SOLUTIONS
MODEL TEST PAPER
AIEEE–2010

BOOKLET CODE (A)

Note: ***(i)*** The test is of 3 **hours** duration.
(ii) The test consists of **90** questions. The maximum marks are **432.**
(iii) There are ***three*** parts in the question paper. The distribution of marks subjectwise in each part is as under for each correct response.
Part A − Physics (144 marks) − Question No. 1 to 2 and 9 to 30 consists FOUR (4) marks each and Question No. 3 to 8 consists EIGHT (8) marks each for each correct response.
Part B − Chemistry (144 marks) − Question No. 31 to 39 and 46 to 60 consists FOUR (4) marks each and Question No. 40 to 45 consists EIGHT (8) marks each for each correct response.
Part C − Mathematics(144 marks) − Question No. 61 to 82 and 89 to 90 consists FOUR (4) marks each and Question No. 83 to 88 consists EIGHT (8) marks each for each correct response.
(iv) Candidates will be awarded marks as stated above for correct response of each question. 1/4th marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an
item in the answer sheet.

PHYSICS

PART − A

1. According to Bohr's theory, the radius of the m^{th} orbit of an atom of atomic number Z is given by

$$r_n = \frac{\varepsilon_0 m^2 h^2}{\pi M e^2 Z} = \frac{m^2}{Z}(0.53)\ \text{Å}$$

According to the problem,

$$(r_n)_{Fm} = \frac{m^2(0.53)\,Å}{100} = (0.53\ Å) \times n$$

For $Z = 100,\ m = 5$

$$\Rightarrow \quad n = \frac{25}{100} = \frac{1}{4}$$ ∴ (d) is correct

2. The decay can be represented as

$$M^{220} \rightarrow M^{216} + \alpha^4$$

Here Atomic Number of parent metal = A = 220

Applying conservation of energy,

$$(K.E.)_\alpha = \frac{(A - 4)}{A}Q$$

$$= \frac{216}{220} \times 5.5 = 5.4\ \text{MeV}$$ ∴ (b) is correct

3. Heat lost by ice = Heat gained by water

Heat gained by water to come from 20°C to 0°C

$$= 5 \times 1 \times 20 = 100\ \text{kcal}$$

Amount of heat lost by ice to come from – 20°C to 0°C

$$= 2 \times 20 \times 0.5 = 20\ \text{kcal}$$

This 20 kcal will be utilized to bring down the temperature of a part of water from 20°C to 0°C. The remaining amount of heat will come from melting of ice.

Amount of heat from melting of ice = 100 – 20 = 80 kcal

$$\text{Mass of ice melted} = \frac{80}{Lf} = \frac{80}{80} = 1\,\text{kg}$$

Final mass of water = 5 + 1 = 6 kg ∴ (b) is correct

4. Using Snell's law,

$$n_1 \sin r = n_2 \sin i$$

i.e. $1 \sin 90° = \mu_{glass} \sin i$

$$\Rightarrow \quad \mu_{glass} = 1 / \sin i$$ ∴ (b) is correct

5. AB is isothermal process and BC is isobaric process. In process AC, pressure decreases and temperature increases, which means volume increases more rapidly. Keeping the above facts in mind (a) is the most suitable P-V diagram.

$\therefore$ (a) is correct

6. Electric lines of force emerge radially outwards from the charge and are not present inside the conductor, as field is zero there.

$\therefore$ (b) is correct

7. Potential energy is

$$U = -MB\cos\theta = -iAB\cos\theta$$

$\therefore$ $U_1 = -iAB\cos 180° = +iAB$

$U_2 = -iAB\cos 90° = 0$

$$U_3 = -iAB\cos 135° = \frac{iAB}{\sqrt{2}}$$

and $U_4 = -iAB\cos(-45°) = \dfrac{iAB}{\sqrt{2}}$

Thus $1 > 3 > 2 > 4$ $\therefore$ (b) is correct

8. To verify Ohm's Law, voltmeter is connected in parallel and ammeter in series.

$\therefore$ (d) is correct

9. Increase in length of aluminium rod,

$$\Delta l_A = l_1 \alpha_A t$$

Increase in length of steel rod

$$\Delta l_S = l_2 \alpha_S t$$

According to given condition,

$$\Delta l_A = \Delta l_S$$

$\therefore$ $l_1 \alpha_A t = l_2 \alpha_S t$

or $\dfrac{l_1}{l_2} = \dfrac{\alpha_A}{\alpha_S}$

or $\dfrac{l_1}{l_1 + l_2} = \dfrac{\alpha_A}{\alpha_A + \alpha_S}$

$\therefore$ (d) is correct

10. **c**

11. Power dissipated $\propto R_{equivalent}$

(I) $R_{eq} = R + R + R = 3R$

(II) $R_{eq} = 2R/3$

(III) $R_{eq} = R/3$

(IV) $R_{eq} = 3R/2$

Thus increasing order of power dissipation is

$I > IV > II > III$ $\therefore$ (a) is correct

12. Since the gravitational field is conservative in nature, therefore work done is path independent. Thus work done in the three cases will be equal

i.e. $W_1 = W_2 = W_3$ $\therefore$ (a) is correct

13. (a) is correct

14. When radius of the wire is changed, it will not change the ratio of the resistance of two arms. Since the null deflection point depends upon the ratio of resistances and not upon the resistance itself, hence null deflection point will not change and will remain x.

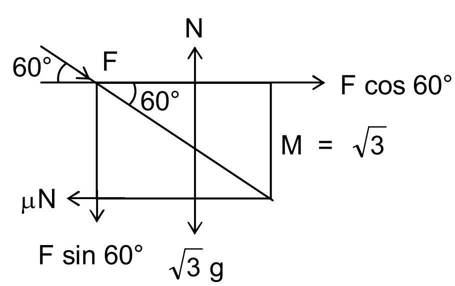
$\therefore$ (d) is correct

15. Since, Mass = volume $\times$ density

and density of nucleus is constant

$\Rightarrow$ $V \propto$ Mass $\therefore$ (b) is correct

16. From the adjoining figure,



$$N = F \sin 60° + \sqrt{3}\,g \quad \text{.......(i)}$$

and $$F \cos 60° = \mu N \quad \text{.......(ii)}$$

From equations (i) and (ii), we get

$$F \cos 60° = \frac{1}{2\sqrt{3}}(F \sin 60° + \sqrt{3}\,g)$$

$$= \frac{1}{2\sqrt{3}}\left(F\frac{\sqrt{3}}{2} + \sqrt{3} \times 10\right) = \frac{F}{4} + 5$$

$$\Rightarrow \quad \frac{F}{2} = \frac{F}{4} + 5$$

$$\Rightarrow \quad \frac{F}{4} = 5$$

$\Rightarrow$ $F = 20\text{ N}$ $\therefore$ (b) is correct

17. The loop will expand due to radially outward magnetic force. $\therefore$ (d) is correct

18. For simple harmonic motion,

$$x = A\cos\omega t$$

On differentiating twice with respect to t, we get

$$\frac{d^2x}{dt^2} = -\omega^2 t$$

Restoring force = $-m\omega^2 x$

Work done $= -\text{P.E.} = \frac{1}{2}m\omega^2x^2$

So when $x = 0$, P.E. $= 0$ *(minimum)*

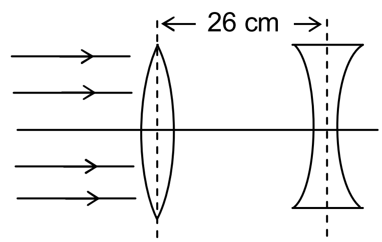
and when $x = \pm A$, P.E. $= \frac{1}{2}m\omega^2A^2$ *(maximum)*

Thus I and III represent the correct graph. $\therefore$ (a) is correct

19. As angular velocity ω is given constant, so angular momentum L will be constant if moment of inertia I is constant, which is about an axis passing through centre.

$\therefore$ (a) is correct

20. For convex lens,



$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{30} = \frac{1}{v} - \frac{1}{\infty}$$

$$\Rightarrow \quad v = 30\text{ cm}$$

This image acts as object for concave lens. Therefore the distance of object from concave lens

$$= 30 - 26 = 4\text{ cm}$$

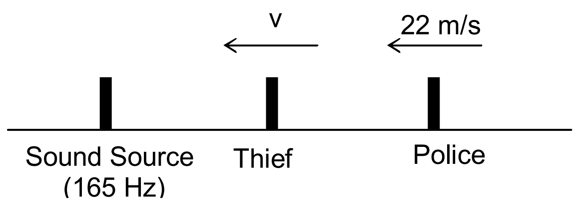
For concave lens,

$$\frac{1}{v} - \frac{1}{4} = \frac{1}{-20} \quad \Rightarrow \quad v = 5\text{ cm}$$

Also, $m = \left|\frac{v}{u}\right| = \left|\frac{h_2}{h_1}\right|$

$\Rightarrow \quad h_2 = 2 \times \frac{5}{4} = 2.5 \text{ cm}$ $\quad \therefore$ (a) is correct

21. The given situation can be represented as



Let f_1 = frequency of the police car heard by motorcyclist

and f_2 = frequency of the siren heard by motorcyclist

$\therefore \quad f_1 = \frac{300 - v}{330 - 22} \times 176$

and $\quad f_2 = \frac{330 + v}{330} \times 165$

ő $\quad f_1 - f_2 = 0$

$\Rightarrow \quad v = 22$ m/s $\quad \therefore$ (b) is correct

22. Plane of motion must be perpendicular to at least one of the component of the magnetic field, and since it is non-circular hence electric field must be there along x-axis.

Only $\vec{E} = a\hat{i}$, $\vec{B} = c\hat{k} + a\hat{i}$ fulfill these two conditions.

$\therefore$ (b) is correct

23. Applying conservation of angular momentum about centre of the rod,

$$\frac{Jl}{2} = I_{cm}\,\omega$$

$$\Rightarrow \quad Mv\frac{l}{2} = 2\left[M\left(\frac{l}{2}\right)^2\right]\omega$$

or $\quad \frac{vl}{2} = 2\,\frac{l^4}{4}\,\omega$

$\Rightarrow \quad \omega = \frac{v}{l}$ $\quad\therefore$ (b) is correct

24. Since, $L + e = \frac{\lambda}{4}$

$\therefore \quad 0.1 + e = \frac{\lambda}{4}$

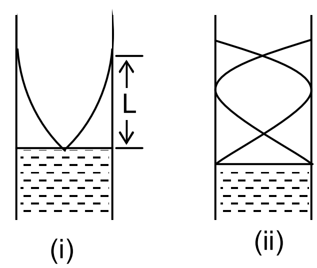
$\Rightarrow \quad 0.35 + e = \frac{3\lambda}{4}$

$\Rightarrow \quad 0.35 + e = \frac{3}{4}(4 + 4e)$

$\Rightarrow \quad 0.35 + e = 0.3 + 3e$

$\Rightarrow \quad 0.05 = 2e$

$\Rightarrow \quad e = 0.025$ $\quad\therefore$ (a) is correct



25. Young's modulus [Y] is

$$\frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta l / L}$$

$$= \frac{20 \times 1}{10^{-6} \times 10^{-4}} = 2 \times 10^{11}\ \text{N/m}^2$$ $\therefore$ (c) is correct

26. As rate of heat transfer,

$$\frac{dQ}{dt} = -ms\frac{dT}{dt} = \sigma eAT^4$$

or $\frac{dQ}{dt} = \sigma eAT^4$

$\therefore$ Emissive power, $E \propto A$

or $E_x < E_y \Rightarrow A_x < A_y$

Thus, $E_x < E_y$; $A_x < A_y$ is the correct relation. $\quad\therefore$ (d) is correct

27. Here, $v = l^3 = (1.2 \times 10^{-2}\ m)^3$

$= 1.728 \times 10^{-6}\ m^3$

Since l has two significant figures, hence v will also have two significant figures i.e. $v = 1.7 \times 10^{-6}\ m^3$.

$\therefore$ (a) is correct

28. (b) $i = E / R$ $\therefore$ (a) is correct

29. 2R $\therefore$ (c) is correct

30. $i_1 = i_0 e^{-t/3RC}$

$i_2 = i_0 e^{-t/RC}$

$\frac{i_1}{i_2} = e^{\frac{2t}{3RC}}$ i.e. increasing with (t) $\therefore$ (b) is correct

CHEMISTRY

PART – B

31. Number of atoms on 24 g of C (12)

$$= \frac{24}{12} \times 6.023 \times 10^{23} = 2 \times 6.023 \times 10^{23} \text{ atoms}$$

Number of atoms in 56 g of Fe (56)

$$= \frac{56}{56} \times 6.023 \times 10^{23} = 6.023 \times 10^{23} \text{ atoms}$$

Number of atoms in 27 g of Al (27)

$$= \frac{27}{27} \times 6.023 \times 10^{23} = 6.023 \times 10^{23} \text{ atoms}$$

Number of atoms in 108 g of Ag (108)

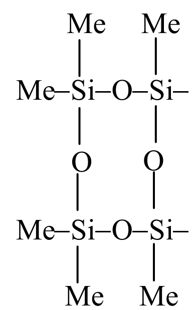
$$= \frac{108}{108} \times 6.023 \times 10^{23} = 6.023 \times 10^{23} \text{ atoms}$$ $\therefore$ (a) is correct

32. Freezing point of a liquid substance is defined as the temperature at which the vapour pressure of its liquid is equal to the vapour pressure of the corresponding solid. The addition of non-volatile solute always lowers the freezing point. At the freezing point, liquid and solid are in equilibrium. On cooling a solution of a non-volatile solute below the freezing point of solution, some of liquid will separate as a solid solvent.

$\therefore$ (a) is correct

33. Me_2SiCl_2 on hydrolysis will produce $Me_2Si(OH)_2$ which ultimately, upon loss of water, will form $Me_2Si{=}O$. But silicon atom, because of its very large size in comparison to oxygen, is unable to form π-bond. Thus, the product of hydrolysis is polymeric in nature.

$$Me_2SiCl_2 + 2H_2O \rightarrow Me_2Si(OH)_2 \rightarrow$$



$\therefore$ (c) is correct

34. When optically active acid reacts with racemic mixture of an alcohol, it forms two types of isomeric esters. In each the configuration of the chiral centre of acid will remain the same. So the mixture will be optically active.

$\therefore$ (a) is correct

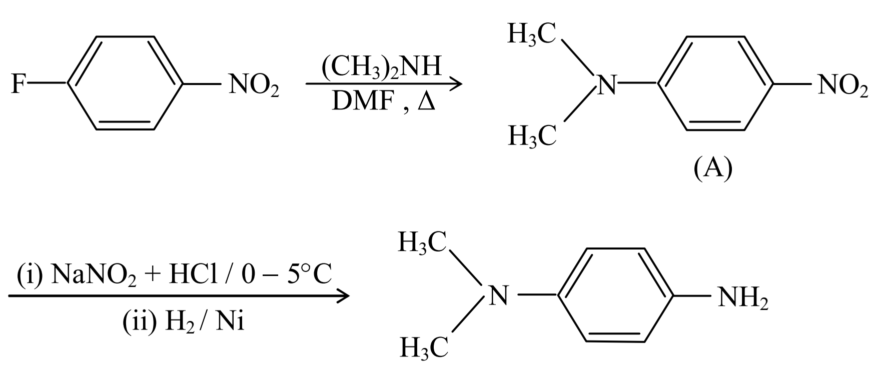
35. SO_2 and H_2S are both reducing agents. H_2S is obtained by the action of acid upon sulphide while SO_2 is obtained by the action of acid upon sulphite. The option (d) is correct because SO_2 is a colourless gas with irritating smell. However H_2S has rotten egg like smell. The reactions involved are

$$\underset{[A]}{SO_3^{2-}} + H_2SO_4 \rightarrow \underset{[B]}{SO_2} + H_2O + SO_4^{2-}$$

$$3SO_2 + K_2Cr_2O_7 + H_2SO_4 \rightarrow K_2SO_4 + Cr_2(SO_4)_3 + H_2O$$

$\therefore$ (d) is correct

36. The given reaction can be completed as



(B)

∴ (a) is correct

37. When MnO_2 is oxidised to K_2MnO_4 in alkaline medium, it acquires a stable oxidation state, i.e., +6 and forms K_2MnO_4 which is purple green in colour

$$2MnO_2 + 4KOH + O_2 \rightarrow 2K_2MnO_4 + 2H_2O$$ ∴ (b) is correct

38. Number of electrons in $NO_3^- = 7 + 24 + 1 = 32$

Number of electrons in $CO_3^{2-} = 6 + 24 + 2 = 32$

Both have same number of electrons. So they are isoelectronic.

Total number of valence electrons in $NO_3^- = 5 + 6 \times 3 + 1 = 24$

Total number of valence electrons in $CO_3^{2-} = 4 + 6 \times 3 + 2 = 24$

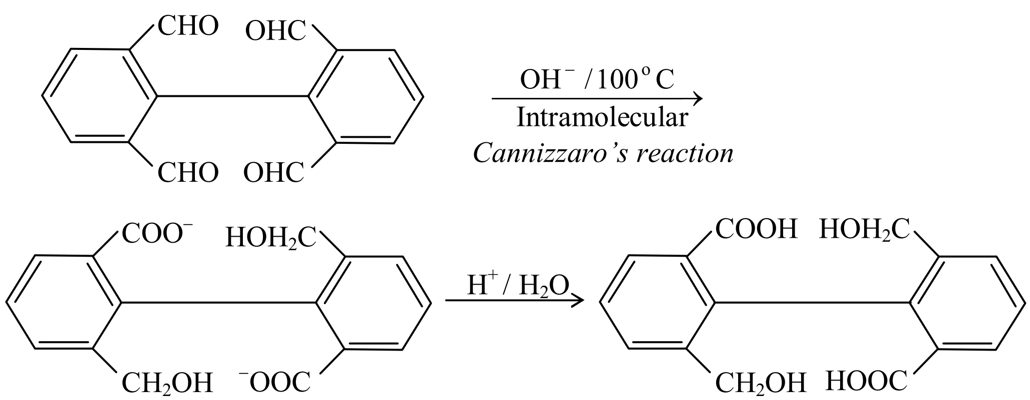
Now, $\frac{24}{8} = 3\ (Q_1) + 0\ (R_1)$

Thus type of hybridisation is sp^2

Central atom in each being sp^2 hybridised, thus they are isostructural also.

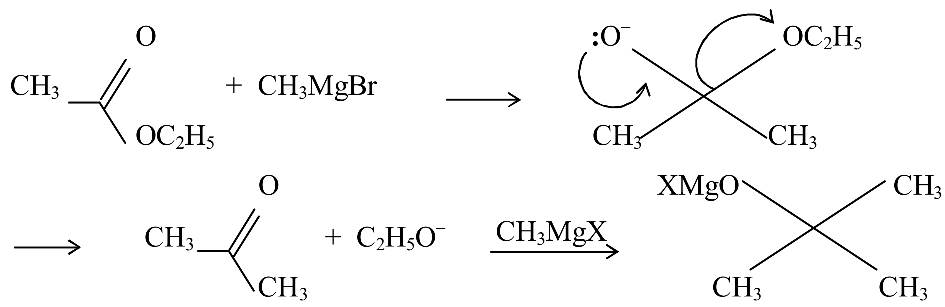
∴ (a) is correct

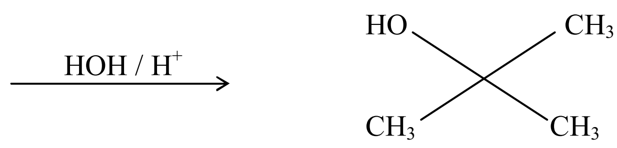
39. The desired reaction can be expressed as



∴ (d) is correct

40. The reaction of Ethyl exter with CH_3MgBr is excess, can be expressed as





∴ (a) is correct

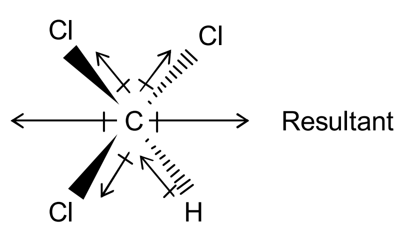
41. Heat of formation is the amount of heat evolved or absorbed when one mole of the compound is formed from its elements. When all the substances are taken in their standard state i.e. 298 K and 1 atm pressure, it is called standard heat of formation (ΔH_f^o).

Thus in given thermochemical equations, (b) represents the standard heat of formation of HF.

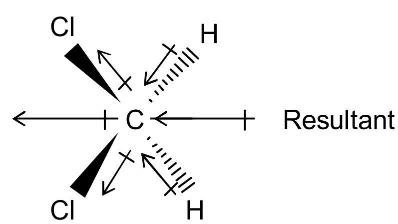
$$\frac{1}{2}H_2(g) + \frac{1}{2}F_2(g) \rightarrow HF(g)\ ;\ \Delta H_f^o,\ \text{standard heat of formation of HF}(g)$$

∴ (b) is correct

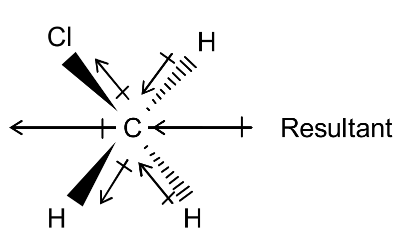
42. The dipole moment of a polar molecule depends on its geometry and shape. A symmetrical molecule is non-polar even though it contains polar bonds. CH_4 being symmetrical has zero resultant dipole moment. The bond dipole moment of C–H bond and that of C–Cl bond reinforce one-another.



$CHCl_3$ (1.03 D)



CH_2Cl_2 (1.62 D)

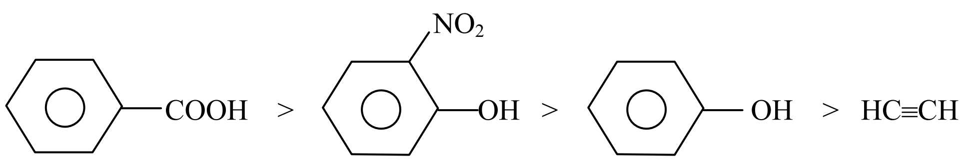


CH_3Cl (1.86 D)

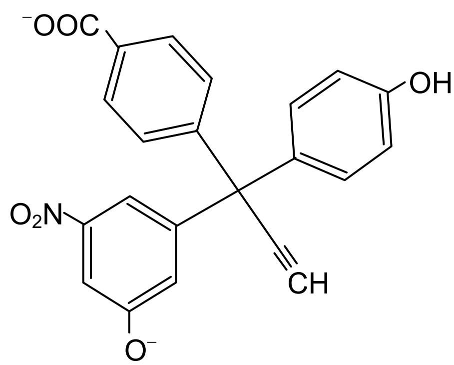
In $CHCl_3$, the resultant of C–H and C–Cl dipoles opposes the resultant of two C–Cl dipoles while in CH_2Cl_2, the resultant of C–H dipoles adds to the resultant of two C–Cl dipoles. In case of CH_3Cl, the resultant of two C–H dipoles adds to the resultant of C–H and C–Cl dipoles . Thus dipole moment of CH_3Cl is highest among the given compounds. The molecule (CCl_4) again becomes symmetrical and dipole moment reduces to zero.

∴ (c) is correct

43. The acidic strength of the attached groups are in the following order.



Two moles of amide ions will attract two moles of most acidic hydrogen and the obtained product will be

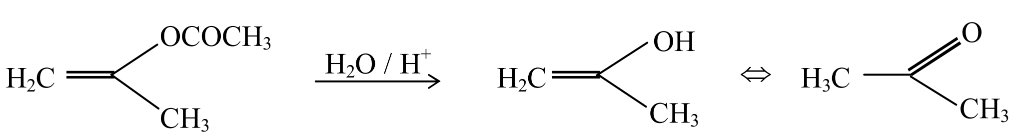


∴ (a) is correct

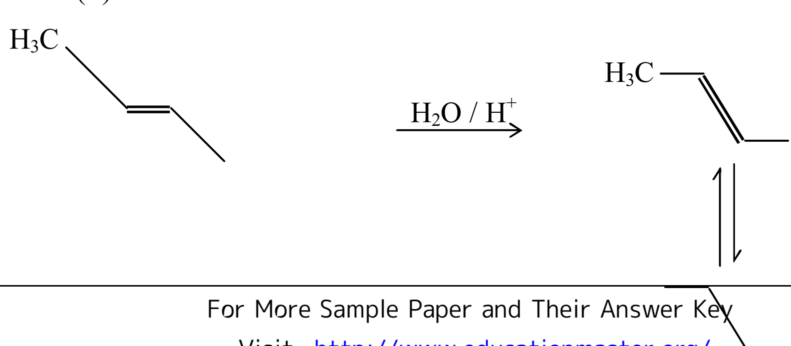
44. The physical adsorption occurs readily at low temperature and increases with decreasing temperature as the adsorption process is exothermic (Le-Chatelier's principle). Since the activation energy in the physical adsorption is more or less zero, the rate of adsorption is not affected even at low temperature.

∴ (b) is correct

45. Acid hydrolysis of P and Q can be expressed as



(P)



OH

$OCOCH_5$

(Q)

H_3C

CHO

Ketone (non-reducing) and aldehyde (reducing) can be distinguished by Fehling's solution as we know that ketone does not react with Fehling's solution.

∴ (c) is correct

46. The rate constant of the first order reaction

$$K = \frac{2.303}{t}\log\frac{N_0}{N}$$

From the given data, we can write

$$K = \frac{2.303}{2\times 10^4}\log\frac{800}{50} = 1.386\times 10^{-4}\ s^{-1}$$

∴ (c) is correct

47. $^{24}_{21}N$ decays into $^{23}_{11}Na$ via radioactive decay. The decay reactions are

$${}_{11}Na^{24} \rightarrow {}_{11}Na^{23} + {}_0n^1$$

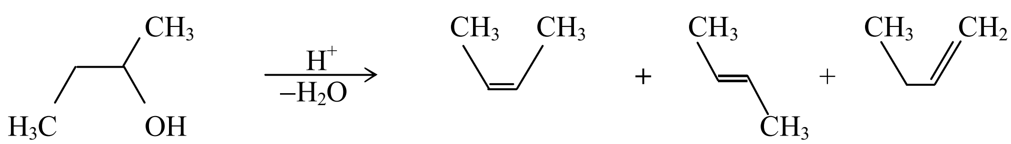
$${}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0 \text{ or } \beta^-$$

Thus the reaction involves β^- emission. ∴ (a) is correct

48. The salt with lower value of K_{sp} precipitates first. Here HgS has the lowest value of K_{sp} hence it will precipitate first.

∴ (c) is correct

49. The desired reaction is

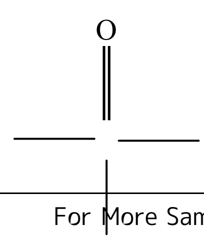


Cis-Butene-2 *(1)* · *Trans-Butene-2* *(2)* *Major* · *Butene-1* *(3)*

Hence only three structures shown above and possible.

∴ (d) is correct

50. The molecular structure of H_3PO_3 is as shown below



H P OH

There are only two –OH groups which means that at the most two hydrogen atoms can be replaced and hence it is dibasic. Here the oxidation number of P is +3. P may also have +5 oxidation state in some compounds. Increase in oxidation number represents that H_3PO_3 can be oxidised and hence it is a reducing agent.

$\therefore$ (c) is correct

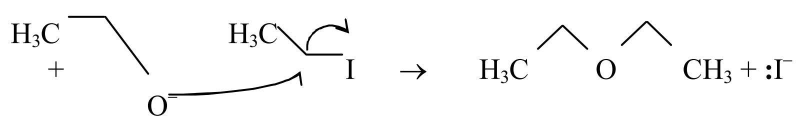
51. After examining the four given structures we can easily see that only (a) has the desired hybridization.

$H_2C{=}CH{-}C{\equiv}N$ *(from left to right)*

sp^2 sp^2 sp sp

$\therefore$ (a) is correct

52. $C_2H_5O^-$ will abstract proton from phenol, converting the phenol into phenoxide ion. This would then make nucleophilic attack on the methylene carbon of alkyl iodide. But $C_2H_5O^-$ is in excess. $C_2H_5O^-$ is better nucleophile than $C_6H_5O^-$ (phenoxide) ion, since in the former the negative charge is localised over oxygen and in the latter it is delocalised over the whole molecular framework. So it is $C_2H_5O^-$ ion that would make nucleophilic attack at ethyl iodide to give diethyl ether (Williamson's synthesis).



Thus the product is $C_2H_5OC_2H_5$ $\therefore$ (b) is correct

53. The extent of deviation of a real gas from an ideal behaviour is expressed in terms of compressibility factor, Z, defined as

$$Z = \frac{PV}{nRT}$$

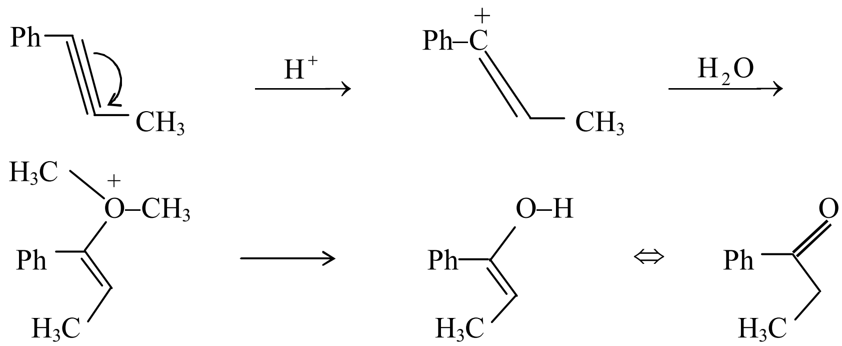
For positive deviation, $Z > 1$. $\therefore$ (a) is correct

54. The $AgNO_3$ and $BaCl_2$ in excess releases Ag^+ and Ba^{++} respectively, in solution in excess and the two salts of mixture release Br^- and SO_4^{--} respectively. The reaction can be represented as

Ag^{+} + Br^{-} → AgBr
Excess *0.01 mole* *0.01 mole*

Ba^{2+} + SO_4^{2-} → $BaSO_4$
Excess *0.01 mole* *0.01 mole* ∴ (d) is correct

55. The desired reaction can be written as



(A)

∴ (a) is correct

56. The central boron atom in H_3BO_3 is electron-deficient. Therefore it accepts a pair of electrons, hence weak Lewis acid. There is no d-orbital of suitable energy in boron atom. So, it can accommodate only one additional electron pair in its outermost shell. Thus it is monobasic weak Lewis acid.

$B(OH)_3 + H_2O \Leftrightarrow [B(OH)_4]^{-} + H^{+}$ ∴ (c) is correct

57. The complete set of reactions of gold extraction can be written as

$$2Au + 4CN^{-} + H_2O + \frac{1}{2}O_2 \rightarrow 2[Au(CN)_2]^{-} + 2OH^{-}$$

$$2[Au(CN)_2]^{-} + Zn \rightarrow [Zn(CN)_4]^{2-} + 2Au$$

Hence on comparison we find

$X = [Au(CN)_2]^{-}$, $Y = [Zn(CN)_4]^{2-}$ ∴ (a) is correct

58. In an electrolytic cell, electrolysis of an electrolyte takes place by passage of electric current through its aqueous solution or molten state. Current flows by the migration of ions towards oppositely charged electrodes. In electrolytic cell, flow of electrons takes place from cathode to anode through internal supply.

∴ (c) is correct

59. (d) is correct

60. (d) is correct

MATHEMATICS

PART – C

61. Here $4x + x + x = 180$

$\Rightarrow$ $6x = 180$

$\Rightarrow$ $x = 30$

From sine rule,

$$\frac{\sin 120}{a} = \frac{\sin 30}{b} = \frac{\sin 30}{c}$$

$\therefore$ $a : a + b + c = (\sin 120) : (\sin 120 + \sin 30 + \sin 30)$

$$= \frac{\sqrt{3}}{2} : \frac{\sqrt{3}+2}{2}$$

$$= \sqrt{3} : \sqrt{3} + 2$$

$\therefore$ (c) is correct

62. The given function I (m, n) is defined as

$$I(m, n) = \int_0^1 t^m (1+t)^n \, dt$$

$$= \left[(1+t)^n \cdot \frac{t^{m+1}}{m+1}\right]_0^1 - \frac{n}{m+1}\int_0^1 (1+t)^{n-1} t^{m+1}$$

$\therefore$ $I(m, n) = \frac{2^n}{m+1} - \frac{n}{m+1} I\,[(n-1), (m+1)]$

$\therefore$ (c) is correct

63. Given parabola is

$$\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$$

Eccentricity $= \sqrt{1 + \tan^2\alpha} = \sec\alpha$

Vertex $= (\pm a, 0) = (\pm \cos\alpha, 0)$

Focus $= (\pm ae, 0) = (\pm 1, 0)$

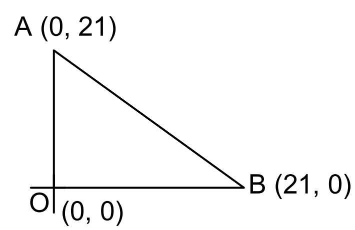
Directrix $= x = \pm\frac{a}{e}$

$\therefore \quad x = \pm \cos^2\alpha$

Thus only abcissa of foci is independent of α $\therefore$ (b) is correct

64. Equation of the line AB can be written as

$$x + y = 21$$



The number of integral solutions to the equation

$$x + y < 21$$

i.e. $x < 21 - y$

Thus, number of integral coordinate

$$= 19 + 18 + \ldots\ldots + 1$$

$$= \frac{19 \times 20}{2} = 190$$ $\therefore$ (a) is correct

65. Volume of parallelopiped formed by vectors

$$= \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 1 - a + a^3$$

$$\frac{dV}{da} = 0 = 3a^2 - 1$$

$\therefore \quad a = \pm \frac{1}{\sqrt{3}}$

Thus V will be minimum at $a = \frac{1}{\sqrt{3}}$ $\therefore$ (c) is correct

66. Equation of the tangent to the given ellipse can be written as

$$x\frac{(3\sqrt{3}\cos\theta)}{27} + y\frac{\sin\theta}{1} = 1$$

$$\Rightarrow \quad \frac{x\cos\theta}{3\sqrt{3}} + \frac{y\sin\theta}{1} = 1$$

Sum of intercepts,

$$l = \frac{3\sqrt{3}}{\cos\theta} + \frac{1}{\sin\theta}$$

$\therefore \quad \dfrac{dl}{d\theta} = 3\sqrt{3}\ \sec\theta\tan\theta - \cot\theta\ \text{cosec}\,\theta = 0$

or $\quad \dfrac{3\sqrt{3}\sin\theta}{\cos^2\theta} - \dfrac{\cos\theta}{\sin^2\theta} = 0$

$\therefore \quad 3\sqrt{3}\tan^3\theta - 1 = 0$

or $\quad \tan^2\theta = \dfrac{1}{3^{3/2}}$

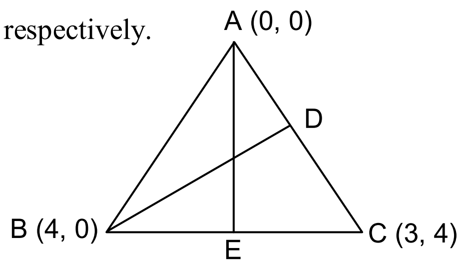
$\Rightarrow \quad \tan\theta = \dfrac{1}{\sqrt{3}}$

$\therefore \quad \theta = \dfrac{\pi}{6}$

$\therefore$ (a) is correct

67. Let A (0, 0), B (4, 0) and C (3, 4) be the triangle.

AE and BD are perpendiculars on BC and AC respectively.



Since $\quad BD \perp AC$

$\therefore \quad$ Slope of BD $= -\dfrac{3}{4}$

Equation of BD is $3x + 4y - 12 = 0$ ……(i)

Similarly $\quad AE \perp BC$

$\therefore \quad$ Slope of AE $= \dfrac{1}{4}$

Equation of AE is $x - 4y = 0$ ……..(ii)

By equations (i) and (ii),

$x = 3, \quad y = \dfrac{3}{4}$

Thus the orthocentre of the triangle is $\left(3, \dfrac{3}{4}\right)$ $\therefore$ (d) is correct

68. Since we know that

$x + \dfrac{1}{x} \geq 2\ ; \quad \forall\ x > 0$

or $\dfrac{x + \dfrac{1}{x}}{2} \geq 1$

$\therefore$ $\dfrac{\sqrt{x^2 + x} + \dfrac{\tan^2\alpha}{\sqrt{x^2 + x}}}{2} \geq \sqrt{\tan^2\alpha}$

$\therefore$ $\sqrt{x^2 + x} + \dfrac{\tan^2\alpha}{\sqrt{x^2 + x}} \geq 2\tan\alpha$ $\therefore$ (c) is correct

69. Since $A^2 = B$

$\therefore$ $A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

Comparing, we get

$\alpha^2 = 1$ or $\alpha = \pm 1$ ....(i)

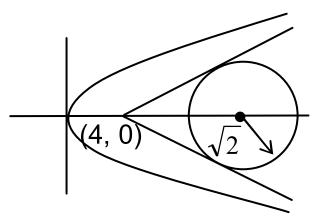
and $\alpha + 1 = 5$ or $\alpha = 4$ ....(ii)

Since the two values of α do not match, thus no real value of α exists

$\therefore$ (d) is correct

70. Let the equation of the chord be

$y = m(x - 4)$



Length of perpendicular dropped from (6, 0) $= \sqrt{2}$

$\therefore$ $\left|\dfrac{m(6-4) - 0}{\sqrt{1 + m^2}}\right| = \sqrt{2}$

$\Rightarrow$ $\dfrac{(2m)^2}{1 + m^2} = 2$ or $\dfrac{4m^2}{1 + m^2} = 2$

$\Rightarrow$ $2m^2 = 2$

$\Rightarrow$ $m = \pm 1$ $\therefore$ (a) is correct

71. Point must lie on both line and plane

$\therefore$ $2(4) - 4(2) + k - 7 = 0$

or $k = 7$

But $a_1a_2 + b_1b_2 + c_1c_2 = 0$

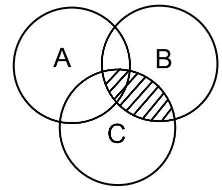
$\therefore$ $2(1) - 1(4) + 2(1) = 0$

Thus $k = 7$ satisfies ∴ (a) is correct

72. Since $P(B \cap C) = P(B) - P(\bar{A} \cap B \cap \bar{C}) - P(A \cap B \cap \bar{C})$

$$\therefore \quad P(B \cap C) = \frac{3}{4} - \left(\frac{1}{3} + \frac{1}{3}\right)$$

$$= \frac{9-8}{12} = \frac{1}{12}$$



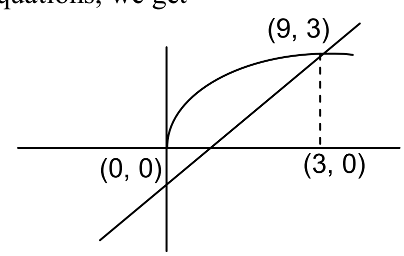
∴ (a) is correct

73. Eliminating x between the two given equations, we get

$$y = \sqrt{2y + 3}$$

$$\Rightarrow \quad y^2 - 2y - 3 = 0$$

$$\Rightarrow \quad (y-3)(y+1) = 0$$



Area bounded by the curve

$$= \int_0^9 \sqrt{x}\, dx$$

$$= \left.\frac{x^{3/2}}{3/2}\right|_0^9 = \frac{2}{3} \times 9^{3/2}$$

$$= \frac{2}{3} \times 27 = 18$$

∴ (b) is correct

74. Given function $f(x) = \int_{x^2}^{x^2+1} e^{-t^2}\, dt$

On differentiation, we get

$$f'(x) = 2x\left[e^{-(x^2+1)^2} - e^{-(x^2)^2}\right]$$

$$= 2x\, e^{-(x^4+1+2x^2)}(1 - e^{2x^2+1})$$

$\therefore \quad f'(x) > 0, \quad \text{for } x \in (-\infty, 0)$ ∴ (b) is correct

75. Let the range of the function be y

Hence $\quad y = \dfrac{x^2 + x + 2}{x^2 + x + 1}$

$\Rightarrow \quad y\,(x^2 + x + 1) = x^2 + x + 2$

$\Rightarrow \quad x = \dfrac{(1-y) \pm \sqrt{(y-1)^2 - 4\,(y-1)\;(y-2)}}{2\,(y-1)}$

or $\quad (y-1)^2 - 4\,(y-1)\,(y-2) > 0$

$\Rightarrow \quad y > 1,\;\; y \le \dfrac{7}{3}$

Thus the range of $f(x)$ is $\left(1, \dfrac{7}{3}\right]$ $\qquad \therefore$ (d) is correct

76. If a function obeys mean value theorem then it must be differentiable in the interval on which it is defined.

The function (a) is not differentiable at $x = 1/2$ $\qquad \therefore$ (a) is correct

77. The given differential equation is

$$(t+1)\,\frac{dy}{dt} - ty = 1$$

$\Rightarrow \quad \dfrac{dy}{dt} - \dfrac{t}{t+1}y = \dfrac{1}{t+1}$

$\therefore \quad \text{I.F.} = e^{-\int \frac{t+1-1}{t+1}dt} = e^{-t + \log_e(t+1)} = e^{-t} \,.\, (t+1)$

Thus the solution is

$\therefore \quad y\,(e^{-t})\,(t+1) = \int e^{-1}dt + c$

or $\quad y = \dfrac{-e^{-t} + c}{e^{-t}(t+1)}$

Since $\quad y(0) = -1$ *(given)*

$\Rightarrow \quad -1 = \dfrac{-1 + c}{1\,(1)}$

or $\quad c = 0$

Thus $y = -\frac{e^{-t}}{e^{-t}(t+1)}$

or $y = \frac{-1}{t+1}$

$\Rightarrow$ $y(1) = -\frac{1}{2}$ $\therefore$ (b) is correct

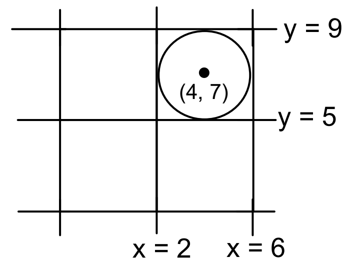
78. Given pair of straight lines is

$y^2 - 14y + 45 = 0$

$\Rightarrow$ $(y-5)(y-9) = 0$

and $x^2 - 8x + 12 = 0$

$\Rightarrow$ $(x-6)(x-2) = 0$

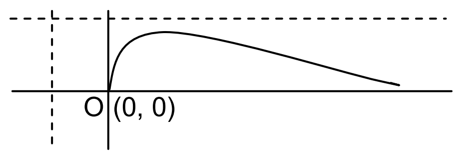


The given square is shown in figure.

Thus centre is (4, 7) $\therefore$ (b) is correct

79. For each value of x, there is one value of f (x) and range $\subset$ [0, 1]

Thus f (x) is one-one not onto, which is also evident from the adjoining figure.



$\therefore$ (b) is correct

80. Desired limit is

$$\lim_{h\to 0} \frac{f(2+2h+h^2) - f(2)}{f(1+h-h^2) - f(1)}$$

$\Rightarrow$ $\lim_{h\to 0} \frac{f'(2)(h+2)}{f'(1)(1-h)} = \frac{6}{4} \cdot 2 = 3$ $\therefore$ (c) is correct

81. Since, $(1+t^2)^{12}(1+t^{12})(1+t^{24}) = (1+t^2)^{12}(1+t^{12}+t^{24}+t^{36})$

Hence required coefficient is $^{12}C_6 + 2$ $\therefore$ (d) is correct

82. $Re(\omega) = \frac{\omega + \overline{\omega}}{2} = \frac{z-1}{z+1} + \overline{\frac{z-1}{z+1}}$

$= \frac{|z|^2 - 1}{|z+1|^2} = 0$ $\therefore$ (d) is correct

83. The coefficient determinant equation gives

$$\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad 1 + a^3 = 0$

$\Rightarrow \quad a = -1$ $\qquad \therefore$ (c) is correct

84. Number of ways of selecting two numbers, such that both numbers are greater than 4 is $^3C_2$.

Total number of ways of selection of two numbers is $^6C_2$

$\therefore$ Required probability $= 1 - \dfrac{^3C_2}{^6C_2} = \dfrac{4}{5}$ $\qquad \therefore$ (a) is correct

85. Given ellipse is

$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$

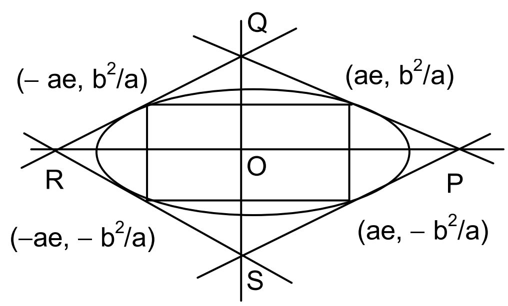
The equation of tangent at (x_1, y_1) can be written as

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

$$e^2 = 1 - \frac{5}{9} = \frac{4}{9}$$

$\Rightarrow \quad e = \dfrac{2}{3}$

one end point of latus rectum (2, 5/3)



Equation of the tangent at (2, 5/3) is

$$\frac{2x}{9} + \frac{y}{3} = 1$$

Area of triangle OPQ $= \dfrac{1}{2} \times \left(\dfrac{9}{2}\right) \times 3 = \dfrac{27}{4}$

Area of quadrilateral PQRS

$= 4 \times$ Area of triangle OPQ $= 4 \times \dfrac{27}{4} = 27$ sq. units.

$\therefore$ (a) is correct

86. The given limit is

$$\lim_{x\to 0} \frac{\sin nx\,[(a-n)\,nx - \tan x]}{x^2} = 0$$

or $$\lim_{x\to 0} \frac{\sin nx}{nx}\left[(a-n)\,n - \frac{\tan x}{x}\right] n = 0$$

$\Rightarrow$ $$an - n^2 - 1 = 0$$

$\Rightarrow$ $$a = n + \frac{1}{n}$$ $\therefore$ (d) is correct

87.(d) is correct

88. (c) is correct

89. (b) is correct

90.(a) is correct