Max. Marks : 432 **Duration : 3 hrs**

Instruction

All questions have single answer correct

Chemistry : Question No. 1 to 24 carry +4 marks for each correct answer and -1 for wrong answer

Qn. 25 to 30 carry +8 marks for each correct answer and -2 for wrong answer

Mathematics : Question to 31 to 54 carry +4 marks for each correct answer and -1 for wrong answer

Qn. 55 to 60 carry +8 marks for each correct answer and -2 for wrong answer

Physics: Question No. 61 to 84 carry +4 marks for each correct answer and -1 for wrong answer

Qn. 85 to 90 carry +8 marks for each correct answer and -2 for wrong answer

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Chemistry

1. The vapour pressure of a solution containing 6g of a solute dissolved in 50g of water at 300 k is 3.5×10^3 pa . Calculate the molar mass of the solute if the vapor pressure of water at 300 k is 3.5×10^3 pa

(a) 4g (b) 40g (c) 80 g (d) 45g

Sol. $\frac{p_1^0 - p_1}{p_1^0} = \frac{W_2}{W_1} \frac{M_1}{M_2}$ $\Rightarrow M_2 = \frac{6\times0.018}{50}\times\frac{3.7\times10^3}{0.2\times10^3}$ = = 40 g/mol (approximately)

2. A single electron system has ionization energy 11137 kJ mol^{-1}. Find the number of protons in the nucleus of the system.

(a) 2 (b) 4 (c) 3 (d) 1

Sol. I.E. = $\frac{Z^2}{n^2} \times 2.17 \times 10^{-18}$ J

$= \frac{11137\times10^3}{6\times10^{23}} = \frac{Z^2}{1^2} \times 2.17 \times 10^{-18}$

Ans. Z = 3

3. Order of acidic proton is

(I) CH_3COCH_3 (II) $(CH_3)_2C{=}CH_2$

(III) $CH_3 - CO - CH_2 - CO - CH_3$ (IV) $(CH_3{-}CO)_3CH$

4. The weight % of free SO_3 in oleum that is labelled as 109% H_2SO_4 by weight.

(a) 2 (b) 4 (c) 3 (d) 1

Sol. 109% H_2SO_4 means 9 g H_2O will combine with all the free SO_3 in 100 g of the oleum to give a total of 109 g H_2SO_4

$\therefore H_2O + SO_3 \rightarrow H_2SO_4$ 9g or 0.5 mole

$\therefore SO_3$ present is also 0.5 mole or 0.5 × 80 = 40 g or

using the formula % of free SO_3 $\frac{80\ (y-100)}{18}$ =; here (y-100) = 9 => $\frac{80\times 9}{18}$ =**40**

5. $CH_3 - CH_3$, $CH_2 = CH_2$ and $CH \equiv CH$ can be distinguished in the laboratory by the use of :

(a) Only Br_2 water (b) Only Baeyer reagent

(c) Only Cu_2Cl_2 / NH_4OH (d) Br_2 water and Cu_2Cl_2 / NH_4OH

6. Consider the equilibrium $LiCl.3NH_{3\,(s)} \rightleftharpoons LiCl.\ NH_{3(s)} + 2NH_{3\,(g)}$ with $K_p = 9(atm)^2$ at 40°C. A 5 litre flask contains 0.1 mole of $LiClNH_3$. How many moles of NH_3 should be added to the flask at this temperature to drive the backward reaction practically to completion?

(a) 0.25 (b) 0.40 (c) 0.371 (d) 0.784

Sol. K_p = = 9 atm^2 $\Rightarrow$ at equilibrium = 3 atm

Let x moles of NH_3 be added

$$LiCl.3NH_{3\,(s)} \rightleftharpoons LiCl.NH_{3(s)} + 2NH_{3(g)}$$

Initial moles 0 0.1 x

Final moles 0.1 0 x - 0.1

under the final condition, x – .0.2 moles of NH_3 should exert a pressure of 3 atom so that $LiCl.3NH_3$ does not decompose.

$x - 0.2 = \frac{3\times 5}{0.082\times 313} \Rightarrow x = 0.784$ moles.

7. K_{sp} of the salt $Li_3Na_3[AlF_6]_2$

(a) 172 S^6 (b) 125 S^4 (c) 162 S^7 (d) 2916 S^8

Sol. $A_xB_y \rightarrow xA^{y+} + yB^{-x}$

– xs ys $\therefore k_{SP} = (xs)^x\ (ys)^y = x^x.y^y.(s)^{x+y}$

$k_{SP} = 3^3.3^3.2^2\ (s)^8 = 3^6.4.(s)^8 = 2916\ s^8$

8. Silver crystallizes in cubic lattice. The density is found to be 10.7 kg/dm^3. If the unit cell length is 4.06Å, The effective number of atoms per unit cell.

(a) 2 (b)1 (c) 4 (d) 3

Sol. $Z = d \times N_0 \times a^3/M$

$= 10.7 \times 10^3 \times 6.023 \times 10^{23} \times (4.06 \times 10^{-10})^3/(108 \times 10^{-3}) = 4$

Thus there are 4 atoms per unit cell. Hence it is FCC

9. A metal M and its compound can given the following observable changes in a sequence of reactions

$$M \xrightarrow[HNO_3]{dilute} X \text{ colourless solution } \xrightarrow[NaOH]{aqueous} Y_{white} \text{ ppt } \xrightarrow[NaOH(aq.)]{excess} Z_{colourless} \xrightarrow[NaOH\ (aq.)]{H_2S}$$ white ppt. The metal M can be

(a) Mg (b) Pb ✓(c) Zn (d) Sn

Sol. $Zn(NO_3)_2 + 2NaOH \rightarrow Zn(OH)_2 \downarrow + 2\ NaNO_3$

$Zn(OH)_2 + 2NaOH \rightarrow Na_2ZnO_2 + 2H_2O$

10. 10^{-2} mole of NaOH was added to 10 litre of water. The pH will change by

✓(a) 4 (b) 3 (c) 11 (d) 7

Sol. Initially pH = 7

finally $[OH]^- = 10^{-3}$ so pOH = 3 pH = 11 so change in (pH) = 4

11. 2-Methylpropene is isomeric with 1-butene , they can be distinguished by

(a) Cu_2Cl_2 / NH_4OH (b) Baeyer reagent

(c) Bromine water ✓(d) Ozonolysis

Sol. d

12. Ammonia can be dried by

(a) Conc. H_2SO_4 (b) P_4O_{10} ✓(c) CaO (d) Anhydrous $CaCl_2$

Sol. Rest all react with ammonia except quicklime(CaO)

13. Acid catalysed hydration of alkenes except ethene leads to the formation of

(a) 1^0 alcohol (b) 2^0 or 3^0 alcohols

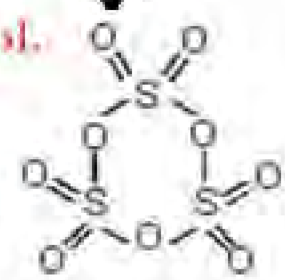
(c) mixture of 1^0 and 2^0 alcohols ✓(d) mixture of 2^0 and 3^0 alcohols

Sol. D

14. In cyclic trimmer structure of sulphur trioxide no of S–o–S bonds.

✓(a) 3 (b) 2 (c) 0 (d) 1

Sol.



15. Identify the correct statement :

(a) BF_3 bond order is 1.33

✓(b) Hydrated $AlCl_3$ is an acidic compound

(c) $AlCl_3$ exists as dimmer with two dative bonds

(d) All of these

Sol. (A) pπ – pπ bonding, resonance

(B) $[Al(H_2O)_6]^{3+} \rightarrow [Al(H_2O)_5]^{2+} + H^+$

(C)

16. $PbO_2 + HCl_{(conc.)} \rightarrow$ [A] gas ; $PbO_2 + HNO_{3\,(conc.)} \rightarrow$ [B] gas

The gases [A] and [B] are respectively :

(a) Cl_2 and O_2 ✔ (b) both are O_2

(c) Cl_2 and NO_2 (d) O_2 & N_2O

Sol. $PbO_2 + 4HCl\ PbCl_2 + Cl_2 + 2H_2O$

$PbO_2 + 2HNO_3 \rightarrow Pb(NO_3)_2 + H_2O + 1/2O_2$

17. The gas (X) bleaches the coloured matter by reduction temporarily while the gas (Y) by oxidation permanently. The gases are respectively :

(a) CO_2 and Cl_2 (b) O_3 and Cl_2

(c) H_2S and CO_2 (d) $SO_2$and CO_2 ✔

Sol. $SO_2 + 2H_2O \rightarrow H_2SO_4 + 2H$

Hence bleaching is through reduction and

$Cl_2 + H_2O \rightarrow 2HCl + [O]$ bleaching is through oxidation

18. The weakest base among the following is

(a) $C_6H_5NH_2$ ✔ (b) $(CH_3)_3N$ (c) NH_3 (d) $(C_2H_5)_2NH$

Sol. Due to resonance and –I effect of C_6H_5- group

19. Match column (I) with column (II) and select the correct answer using codes given below in the lists.

Column- I	Column- II
(i) Cyanide process	(a) Extraction of Al
(ii) Self reduction	(b) Extraction of Ag
(iii) Electrolytic reduction	(c) Extraction of Pb
(iv) Carbon reduction	(d) Extraction of Sn

(a) (i) - (b) , (ii) - (c), (iii) - (a), (iv) - (d) ✔

(b) (i) - (b) , (ii) - (d), (iii) - (a), (iv) - (c)

(c) (i) - (d) , (ii) - (a), (iii) - (c), (iv) - (b)

(d) (i) - (c) , (ii) - (b), (iii) - (d), (iv) - (a)

Sol. [A]

Cyanide process → Ag; Self reduction → Pb; Electrolytic reduction → Al, Carbon reduction → Sn.

20. The ion that can be precipitated by H_2S but not by HCl is

(a) Pb^{2+} (b) Cu^{+} (c) Ag^{+} (d) Sn^{2+} ✔

Sol. (A) (B) (C) can be precipitated as insoluble chlorides and sulphides respectively.

(D) $Sn^{2+} + 2HCl\ \ SnCl_{2\,(soluble)} + 2H^{+}$. It can be precipitated only by H_2S as its sulphide (brown).

21. The reddish brown precipitate formed by NH_3 with Nesler's reagent is

(a) $HgINH_2$ (b)HgI_2 (c) $HgIONH_2$ (d) $HgOHgNH_2$ ✔

Sol. $2K_2(HgI_4) + NH_3 + 3KOH\ \ HgOHgNH_2I_{(C)} + 7KI + 2H_2O$

22. The pair of amphoteric hydroxides is :

(a) $Be(OH)_2$, $Zn(OH)_2$ ✓ (b) $Al(OH)_3$, LiOH

(c) $B(OH)_3$, $Be(OH)_2$ (d) $Be(OH)_2$, $Mg(OH)_2$

Sol. Amphoteric hydroxides are $Be(OH)_2$, $Al(OH)_3$, $Zn(OH)_2$

23. When KO_2 is added to water, the solution is basic because it contains a significant concentration of

(a) O_2^{2-} (b) O^{3-} (c) OH^- ✓ (d) K^+

Sol. $2KO_2 + 2H_2O \rightarrow 2KOH + H_2O_2 + O_2$

24. Which of the following pair of complexes have the same EAN of the central metal atoms/ ions?

(a) $[Cu(NH_3)_4] SO_4$ and $K_3 [Fe(CN)_6]$

(b) $K_3 [Co(en)_3]$ and $[Ni(CO)_4]$

(c) $K_4 [Fe(CN)_6]$ and $[Co(H_2O)_6] Cl_3$

(d) All of these ✓

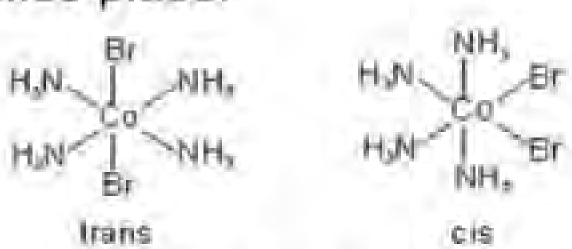
Sol. Ans ;d

25. The complex $[Co(NH_3)_4 Br_2] Cl$ and $[Co(NH_3)_4 BrCl] Br$ can show isomerism :

(a) Ionisation and geometrical ✓ (b) Geometrical and optical

(c) Ionisation and optical (d) only optical

Sol. Exchange of ions between coordination sphere and ionisation sphere takes place.



No optical isomersim, as there are more than two identical ligands (i.e. NH_3)

26. Which of the following represent(s) the correct order of electron affinities?

(I) F > Cl > Br > □ (II) C < N < O < F

(III) N < C < O < F (IV) C < Si > P > N

(a) I and II (b) III and IV ✓ (c) III and II (d) I and III

Sol. b

27. Which is most covalent in nature ?

(a) NaCl (b) $MgCl_2$ (c) $AlCl_3$ ✓ (d) $CaCl_2$

Sol. Higher charge density on Al leads to higher polarising power of Al^{3+} cation. So $AlCl_3$ is most covalent in nature.

28. **Assertion:** Tertiary halolkanes are more reactive than primary haloalkanes towards elimination reactions.

Reason: The + I effect of the alkyl group weakens the C – X bond.

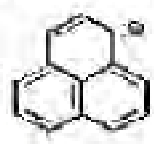
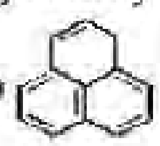
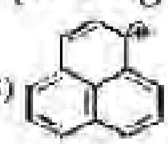
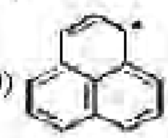
(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

✔(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.

(C) If Assertion is true, but Reason is false.

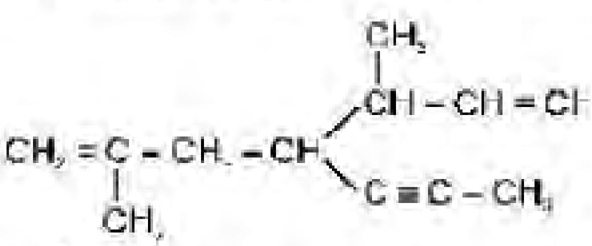
(D) If Assertion is false, but the Reason is true.

29. Which of the following is fully aromatic (all rings aromatic) in nature

✔(A)  (B)  (C)  (D) 

Sol. 'A' has 14 π electrons

30. The correct IUPAC name of the following compound is



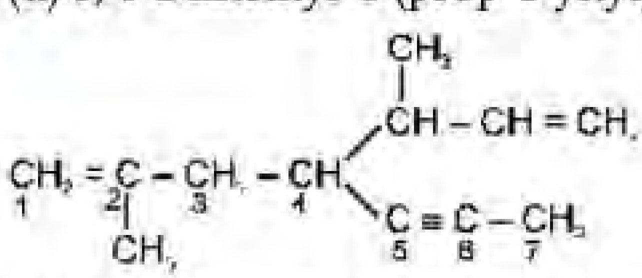
✔(a) 2, 5 dimethyl-4-(prop-1-ynyl)hepta-1, 6-diene

(c) 3-methyl-4-(2-methylprop-2-enyl) hept-1-en-5-yne

(c) 2-methyl-4-(1-methylprop-2-enyl) hept-1-en-5-yne

(d) 3, 6-Dimethyl-4-(prop-1-ynyl) hepta-1, 6-diene

Sol.

(A) 

2, 5 dimethyl-4-(prop-1-ynyl)hepta-1, 6-diene

MATHEMATICS

31. For $x > 0, \lim_{x\to 0}\left[(\sin x)^{\frac{1}{x}} + \left(\frac{1}{x}\right)^{\sin x}\right]$ is

(a) 0 (b) –1 ✔(c) 1 (d) 2

Sol. $P = \lim_{x\to 0}(\sin x)^{1/x} = 0 \quad \{\because \sin x \cong x \text{ as } x \to 0\}$

$Q = \lim_{x\to 0}\left(1/x\right)^{\sin x}$

$\Rightarrow \ln Q = \lim_{x\to 0}(-\sin x \ \ln x)$

$= \lim_{x\to 0}\left[\left(\frac{-\sin x}{x}\right)x . \ln x\right]$

$= -\lim_{x\to 0}(x \ln x) = 0 \Rightarrow Q = 1$

$\therefore \lim_{x\to 0}\left[(\sin x)^{1/x} + \left(\frac{1}{x}\right)^{\sin x}\right] = P + Q = 0 + 1 = 1$

32. If $0<\theta<2\pi$ then the interval of values of θ for which $2\sin^2\theta-5\sin\theta+2>0$, is

✓(a) $\left(0,\frac{\pi}{6}\right)\cup\left(\frac{5\pi}{6},2\pi\right)$ (b) $\left(\frac{\pi}{8},\frac{5\pi}{6}\right)$ (c) $\left(0,\frac{\pi}{8}\right)\cup\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$ (d) $\left(\frac{41\pi}{48},\pi\right)$

Sol. $2\sin^2\theta-5\sin\theta+2>0$

$\Rightarrow(\sin\theta-2)(2\sin\theta-1)>0$

$\Rightarrow\sin\theta\, 1/2$ or $\sin\theta>2$

But $\sin\theta>2$ is impossible

$\sin\theta<1/2\Rightarrow\alpha\in\left(0,\pi/6\right)\cup\left(\frac{5\pi}{6},2\pi\right)$

33. A plane passes through (1,-2,1) and is perpendicular to two planes $2x-2y+z=0$ and $x-y+2z=4$, the distance of the plane from the point (1,2,2) is

(a) 0 (b)1 (c) $\sqrt{2}$ ✓(d) $2\sqrt{2}$

Sol. Let a,b,c be the d.r's of normal to the plane

Equation of plane is $a(x-1)+b(y+2)+c(z-1)=0$

Also $2a-2b+c=0$ and $a-b+2c=0\Rightarrow c=0$ and $a=b$

$\therefore$ Equation of plane is $x+y=1=0$

$\therefore$ Distance from (1,2,2) to $x+y=1=0$ is

$D=\frac{|1+2+1|}{\sqrt{1+1}}=\frac{4}{\sqrt{2}}=2\sqrt{2}$

34. Let $\vec{a},\vec{b},\vec{c}$ be three non-coplanar vectors. Define $\vec{a}',\vec{b}',\vec{c}'$ as $\vec{a}'=\frac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]},\vec{b}'=\frac{\vec{c}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]},\vec{c}'=\frac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$.Which one of the following is false

(a) $\vec{a}.\vec{a}'+\vec{b}.\vec{b}'+\vec{c}.\vec{c}'=3$ (b) $\sum\overline{a}'.\left(\vec{b}\times\vec{c}\right)=\frac{3}{[\vec{a}\vec{b}\vec{c}]}$

(c) $[\vec{a}'\ \vec{b}'\ \vec{c}']=\frac{1}{[\vec{a}\ \vec{b}\ \vec{c}]}$ ✓(d) $[\vec{a}'\times\vec{b}'\ \ \vec{b}'\times\vec{a}'\ \ \vec{c}'\times\vec{a}']=\frac{2}{[\vec{a}\ \vec{b}\ \vec{c}]^2}$

Sol. (1) $\vec{a}.\vec{a}'+\vec{b}.\vec{b}'+\vec{c}.\vec{c}'=\vec{a}.\frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}+\vec{b}.\frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}+\vec{c}.\frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}=1+1+1=3$

(2) $\vec{a}'.(\vec{b}'\times\vec{c}')+\vec{b}'.(\vec{c}'\times\vec{a}')+\vec{c}'.(\vec{a}'\times\vec{b}')$

$$\sum\frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}.\frac{(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})}{(\vec{a}\vec{b}\vec{c})^3}=\frac{1}{[\vec{a}\vec{b}\vec{c}]^3}\sum[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=\frac{3[\vec{a}\vec{b}\vec{c}]^2}{[\vec{a}\vec{b}\vec{c}]^3}$$

35. Let $f_n(x)=\frac{1}{n}\left(\sin^n x+\cos^n x\right)$ for $n=1,2,\dots\dots$ then $f_4\left(\frac{3\pi}{16}\right)-f_6\left(\frac{3\pi}{16}\right)$ equals.

(a) $\frac{2}{\sqrt{2}-1}$ ✓(b) $\frac{1}{12}$ (c) $\frac{4}{\sqrt{3}-1}$ (d) 0

Sol. $f_4(x)-f_6(x)=\frac{1}{4}\left[\sin^4 x\cos^4 x\right]-\frac{1}{6}\left[\sin^6 x\cos^6 x\right]$

$=\frac{1}{4}\left[\left(\sin^2 x\right)^2+\left(\cos^2 x\right)^2\right]-\frac{1}{6}\left[\left(\sin^2 x\right)^3+\left(\cos^2 x\right)^3\right]$

$=\frac{1}{4}\left[1-2\sin^2 x\cos^2 x\right]-\frac{1}{6}\left[1-3\sin^2 x\cos^2 x\left(\sin^2 x\cos^2 x\right)\right]$

$=\frac{1}{4}-\frac{1}{2}\sin^2 x\cos^2 x-\frac{1}{6}+\frac{1}{2}\sin^2 x\cos^2 x=\frac{1}{12}$

$\therefore f_4\left(\frac{3\pi}{16}\right)-f_6\left(\frac{3\pi}{16}\right)=\frac{1}{12}$

36. If $f(x)$ is a polynomial satisfying $f(x)=\frac{1}{2}\begin{vmatrix} f(x) & f\left(\frac{1}{x}\right)-f(x) \\ 1 & f\left(\frac{1}{x}\right) \end{vmatrix}$ and f(2)=17 then $\int_{-1}^{1} f(x)dx$ equals

(a) 0 (b) $\frac{5}{12}$ ✓(c) $\frac{12}{5}$ (d) $\frac{6}{5}$

Sol. Let x_1, x_2, x_3, x_4 be the number of white ,red ,blue & green to be selected

$\therefore x_1+x_2+x_3+x_4=10$ ......(1)

Number of solution of (1)

$=$ coeff of $x^{10} in (1-x)^{-4}$

$=$coeff of $x^{10} in \left(1+{}^4C_1x+{}^5C_2x^2+{}^6C_3x^3+....\right)$

$={}^{13}C_{10}={}^{13}C_3=286$

37. The number of ways of selecting 10 balls out of an unlimited number of white, red, blue and green ball is

(a) 270 (b) 280 ✓(c) 286 (d) 90

Sol. Let x_1, x_2, x_3, x_4 be the number of white ,red ,blue & green to be selected

$\therefore x_1+x_2+x_3+x_4=10$ ......(1)

Number of solution of (1)

$=$ coeff of $x^{10} in (1-x)^{-4}$

$=$coeff of $x^{10} in \left(1+{}^4C_1x+{}^5C_2x^2+{}^6C_3x^3+....\right)$

$={}^{13}C_{10}={}^{13}C_3=286$

38. A mapping is selected at random from set $A=\{1,2,......,10\}$ into itself. The probability that mapping selected is an injective is

(a) $\frac{10}{10^9}$ ✓(b) $\frac{9!}{10^9}$ (c) $\frac{9}{10!}$ (d) $\frac{10^9}{9!}$

Sol. $A=\{1,2,3,....10\}$

$n(s)=\neq$ function from A to $A=10^{10}$

$n(s)=\neq$ injective function from A to $A=10^!$

$\therefore p(E)=\frac{n(E)}{n(S)}=\frac{10!}{10^{10}}=\frac{9!}{10^9}$

39. What is the probability that the two squares chosen randomly on a chess board, share a side

✓(a) $\frac{1}{18}$ (b) $\frac{13}{254}$ (c) $\frac{105}{288}$ (d) $\frac{13}{96}$

Sol. $m(S)=\neq$ way of selections two squares from a chess board

$={}^{64}C_2$

$=n(E)=\neq$ way of selections two consecutive squares from rows or columns'

$=(7+7+.....8\,times)+(7+7+......8\,times)$

$56+56=112$

$\therefore p(E)=\frac{n(E)}{n(s)}=\frac{112\times2}{64\times63}=\frac{1}{18}$

40. A man throws a fair coin a number of times and gets 2 points for each head he throws and 1 point for each tail he throws. The probability that he gets exactly 6 points is

✔(a) $\frac{43}{64}$ (b) $\frac{23}{32}$ (c) $\frac{13}{24}$ (d) $\frac{83}{128}$

Sol. To get six points exactly, the must get 3 heads (or) 2heads 2 Tails (or) 1 head 4 tails (or) 6 Tails.

Probability of getting 3 heads = $\left(\frac{1}{2}\right)^3$

Probability of getting 2H2T= $^4C_2\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

Probability of getting 1H4T= $^5C_1\left(\frac{1}{2}\right)^1 - \left(\frac{1}{2}\right)^4$

Probability of getting 6T= $\left(\frac{1}{2}\right)^6$

P(getting exactly 6 points)

$= \frac{1}{8} + 6.\frac{1}{4}.\frac{1}{4} + 5.\frac{1}{2}.\frac{1}{16} + \frac{1}{64} = \frac{43}{64}$

41. If $0 \le [x] < 4,\ -2 \le [y] < 3,\ -1 \le [z] < 5$ where $[\bullet]$represents greatest integer function then the maximum value of Δ where $\Delta = \begin{vmatrix} [x]+2 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ equals

(a) 13 (b) 15 ✔(c) 17 (d) 19

Sol. R→R₁-R₂; R₂→R₂-R₃

$\Delta = \begin{vmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ [x] & [y] & [z]+1 \end{vmatrix}$

2[z]+2+2[y]+[x]

For Δ_{max}, [x]=3; [y]=2; [z]=4.

Δ_{max}=8+2+4+3=17.

42. The total number of terms in the expansion of $(a+b+c+d)^{20}$ is

(a) 21 (b) 1021 ✔(c) 1771 (d) 42

Sol. # of terms of (a+b+c+d+^{20}

= # of non-negative solution of $x_1+x_2+x_3+x_4=20$.

$= {}^{20+4-1}C_{4-1} = {}^{23}C_3 = 1771$

43. $ABCD$ is a square of area 256, F is point on AD, E is point on extended AB such that CE Perpendicular CF. Area of ΔCEF is 200, then BE is equal to

✔(a) 12 (b) 14 (c) 15 (d) 20

Sol. Area of ABCD = 256⇒ side=16

$\Delta CDF \cong BCE \quad \therefore CF=CE=K$

Area of ΔCEF=200 (given)

$\Rightarrow \frac{1}{2}k^2 = 200.$

$\Rightarrow k = 20.$

we have BC = 16; CE = 20

$BE = \sqrt{CE^2 - BC^2} = \sqrt{20^2 - 16^2} = 12.$

44. If $f(x)$ and $g(x)$ are two functions with $g(x)=x-\frac{1}{x}$ and $(fog)(x)=x^3-\frac{1}{x^3}$, then $f'(x)=$

(a) $3x^2+3$ ✓ (b) $x^2-\frac{1}{x^2}$ (c) $1+\frac{1}{x^2}$ (d) $3x^2+\frac{3}{x^4}$

Sol. $g(x)=x-\frac{1}{x}$

$$fog(x)=x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)^3+3x\frac{1}{x}\left(x-\frac{1}{x}\right)$$

$$\Rightarrow f(x)=x^3+3x \quad \left\{\because a^3-b^3=(a-b)^3+3ab(a-b)\right\}$$

$$\Rightarrow f^1(x)=3x^2-3.$$

45. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set then the values n and m are

(a) $(6,3)$ (b) $(3,6)$ ✓ (c) $(2^3,2^6)$ (d) $(2^5,2^3)$

Sol. # subsets of first set $=2^m$

subsets of second set $=2^n$ Given that $2^m=56+2^n \Rightarrow m=6,\ n=3$

46. The locus of the points in the Argand plane representing the complex number $z=x+iy$ for which

(i) $|z-1|+|z+1|=3$ and (ii) $|z-(1+i)|-|z-(1-i)|=1$ are

(a) (A hyperbola, an ellipse) (b) (an ellipse, a hyperbola) ✓

(c) (a parabola, an ellipse) (d) (a parabola, a circle)

Sol. (i) Let P, A, B be there points z, 1, -1

$|z-1|+|z+1|=3 \Rightarrow PA+PB=3$ represents an ellipse with foci $(1,0)$ and $(-1,0)$

(ii) Let P, A, B be the points $z, 1+i, 1-i$ $|z-(1+i)|-|z-(1-i)|=1 \Rightarrow |PA-PB=1|$ represents a hyperbole with foci $(1,-1)$ and $(1,1)$

47. $(a,b)R(c,d) \Leftrightarrow a+d=b+c$ for all $(a,b),(c,d)\in N\times N$ is

(a) Equivalence relation ✓ (b) symmetric but not transitive

(c) reflexive but not symmetric (d) reflexive symmetric not transitive

Sol. Let $(a,b)\in N\times N$ $a+b=a+b \Rightarrow (a,b)R(a,b)$ $\therefore R$ is reflexive

Let $(a,b),(c,d)\in N\times N$ $(a,b)R(c,d)\Rightarrow a+d=b+c \Rightarrow c+b=d+a \Rightarrow (c,d)R(a,b)$

$\therefore$ 'R' is symmetric Let $(a,b),(c,d),(e,f)\in N\times N$ Let

$(a,b)R(c,d)\Rightarrow a+d=b+c\Rightarrow a-c=b-d$..........(1)

$(c,d)R(e,f)\Rightarrow c+f=d+e$..........(2)

Adding (1) and (2) we get $a+f=b+e\Rightarrow (a,b)R(e,f)$ $\therefore R$ is transitive

Hence 'R' is Equivalence relation

48. If $I_1=\int_0^{\pi/2}\frac{x}{\sin x}dx$ and $I_2=\int_0^1\frac{\tan^{-1}x}{x}dx$, then $\frac{I_1}{I_2}=$

(a) 1 (b) $\frac{1}{2}$ (c) 2 ✓ (d) $\frac{\pi}{2}$

Sol. $I_2=\int_0^1\frac{\tan^{-1}x}{x}dx$ put $\begin{matrix}x=\tan\theta \\ dx=\sec^2\theta\, d\theta\end{matrix}$ $\begin{matrix}x=0\Rightarrow\theta=0 \\ x=1\Rightarrow\theta=\pi_4\end{matrix}$

$$=\int_0^{\pi_4}\frac{\theta}{\tan\theta}\sec^2\theta\, d\theta$$

$$=\int_0^{\pi_4}\frac{\theta}{\sin\theta\cos\theta}d\theta \quad =\int_0^{\pi_4}\frac{2\theta}{\sin 2\theta}d\theta \quad \begin{matrix}\text{put } 2\theta=t \\ \Rightarrow d\theta=\frac{dt}{2}\end{matrix}$$

$$=\int_0^{\pi_2}\frac{t}{\sin t}\ \frac{dt}{2} \quad \begin{matrix}\theta=0\Rightarrow t=0 \\ \theta,\pi_4\Rightarrow t=\frac{\pi}{2}\end{matrix} \quad =\frac{1}{2}I_1 \quad \Rightarrow 2=\frac{I_1}{I_2}$$

49. If α and β are the roots of the equation $x^2+mx+1=0$ and γ,δ that of $x^2+nx+1=0$ then the value of $(\alpha-\delta)(\alpha-\gamma)(\beta-\delta)(\beta-\gamma)=$

(a) $(m-n)^2$ ✓ (b) m^2-n^2 (c) m^2+n^2 (d) $(m+n)^2$

Sol. α,β are the roots of $x^2+mx+1=0\Rightarrow\alpha+\beta=-m;\ \alpha\beta=1$

γ,δ are the roots of $x^2+nx+1=0\Rightarrow\gamma+\delta=-n=\delta=1$

Now $(\alpha-\delta)(\alpha-\gamma)(\beta-\delta)(\beta-\gamma)=\left(\alpha^2-(\delta+\gamma)\alpha+\delta\alpha\right)\left(\beta^2-(\delta+\gamma)\beta+\delta\gamma\right)$

$=\left(\alpha^2+n\alpha+1\right)\left(\beta^2+n\beta+1\right)=(-m\alpha+n\alpha)(-m\beta+n\beta)\left\{\because-\alpha^2+m\alpha+1=0 \text{ and } \beta^2+m\beta+1=0\right\}$

$=(n-m)^2\alpha\beta=(n-m)^2=(m-n)^2$

50. If $a_n = \sum \frac{n^2}{n!}$ then $\sum_{n=1}^{\infty} a_n =$

(a) $17e$ (b) $\frac{8e}{3}$ (c) $3e$ ✔(d) $\frac{17e}{6}$

Sol. $a_n = \frac{\Sigma n^2}{n!} = \frac{n(n+1)(2n+1)}{6n!}$; $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{n(n+1)(2n+1)}{6n!}$

$= \frac{1}{6}\Sigma \frac{2n^2+3n+1}{(n-1)!}$ $= \frac{1}{6}\sum_{n=1}^{\infty} \frac{2(n^2+1)+3(n-1)+6}{(n-1)!}$

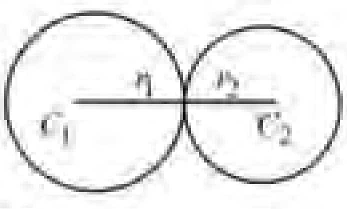
$= \frac{1}{6}\sum_{n=1}^{\infty}\left[\frac{2(n+1)}{(n-2)!} + \frac{3}{(n-2)!} + \frac{6}{(n-1)!}\right]$ $= \frac{1}{6}\left\{\sum_{n=1}^{\infty}\left(\frac{2(n-2)}{(n-2)!} + \frac{6}{(n-2)!} + \frac{3}{(n-2)!} + \frac{6}{(n-1)!}\right)\right\}$

$= \frac{1}{6}\sum_{n=1}^{\infty}\left(\frac{2}{(n-3)!} + \frac{9}{(n-2)!} + \frac{6}{(n-1)!}\right) = \frac{1}{6}(2e+9e+6e)\left[\because \frac{1}{(-ve)!} = 0\right] = \frac{17e}{6}$

51. Two circles touch externally. The sum of their areas is 130π sq. cm. The distance between their centres is 14 cm. Then the radii of the circles are in the ratio

(a) 11:31 (b) 31:11 (c) 3:1 ✔(d) 11:3

Sol. $r_1 + r_2 = 14$: 130π = sum of area : $= \pi(r_1^2 + r_2^2) \Rightarrow r_1^2 + r_2^2 = 130$



$\Rightarrow (r_1+r_2)^2 - 2r_1r_2 = 130 \Rightarrow 196 - 130 = 2r_1r_2 \Rightarrow 66 = 2r_1r_2$

$r_1 - r_2 = \sqrt{(r_1+r_2)^2 - 4r_1r_2} = \sqrt{196-132} = 8$ $r_1 + r_2 = 14$ and $r_1 - r_2 = 8$

$\Rightarrow r_1 = 11;\ r_2 = 3$ $\therefore r_1 : r_2 = 11:3$

52. If the system of equation $x+2y-3z=1$, $(p+2)z=3$, $(2p+1)y+z=2$ is inconsistent, then the value of p is

(a) -2 (b) $\frac{-1}{2}$ (c) 0 ✔(d) 2

Sol. $A:B = \begin{pmatrix} 1 & 2 & -3 & : & 1 \\ 0 & 2p+1 & 1 & : & 2 \\ 0 & 0 & p+2 & : & 3 \end{pmatrix}$ For inconsistency Rank $(A:B) \neq$ Rank $(A) \Rightarrow p+2=0 \Rightarrow p=-2$

53. If $g(x)$ is inverse of $f(x)$ then $g''(f(x)) =$

✔(a) $-\frac{f''(x)}{(f'(x))^3}$ (b) $-\frac{f''(x)}{(f(x))^3}$ (c) $\frac{f''(x)}{(f'(x))^3}$ (d) 0

Sol. $g(f(x)) = x$; Diff, w.r.t 'x' we get $g'(f(x)).f'(x) = 1 \Rightarrow g'(f(x)) = \frac{1}{f'(x)}$

Diff w.r.t 'x' we get $g''(f(x)).f'(x) = -\frac{f''(x)}{(f'(x))^2} \Rightarrow g''(f(x)) = -\frac{f''(x)}{(f'(x))^3}$

54. Given $\int_0^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} = \frac{\pi}{2(a+b)(b+c)(c+a)}$ then $\int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$ is

✔(a) $\frac{\pi}{60}$ (b) $\frac{\pi}{20}$ (c) $\frac{\pi}{40}$ (d) $\frac{\pi}{80}$

Sol. $\int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)} = \int_0^{\infty} \frac{x^2 dx}{(x^2-0)(x^2+2^2)(x^2+9^2)} = \frac{\pi}{2(0+2)(2+3)(3+0)} = \frac{\pi}{60}$

55. The solution curve of $\frac{dy}{dx} = \frac{y^2-2xy-x^2}{y^2+2xy-x^2}, y(-1)=1$ is

(a) Parabola (b) ellipse (c) circles ✔(d) Straight line

Sol. $\frac{dy}{dx} = \frac{y^2-2xy-x^2}{y^2+2xy-x^2}$ Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \Rightarrow x\frac{dv}{dx} = \frac{-(v^2+1)(v+1)}{v^2+2v-1}$

$= \int \frac{v^2+2v-1}{(v^2+1)(v+1)} dv = \int -\frac{dx}{x}$ $\Rightarrow \int\left(\frac{2v}{v^2+1} + \frac{-1}{v+1}\right) dv = \int -\frac{dx}{x}$

$= \lim\left(\frac{v^2+1}{v+1}\right) = -\lim x - \lim c \Rightarrow \frac{v^2+1}{v+1} = \frac{1}{cx} \Rightarrow \frac{x^2+y^2}{x+y} = \frac{1}{c}$

$= c(x^2+y^2) = (x+y)$ $y(-1)=1 \Rightarrow c(1+1) = (0) \Rightarrow c = 0$

$\therefore$ Eqi of curve is $x+y=0$, which a straight line

56. Given an isosceles triangle ,whose one angle is 120° ,and radius of its incircle is $\sqrt{3}$, then the area of the triangle in sq units is

(a) $7+12\sqrt{3}$ (b) $12-7\sqrt{3}$ ✔(c) $12+7\sqrt{3}$ (d) 4π

Sol. Area $= \Delta = \frac{1}{2}bc\sin A$

$= \frac{1}{2}b^2\sin 120 \{\because b = c\} = \frac{\sqrt{3}}{4}b^2$ ....(1)

$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{a}{\sin 120} = \frac{b}{\sin 30}$

$\Rightarrow a = \sqrt{3}b$

Also $\Delta = rs \Rightarrow \Delta = \sqrt{3}s$

$\because$ Now $2s = a+b+c \Rightarrow s = \frac{b}{2}(\sqrt{3}+2)$

$\therefore \Delta = \frac{\sqrt{3}b}{2}(\sqrt{3}+2)$.... ....(2)

Solving (1) and (2) we get $b = 4+2\sqrt{3}$ and $\Delta = 12+7\sqrt{3}$

57. The number of the solutions of the following systems of equations $\{x\}+y+[z]=3.1$, $x+[y]+\{z\}=2.4$ $[x]+\{y\}+z=1.3$ (where $[x]$ and $\{x\}$ denote the integral and fractional part of x respectively)

(a) Exactly two (b) Infinite

(c) exactly one (d) zero

Sol. $\{x\}+y+[z]=3.1$ ....(1)

$x+[y]+\{z\}=2.4$ ....(2)

$[x]+\{y\}+z=1.3$ ....(3)

Adding we get $2(x+y+z)=6.8 \Rightarrow x+y+z=3.4$

$(4)-(1) \Rightarrow x-\{x\}+z-[z]=0.3$

$\Rightarrow [x]+\{z\}=0.3 \Rightarrow [x]=0$ and $\{z\}=0.3$

$(4)-(2) \Rightarrow y-[y]+z-\{z\}=1.0$

$\Rightarrow \{y\}+[z]=1.0$

$\Rightarrow \{y\}=0$ *and* $[z]=1$

$(4)-(3) \Rightarrow x-[x]+y-\{y\}=2.1$

$\Rightarrow \{x\}+[y]=2.1$

$\Rightarrow \{x\}=0.1$ *and* $[y]=2$

$\therefore x=[x]+\{x\}=0.1$

$y=[y]+\{x\}=2.0$

$z=[z]+\{z\}=1.3$

58. A circle touches $x=2$ and intersects $x^2+y^2=1$ orthogonally locus of centre of the circle is

(a) pair of lines (b) ellipse (c) hyperbola (d) parabola

Sol. Let $x^2+y^2+2gx+2fy+c=0$ intersect $x^2+y^2-1=0$ orthogonally. There fore $c=1$

The circle touches $x=2$

$\Rightarrow r=\perp^{lar}$ distance from C to $x-2=0$

$\Rightarrow \sqrt{g^2+f^2-1}=\frac{|-g-2|}{1}$

$\Rightarrow f^2-1=4+4g$

$\therefore$ Locus of center C=(-g, -f) is

$y^2-1=4-4x \Rightarrow y^2=5-4x$ a parabola

59. The remainder when $13^{99}+19^{93}$ is divided by 81 is

(a) 29 (b) 37 (c) 49 (d) 57

Sol. $13^{99}=(9+4)^{99}=9^{99}+{}^{99}C_1.9^{98}.4+.....+{}^{99}C_{98}.99.4^{98}+4^{99}$

$\therefore 13^{99}\equiv 4^{99} \pmod{81}$

$\equiv (9-1)^{66} \pmod{81}$

$\equiv -66.9+1 \pmod{81}$

$\equiv -26 \pmod{81}$

$19^{93}=(18+1)^{93}={}^{93}C_0 18^{93}+{}^{93}C_1 18^{92}+.....+{}^{93}C_{92}18+1$

$19^{93}\equiv 93.18+1 \pmod{81}$

$\equiv 90.18+3.18+1 \pmod{81}$

$\equiv 3.18+1 \pmod{81}$

$\equiv 55 \pmod{81}$

$\therefore 13^{99}+19^{93}\equiv -26+55 \pmod{81}$

$\equiv 29 \pmod{81}$

60. $\int_0^{\pi/4}\left(\frac{x}{x\sin x+\cos x}\right)^2 dx=$

(a) $\frac{3+\pi}{4-\pi}$ (b) $\frac{4-\pi}{3+\pi}$ (c) $\frac{4-\pi}{4+\pi}$ (d) $\frac{4+\pi}{4-\pi}$

Sol. $\frac{d}{dx}=\left[\frac{1}{x\sin x+\cos x}\right]=\frac{-x\cos x}{(x\sin x+\cos x)^2}$

Now $I=\int_0^{\frac{\pi}{4}}\left(\frac{x}{x\sin x+\cos x}\right)^2 dx$

$=\int_0^{\frac{\pi}{4}} x\sec x\cdot\frac{x\cos x}{(x\sin x+\cos x)^2}dx$

$=\int_0^{\frac{\pi}{4}} x\sec x\frac{d}{dx}\left(-\frac{1}{x\sin x+\cos x}\right)dx$

Integration by parts

$=\left[\frac{-x\sec x}{x\sin x+\cos x}\right]_0^{\pi/4}+\int_0^{\pi/4}\frac{x\sec x\tan x+\sec x}{x\sin x+\cos x}dx$

$=\frac{\sqrt{2}\frac{\pi}{4}}{\frac{1}{\sqrt{2}}\frac{\pi}{4}+\frac{1}{\sqrt{2}}}+\int_0^{\pi/4}\sec^2 x dx$

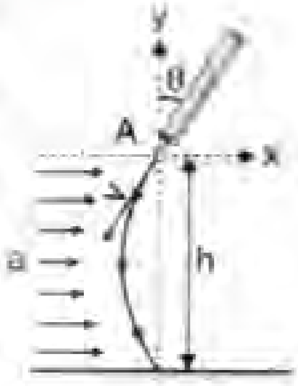
$-\frac{2\pi}{\pi+4}+[\tan x]_0^{\frac{\pi}{4}}$

$I=\frac{-2\pi}{\pi+4}+1$

$=\frac{-2\pi+\pi+4}{+\pi+4}=\frac{4-\pi}{4+\pi}$

PHYSICS

61. A particle is ejected from the tube at A with a velocity v at an angle θ with the vertical $y-$axis. A strong horizontal wind gives the particle a constant horizontal acceleration a in the $x-$direction. If the particle strikes the ground at a point directly under its released position and the downward $y-$acceleration is taken as g then



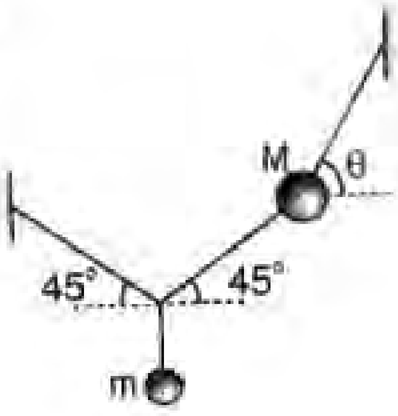
(a) $h=\frac{2v^2\sin\theta\cos\theta}{a}$ (b) $h=\frac{2v^2\sin\theta\cos\theta}{g}$

(c) $h=\frac{2v^2}{g}\sin\theta\left(\cos\theta+\frac{a}{g}\sin\theta\right)$ (d) $h=\frac{2v^2}{a}\sin\theta\left(\cos\theta+\frac{g}{a}\sin\theta\right)$

Sol. Since $0=(v\sin\theta)t+\frac{1}{2}(-a)t^2 \Rightarrow t=\frac{2v\sin\theta}{a}$

Also $h=(v\cos\theta)t+\frac{1}{2}gt^2 \Rightarrow h=\frac{2v^2}{a}\sin\theta\left(\cos\theta+\frac{g}{a}\sin\theta\right)$

62. Two masses m and M are attached with strings as shown. For the system to be in equilibrium we have



(a) $\tan\theta=1+\frac{2M}{m}$ (b) $\tan\theta=1+\frac{2m}{M}$

(c) $\tan\theta=1+\frac{M}{2m}$ (d) $\tan\theta=1+\frac{m}{2M}$

Sol. Ans : a

$mg=2T\sin 45^\circ$

$\Rightarrow mg=\sqrt{2}T$ ....(1)

$T_1\cos\theta-T\cos 45^\circ$

$T_1\cos\theta-\frac{T}{\sqrt{2}}=\frac{mg}{2}$ $\left\{\because T=\frac{mg}{\sqrt{2}}\right\}$

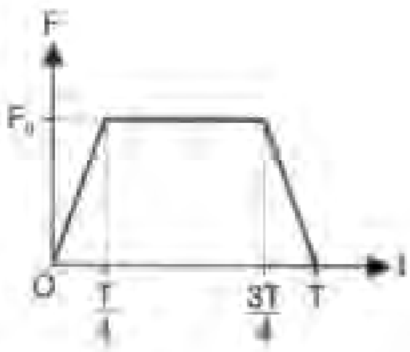
Further, $mg+T\cos 45-T_1\sin\theta$

$\Rightarrow T_1\sin\theta-Mg+\frac{mg}{\sqrt{2}}\frac{1}{\sqrt{2}}$

$\Rightarrow T_1\sin\theta-Mg+\frac{mg}{2}$ ....(2)

$\Rightarrow \tan\theta=\frac{Mg+\frac{mg}{2}}{\frac{mg}{2}}-1+\frac{2M}{m}$ (divide (2) by (1))

63. A particle of mass m moving with a velocity v makes an elastic one dimensional collision with a stationary particle of mass m establishing a contact with it for extremely small time T. Their force of contact increases from zero to F_0 linearly in time $\frac{T}{4}$, remains constant for a further time $\frac{T}{2}$ and decreases linearly from F_0 to zero in further time $\frac{T}{4}$ as shown. The magnitude possessed by F_0 is



(a) $\frac{mu}{T}$ (b) $\frac{2mu}{T}$ (c) $\frac{4mu}{3T}$ (d) $\frac{3mu}{4T}$

Sol. Ans : c

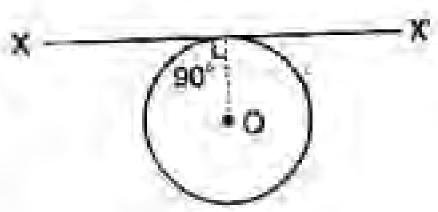
Impulse = Area of Trapezium

$=\frac{1}{2}\left(T+\frac{T}{2}\right)F_0=\frac{3TF_0}{4}$

According to impulse momentum Theorem impulse = Change in momentum

$\Rightarrow \frac{3TF_0}{4}=mu \Rightarrow F_0\frac{4mu}{3T}$

64. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is



(a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$ (c) $\frac{5\rho L^3}{16\pi^2}$ ✓(d) $\frac{3\rho L^3}{8\pi^2}$

Sol. Ans : d

Mass of the loop $= M = L\rho$

Further if r is the radius of the loop, then $2\pi r = L$

$\Rightarrow r = \frac{L}{2\pi}$

Momentum of inertia about xx' is $I = \frac{3}{2}Mr^2$

$\Rightarrow I = \frac{3}{2}(L\rho)\frac{L^2}{(2\pi)^2} = \frac{3\rho L^3}{8\pi^2}$

65. A projectile is fired upwards from the surface of the earth with a velocity kv_e where v_e is the escape velocity and $k<1$. If r is the maximum distance from the centre of the earth to which it rises and R is the radius of the earth, then r is

(a) $\frac{R}{K^2}$ (b) $\frac{2R}{1-k^2}$ (c) $\frac{2R}{k^2}$ ✓(d) $\frac{R}{1-k^2}$

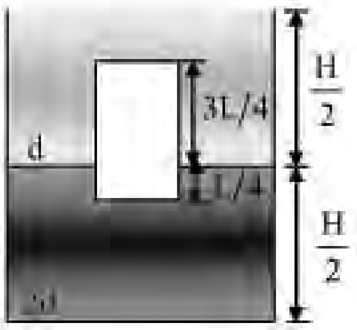
Sol. Ans : d

$\begin{pmatrix}\text{Total Mechanical}\\ \text{Energy}\end{pmatrix}_{surface} = \begin{pmatrix}\text{Total Mechanical}\\ \text{Energy}\end{pmatrix}$

$\Rightarrow \frac{GMm}{R} + \frac{1}{2}m(kv_0)^2 = -\frac{GMm}{r} + 0$ where $v_0 = \sqrt{\frac{2GM}{R}}$

$\Rightarrow r = \frac{R}{1-k^2}$

66. A container of a large uniform cross-sectional area A resting on a horizontal surface holds two immiscible, non-viscous and incompressible liquids of densities d and $2d$ each of height $\frac{H}{2}$ as shown. The lower density liquid is open to atmosphere. A homogeneous solid cylinder of length $L\left(<\frac{H}{2}\right)$, cross-sectional area $\frac{A}{5}$ is immersed such that it floats with its axis vertical to the liquid-liquid interface with length $\frac{L}{4}$ in denser liquid. If D is the density of the solid cylinder then



(a) $D=\frac{3d}{2}$ (b) $D=\frac{d}{2}$ (c) $D=\frac{2d}{3}$ ✓(d) $D=\frac{5d}{4}$

Sol. For equilibrium

$\begin{pmatrix}\text{Weight of}\\ \text{cylinder}\end{pmatrix} = \begin{pmatrix}\text{Total Upward}\\ \text{Thrust}\end{pmatrix}$

on

$\begin{pmatrix}\text{Weight of}\\ \text{cylinder}\end{pmatrix} = \begin{pmatrix}\text{Total weight of}\\ \text{liquid displaced}\end{pmatrix}$

$\Rightarrow L\left(\frac{A}{5}\right)Dg = \frac{3L}{4}\left(\frac{A}{5}\right)gd + \frac{L}{4}\left(\frac{A}{5}\right)2gd \Rightarrow D = \frac{3d}{4} + \frac{1}{2}d \Rightarrow D = \frac{5}{4}d$

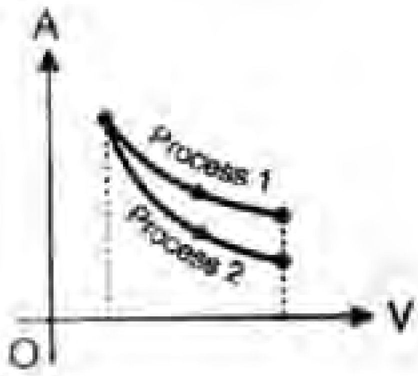
67. A column of liquid is contained in a horizontal tube which is open at both ends. The change in temperature does not alter the length of this liquid column in the tube. If a be the coefficient of linear expansion of the material of the tube and g be the coefficient of volume expansion of the liquid then

(a) g = 3a (b) g = 2a (c) g = a (d) $g = \frac{1}{2}a$

Sol. Ans : b

Since the length of the liquid column does not alter in the tube, hence we have superficial expansion and so $\gamma = 2\alpha$, instead of $\gamma = 3\alpha$

68. The indicator diagram for two processes 1 and 2 carried on an ideal gas is shown in figure. If m_1 and m_2 be the slopes $\left(\frac{dP}{dV}\right)$ for Process 1 and Process 2 respectively, then



(a) $m_1 = m_2$ (b) $m_1 > m_2$

(c) $m_1 < m_2$ (d) $m_2 C_v = - m_1 C_p$

Sol. Ans : c

During expansion an isotherm lies above an adiabat

Also $\begin{pmatrix}\text{Slope of} \\ \text{an adiabat}\end{pmatrix} = \gamma \begin{pmatrix}\text{Slope of an} \\ \text{isotherem}\end{pmatrix}$

$\Rightarrow m_2 = \frac{C_p}{C_v}(m_1)$

$\Rightarrow m_2 C_v = m_1 C_p$. Since $\gamma = 1 \Rightarrow m_2 = m_1$

69. In the figure shown, the spring is light and has a force constant k. The pulley is light and smooth and the string is light. The suspended block has a mass m . On giving a slight displacement vertically to the block in the downward direction from its equilibrium position the block executes S.H.M. on being released with time period T.

(a) $T = 2p\sqrt{\frac{m}{k}}$ (b) $T = 2p\sqrt{\frac{m}{2k}}$

(c) $T = 2p\sqrt{\frac{2m}{k}}$ (d) $T = 4p\sqrt{\frac{m}{k}}$

Sol. Ans : d

Let the extension in the spring be x_0 at equilibrium if F_0 be the tension in the string then $F_0 = kx_0$

Further if T_0 is the tension in the thread then $T_0 mg$ and $2T_0 = kx_0$

Let the mass m be displaced through a slight displacement x downwards. Let the new tension in the string and spring be T and F respectively

$\Rightarrow F = k\left(x_0 + \frac{x}{2}\right)$ and $F = 2T$

$\Rightarrow 2T = k\left(x_0 + \frac{x}{2}\right) \Rightarrow (T - T_0) = \frac{kx}{4}$ $\{\because kx_0 = 2T_0\}$

$\Rightarrow$ Time period $= 2\pi\sqrt{\frac{m/k}{4}} = 4\pi\sqrt{\frac{m}{k}}$

70. The displacement due to the wave moving in the positive $x-$ direction is given by $y=\frac{1}{1+x^2}$ at time $t=0$ and by $y=\frac{1}{\left[1+(x-1)^2\right]}$ at $t=2s$, where x and y are in metre. The velocity of wave in ms^{-1} is

(a) 0.5 ✓ (b) 1 (c) 2 (d) 4

Sol. Ans : a

$x - vt = x - t$

$\Rightarrow vt - t \Rightarrow v = 0.5\ ms^{-1}$

71. Three point charges q, $2q$ and $8q$ are to be placed on a straight line $9cm$ long. The system possesses minimum potential energy when

(a) $2q$ and q lie at ends with $8q$ at

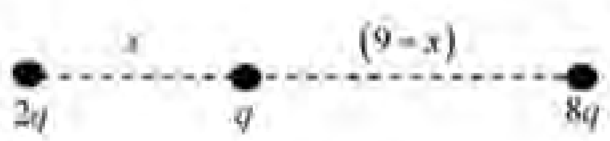
(b) $2q$ and $8q$ lie at ends with q at $6cm$ from $8q$ ✓

(c) q and $8q$ lie at ends with $2q$ at $6cm$ from q

(d) none of these

Sol. The system will posses minimum potential energy when charges with larger values are placed, the farthest. Also minimum potential energy implies stability (i.e equilibrium)

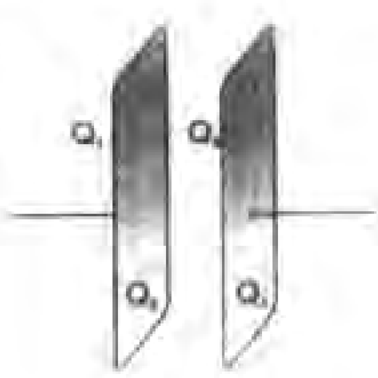
$\Rightarrow \frac{2q^2}{x^2} = \frac{8q^2}{(9-x)^2} \Rightarrow \frac{9-x}{x} = \pm 2$



$9 - x = 2x \Rightarrow 3x = 9 \Rightarrow x = 3$

The other values is rejected as equilibrium point must lie on the line joining the two charges between them.

72. An isolated parallel plate capacitor of capacitance C has four surfaces with charges Q_1, Q_2, Q_3 and Q_4 as shown in figure. The potential difference between the plates is



(a) $\frac{Q_1+Q_2+Q_3+Q_4}{2C}$ (b) $\frac{Q_2+Q_3}{2C}$

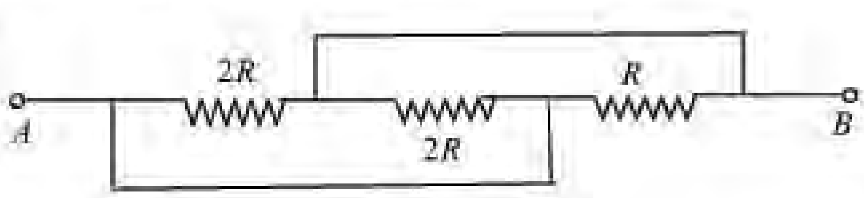
(c) $\frac{Q_2-Q_3}{2C}$ ✓ (d) $\frac{Q_1+Q_4}{2C}$

Sol. Ans : c

$Q_2 = -Q_3$ (By induction)

Further, $\Delta V = \frac{Q_2}{C} = \frac{2Q}{2C} = \frac{Q_2-(-Q_2)}{2C} \Rightarrow \Delta V = \frac{Q_2-Q_3}{2C}$

73. The equivalent resistance between points A and B in the circuit shown is



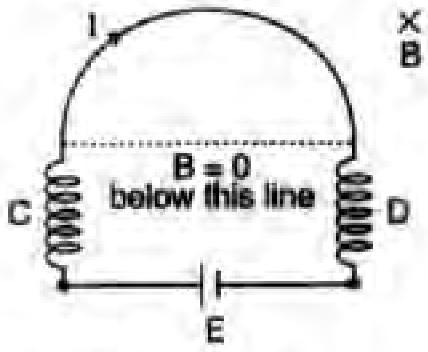
(a) R (b) R/2 ✓

(c) R/4 (d) R/8

Sol. Ans : b

$\frac{1}{R_{AB}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{R}$

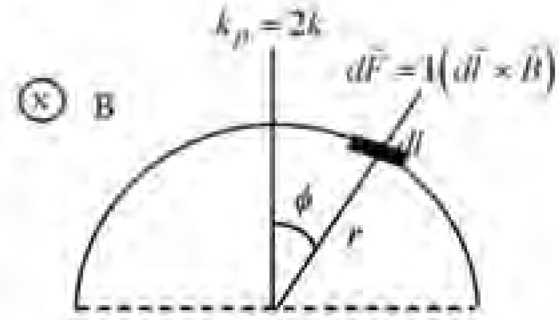
74. A wire of resistance R in the form of a semicircle lies on the top of a smooth table. A uniform magnetic field B is confined to the region as shown. The ends of the semicircle are attached to springs C and D whose other ends are fixed. If r is the radius of the semicircle and k is the force constant for each spring, then the extension x in each spring is



(a) $x=\frac{2EBr}{kR}$ (b) $x=\frac{2EBR}{kr}$

(c) $x=\frac{EBr}{kR}$ (d) $x=\frac{EBr}{2kR}$

Sol. Since both springs are identical and are connected in parallel, so net force constant is



Force applied on the semicircular wire of radius r is calculated as follows

$F_m=\int dF\cos\phi$ {∵ all sine components cancel each other. So as to give no contribution to the net magnitude field F_m}

$\Rightarrow F_m=2IBr\int_0^{\frac{\pi}{2}}\cos\phi d\phi$ $\left\{\because \int_0^{\pi}\cos\phi d\phi-2\int_0^{\pi/2}\cos\phi d\phi\right\}$

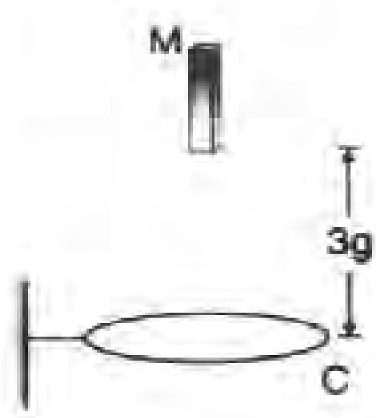
$\Rightarrow F_m=2IBr$

Further, $F_m=k_0x$

$\Rightarrow 2IBr=(2k)_x$

$\Rightarrow 2\left(\frac{E}{R}\right)Br-2kx\Rightarrow x=\frac{EBr}{kR}$

75. A bar magnet M is allowed to fall towards a fixed conducting ring C. If g is the acceleration due to gravity, v is the velocity of the magnet at $t=2\,s$ and s is the distance travelled by it in the same time then,



(a) $v>2\,g$ (b) $v<g$

(c) $s>2\,g$ (d) $s<2\,g$

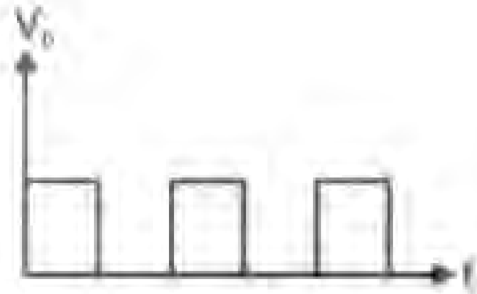
Sol. Ans : d

$v<g(t)$

$\Rightarrow v<2g$ (due to Lenz's law]

Also, $s<\frac{1}{2}gt^2\Rightarrow s=2g$ (due to Lenz's law]

76. The rms value of the potential difference shown is



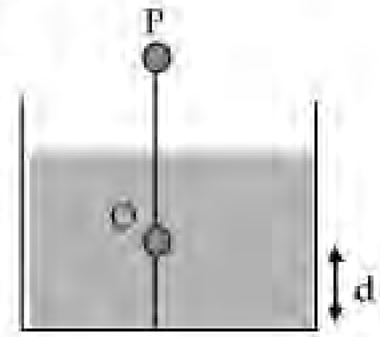
(a) V_0 (b) $\frac{V_0}{\sqrt{2}}$

(c) $\frac{V_0}{2}$ (d) $2V_0$

Sol. Ans : b

$V_{rms}=\sqrt{\frac{0^2+V_0^2}{2}}=\frac{V_0}{\sqrt{2}}$

77. A tank contains a transparent liquid of refractive index n the bottom of which is made of a mirror as shown. An object O lies at a height d above the mirror. A person P vertically above the object sees O and its image in the mirror and finds the apparent separation to be



(a) $2nd$ (b) $\frac{2d}{n-1}$

(c) $\frac{2d}{n}$ (d) $\frac{d}{n}(1+n)$

Sol. Ans : c

Apparent depth = $\frac{d}{n}$

Hence apparent separation = $\frac{2d}{n}$

78. Four lenses are made from same type of glass. The radius of curvature of each face is given. Out of these, the lens having the greatest positive power is

(a) 10 cm convex and 15 cm convex.

(b) 20 cm convex and 30 cm concave.

(c) 15 cm convex and plane.

(d) 5 cm convex and 10 cm concave.

Sol. Ans : a

$$P=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

79. If Young's interference experiment is performed using two separate identical sources of light instead of using two slits and one bulb, then

(a) interference fringes will be brighter

(b) interference fringes will be coloured

(c) interference fringes will be darker

(d) no fringes will be obtained

Sol. Ans : d

For interference to take place, we must keep in mind the following conditions

(i) The two sources must be coherent i.e. must have a constant phase difference

(ii) They must be derived from the same parent source i.e. two independent sources of light can nevery produce interference

(iii) The sources must either have same amplitude or nearly the same amplitude.

80. A sensor is exposed for time 't' to a lamp of power 'P' placed at a distance 'l' The sensor has an opening that is '4d' in diameter. Assuming all energy of the lamp is given off as light, the number of photons entering the sensor if the wavelength of light is λ is

(a) ✓ $N=\frac{P\lambda d^2 t}{hcl^2}$ (b) $N=\frac{4P\lambda d^2 t}{hcl^2}$ (c) $N=\frac{P\lambda d^2 t}{4hcl^2}$ (d) $N=\frac{P\lambda d^2 t}{16hcl^2}$

Sol. Ans : a

$E=\frac{hc}{\lambda}$

Number of photons emitted is $\frac{Pt}{\left(\frac{hc}{\lambda}\right)}=n_0$

$\Rightarrow n_0=\frac{P\lambda t}{hc}$

Since the radiation is spherically symmetric, so total number of photons entering the sensor is n_0 times the ratio of aperture area to the area of a sphere of radius r

$\Rightarrow N-n_0\frac{\pi(2d)^2}{4\pi l^2}=\frac{P\lambda t}{hc}\frac{d^2}{l^2}$

81. If elements of quantum number greater than n were not allowed, the number of possible elements in nature would have been

(a) $\frac{1}{2}n(n+1)$ (b) $\left\{\frac{n(n+1)}{2}\right\}^2$ (c) $\frac{1}{6}n(n+1)(2n+1)$ (d) ✓ $\frac{1}{3}n(n+1)(2n+1)$

Sol. Ans : d

For each principal quantum number n_0 number of electrons permitted equals the number of elements corresponding to the quantum number

$\Rightarrow\begin{pmatrix}\text{Total Number}\\ \text{of Electrons}\end{pmatrix}=\sum 2n^2=\frac{n(n+1)(2n+1)}{3}$

82. Which of the following processes represents a γ – decay?

(a) ${}^{A}X_Z+\gamma\rightarrow{}^{A}X_{Z-1}+a+b$ (b) ${}^{A}X_Z+{}^{1}n_0\rightarrow{}^{A-3}X_{Z-2}+c$

(c) ✓ ${}^{A}X_Z\rightarrow{}^{A}X_Z+f$ (d) ${}^{A}X_Z+e_{-1}\rightarrow{}^{A}X_{A-1}+g$

Sol. Ans : c

83. In a n-p-n transistor circuit, the collector current is 10 mA. If 90% of the electrons emitted reach the collector then the

(a) emitter current will be 9 mA (b) emitter current will be 10 mA

(c) ✓ base current will be 1 mA. (d) base current will be -1 mA

Sol. Ans : c

$I_0=10mA$

As I_0 is 90% of the emitter current I_0

So $I_0=\frac{I_0}{0.9}=11mA$

Since $I_a=I_b+I_c$

$\Rightarrow I_b=I_a-I_c\Rightarrow I_c=1mA$

84. Hot water cools from $60\,^\circ C$ to $50\,^\circ C$ in the first 10 minute and to $42\,^\circ C$ in the next 10 minute. The temperature of the surroundings is

(a) $5\,^\circ C$ (b) ✓ $10\,^\circ C$ (c) $15\,^\circ C$ (d) $20\,^\circ C$

Sol. Ans : b

According to Newton's Law of cooling

Rate of cooling = Average excess temperature

$\frac{dT}{dt}=\begin{pmatrix}\text{Average}\\ \text{Temperature}\end{pmatrix}-\begin{pmatrix}\text{Ambient}\\ \text{Temperature}\end{pmatrix}$

$\Rightarrow\frac{60-50}{10}\propto\left(\frac{60+50}{2}-T_0\right)$

$\Rightarrow 55-T_0-k$ ....(1)

(where k is a constant proportionally)

Also $\frac{50-42}{10}\propto\left(\frac{50+42}{2}-T_0\right)\Rightarrow 46-T_0=\frac{8}{10}k$

$\Rightarrow 46-T_0=\frac{8}{10}(55-T_0)\Rightarrow 460-10T_0=440-8T_0\Rightarrow 2T_0=20\Rightarrow T_0=10^\circ C$

85. If a number of little droplets of a liquid of density ρ, surface tension T and specific heat C, each of radius r, coalesce to form a single drop of radius R, the rise in temperature will be

(a) $\frac{3T}{\rho c}\left(\frac{1}{r}+\frac{1}{R}\right)$ (b) ✓ $\frac{3T}{\rho c}\left(\frac{1}{r}-\frac{1}{R}\right)$ (c) $\frac{3T}{2\rho c}\left(\frac{1}{r}+\frac{1}{R}\right)$ (d) $\frac{3T}{2\rho c}\left(\frac{1}{r}-\frac{1}{R}\right)$

Sol. Ans : b

86. A cord of mass m, length L, area of cross-section A and Young's modulus Y is hanging from a ceiling with the help of a rigid support. The elongation developed in the wire due to its own weight is

(a) ZERO (b) $\frac{mgL}{AY}$ (c) $\frac{mgL}{2AY}$ (d) $\frac{2mgL}{AY}$

Sol. Ans : c

Since the entire weight mg of the wire is concentrated at its centre of mass which lies at the geometrical centre of the wire. So the effective original length of the wire is $\frac{L}{2}$. According to Hooke's Law

$$Y = \frac{\text{Stress}}{\text{strain}} \Rightarrow Y = \frac{\frac{A}{\Delta L}}{L/2} \Rightarrow \Delta L = \frac{mgL}{2AY}$$

87. Two open pipes A and B are sounded together such that beats are heard between the first overtone of A and second overtone of B. If the fundamental frequency of A and B is 256 Hz and 170 Hz respectively, then the beat frequency heard is

(a) 4 Hz (b) 3 Hz (c) 2 Hz (d) 1 Hz

Sol. Ans : c

Beat frequency $= 2(256) - 3(170) = 512 - 510 = 2 Hz$

88. A ball is bouncing down a flight of stairs. The coefficient of restitution is e. The height of each step is d and the ball bounces one step at each bounce. After each bounce the ball rebounds to a height h above the next lower step. Neglect width of each step in comparison to h and assume the impacts to be effectively head on

(a) $\frac{h}{d} = 1 - e^2$ (b) $\frac{h}{d} = 1 - e$ (c) $\frac{h}{d} = \frac{1}{1 - e^2}$ (d) $\frac{h}{d} = \frac{1}{1 - e}$

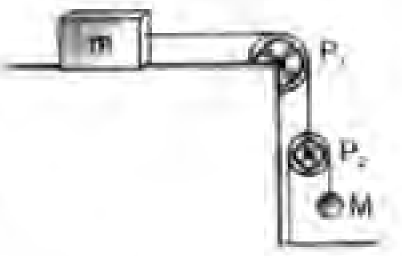
Sol. Ans : c

The ball falls a distance h from the highest point from rest and rebounds to a height $(h - d)$. By definition

$$e = \frac{\text{velocity after collision}}{\text{velocity before collision}} \Rightarrow e = \frac{\sqrt{2g(h-d)}}{\sqrt{2gh}} \Rightarrow \frac{h-d}{h} = e^2$$

$$\Rightarrow 1 - \frac{d}{h} = e^2 \Rightarrow \frac{h}{d} = \frac{1}{1 - e^2}$$

89. In the pulley arrangement shown, the pulley P_2 is movable. Assuming coefficient of friction between m and surface to be μ, the minimum value of M for which m is at rest is

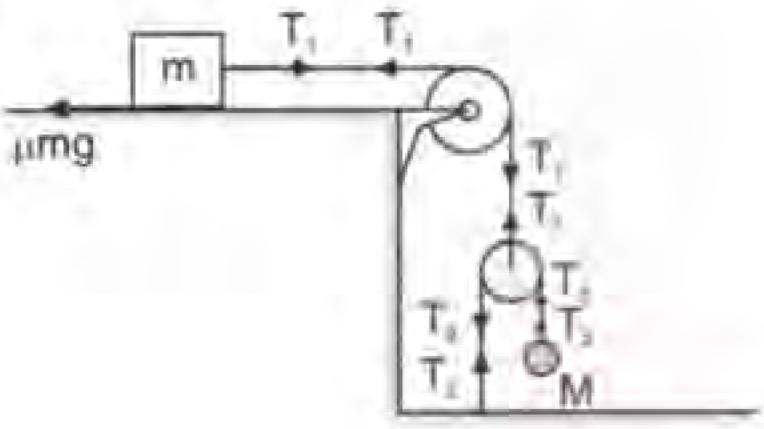


(a) $M = \frac{\mu m}{2}$ (b) $m = \frac{\mu M}{2}$

(c) $M = \frac{m}{2\mu}$ (d) $m = \frac{M}{2\mu}$

Sol. Ans : a

On displacing m through x towards right, M gets displaced downward by $2x$



For equilibrium

$T_0 = \mu mg$ ....(1)

$T_1 = 2T_0$ ....(2)

$T_3 = Mg$ ....(3)

$$\Rightarrow 2(Mg) = \mu mg \Rightarrow M = \frac{1}{2}\mu m$$

90. The dimensional formula for thermal resistance is

(a) $ML^2T^{-3}K^{-1}$ (b) $ML^2T^{-2}A^{-1}$ (c) $ML^2T^{-3}K^{-2}$ (d) $M^{-1}L^{-2}T^3K$

Sol. Ans : d

Thermal Resistance $= \frac{\text{Temperature Difference}}{\text{Thermal Current}}$

Thermal Resistance $= \frac{\text{Temperature Difference}}{\text{Rate of flow of heat}} = \frac{\Delta T}{\left(\frac{\Delta Q}{\Delta t}\right)}$

$$[\text{Thermal Resistance}] = \frac{K}{\left(\frac{ML^2T^{-2}}{T}\right)} = M^{-1}L^{-2}T^3K$$