \ p < 0$, $p^2 < 0$ (absurd).

42. (D)

$$a \sin\theta = b\cos\theta = \frac{2c\tan\theta}{1-\tan^2\theta} \qquad \text{where, } \theta = x^2$$

$$\tan\theta = \frac{b}{a} \qquad \text{and} \qquad \frac{b}{\sec\theta} = \frac{2c\tan\theta}{1-\tan^2\theta}$$

Squaring both sides we get,

$$b^2(1+\tan^4\theta - 2\tan^2\theta) = (4c^2\tan^2\theta)(1+\tan^2\theta)$$

$$b^2\left(1+\frac{b^4}{a^4}-2\frac{b^2}{a^2}\right) = 4\frac{c^2b^2}{a^2}\cdot\left(1+\frac{b^2}{a^2}\right)$$

$$b^2\left(\frac{a^4+b^4-2a^2b^2}{a^4}\right) = \frac{4c^2b^2(a^2+b^2)}{a^4} \quad \text{or} \quad (a^2-b^2)^2 = 4c^2(a^2+b^2)$$

43. (A)

$$a\left(\tan A - \tan\frac{A+B}{2}\right) + b\left(\tan B - \tan\frac{A+B}{2}\right) = 0$$

$$\text{or,} \quad a\left(\frac{\sin A}{\cos A} - \frac{\sin\frac{A+B}{2}}{\cos\frac{A+B}{2}}\right) + b\left(\frac{\sin B}{\cos B} - \frac{\sin\frac{A+B}{2}}{\cos\frac{A+B}{2}}\right) = 0$$

$$\text{or,} \quad a \cdot \frac{\sin\left(\frac{A-B}{2}\right)}{\cos A\cos\frac{A+B}{2}} - b \cdot \frac{\sin\left(\frac{A-B}{2}\right)}{\cos B\ \cos\frac{A+B}{2}} = 0$$

$$\text{or,} \quad a \cdot \frac{\sin\left(\frac{A-B}{2}\right)}{\cos A} = b \cdot \frac{\sin\left(\frac{A-B}{2}\right)}{\cos B}$$

Either, $\sin\frac{A-B}{2} = 0 \Rightarrow A = B$

or, $\frac{a}{\cos A} = \frac{b}{\cos B}$ or, $\frac{a.2bc}{b^2+c^2-a^2} = \frac{2ac.b}{a^2+c^2-b^2}$

or, $b^2 = a^2$

$\therefore$ $a = b$

$\therefore$ The Δ is isoceles.

44. (A)

$\sin(\alpha+\beta) = 1 \Rightarrow \alpha+\beta = \frac{\pi}{2}$

$\sin(\alpha-\beta) = \frac{1}{2} \Rightarrow \alpha-\beta = \frac{\pi}{6}$

$\alpha = \frac{\pi}{3}, \quad \beta = \frac{\pi}{6}$

Now, $\tan(\alpha+2\beta)\tan(2\alpha+\beta) = \tan\left(\frac{2\pi}{3}\right)\tan\left(\frac{5\pi}{6}\right) = \left(-\cot\frac{\pi}{6}\right)\left(-\cot\frac{\pi}{3}\right) = 1$

45. (C)

$$\cos\left[\frac{1}{2}\cos^{-1}\left(\cos\left(-\frac{14\pi}{5}\right)\right)\right] = \cos\left[\frac{1}{2}\cos^{-1}\left(-\cos\frac{\pi}{5}\right)\right]$$

$$= \cos\left(\frac{\pi}{2}-\frac{\pi}{10}\right) = \sin\frac{\pi}{10} = \cos\left(\frac{\pi}{2}-\frac{\pi}{10}\right) = \cos\frac{2\pi}{5}$$

46. (C)

See the figure since, $OP \perp PY$

$\therefore$ $\angle APY = 90° - \theta$, where $\angle OPA = \theta$

$\therefore$ $\angle PAY = \theta$

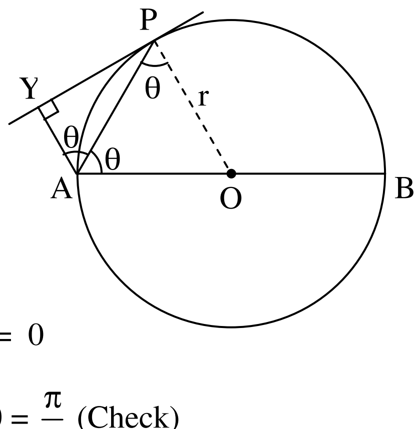
Now, in ΔOPA,

$AP^2 = r^2 + r^2 - 2 . r . r\cos(\pi - 2\theta) = 4r^2\cos^2\theta$

$\therefore$ $AP = 2r\cos\theta$

$\Rightarrow$ $PY = AP\sin\theta = r\sin 2\theta$

and $AY = AP\cos\theta = 2r\cos^2\theta$



$\therefore$ Area of ΔAPY, $\Delta = \frac{1}{2}.PY.AY = r^2\sin 2\theta\cos^2\theta$

$$\frac{d\Delta}{d\theta} = r^2[2\cos 2\theta\cos^2\theta - \sin^2 2\theta] = 0$$

$\Rightarrow \theta = \frac{\pi}{2}, \frac{\pi}{6}$ $\quad \because \theta \ne \frac{\pi}{2}$. Also, Δ is maximum at $\theta = \frac{\pi}{6}$ (Check)

$\therefore$ $\Delta_{max} = r^2\frac{\sqrt{3}}{2}.\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3\sqrt{3}r^2}{8}$

47. (B)

We have, $e^{-\pi/2} < \theta < \pi/2$

$\Rightarrow -\frac{\pi}{2}\log e < \log\theta < \log\frac{\pi}{2}$

Now, $\log_e e = 1$ and $\dfrac{\pi}{2} < e$, $\therefore \log\dfrac{\pi}{2} < \log e = 1$

$\therefore -\dfrac{\pi}{2} < \log\theta < 1 < \dfrac{\pi}{2}$

$\Rightarrow$ log θ lies in 1st and 4th quadrant

$\therefore$ cos (log θ) is positive …… (1)

Now, $0 < \cos\theta < 1 \Rightarrow \log\cos\theta < \log 1 = 0$

$\Rightarrow$ log cos θ is negative …… (2)

From (1) and (2), we conclude cos log θ > log cos θ

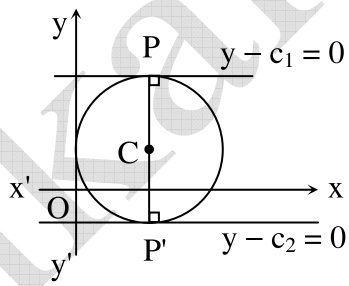
48. (B)

Let the lines be $y = c_1$ and $y = c_2$ [$\because$ the lines are parallel to x−axis]

From the equation of circle, $x^2 + y^2 - 6x - 4y - 12 = 0$

centre (3, 2) and radius = 5

The perpendicular drawn from centre to the lines are CP and CP'



$$CP = \frac{2 - c_1}{\sqrt{1}} = \pm 5 \quad \Rightarrow \quad 2 - c_1 = \pm 5$$

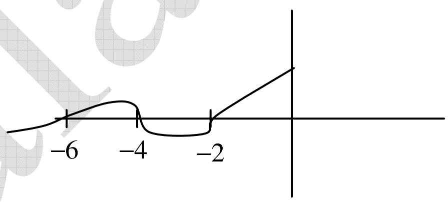
$\therefore$ $c_1 = 7$ and $c_1 = -3$

Hence the lines are $y - 7 = 0$ and $y + 3 = 0$

$\therefore$ Pair of lines is $(y - 7)(y + 3) = 0$, i.e., $y^2 - 4y - 21 = 0$.

49. (D)

$$\frac{(]x[+2)(]x[+4)}{(]x[+6)} > 0$$



$\Rightarrow$ $-6 <]x[< -4$ or $]x[> -2$

$\therefore$ $]x[= -5, -1, 0$

$\therefore$ $x = -5, -1, 5, 1, 0$

50. (B)

$$-\left|x + \frac{1}{x}\right| = \left|x + \frac{1}{x}\right| \quad \Rightarrow \quad 2\left|x + \frac{1}{x}\right| = 0$$

which is not possible, as $\left|x + \dfrac{1}{x}\right| \ge 2$

51. (B)

$g(x) \equiv f(x) - x^3 + 1 = 0$ has at least 3 roots in [1, 4]

$\Rightarrow$ $f'(x) = 3x^2$ has at least 1 root each in (1, 2) and (2, 4).

52. (C)

f(x) − x retains it's sign, so consider f(x) − x > 0.

$\Rightarrow f^2(x) - f(x) > 0$

$f^3(x) - f^2(x) > 0$

:

:

$f^n(x) - f^{n-1}(x) > 0 \quad \Rightarrow \quad f^n(x) - x > 0$

53. (D)

$$P_k = \frac{1-x^{k+1}}{1-x}$$

$$P_1 P_2 P_3.....P_n = \frac{(1-x^2)(1-x^3)(1-x^4).....(1-x^{n+1})}{(1-x)^n}$$

No. of terms $= 1 + \text{max. power of x}$

$$= 1+\frac{n(n+1)}{2} = \frac{n^2+n+2}{2}$$

54. (D)

$$(1+x)(1+x+x^2)....(1+x+.....+x^n) = a_0 + a_1x + a_2x^2 + a_3x^3 + ...$$

Put, x = 1,

$$2\times3\times4\times.....(n+1) = a_0 + a_1 + a_2 + a_3 + ...$$

Put x = −1,

$$0 = a_0 - a_1 + a_2 - a_3 + ...$$

$$(n+1)! = 2[a_0 + a_2 + ...] \qquad \Rightarrow a_0 + a_2 + ... = \frac{(n+1)!}{2}$$

55. (B),(C),(D)

(A) f (0) $\rightarrow$ undefined

$$f(0^-) = \lim_{h\to0} \frac{1}{1+2^{-\cot h}} = 1$$

$$f(0^+) = \lim_{h\to0} \frac{1}{1+2^{\cot h}} = 0$$

$\Rightarrow$ non-removable discontinuity at x = 0.

(B) f (0) $\rightarrow$ not defined

$$f(0^-) = \lim_{h\to0} \cos\left(\frac{\sin h}{-h}\right) = \cos 1$$

$$f(0^+) = \lim_{h\to0} \cos\left(\frac{\sin h}{h}\right) = \cos 1$$

$\Rightarrow$ removable discontinuity at x = 0.

(C) f (0) $\rightarrow$ not defined

$$f(0^-) = \lim_{h\to0} h\,.\,\sin\frac{\pi}{h} = 0$$

$$f(0^+) = \lim_{h\to0} h.\sin\frac{\pi}{h} = 0$$

$\Rightarrow$ removable discontinuity at x = 0.

(D) $$f(0^-) = \lim_{h\to0} \frac{1}{\ln h} = 0$$

$$f(0^+) = \lim_{h\to0} \frac{1}{\ln h} = 0$$

$\Rightarrow$ removable discontinuity at x = 0.

56. (A), (B), (C)

Let the roots be α, β, γ

$$\sum \alpha = -a$$

$$|a| = \left|\sum \alpha\right| \le \sum |\alpha| = 3$$

$$b = \sum \alpha\beta$$

$$|b| = \left|\sum \alpha\beta\right| \le \sum |\alpha||\beta| = 3$$

$$c = \alpha\beta\gamma \quad \Rightarrow \quad |c| = \Pi|\alpha| = 1$$

57. (A),(B),(C),(D)

We have, $f(x) = |\,[x]\,x\,|$ in $-1 < x \le 2$

$$\Rightarrow \; f(x) = \begin{cases} -x, & -1 < x < 0 \\ 0, & 0 \le x < 1 \\ x, & 1 \le x < 2 \\ 2x, & x = 2 \end{cases}$$

It is evident from the graph of this function that it is continuous but not differentiable at $x = 0$. Also, it is discontinuous at $x = 1$ and non−differentiable at $x = 2$.

58. (A), (B), (C), (D)

We have, $\dfrac{P-Q}{p-q} = \dfrac{Q-R}{q-r} = \dfrac{R-P}{r-p} = d$ = common difference.

$\therefore$ $(q - r)(P - Q) = (p - q)(Q - R)$

$qP - qQ - rP + rQ = pQ - pR - qQ + qR$

$\therefore$ $pQ + qR + rP = pR + rQ + qP$ ……(A)

Simplifying, $\begin{vmatrix} P & Q & R \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$

$= P(q - r) - Q(p - r) + R(p - q)$ ……(B)

$= 0$ from (A)

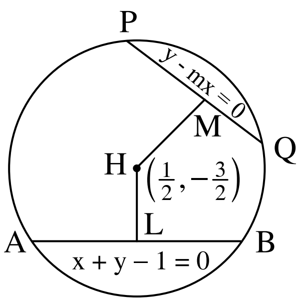
(B) can be written as $\Sigma P(q - r)$

$\therefore$ $\Sigma P(q - r) = 0$ ……(C)

(B) readjusting,

$\Sigma P(q - r) = \Sigma p(Q - R) = 0.$ ……(D)

59. (A), (B)



Let H be the centre of circle.

$$H \equiv \left(\frac{1}{2}, -\frac{3}{2}\right) \text{ and radius} = \sqrt{\frac{1}{4} + \frac{9}{4}} = \sqrt{\frac{5}{2}}$$

Given AB = PQ

$\Rightarrow$ HL = HM

$$\Rightarrow \quad \left|\frac{\frac{1}{2} - \frac{3}{2} - 1}{\sqrt{2}}\right| = \left|\frac{-\frac{3}{2} - \frac{m}{2}}{\sqrt{1+m^2}}\right|$$

$$\Rightarrow \quad \sqrt{2} = \frac{3+m}{2\sqrt{1+m^2}} \quad \Rightarrow \; 8(1 + m^2) = 9 + m^2 + 6m$$

$\Rightarrow$ $7m^2 - 6m - 1 = 0$

$\Rightarrow$ $m = \dfrac{6 \pm \sqrt{36+28}}{14} = 1, -\dfrac{1}{7}$

$\therefore$ Equation of L_1 can be

$y - x = 0$ or $y + \dfrac{1}{7}x = 0$

i.e., $x + 7y = 0$

$\therefore$ correct option is (A) or (B)

60. (A),(C)

Since $f'(x) > 0$ $\Rightarrow$ f (x) is increasing function

and $g'(x) < 0$ $\Rightarrow$ g (x) is decreasing function.

Since $x + 1 > x$

$\therefore$ $g(x+1) < g(x)$ ($\because$ g is decreasing)

$\therefore$ $f\{g(x+1)\} < f\{g(x)\}$ {$\because$ f is increasing}

or $f\{g(x)\} > f\{g(x+1)\}$

Alternate (A) is correct.

Now : $x + 1 > x$

$f(x+1) > f(x)$ {$\because$ f is increasing}

$\Rightarrow$ $g\{f(x+1)\} < g\{f(x)\}$ {$\because$ g is decreasing}

or $g\{f(x)\} > g\{f(x+1)\}$

Alternate (C) is correct.

Now replacing x by $x - 1$ in alternate (A) and (C) then

$f\{g(x-1)\} > f\{g(x)\}$

and $g\{f(x-1)\} > g\{f(x)\}$

Hence alternates (B) and (D) are wrong.

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