1. **Units**

(a) All physical quantities that we study in physics can be represented as a resultant of some basic or fundamental physical quantities and the units of measurement of these fundamental physical quantities are known as fundamental units.

(b) Fundamental units are well defined, independent of each other and cannot be resolved into any other units.

S.no	Fundamental physical Quantity	Fundamental Unit	Symbol Used.
1.	Mass	Kilogram	kg
2.	Length	Metre	m
3.	Time	Sec.	s
4.	Temperature	Kelvin	K
5.	Electric current	Ampere	A
6.	Luminous Intensity	Candela	Cd
7.	Quantity of matter	Mole	mol

(c) Most of the physical quantities that we shall study in mechanics can be represented in terms of three fundamental quantities namely **Mass**, **length** and **time** which we shall use most often.

2. **System of Units**

(a) **CGS System:** This system was established in France and is based on **centimeter, gram** and **second** as the fundamental units of length, mass and time respectively. In this system, unit of force is dynes, unit of energy is ergs, and so on.

(b) **FPS System:** This system is known as British system of units. It is based on **foot, pound** and **second** as the fundamental units of length, mass and time respectively. In this system, unit of force is poundal, unit of energy is foot-poundal, and so on.

(c) **MKS System:** This system was also established in France. It is based on **metre**, **kilogram** and **second** as the fundamental units of length, mass and time respectively. In this system, unit of force is Newton, unit of energy is Joule, and so on.

3. **International System of Units (S.I)**

(a) The units of mass, length and time can be used to obtain the units of physical quantities in mechanics only. These three fundamental units are not sufficient to obtain the units of physical quantities which come across in other branch of physics like optics, electricity, heat, etc.

(b) The **general conference of weights and measures** held in 1971 decided a new system of units which is known as the **International System of Units**. It is abbreviated as S.I from the French name ***Le Systerne International d' Unites***.

(c) It is based on the seven fundamental units mentioned earlier.

4. **Dimensional Analysis**

(a) We know that derived units of all physical quantities can be obtained from the seven fundamental units. Thus representing mass as $[M]$, length as $[L]$, and time as $[T]$, all physical quantities in mechanics can be expressed in terms of $[M]$, $[L]$, and $[T]$.

(b) ***The dimensions of a physical quantity are the powers to which the fundamental quantities must be raised to represent the given physical quantity.***

For example, $$[\text{Density}] = \frac{[\text{Mass}]}{[\text{Volume}]} = \frac{[M]}{[L^3]} = [ML^{-3}] \text{ or } [M^1L^{-3}T^0]$$

So, the dimensions of density are 1 in mass, - 3 in length and 0 in time.

The dimensional formula of density is thus represented as $[ML^{-3}]$ or $[ML^{-3}T^0]$

(c) The constants such as π, $1/2$ or trigonometric functions such as $\sin\theta$, etc. have no units and dimensions.

5. **Applications of Dimensional Analysis**

(a) **To convert a physical quantity from one system to another.**

(b) **To check the accuracy of a given equation of formula**

- This is based on the principle of homogeneity. According to this principle, the dimensions of each term on both sides of an equation must be the same. (The simple reason for this principle is that mass can be added to mass to give mass and not to length or time)
- If the dimensions of each term on both sides are same, the equation is dimensionally correct, otherwise not.

(c) **To deduce relation among physical quantities.**

- This is also based on the principle of homogeneity. If one knows the quantities on which a particular physical quantity depends and if one guesses that this dependence is of product type, the method of dimensional analysis may be helpful in the derivation of the relation.

- **Example:** Derive an expression for the time period (T) of a simple pendulum which may depend upon the mass (m) of the bob, length (ℓ) of the pendulum and acceleration due to gravity (g).

  Solution: Let $T = km^x \ell^y g^z$ where k is a dimensionless constant.

  Writing the equation in dimensional form, we have $[M^0L^0T^1] = [M]^x [L]^y [LT^{-2}]^z = [M^x L^{y+z} T^{-2z}]$

  Equating exponents of M, L and T on both sides, we get $x = 0$, $y + z = 0$, $-2z = 1$.

  Solving the equations, we get $x = 0$, $y = 1/2$, $z = -1/2$. Hence, $T = km^0 \ell^{1/2} g^{-1/2} = k\sqrt{\ell/g}$

- **Example:** When mass (m) is converted into Energy (E), the energy produced depends upon the mass and speed of light (c). Deduce the formula of E in terms of m and c.

  Solution: Let $E = K\ m^x c^y$ where k is a dimensionless constant.

  Writing the equation in dimensional form, we have $[ML^2T^{-2}] = [M]^x [LT^{-1}]^y = [M^x L^y T^{-y}]$

  Equating exponents of M, L and T on both sides, we get $x = 1$, $y = 2$. Hence, $E = kmc^2$.

6. Errors in Measurement

(a) All measurements are carried out with the help of instruments.
(b) What we need to understand is that it is never possible to make an absolutely accurate measurement of any physical quantity.
(c) An error in measurement may be defined as the difference between the actual value and the measured value of the physical quantity.

7. Some important terms related to Random Errors

(a) **Measured value:** It is the value obtained in a measurement and is denoted by a_i

(b) **Arithmetic Mean:** Let the values obtained in several measurements be $a_1, a_2, a_3, ...a_n$.

Then, the arithmetic mean of these values is given by $a_m = \dfrac{a_1 + a_2 + a_3 + ...a_n}{n} = \sum_{i=1}^{n} \dfrac{a_i}{n}$

(c) **True value:** Actual value is something which nobody knows because no instrument is dead accurate. Every instrument has a least count which gives the degree of its accuracy. So arithematic mean of all the measured values is taken as the true value. It is denoted by a_m

(d) **Absolute Error :** It is the magnitude of the difference between true value and the measured value of a physical quantity. This is denoted by $|\Delta a_i|$ and is always positive. The error in the individual measured value is denoted by a_i and is given by $\Delta a_i = a_m - a_i$

(e) **Mean Absolute Error :** It is the arithmetic mean of all of the aboslute errors. It is denoted by Δa_m or $\overline{\Delta a}$

Let the absolute errors of the individual measurements be $|\Delta a_1|, |\Delta a_2|, |\Delta a_3|, ...|\Delta a_n|$

where,
$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
....
$$\Delta a_n = a_m - a_n$$

Then, the mean absolute error is given by $\Delta a_m = \dfrac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + ...|\Delta a_n|}{n} = \sum_{i=1}^{n} \dfrac{|\Delta a_i|}{n}$

(f) **Relative Error or Fractional Error :** Relative error or Fractional error $= \dfrac{\Delta a_m}{a_m}$

(g) **Percentage error :** Percentage error $= \dfrac{\Delta a_m}{a_m} \times 100$

(h) If we do a single measurement, we generally get a value (a) in the range $a_m \pm \Delta a_m$ i.e., $a_m - \Delta a_m \le a \le a_m + \Delta a_m$.

8. Propagation or Combination of Errors

(a) Experimental determination of a physical quantity (e.g., velocity) may involve the measurement of a large number of various other physical quantities (e.g., displacement, time) Also, various mathematical operation like addition, subtraction, multiplication, division, etc. may be involved.
(b) The errors in individual physical quantities combine which depends upon the type of mathematic operation performed on these physical quantities.

(c) **Error of a sum or a difference**

- ***When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.***
- If $Z = A + B$, then $\Delta Z = \Delta A + \Delta B$

  For example, if $A \pm \Delta A = 100 \pm 2$ and $B \pm \Delta B = 20 \pm 1$ then, $Z \pm \Delta Z = (30 + 20) \pm (2 + 1) = 50 \pm 3$
- If $Z = A - B$, then $\Delta Z = \Delta A + \Delta B$

  For example, if $A \pm \Delta A = 30 \pm 2$ and $B \pm \Delta B = 20 \pm 1$ then, $Z \pm \Delta Z = (30 - 20) \pm (2 + 1) = 10 \pm 3$

(d) **Error of a product or a quotient**

- ***When two quantities are multiplied or divided, the percentage error in the result is the sum of the percentage errors in the individual quantities.***
- If $Z = AB$, then $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$

  For example, if $A \pm \Delta A = 100 \pm 2$ and $B \pm \Delta B = 10 \pm 0.1$

  then, $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B} = \frac{2}{100} + \frac{0.1}{10} = 0.02 + 0.01 = 0.03$

  $\Rightarrow$ $Z = 100 \times 10 = 1000$ and $\Delta Z = 0.03 \times 1000 = 30$ $\therefore$ $(100 \pm 2) \times (10 \pm 0.1) = 1000 \pm 30$
- If $X = \frac{A}{B}$, then $\frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$

  For example, if $A \pm \Delta A = 100 \pm 2$ and $B \pm \Delta B = 10 \pm 0.1$,

  then, $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B} = \frac{2}{100} + \frac{0.1}{10} = 0.02 + 0.01 = 0.03$

  $\Rightarrow$ $Z = 100^2/10 = 10$ and $\Delta Z = 0.03 \times 10 = 0.3$ $\therefore$ $(100 \pm 2)/10 \pm 0.1 = 10 \pm 0.3$

(e) **Error in quantity raised to some power**

- When a quantity is expressed as $Z = A^a B^b C^c$,

  then, $\frac{\Delta Z}{Z} = \left|a\frac{\Delta A}{A}\right| + \left|b\frac{\Delta B}{B}\right| + \left|c\frac{\Delta C}{C}\right|$ where a, b, c are numbers which could be positive or negative.
- **For example**, if $Z = \frac{A^2}{B}$, $A \pm \Delta A = 100 \pm 0.5$ and $B \pm \Delta B = 10 \pm 0.02$ then, $Z = A^2 B^{-1}$

  and hence, $\frac{\Delta Z}{Z} = \left|2\frac{\Delta A}{A}\right| + \left|-1 \times \frac{\Delta B}{B}\right| = \left|2 \times \frac{0.5}{100}\right| + \left|-1 \times \frac{0.1}{10}\right| = 0.01 + 0.01 = 0.02$

  $\Rightarrow$ $Z = 100^2/10 = 1000$ and $\Delta Z = 0.02 \times 1000 = 20$ $\therefore$ $\frac{(100 \pm 0.5)^2}{(10 \pm 0.1)} = 1000 \pm 20$

9. Significant Figures

(a) Significant figures indicate the precision of measurement which depends on the least count of the measuring instrument.

(b) A change of unit of a physical quantity does not change the number of significant digits or figures in a measurement. e.g., 432cm has 3 significant digits.

The same measurement expressed as either 4.32m or $4320\,\text{mm}$ or $0.00432\,\text{km}$ also has 3 significant digits.

(c) All the non-zero digits are significant.

(d) All zero between two non-zero digits are significant no matter where the decimal point is (if it exists)

(e) If the number is less than 1, the zeros to the right of decimal point but to the left of the first non-zero digit are not significant.

(f) If the number does not have a decimal point, the trailing zeros in the number are not significant.

(g) The trailing zeros in a number with a decimal point are significant.

10. Rules for Arithmetic operations with Significant figure

(a) In addition or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places.

(b) In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.

11. Rounding off the uncertain Digits

(a) The result of a computation is generally an approximate value and must be rounded off to the appropriate significant figures.

(b) The rule is that if the digit to be dropped has value more than 5, the preceding digit is raised by 1 and if the digit has value less than 5, the preceding digit is left as it is.

(c) However, if the digit to be dropped has a value 5, then the preceding digit is raised by 1 if odd and left as it is if even.

Example: Round off the following numbers to 3 significant figures 4.627, 4.624, 4.637, 4.634, 4.625, 4.635

Solution:

Number before rounding off	4.627	4.624	4.637	4.634	4.625	4.635
Number after rounding off	4.63	4.62	4.64	4.63	4.62	4.64

12. Scalars

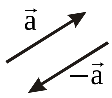
(a) **Scalar:** It is a physical quantity that has magnitude but no direction. Examples of scalar are mass, density, distance, speed, time, temperature, energy, etc.

(b) The rules for combining scalars are the rules of ordinary algebra. Scalars can be added, substracted, multiplied and divided just as ordinary numbers.

13. Vectors

(a) **Vector:** It is a physical quantity that has both a magnitude and a direction.
Examples of vector are displacement, velocity, acceleration, momentum, force, etc.

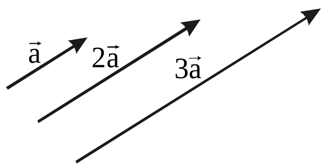
(b) In diagrams, vector is denoted by an arrow, the length of which is proportional to the magnitude of the vector. The direction of the arrow head indicates the direction of vector.



(c) A vector with a minus sign, i.e., $-\vec a$ is a vector with same magnitude as $\vec a$ but points in opposite direction

(d) Vector $\vec a$ when multiplied by a scalar k results in a vecter denoted by $k\vec a$

Vector $k\vec a$ has direction same as $\vec a$ but magnitude equal to k times the magnitude of $\vec a$



(e) **Modulus of vector:** It is simply the magnitude of a vector.

It is written as $|\vec a|$ or simply a and is read as 'mod of $\vec a$'.

(f) **Unit vector:** It is a vector whose modulus is unity.

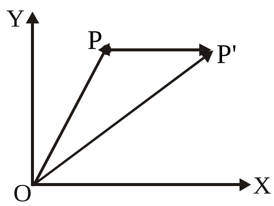
To obtain a unit vector in the direction of vector $\vec a$, we simply divide $\vec a$ by its magnitude $|\vec a|$. $\therefore \ \hat a = \dfrac{\vec a}{|\vec a|}$

14. Position and displacement vectors

(a) To describe the position of an object moving in a plane, we first choose some convenient point, say O as origin.

(b) If P and P' are the positions of the object at time t and t' respectively, then the position vectors of points P and P' are given by vectors $\overrightarrow{OP}$ and $\overrightarrow{OP'}$ respectively.

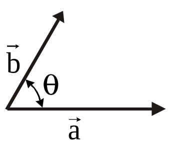
(c) The displacement vector corresponding to the motion of the object from point P to P' is given by $\overrightarrow{PP'}$.



15. Addition and Subtraction of vectors

(a) Consider two vectors $\vec a$ and $\vec b$ inclined at an angle θ to each other.

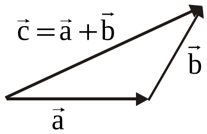
The vector $\vec c$ termed as the sum of vectors $\vec a$ and $\vec b$ is given by $\vec c = \vec a + \vec b$.



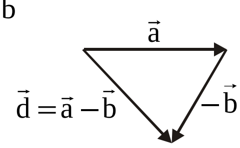
(b) **Triangle law of addition**

- To find the sum $\vec a + \vec b$, place the tail of $\vec b$ at the head of $\vec a$.

  Then the sum $\vec c = \vec a + \vec b$ is defined as the vector from tail of $\vec a$ to the head of $\vec b$



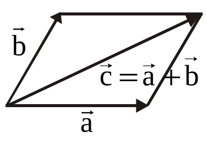
- To find the difference $\vec d = \vec a - \vec b$, we simply add $(-\vec b)$ to $\vec a$ as shown.



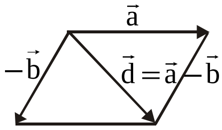
(c) **Parallelogram law of addition**

- To find the sum of $\vec a + \vec b$, place the tails of both $\vec a$ and $\vec b$ at a common point.

  With $\vec a$ and $\vec b$ as two sides, complete the parallelogram.
  Then, the diagonal from the common point is the sum $\vec c = \vec a + \vec b$



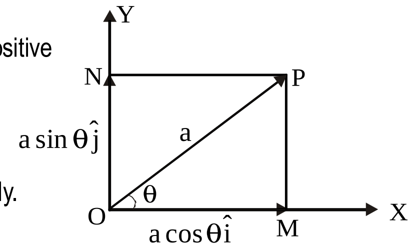
- To find the difference $\vec d = \vec a - \vec b$, we simply add $(-\vec b)$ to $\vec a$



(d) **Magnitude of sum and difference of vectors**

- The magnitude of the sum of vectors $\vec{a}$ and $\vec{b}$ is given by $|\vec{a}+\vec{b}| = \sqrt{a^2+b^2+2ab\cos\theta}$
- The magnitude of the difference of vectors $\vec{a}$ and $\vec{b}$ is given by $|\vec{a}-\vec{b}| = \sqrt{a^2+b^2-2ab\cos\theta}$

16. Resolution of vectors



(a) Consider a vector $\vec{a}$ having magnitude $|\vec{a}|$ or a which makes an angle θ with the positive X- axis. Let $\hat{i}$ and $\hat{j}$ represent unit vectors along X and Y axes respectively.

(b) The vector $\vec{a}$ can be represented as $\vec{a} = a_x\hat{i} + a_y\hat{j}$

a_x and a_y are known as components of vector $\vec{a}$ along X and Y axes respectively.

where $a_x = a\cos\theta$ and $a_y = a\sin\theta$

(c) Conversely, if we know the components of a vector i.e., a_x and a_y, we can find the magnitude and direction of the vector.

$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2}$ and $\theta = \tan^{-1}(a_y/a_x)$

(d) In three dimensions, we have three components of a vector along X, Y and Z axes and the vector $\vec{a}$ is represented as

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

17. Vector Addition-Analytical Method

(a) Although the graphical method of adding vectors is commonly used, it is generally easier to add vectors by simply adding their respective components.

(b) Consider two vectors $\vec{a}$ and $\vec{b}$ given by $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ and $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

(c) The sum of these two vectors is given by $\vec{c} = \vec{a} + \vec{b} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j} + (a_z + b_z)\hat{k}$

18. Dot products of two vectors (Scalar product)

(a) The dot product of two vectors $\vec{a}$ and $\vec{b}$ is defined by $\vec{a}\cdot\vec{b} = ab\cos\theta$

where θ is the angle between vectors $\vec{a}$ and $\vec{b}$ when placed tail to tail.

(b) $\vec{a}\cdot\vec{b}$ is a scalar and hence dot product is also called scalar product.

(d) If $\vec{a}$ and $\vec{b}$ are parallel, $\theta = 0°$ and hence $\vec{a}\cdot\vec{b} = ab$

(e) If $\vec{a}$ and $\vec{b}$ are perpendicular, $\theta = 90°$ and hence $\vec{a}\cdot\vec{b} = 0$

(f) Since, $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors perpendicular to each other, we have $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$. Also, $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

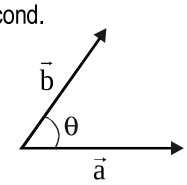
(g) Accordingly, $\vec{a}\cdot\vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\cdot(b_x\hat{i} + b_y\hat{j} + b_z\hat{k}) = a_xb_x + a_yb_y + a_zb_z$

19. Cross product of two vectors (Vector product)

(a) The cross product of two vectors $\vec{a}$ and $\vec{b}$ is defined by $\vec{a}\times\vec{b} = ab\sin\theta\ \hat{n}$

where, θ is the angle between vectors $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is a unit vector perpendicular to the plane containing vectors $\vec{a}$ and $\vec{b}$.

(b) There are two directions perpendicular to any plane. The ambiguity is resolved by right hand thumb rule. Let your fingers point in the direction of first vector and curl around (via the smaller angle) towards second. Then your thumb indicates the direction of $\hat{n}$.



(c) In the figure shown, $\vec{a}\times\vec{b}$ points out of the page and $\vec{b}\times\vec{a}$ points into the page

(d) $\vec{a}\times\vec{b} = -\ \vec{b}\times\vec{a}$

(e) If $\vec{a}\parallel\vec{b}$, $\vec{a}\times\vec{b} = 0$

(f) $\vec{a}\times\vec{a} = 0$, $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

(g) $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$ Also, $\hat{j}\times\hat{i} = -\hat{k}$, $\hat{k}\times\hat{j} = -\hat{i}$, $\hat{i}\times\hat{k} = -\hat{j}$

(h) Accordingly, $\vec{a}\times\vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\times(b_x\hat{i} + b_y\hat{j} + b_z\hat{k}) = (a_yb_z - a_zb_y)\hat{i} + (a_zb_x - a_xb_z)\hat{j} + (a_xb_y - a_yb_x)\hat{k}$

1. Motion in one dimension (Rectilinear motion)

(a) To describe the motion of an object, we introduce four important quantities namely **position, displacement, velocity** and **acceleration**.

(b) For a restricted motion confined to move only in a straight line, there are only two directions, one is designated as positive and the other negative.

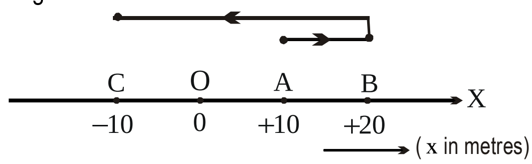
2. Position

(a) Since the motion is along a straight line, we choose an axis say $X-$ axis so that it coincides with the path of the object.

(b) **Position** : It is the location ($X-$ coordinate) of the particle with reference to a conveniently chosen origin, say O.

(c) Position to the right of O is taken as positive and to the left of O is taken as negative.

(d) In the figure shown, the position of the object at points A, B and C are respectively $+10m, +20m$ and $-10m$



3. Distance and Displacement

(a) **Distance**: It is the length of the actual path traversed by an object during motion in a given interval of time. Distance has only magnitude and hence is a scalar.

(b) **Displacement**: It is the shortest distance between the starting and end positions of an object during motion in a given interval of time. Displacement has both magnitude and direction and hence is a vector.

(c) Suppose an object moves from position $A(x=+10\,m)$ to $B(x=+20\,m)$ and then to $C(x=-10\,m)$
Then the distance travelled by the object from A to B is $10m$ and from B to C is $30m$.
The total distance travelled from A to C is $40m$.
The displacement is given by

$$\Delta x = \text{(Coordinate of end position, } C) - \text{(Coordinate of starting position, } A) = (-10m) - (+10m) = -20m$$

where, Δx denotes the change in position.

	Path		
	$A-B$	$B-C$	$A-C$
DIstance	$10m$	$30m$	$40m$
Displacement	$10m$	$-30m$	$-20m$

(d) If the particle is at x_1 at time t_1 and at x_2 at time t_2, then the displacement of the object is $\Delta x = x_2 - x_1$

(e) If $x_2 > x_1$, Δx is positive and hence displacement is positive.
If $x_2 < x_1$, Δx is negative and hence displacement is negative.

4. Speed and Velocity

(a) **Speed** : It is defined as the rate of change of position with respect to time of the object.
$$\text{Speed} = \frac{\text{Distance travelled}}{\text{time taken}}$$

(b) **Average speed** : $\text{Average speed} = \dfrac{\text{Total distance travelled}}{\text{total time interval}}$

(c) **Average velocity** : $v_{av} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{\Delta x}{\Delta t}$

(d) **Instantaneous velocity** : $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$

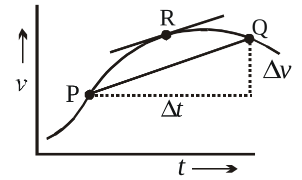
5. Acceleration

(a) When the velocity of the object changes continuously as the motion proceeds, the body is said to have accelerated motion.

(b) **Average acceleration** : $a_{av} = \dfrac{v_2 - v_1}{t_2 - t_1} = \dfrac{\Delta v}{\Delta t}$

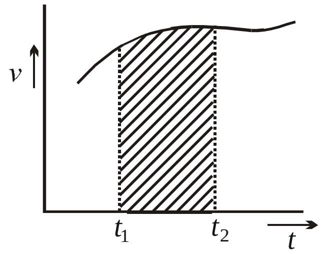
(c) **Instantaneous acceleration** : $a = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$

6 Relation among Displacement, Velocity and Acceleration in Velocity-Time graph

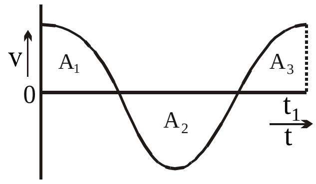


(a) The average acceleration between two given points P and Q in time interval Δt is equal to the slope of the chord connecting the points on a velocity-time graph.

(b) The instantaneous acceleration at a given point say R is the slope of the tangent drawn to the velocity-time graph.



(c) Given a velocity versus time graph, the displacement during an interval between times t_1 and t_2 is the area bounded by the velocity curve, the two vertical lines at $t = t_1$ and $t = t_2$ and the X - axis.
The shaded portion shown in the figure is the bounded area and hence displacement.



(d) The area above X - axis is positive and below it is negative.

(e) If the area is summed up without taking signs into consideration, it gives distance covered in time interval $t = 0$ to $t = t_1$

Distance $= A_1 + A_2 + A_3$

Displacement $= A_1 - A_2 + A_3$

7. Equations of motion for uniform accelerated motion

(a) For a uniformly accelerated motion, the variables u, v, a, s and t are connected by the following relations

$$v = u + at, \qquad s = ut + \frac{1}{2}at^2, \qquad v^2 - u^2 = 2as$$

(b) It should be noted that all of these equations contain exactly four variables as shown in the table below.

Equation	*Contains*				
	u	v	a	s	t
$v = u + at$	yes	yes	yes	no	yes
$s = ut + \frac{1}{2}at^2$	yes	no	yes	yes	yes
$v^2 - u^2 = 2as$	yes	yes	yes	yes	no

In most problems, in uniformly accelerated motion, three parameters are given and fourth or fifth or both are to be found.
Depending upon convenience, one can choose any one or two of the three relations given in the table to calculate the unknown parameters.

(c) ***Sometimes representing motion on a velocity versus time graph can help in solving problems faster than solving them by conventional methods.***

(d) ***It should also be noted that the variables u, v, a and s are vectors and t is a scalar. This aspect is used while solving problems.***

(e) **Distance covered in n^{th} second :**

(Distance covered in n^{th} second) =(Distance covered in n seconds) - (Distance covered in $n - 1$ seconds)

$$\therefore s_n = \left(un + \frac{1}{2}an^2\right) - \left(u(n-1) + \frac{1}{2}a(n-1)^2\right) = u + \frac{a}{2}(2n-1)$$

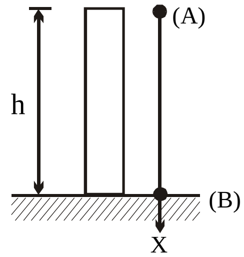
8. Approach to Solving Problems

Step- I:	Define initial and final points between which the equations of motion are to be written
Step- II:	With initial point as origin, define positive direction and call it X - axis.
Step- III:	Make a table with 5 variables u, v, a, s and t written in horizontal line and fill up the table with appropriate values.
Step-IV:	Count the no. of unknown variables and write as many no. of eqns. Solve the eqns to get the values of unknown vairables.

Example 1 : A stone is dropped from a tower of height h. Calculate the velocity and time after which the stone strikes the ground.

Solution : Here, Initial point - A, Final point - B.

Let origin be at A and X-axis be downwards.



Initial Velocity (u)	Final Velocity (v)	Acceleration (a)	Displacement (s)	Time Interval (t)
0	v	g	h	t

Unknown variables are u and t

Equations: $v^2 - u^2 = 2as$ and $v = u + at$

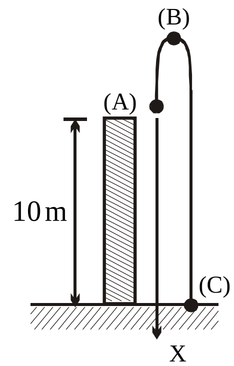
i.e, $v^2 - 0^2 = 2gh$ and $v = 0 + gt$

Hence, $\boxed{v = \sqrt{2gh}}$ and $\boxed{t = \sqrt{\frac{2h}{g}}}$

Example 2 : A stone is thrown vertically upward with a speed $5\ m/s$ from the top of a building $10m$ high. Find
(a) the time after which it strikes the ground.
(b) the velocity with which it strikes the ground. (Take $g = 10\ m/s^2$)

Solution: Here, Initial point - A, Final point - C

Let origin be at A and X - axis be downwards



	Initial Velocity (u)	Final Velocity (v)	Acceleration (a)	Displacement (s)	Time Interval (t)
Path A to C via B	$-5m/s$	v	$10m/s^2$	$10m$	t

Note that s is displacement and hence a vector.

The displacement from point A to C via B is $10m$.

Unkonwn variables are v and t

Equations: $v^2 - u^2 = 2as$ and $v = u - at$

i.e., $v^2 - (-5)^2 = 2 \times 10 \times 10$ and $v = -5 + 10 \times t$

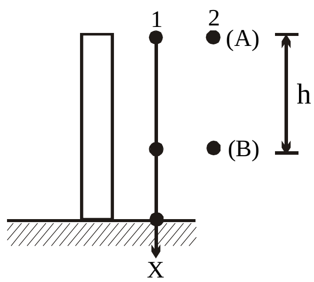
Hence, $\boxed{v = 15m/s}$ and $\boxed{t = 2sec}$

Note: X - axis can also be taken upwards. Accordingly, the signs of vectors u, v, a and s gets reversed. However, the final results remain the same. Students should try to solve the same problem themselves by taking X - axis upwards.

Example 3 : A stone is dropped from the top of a building and 1 second later, another stone is thrown downwards with a velocity of $20m/s$. How far below the top, will the second stone over take the first ? (Take $g = 10m/s^2$)

Solution: Here, Initial point - A Final point - B (where the two stones meet)

Let origin be at A and X - axis be downwards.



	Initial Velocity (u)	Final Velocity (v)	Acceleration (a)	Displacement (s)	Time Interval (t)
Stone 1	0	–	$10m/s^2$	h	t
Stone 2	$20m/s$	–	$10m/s^2$	h	$t - 1$

Unknown variables are h and t

Eqn for stone 1: $s = ut + \frac{1}{2}at^2$ i.e., $h = 0 + \frac{1}{2} \times 10t^2 = 5t^2$ ... (i)

Eqn for stone 2: $s = ut + \frac{1}{2}at^2$ i.e., $h = 20(t-1) + \frac{1}{2} \times 10(t-1)^2 = 20(t-1) + 5(t-1)^2$ ... (ii)

From (i) and (ii), $5t^2 = 20(t-1) + 5(t-1)^2$ $\Rightarrow$ $20(t-1) + 5(-2t+1) = 0$

$\Rightarrow$ $10t - 15 = 0$ or $t = 1.5$ sec $\Rightarrow h = 5t^2 = 5(1.5)^2$ Hence, $\boxed{h = 11.25m}$

Example 4: A stone is dropped from the top of a tower of height h. Simultaneously, another stone is projected upwards from bottom. They meet at a height $2h/3$ from the ground level. If $h = 60\,m$, find the initial velocity of the lower stone.

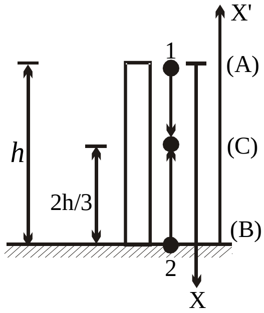
(Take $g = 10\,m/s^2$)

Solution: Here, Initial point - A for stone 1 and B for stone 2. Final point - C for both stones.

Stone 1 Let A be origin and X - axis be downwards

Stone 2 Let B be origin and X'- axis be upwards

Note: For different particles, different axes and different directions may be considered.



	Initial Velocity (u)	Final Velocity (v)	Acceleration (a)	Displacement (s)	Time Interval (t)
Stone 1	0	–	g	$h/3$	t
Stone 2	u	–	$-g$	$2h/3$	t

Unknown variables are u and t

Equation for stone 1: $s = ut + \frac{1}{2}at^2$ i.e., $\frac{h}{3} = 0 + \frac{1}{2}gt^2$ ....(i)

Equation for stone 2: $s = ut + \frac{1}{2}at^2$ i.e., $\frac{2}{3}h = ut - \frac{1}{2}gt^2$ ....(ii)

From (i), $t = \sqrt{\frac{2h}{3g}} = \sqrt{\frac{2\times 60}{3\times 10}} = \boxed{2\ sec}$

From (ii), $u = \frac{2h}{3t} + \frac{1}{2}gt = \frac{2\times 60}{3\times 2} + \frac{1}{2}\times 10\times 2 = \boxed{30\ m/s}$

9. Motion in two dimensions with constant acceleration.

(a) We shall now consider the special case of motion in a plane (two dimension) with constant acceleration.

(b) As the particle moves, the acceleration $\vec{a}$ does not vary either in magnitude or direction.

Consequently, the components of acceleration a_x and a_y remain constant.

(c) We then have a situation which can be described as the sum of two component motion occurring simultaneously and each motion with constant acceleration along each of the two perpendicular directions.

(d) The equations of motion, i.e.,

$v = u + at$, $\quad s = ut + \frac{1}{2}at^2$, $\quad v^2 - u^2 = 2as$

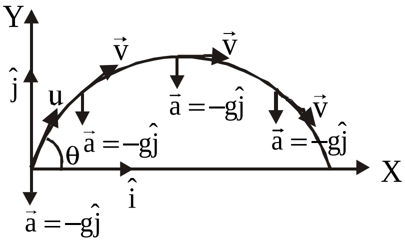
can be applied separately for X - axis and Y - axis.

10. Projectile Motion

(a) The most common example of a particle moving with uniform acceleration is the motion of particle near the surface of earth. Such a motion is called projectile motion.

(b) Consider a particle projected with initial velocity $\vec{u}$ which is directed in direction which makes a certain angle θ with the horizontal. Such a particle has two dimensional motion and will move in a curve as we know from experience

(c) If we choose X - axis along the horizontal and Y - axis along the vertical with the origin at the initial position of the particle, the motion of the particle will be along the curve as shown.



11. Important points in projectile motion

(a) The acceleration of the particle is constant. Its magnitude is $g = 9.8\,ms^{-2}$ and direction of acceleration is directed along the vertical downward direction. In the chosen coordinate system, shown in the figure, $\vec{a} = -g\ \hat{j} = -9.8\ \hat{j}\ \ ms^{-2}$

(b) The velocity vector changes with time both in magnitude and direction. The direction of velocity is always along the tangent to the curve.

(c) The acceleration being in the vertical direction, the horizontal component of velocity remains constant.

(d) When the particle is at the highest point, velocity is directed towards the horizontal. This means that at the highest point, $v_y = 0$

(e) When the particle again hits the ground, its y - coordinate is zero.

12. Approach to solving problems

Step-I: Define initial and final points between which the equations of motion are to be written

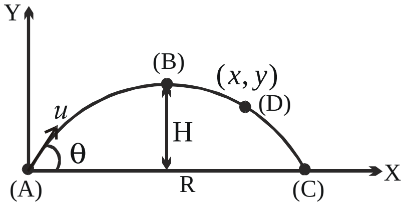
Step-II: With initial point as origin, define two mutually perpendicular axes. Call them $X-$ axis and $Y-$ axis.

Step-III: Make a table with 5 variables u, v, a, s and t written in horizontal row and axes X and Y in vertical column. Fill up the table with appropriate values.

Step-IV: Count the no. of unknown variables and write as many no. of eqns. Solve the eqns to get the values of unknown variables.

13. Particle projected at some angle to horizontal

(a) Consider a particle projected with velocity u at an angle θ to horizontal. We shall derive an expression for



- Horizontal range (R)
- Time of flight (T)
- Maximum height attained (H)

(b) Let origin be point A and X and Y axes as shown.

(c) **Derivation for Range and Time of flight**

Here, Inital point is A and Final point is C

	Axis	Initial Velocity (u)	Final Velocity (v)	Acceleration (a)	Displacement (s)	Time Interval (t)
Path A to C	X	$u\cos\theta$	-	0	R	T
	Y	$u\sin\theta$	-	$-g$	0	T

Unknown variables are R and T

Equation along $X-$ axis $s = ut + \frac{1}{2}at^2$ i.e., $R = u\cos\theta T$ .... (i)

Equation along $Y-$ axis $s = ut + \frac{1}{2}at^2$ i.e., $0 = u\sin\theta T - \frac{1}{2}gT^2$ .... (ii)

From eqn (ii), we obtain $T = \frac{2u\sin\theta}{g}$

Putting the value of T in eqn (i), we get $R = u\cos\theta \times \frac{2u\sin\theta}{g}$ $\therefore$ $R = \frac{u^2\sin 2\theta}{g}$

(d) **Derivation for Maximum height attained**

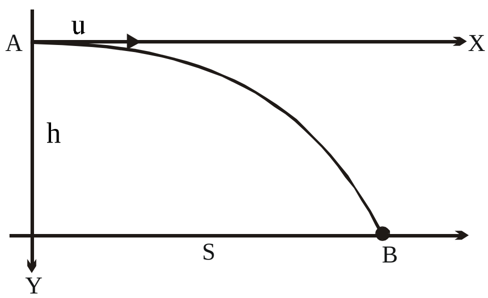
Here, Inital point is A and Final point is B

	Axis	Initial Velocity (u)	Final Velocity (v)	Acceleration (a)	Displacement (s)	Time Interval (t)
Path A to B	Y	$u\sin\theta$	0	$-g$	H	-

Equation is $v^2 - u^2 = 2as$ i.e., $0^2 - (u\sin\theta)^2 = 2(-g)H$ $\therefore$ $H = \frac{u^2\sin^2\theta}{2g}$

14. Particle projected horizontally from some height

(a) Consider a particle projected horizontally with velocity u from a point at height h above the ground. We shall derive an expression for



- Horizontal distance (s) at which the particle will strike the ground
- Time taken (t) to strike the ground.

(b) Here, Initial point is A and Final point is B

Let origin be at point A with X and Y axes as shown.

Axis	Initial Velocity (u)	Final Velocity (v)	Acceleration (a)	Displacement (s)	Time Interval (t)
X	u	-	0	s	t
Y	0	-	g	h	t

Unknown variables are s and t

Equations along X - axis, $s = ut + \frac{1}{2}at^2$ i.e., $s = ut$ ....(i)

Equations along Y - axis, $s = ut + \frac{1}{2}at^2$ i.e., $h = \frac{1}{2}gt^2$ ....(ii)

From eqn (ii), we get, $\boxed{t = \sqrt{\frac{2h}{g}}}$ From eqn (i), we get, $\boxed{s = u\sqrt{\frac{2h}{g}}}$

15. Relative velocity in two dimensions

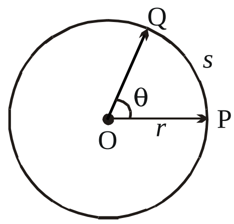
(a) Consider two objects A and B moving with velocities $\vec{v}_A$ and $\vec{v}_B$ (with respect to some common frame of reference, say ground)

(b) Then velocity of object B relative to object A, $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

Similarly, the velocity of object A relative to object B, $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

(c) Therefore, $\vec{v}_{BA} = -\vec{v}_{AB}$ and $|\vec{v}_{AB}| = |\vec{v}_{BA}|$

16. Uniform Circular Motion



(a) Consider a particle moving in a circle of radius r with its centre at O. Since, the distance of the particle from origin O remains constant, its position can be uniquely specified by angle θ.

(b) **Angular displacement :** $\theta = \frac{PQ}{r} = \frac{s}{r}$ and is expressed in radians.

(c) **Angular velocity:** $\omega = \underset{\Delta t \to 0}{Lt} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$ **For a uniform circular motion, α is a constant.**

(d) **Angular acceleration:** $\alpha = \underset{\Delta t \to 0}{Lt} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$ **For a uniform circular motion, α is zero.**

(e) The relation between particle speed (v) and angular velocity (α) is given by $v = r\alpha$

17. Relationship between linear and angular variables in Circular Motion

Linear Motion	**Circular Motion**	**Relation**
Distance - s	Angular Displacement - θ	$s = r\theta$
Velocity - v	Angular velocity - α	$v = r\alpha$
Tangential Acceleration - a_t	Angular Acceleration - α	$a_t = r\alpha$

Equations for linear motion with constant acceleration and initial velocity u

$v = u + at$ $s = ut + (1/2)at^2$ $v^2 - u^2 = 2as$

Equations for circular motion with constant angular acceleration and initial angular velocity ω_i

$\omega = \omega_i + \alpha t$ $\theta = \omega_i t + (1/2)\alpha t$ $\omega^2 - \omega_i^2 = 2\alpha\theta$

18. Acceleration of particle in Uniform Circular Motion

(a) Consider a particle moving in a circle of radius r with a constant speed v. Since, its velocity (which is in the direction of tangent to the circle) changes its direction continuously, the particle is said to be undergoing an acceleration.

(b) This acceleration is directed towards the centre of the circle known as **centripetal acceleration.**

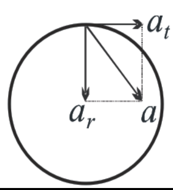
Its magnitude is given by $a_r = \frac{v^2}{r} = \omega^2 r$

19. Acceleration of particle in non-uniform circular motion

(a) Consider a particle moving in a circle of radius r whose speed v changes at a uniform rate.

(b) The acceleration in this case is the vector sum of

centripetal acceleration, $a_r = \frac{v^2}{r}$ and tangential acceleration, $a_t = \frac{dv}{dt}$



(c) The total acceleration is given by $a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$

1. Newton's laws of motion

(a) **First law of motion :** This law states that a body continues to remain in its state of rest or of uniform motion in a straight line, unless it is compelled to change the state by external forces. It is also known as **law of inertia.**

(b) **Second law of motion :** This law states that the rate of change of momentum of a body is equal to the net external force acting on it and takes place in the direction in which the force acts. i.e., $F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$

(i) If mass of an object is constant, then $\frac{dm}{dt} = 0$ and $F = m\frac{dv}{dt} = ma$.

(ii) If mass of the body into consideration is not a constant, then $v\frac{dm}{dt} \neq 0$.

Some examples of variable mass situations are

- Rocket propulsion where the rocket moves forward due to thrust gained by ejecting exhaust gases backwards. The mass of the rocket thus reduces continuously.
- A truck full of sand moving with sand continuously spilling out through a hole.
- A truck moving with rain water continuously getting collected into the truck.

(c) **Third law of motion :** This law states that to every action, there is always an equal and opposite reaction.

2. Inertial and Non-Inertial frames

(a) **Inertial frame:** A frame of reference relative to which Newton's laws of motion are valid is called an ***inertial frame.*** Such a frame has a zero acceleration, i.e., such a frame is either stationary or moves with a uniform velocity.

(b) **Non-Inertial frame:** A frame of reference relative to which Newton's laws are not valid is called a ***non-inertial frame.*** Such a frame has a non zero acceleration. A lift that is moving up or down but accelerating or retarding is a non inertial frame.

3. Common forces acting on a body

In most cases that we encounter in everyday life, some of the forces that act on the bodies are

- Weight
- Normal Reaction
- Tension
- Friction

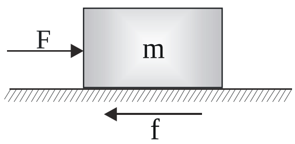
(a) **Weight :** It is the gravitational pull of the earth and is given by $W = mg$
The direction of the force acting on the body is vertically downwards.

(b) **Normal Reaction :** When two surfaces are in contact, they press each other with a certain force which is denoted by N or R.
The direction of the normal force is always perpendicular to the surface and tends to push the body away from the surface.

(c) **Tension :** When a cord (string, rope or cable) is attached to a body and pulled taut, the cord is said to be under tension.
It pulls on the body with force T and it 's direction is away from the body and along the cord at the point of attachment.
Unless otherwise stated, a cord is assumed to be massless and unstretchable. Under these conditions, it exists only as a connection between two bodies and it pulls both bodies at each end with same magnitude, T.

(d) **Friction :** If we move or try to move a body over the surface of another body, the motion is resisted by bonding between the surfaces of two bodies. This resistance is called the frictional force or the force of friction.
The force of friction acts along the surface.

Let F be a variable horizontal force that attempts to slide a block of mass m kept stationary on a rough horizontal surface. Let f be the resulting frictional force as shown in the figure.



(i) If F is too small to cause actual sliding, then the frictional force that acts on the body is known as **static frictional force** and is denoted by f_s.

(ii) Static frictional force, f_s is parallel to the surface and equal in magnitude but opposite to F.

(iii) Static frictional force, f_s has a maximum value beyond which it cannot increase. The maximum value is given by $(f_s)_{max} = \mu_s N$

where, μ_s = coefficient of static friction N = Normal force with which the two surfaces press each other.

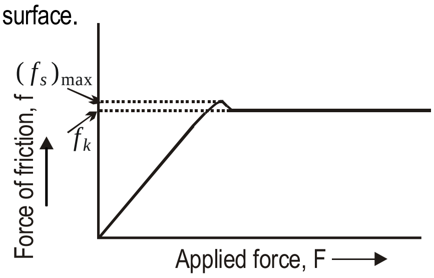
(iv) If the magnitude of F exceeds $(f_s)_{max}$, then the body begins to slide along the surface.

(v) If the body actually starts moving along the surface, then the force of friction is called the ***kinetic friction*** or ***dynamic friction*** and is denoted by f_k.

(vi) Kinetic friction, f_k has a constant value and is independent of the velocity or acceleration of the body. It is given by $f_k = \mu_k N$

where, μ_k = coefficient of kinetic friction

N = Normal force with which the two surfaces press each other



(vii) The value of f_k is slightly less then $(f_s)_{max}$. Hence, The value of μ_k is slightly less than μ_s .

(viii) Therefore, we have $0 \le f_s \le \mu_s N$ and $f_k = \mu_k N$

(ix) The maximum value of static friction, $(f_s)_{max}$ is also known as limiting friction. Hence, ***limiting friction*** is the maximum opposing force that comes into play when one body is just at the verge of moving over the surface of other body.

(x) The force of friction is independent of the area of contact of the two surfaces. It depends upon
- Normal force with which the two surfaces press each other.
- Coefficient of friction.

(xi) The coefficient of friction (μ_s or μ_k) depends upon
- Material of the two surfaces in contact
- Roughness of the two surfaces in contact

(xii) The force of friction on a body always acts opposite to the direction of intended motion.

(xiii) According to Newton's third law of motion, the frictional force acting on the two bodies in contact is opposite to each other in direction.

(xiv) If the relative velocity between two surfaces is zero, then static friction acts. Otherwise, kinetic friction acts.

(xv) If in a given problem, only μ is mentioned, we may consider $\mu_s = \mu_k = \mu$.

4. Free body diagram (FBD)

A free body diagram is a stripped down diagram (of a body under consideration) in which we show
- all forces acting on the body with due consideration to the directions in which they point.
- direction of acceleration of the body.

5. Summary of forces to be considered while making F.B.D

Type of Force	Magnitude of Force	Direction of Force
Gravitation	mg	Vertically downwards
Normal or Reaction	N or R	Perpendicular to the surface. (It pushes the body away from the surface)
Tension	T	Away from the body in line with the cord (It pulls the body)
Friction	f (see note)	Parallel to surface and opposite to the direction of intended motion
Any other force given in the problem	As given	As given

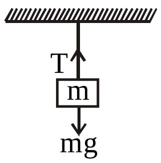
Note : $f = \mu_k N$ if the body slides along the surface

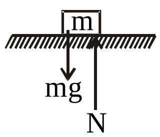
$f = \mu_s N$ if the body does not slide but a slightly greater force will make it slide (limiting case)

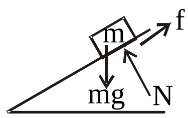
$f < \mu_s N$ if the body does not slide and a slightly greater force will also not make the body slide

6. Some examples of F.B.D

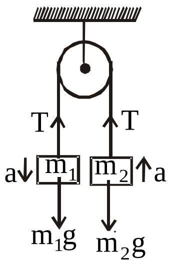
(a) **Blocks in equilibrium**



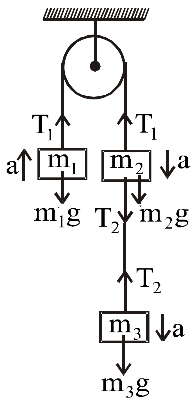


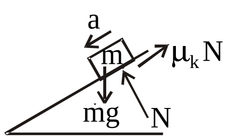


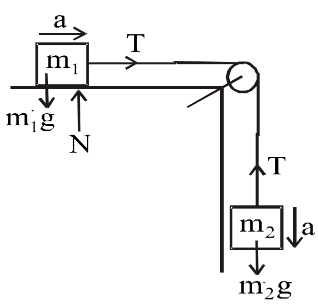
(b) **Blocks moving with acceleration**



$m_1 > m_2$







7. Approach to solving problems in Mechanics

Step- I : Choose the body. A body to which Newton's laws are applied may be a particle, a block or a combination of blocks connected by a cord, etc.

Step- II : Identify the forces that act on the body and draw a free body diagram showing
- all forces acting on the body
- direction of acceleration of the body if any.

Step- III: Choose axis. It is convenient to choose one of the axis (say $X-$ axis) along the direction in which the body is likely to have acceleration. A direction perpendicular to this may be chosen as the other axis (say $Y-$ axis). Resolve all forces along the two axes.

Step-IV : Write down separate equations along the two chosen axis.
If a is the acceleration of the body, then by Newton's second law of motion,
- along x-axis, we have $\Sigma F_x = ma$
- along y-axis, we have $\Sigma F_y = 0$

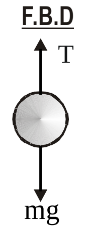
Solve the equations to obtain the desired result.

8. Some Solved Examples

Example 1: A body of mass $6\ kg$ is supported by a cord. Find the tension in the cord. (Take $g = 10ms^{-2}$)

Solution: Since, the body is at rest, its acceleration is zero.

$\Rightarrow \quad T - mg = 0 \qquad \therefore \quad T = mg = 6\times 10 = 60N$

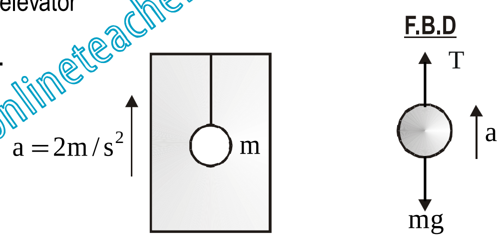


Example 2: A body of mass $6\ kg$ is supported by a cord inside an elevator which is moving up with a constant acceleration $2\,m/s^2$.

Find the tension in the cord. (Take $g = 10ms^2$)

Solution: The equation is $\quad T - mg = ma$

$\therefore \quad T = m(g + a) = 6(10 + 2) = 72N$

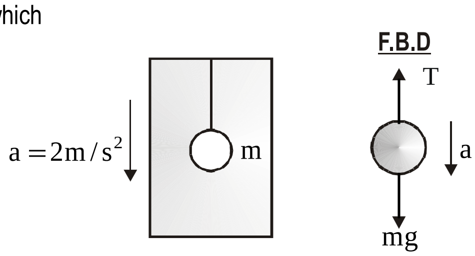


Example 3: A body of mass $6\ kg$ is supported by a cord inside an elevator which is moving down with a constant acceleration $2\ m/s^2$.

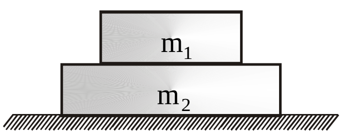
Find the tension in the cord. (Take $g = 10m/s^2$)

Solution: The equation is $\quad mg - T = ma$

$\therefore \quad T = m(g - a) = 6(10 - 2) = 48N$



Example 4: The figure shows two blocks at rest on a horizontal surface.
Find the contact forces acting on each of the blocks.



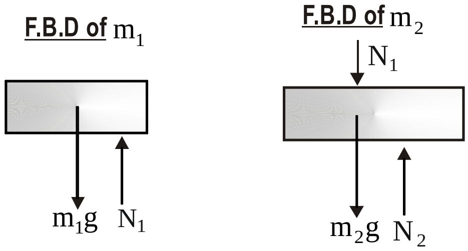
Solution: Let the contact force between m_1 and m_2 be N_1 and that between m_2 and the horizanal surface be N_2 Since, the masses are at rest, their accelerations are zero.

$\Rightarrow \quad m_1 g - N_1 = 0 \qquad$ (from F.B.D of m_1) .... (i)

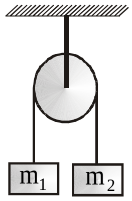
$m_2 g + N_1 - N_2 = 0 \qquad$ (from F.B.D of m_2) .... (ii)

From (i), $\quad N_1 = m_1 g$

From (ii), $\quad N_2 = N_1 + m_2 g = (m_1 + m_2)g$



Example 5: A light and inextensible string passes over a light and frictionless pulley as shown. Calculate the acceleration of the system and tension in the string



Solution: Let mass m_2 be accelerated downwards with acceleration a.

Then, the mass m_1 accelerates upwards with acceleration a.

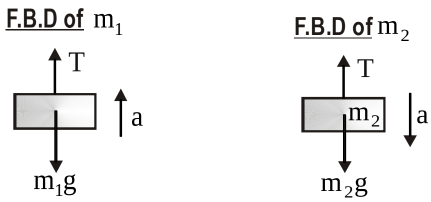
Let T be the tension in the string.

The equations are

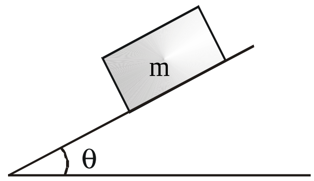
$T - m_1g = m_1a$ and $m_2g - T = m_2a$

On solving the two equations, we get

$$a = \frac{m_2 - m_1}{m_1 + m_2}g \quad \text{and} \quad T = \frac{2m_1m_2}{m_1 + m_2}g$$



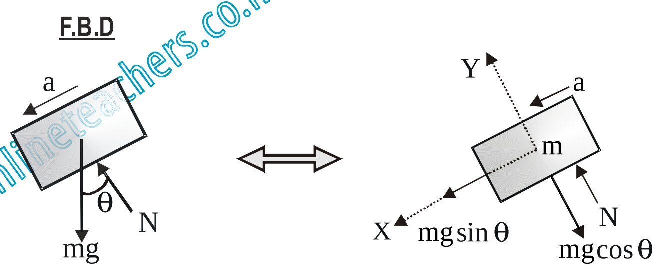
Example 6: A block of mass m is lying on a smooth inclined plane, inclined at an angle θ with the horizontal. Find the acceleration of the block



Solution: We first make a free body diagram of the block. Since the acceleration is along the inclined surface, let us resolve the forces along the axis as shown.

Along $X-$ axis, $mg\sin\theta = ma$

. $a = g\sin\theta$



Example 7: In the above problem, if the inclined planeis rough having coefficient of kinetic friction μ_k, find the acceleration of the block.

Solution: Along $X-$ axis,

$mg\sin\theta - \mu_k N = ma$ .... (i)

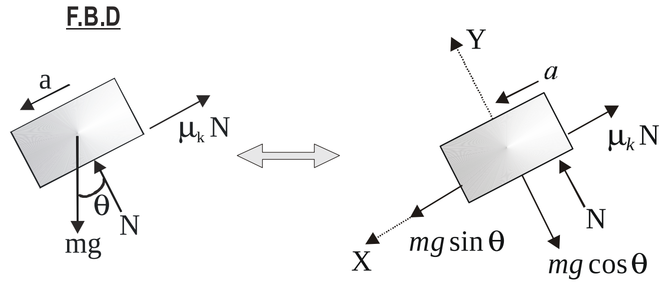
Along $Y-$ axis,

$N - mg\cos\theta = 0$ or $N = mg\cos\theta$

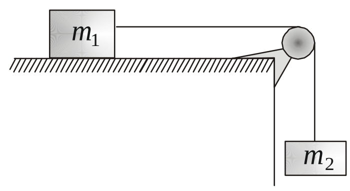
Substituting, the value of N in (i),

we have $mg\sin\theta - \mu_k mg\cos\theta = ma$

. $a = g(\sin\theta - \mu_k\cos\theta)$



Example 8: Figure shows a block of mass m_1 on a smooth horizontal surface pulled by a massless string which is attached to a block of mass m_2 hanging over a light frictionless pulley as shown. Find the acceleration of the system and tension in the string.

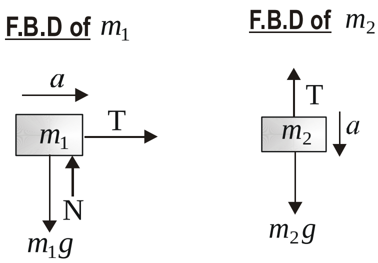


Solution: Note that tension in the string at both ends is same and also, the acceleration of both blocks is same.

The equations are $T = m_1a$ and $m_2g - T = m_2a$

On solving the two equations, we get

$$a = \frac{m_2}{m_1 + m_2}g \quad \text{and} \quad T = \frac{m_1m_2}{m_1 + m_2}g$$



Example 9: In the above problem, if the horizontal surface is rough having coefficient of kinetic friction μ_k, find the acceleration of the system and tension in the string.

Solution: From F.B.D of m_1, we have

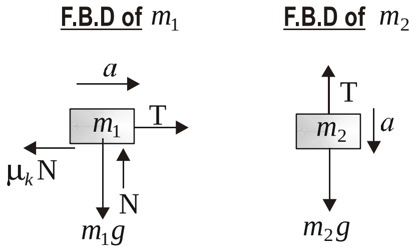
$N - m_1 g = 0$ or $N = m_1 g$

$T - \mu_k N = m_1 a$ or $T - \mu_k m_1 g = m_1 a$ ...... (i)

From F.B.D of m_2, we have $m_2 g - T = m_2 a$ ...... (ii)

From (i) and (ii), we get

$$a = \frac{(m_2 - \mu_k m_1)}{m_1 + m_2} g \quad \text{and} \quad T = \frac{(1+\mu_k) m_1 m_2}{m_1 + m_2} g$$

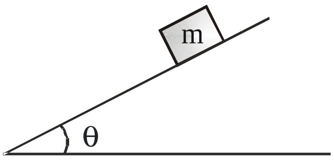


9. Angle of the inclined plane at which a body just starts sliding

Consider a body of mass m kept on an inclined plane.

Let μ_s be the coefficient of static friction between the body and the plane.

If we keep increasing the angle θ of the plane, at a certain value of angle θ, the body just starts sliding. This angle is given by $\theta = \tan^{-1}(\mu_s)$.



This can be obtained by making a FBD.

Since, **it is a limiting case**,

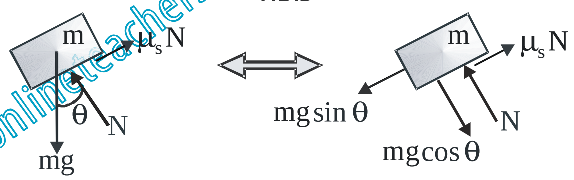
friction is equal to $\mu_s N$ and acceleration of the body is zero.

From FBD, we get

$\mu_s N = mg\sin\theta$ and $N = mg\cos\theta$

$$\Rightarrow \frac{\mu_s N}{N} = \frac{mg\sin\theta}{mg\cos\theta} \quad \text{or} \quad \mu_s = \tan\theta$$

$$\therefore \theta = \tan^{-1}(\mu_s)$$



10. Uniform Circular Motion

(a) In the last chapter, we have seen that a body moving in a circle (of radius r) with uniform speed v is accelerated towards the centre given by $a = v^2/r$

(b) This acceleration is known as centripetal acceleration and the force responsible for this is known as centripetal force and is given by $F = ma = mv^2/r$.

(c) For a stone rotating in a circle by a string, the centripetal force is provided by the tension in the string.

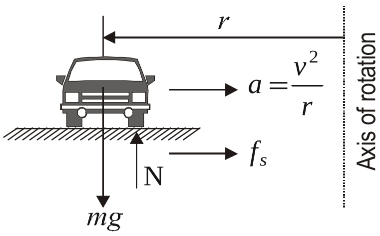
(d) For a planet revolving around the sun, the centripetal force is provided by the gravitational force on the planet due to the sun.

(e) For a car taking a circular turn on a horizontal road, the centripetal force is provided by the force of friction.

11. Circular motion of a vehicle on a level road

(a) Consider a vehicle of mass m moving at a speed v and making a turn on a circular path of radius r. The external forces acting on the vehicle are

- Weight, mg
- Normal force, N
- Friction, f_s



The equations from F.B.D are $N - mg = 0$ and $f_s = ma = \frac{mv^2}{r}$

(b) The only horizontal force that acts towards the centre is the friction, f_s. This is static friction and is self adjustable.

(c) The tyres have a tendency to skid outwards and the frictional force, f_s apposes the skidding tendency and acts towards the centre.

(d) Thus, for a safe turn, $f_s \le \mu_s N$ $= \frac{mv^2}{r} \le \mu_s mg$ $\therefore v^2 \le \mu_s rg$

Hence, the maximum safe speed an a level road so that vehicle do not skid is given by $v_{max} = \sqrt{\mu_s rg}$

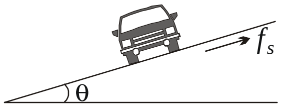
12. Circular motion of a vehicle on a banked road

(a) We have seen that on a level road, the maximum safe speed is given by $v_{max} = \sqrt{\mu_s rg}$

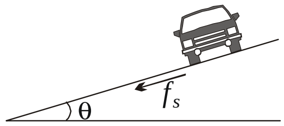
(b) To avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is somewhat lifted up as compared to the inner part. This increases the maximum safe speed.

(c) Let the surface of road make an angle θ with the horizontal.

(d) Clearly, if the speed of vehicle is zero or very low, the tendency of the vehicle is to skid downwards and the direction of friction f_s will be upwards as shown.

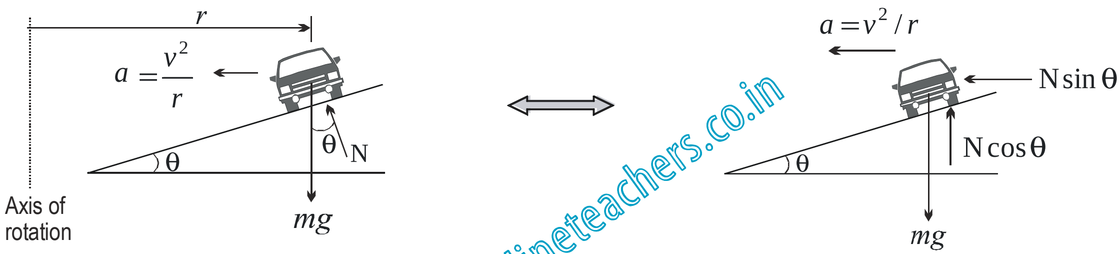


(e) If the speed of vehicle is very high, the tendency of the vehicle is to skid upwards and the direction of friction, f_s will be downwards as shown.



(f) **At a certain value of speed, the frictional force is neither upwards nor downwards (its value is zero). Such a speed is known as the speed for which there is no dependence on friction**

F.B.D



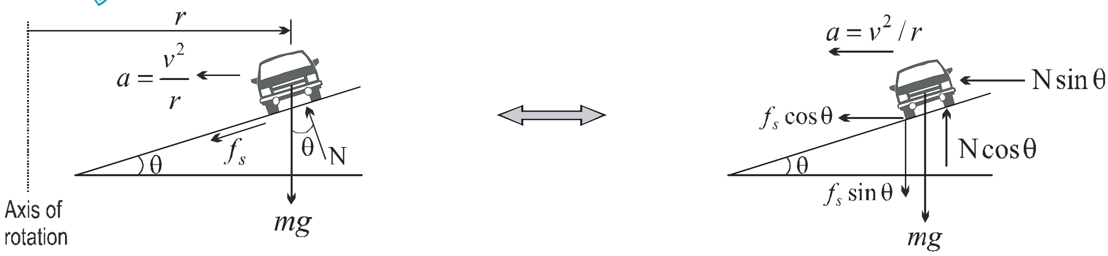
From F.B.D, the equations are $N\cos\theta - mg = 0$ and $N\sin\theta = \frac{mv^2}{r}$

Eliminating N, we get $\tan\theta = \frac{v^2}{rg}$

Hence, the speed for which there is no dependence on friction is given by, $\boxed{v = \sqrt{rg\tan\theta}}$,

(g) To find the **maximum safe value of speed,** the frictional force, $f_s = \mu_s N$ acts downwards.

F.B.D



From F.B.D, the equations are $N\cos\theta - f_s\sin\theta - mg = 0$ and $N\sin\theta + f_s\cos\theta = \frac{mv^2}{r}$

Since, for v_{max}, $f_s = \mu_s N$, we have

$N\cos\theta - \mu_s N\sin\theta = mg$ and $N\sin\theta + \mu_s N\cos\theta = \frac{mv_{max}^2}{r}$

Dividing 2nd equation by 1st equation, we get

$$\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = \frac{v_{max}^2}{rg} \qquad \Rightarrow v_{max}^2 = rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}\right)$$

Hence, the maximum safe speed an a banked road is given by $\boxed{v_{max} = \sqrt{rg\left(\frac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}\right)}}$

(h) **Therefore, the maximum safe speed on a banked road is more than that on a level road.**

WORK, ENERGY AND POWER

1. Work

(a) **The work done by a force is defined by the product of the component of force in the direction of displacement and the magnitude of the displacement.**

(b) No work is done on a body by a given force if

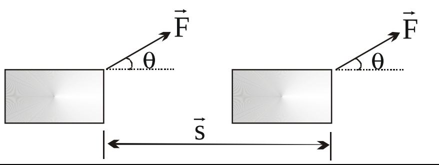
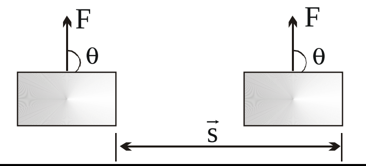
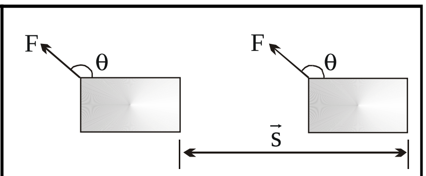
- The displacement of the body is zero
- The force itself is zero
- The force and displacement are mutually perpendicular to each other.

(c) Work done is a scalar

2. Work done by a constant force

(a) **The work done by a constant force** $\vec{F}$ **on a body which undergoes displacement** $\vec{s}$ **is given by** $W = \vec{F}.\vec{s} = Fs\cos\theta$ **where** θ **is the angle between vectors** $\vec{F}$ **and** $\vec{s}$ **.**

(b)

		
θ is acute	$\theta = 90°$	θ is obtuse
W is positive	W is zero	W is negative
Force tries to increase the speed of the body	there is no change in speed of the body due to the force	Force tries to decrease the speed of the body

(c) If $\theta = 0^0$, $W = Fs$. If $\theta = 180^0$, $W = -Fs$. If $\theta = 90^o$, $W = 0$

3. Work done by a variable force

(a) A constant force is rare. It is the variable force which is more commonly encountered.

(b) **If the force changes its direction or magnitude while the body is moving, then the work done by the force is given by**

$W = \int \vec{F}.\overrightarrow{ds} = \int F\cos\theta \, ds$ **where the integration is performed along the path of the body.**

(c) **If the motion is one dimensional, then we can orient the** X - **axis along the direction of motion and the equation can be expressed as** $$W = \int_{x_i}^{x_f} F(x)\, dx$$

where $F(x)$ is a variable force as a function of x and is also the component of force along the direction of motion i.e, X - axis

x_i and x_f are the coordinates of the initial position and final position respectively of the body.

4. Kinetic Energy

(a) **The kinetic energy of a body is defined as the energy possessed by the body by virtue of its motion.**

(b) It may also be defined as a measure of the work the body can do by virtue of its motion and is given by $K = \frac{1}{2}mv^2$

5. Work Energy Theorem

(a) **According to work energy theorem, the work done by the net force in displacing a body measures the change in kinetic energy of the body.** i.e., $W = \Delta KE$. Hence, Work done = Change in Kinetic Energy

(b) This equation is also true when a variable force is applied on the body.

(c) This equation is also true when more than one force is applied on the body. In this case, the change in kinetic energy of the body is equal to work done by the resultant force or the sum of work done by each individual force.

(d) When the force is applied in the direction of motion of the body, a positive work is done by the force and the body accelerates. This increases the kinetic energy of the body.

(e) When the force is applied opposite to the direction of motion of the body, a negative work is done by the force and the body deccelerates. This decreases the kinetic energy of the body.

6. Potential Energy

(a) **Potential energy of a body is defined as the energy possessed by the body by virtue of its position or configuration.**

(b) When work is done on a system and the system preserves this work in such a way that it can be subsequently recovered back in form of kinetic energy, the system is said to be capable of possessing potential energy.

For example, when a ball is raised to some height, the work done on the ball gets stored in form of potential energy which can be recovered back in form of kinetic energy when the ball is dropped from the height to which it was raised.

(c) In short, the energy that can be stored is potential energy.

(d) The body possessing potential energy when left to itself, the stored energy gets released in the form of kinetic energy.

(e) The potential energy can be of various types. Some of these are

- Gravitational potential energy
- Elastic potential energy
- Electrostatic potential energy

7. Gravitation Potential Energy

(a) **It is the energy possessed by earth and block system which is by virtue of the block's position with respect to the earth.**

(b) Consider a block of man m near the surface of earth and raised through a height h above the earth's surface.

For $h << \text{radius of earth}$, the gravitational force acting on the block is mg and hence the potential energy of the earth block system increases by mgh.

(c) If the block descends by height h, the potential energy decreases by mgh.

(d) Since the earth remains almost fixed, the potential energy of the earth-block system may be called as the potential energy of the block only.

(e) The absolute value of potential energy is not physically of any significance. It is the difference of potential energy between two points which is important.

(f) The point where the absolute potential energy is taken as zero is a matter of our choice.

In case of gravitational potential energy (as we shall see in the chapter Gravitation), a convenient choice of taking gravitational potential energy zero is when the block is at an infinite distance from the earth.

(g) However, we shall choose any convenient reference position of the block and shall call the gravitational potential energy to be zero in this position. The potential energy at a height h above this position is taken as mgh.

8. Potential energy of a Spring

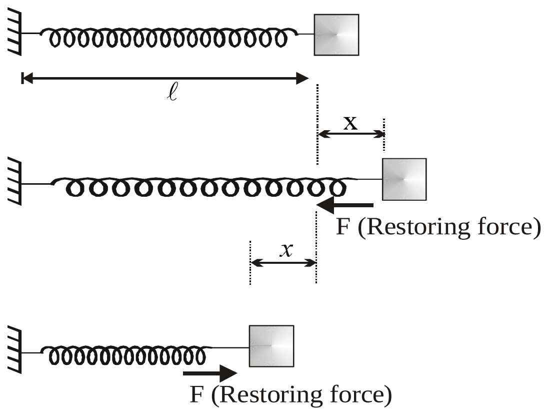
(a) Consider a spring block system as shown in the figure where the natural length of the spring is ℓ.

(b) As the spring is stretched by amount x, a restoring force F gets developed in the spring that tends to pull the block to its original position.

Spring in natural state

Spring in stretched position

Spring in compressed position



(c) As the spring is compressed by amount x, a restoring force F gets developed in the spring that tends to push the block to its original position.

(d) In either case, the restoring force F is proportional to x

i.e, $F \propto x$ or $F = kx$ where k is known as the spring constant and may be defined as the restoring force developed in the spring per unit amount of stretch or compression.

(e) As the spring is elongated or compressed from its natural state, work is done against the restoring force of the spring and the work done is stored in the spring as elastic potential energy.

(f) The work done or the change in elastic potential energy of the spring is given by $W = \int_0^x F.dx = \int_0^x kx\,dx = \frac{1}{2}kx^2$

(g) It is customary to choose the potential energy of the spring in its natural length to be zero.

(h) With this choice, ***the potential energy of the spring is*** $U = \frac{1}{2}kx^2$ ***where*** x ***is the elongation or compression of the spring.***

9. Conservative Force

(a) ***A force is said to be conservative if the work done by the force depends only on the initial and final positions of the body and not on the path taken.***

(b) The work done by a conservative force during a round trip of a system is always zero.

(c) ***If conservative forces act on a body, then during the motion of the body, the sum of its kinetic and potential energy always remain conserved.***

(d) Examples of conservative forces are gravitational force, spring force, etc.

10. Non-Conservative Force

(a) A force is said to be non-conservative if the work done by the force between two given position of the body depends upon the path taken by the body.

(b) The work done by a non-conservative force during a round trip of a system is non zero.

(c) If a non-conservative force acts on a body, then during the motion of the body, the sum of its kinetic and potential energy does not remain conserved.

(d) Examples of non-conservative forces are the force of friction, viscous force, etc.

11. Relation between Conservative Force and Potential Energy

(a) ***Potential energy can be defined only for conservative forces.*** It does not exist for non-conservative forces.

(b) For every conservative force $F(x)$ (that depends upon the position x), there is an associated potential energy function $U(x)$.

(c) The relation between the two is given by $F(x) = \frac{-dU(x)}{dx}$

12. Conservation of Mechanical Energy

(a) **The mechanical energy (E) of a body is defined as the sum of its Kinetic energy (K) and potential energy (U).**

$E = K + U$

(b) **The principle of conservation of mechanical energy states that the total mechanical energy of a system is conserved if all forces doing work acting on the body are conservative.**

13. Power

(a) **Power is defined as the rate at which work is done.**

(b) **The average power of a force is defined as the ratio of work done (W) by the force to the total time taken (t).** $P_{av} = \frac{W}{t}$

(c) **The instantaneous power,** $P = \frac{dW}{dt}$

(d) The work done dW by a force F for a small displacement ds is given by $dW = \vec{F}.\vec{ds}$.

The instantaneous power can therefore be expressed as $P = \frac{dW}{dt} = \vec{F} \cdot \frac{\vec{ds}}{dt} = \vec{F} \cdot \vec{v}$.

where $\vec{v}$ is the instantaneous velocity when the force is $\vec{F}$.

1. Centre of Mass

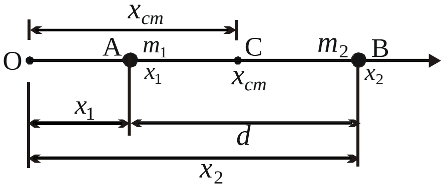
Centre of Mass of a body or a system of particles may be defined as a point at which the entire mass of the body/system of particles is supposed to be concentrated.

2. Centre of mass of two particle system

(a) Consider a system of two particles of masses m_1 and m_2 separated by a distance d.

We may set up the coordinate axes such that X - axis passes through the two particles.

(b) Let the origin be at O such that the coordinates of m_1 and m_2 are x_1 and x_2 respectively.

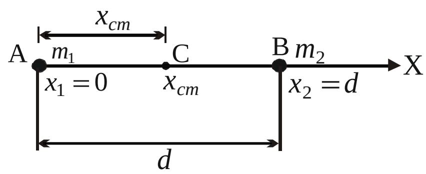


(c) If C is the location of centre of mass whose coordinate is x_{cm}, then x_{cm} is given by $x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$

(d) Note that the location of centre of mass, i.e, distance AC is independent of location of origin since it depends only upon m_1, m_2 and d.

(e) For convenience, we may choose the origin at m_1 so that the coordinates of m_1 and m_2 are $x_1 = 0$ and $x_2 = d$ respectively.

Now, $AC = x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{m_1 \times 0 + m_2 \times d}{m_1 + m_2} = \frac{m_2 d}{m_1 + m_2}$



Similarly, $BC = \frac{m_1 d}{m_1 + m_2}$

(f) Note that $AC : BC = m_2 : m_1$ This shall be used to solve problems fast.

3. Centre of mass of a system of N particles

(a) Consider a collection of N particles of masses $m_1, m_2, m_3, ...m_N$.

Let the mass of the i^{th} particle be m_i and its coordinates with reference to the chosen axes be x_i, y_i, z_i

(b) If the coordinates of centre of mass are x_{cm}, y_{cm}, z_{cm}, then,

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + ...m_Nx_N}{m_1 + m_2 + ...m_N} = \frac{\Sigma m_i x_i}{M}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + ...m_Ny_N}{m_1 + m_2 + ...m_N} = \frac{\Sigma m_i y_i}{M}$$

$$z_{cm} = \frac{m_1z_1 + m_2z_2 + ...m_Nz_N}{m_1 + m_2 + ...m_N} = \frac{\Sigma m_i z_i}{M}$$

where, $M = m_1 + m_2 + ...m_N = \Sigma m_i =$ Total mass of the system.

4. Velocity of centre of mass of the system of N particles

(a) Let the velocities of N particles of masses $m_1, m_2, m_3, ...m_N$ be $\vec{v}_1, \vec{v}_2, \vec{v}_3 .....\vec{v}_N$ respectively.

(b) If $\vec{v}_{cm}$ is the velocity of centre of mass, then $\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ...m_N\vec{v}_N}{m_1 + m_2 + ...m_N} = \frac{\Sigma m_i \vec{v}_i}{M}$

5. Acceleration of centre of mas of the system of N Partides

(a) Let the accelerations of N Particles of masses $m_1, m_2, m_3.............m_n$ be $\vec{a}_1, \vec{a}_2, \vec{a}_3.............\vec{a}_n$ respectively.

(b) If $\vec{a}_{cm}$ is the acceleration of centre of mass, then $\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...m_n\vec{a}_n}{m_1 + m_2 + ...m_n} = \frac{\sum \vec{m}_i \vec{a}_i}{M}$

(c) Hence, $M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + ...m_N\vec{a}_n = \vec{F}_1 + \vec{F}_2 + ...\vec{F}_N = \vec{F}_{ext}$

where, $\vec{F}_{ext}$ represents the sum of all external forces acting on the particles of the system.

(d) This equation shows that the **centre of mass of a system of particles moves as if all the mass of the system was concentrated at the centre of mass and all the external forces were applied at that point.**

6. Momentum conservation and centre of mass motion

(a) The linear momentum of i^{th} particle is defined as $\vec{p}_i = m_i \vec{v}_i$

(b) The momentum of N particle system is the vector sum of the momenta of N particles, i.e., $\vec{p} = \Sigma \vec{p}_i = \Sigma m_i \vec{v}_i$

(c) **If no external force acts on the system, the momentum $\vec{p}$ is a constant, i.e., the linear momentum of the system remains conserved.**

Hence, $\vec{p} = \Sigma m_i \vec{v}_i = \vec{M}v_{cm} = \text{constant}$ $\left\{ \text{Since, } \vec{v}_{cm} = \dfrac{\sum m_i \vec{v}_i}{M}, \quad \sum m_i \vec{v}_i = M\vec{v}_{cm} \right\}$

. $\vec{v}_{cm} = \text{constant}$

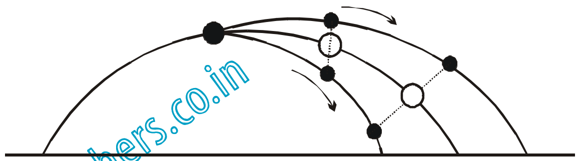
(d) Hence, if no external force acts on the system, $\vec{v}_{cm}$ is a constant i.e., the centre of mass of an isolated system has a constant velocity.

7. Examples of motion of centre of mass

(a) We have seen that in absence of any external force, the centre of mass of a system moves with a constant velocity. If the centre of mass is initially at rest, it would continue to be at rest.

(b) **Example 1:** It follows that when a radioactive nucleus initially at rest decays, its fragments may fly off in different directions but at any given instant, the centre of mass of the fragments remain at the same position where it was before the decay, i.e, the centre of mass remains at rest.

(c) **Example 2:** Consider a bomb thrown at an angle and moving along a parabola explode in mid air and split in two parts (as shown in figure) so that each part after explosion follows its own parabolic path.
However, the centre of mass of the two parts at any given instant shall be at the same position as the bomb if the explosion would not have been there. i.e., the centre of mass continue to move along the original parabola.



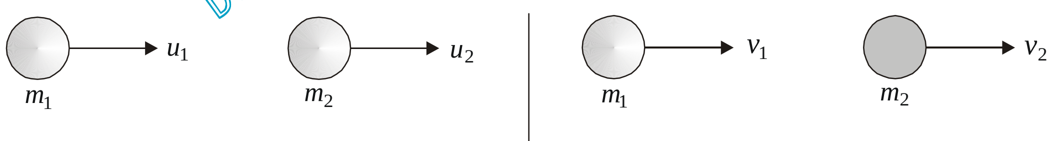
8. Collisions

(a) In common language, a collision between two bodies is said to occur when they crash into each other. e.g, collision between two vehicles, between billiard balls, marbles, carrom coins, etc.

(b) **We define collision as an event in which two or more colliding bodies exert relatively strong forces an each other but for a relatively short time.**

(c) **In all collisions, the total linear momentum is conserved.**

Consider two particles of mass m_1 and m_2, collide. Let their velocities be u_1 and u_2 before collision and v_1 and v_2 after collision respectively.



Before collision **After collision**

The total linear momentum before collision $= m_1u_1 + m_2u_2$

The total linear momentum after collision $= m_1v_1 + m_2v_2$

Then, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

(d) **The total kinetic energy of the system, however may or may not be conserved.** The impact of particles during collision may generate heat, sound and permanent deformation which consumes a part of the initial kinetic energy which results in a decreased final kinetic energy.

(e) The collisions between particles are broadly divided into two types. **Elastic collisions** and **Inelastic collisions**

(f) Since, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$, we can write $\dfrac{m_1u_1 + m_2u_2}{m_1 + m_2} = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2}$

$= \quad u_{cm} = v_{cm}$

Hence, the velocity of centre of mass before collision is equal to velocity of centre of mass after collision.
In fact, **for any collision, the velocity of centre of mass before, during and after collision remains the same.**

9. <u>Elastic Collisions</u>

(a) An elastic collision is one in which in addition to total linear momentum, the total kinetic energy is also conserved.

(b) **The basic characteristics of an elastic collision are**

(i) **The total linear momentum of the system is conserved**

(ii) **The total kinetic energy of the system is conserved**

(c) A body is said to be elastic if on application of deforming forces, the body deforms and subsequently on removal of these forces, the body reforms back to its original shape and size like in case of a spring.

(d) As the name suggests, in elastic collision, the deformation of particles that occur during collision is relieved and hence a part of initial kinetic energy that is consumed in deformation gets released. This results in no loss of kinetic energy.

10. <u>Inelastic Collisions</u>

(a) An inelastic collision is one in which the linear momentum is conserved but the total kinetic energy is not conserved.

(b) **The basic characteristics of an inelastic collision are**

(i) **The total linear momentum of the system is conserved**

(ii) **The total kinetic energy of the system is not conserved**

(c) In an inelastic collision, the deformation of particles that occur during collision is not completely relieved and hence a part of initial kinetic energy is permanently consumed in permanent deformation resulting in a loss of kinetic energy.

(d) In most of the cases, the deformation is partly relieved.

However, if there is absolutely no relieving of deformation, the collision is called **perfectly inelastic collision. In the case of perfectly inelastic collision,**

(i) **the colliding particles stick together and move together with same velocity after collision.**

(ii) **the loss of kinetic energy is maximum.**

11. <u>Coefficient of Restitution or Coefficient of Resilience</u>

(a) In real practice, the collisions between all real objects are neither perfectly elastic nor they are perfectly inelastic.

(b) To quantitatively measure the degree of elasticity of a collision, we introduce a term called coefficient of restitution. It is also called coefficient of resilience and is denoted by letter $'e'$.

(c) **Coefficient of restitution is defined as the ratio of relative velocity of separation of particles after collision to the relative velocity of approach before collision.**

$$e = \frac{\text{relative velocity of separation (after collision)}}{\text{relative velocity of separation (after collision)}} \qquad \text{i.e.,} \quad e = \frac{v_2 - v_1}{u_1 - u_2}$$

(d) $e = 1$ **for perfectly elastic collision**

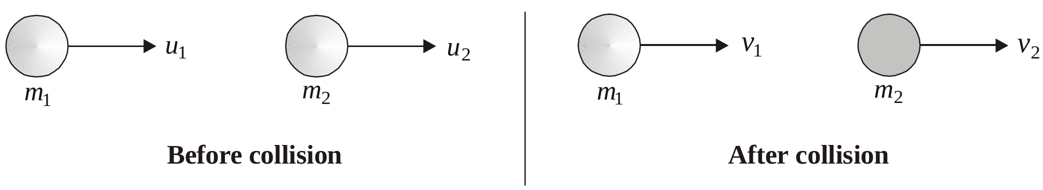
$e = 0$ **for perfectly inelastic collision**

$0 < e < 1$ **for all other inelastic collisions**

12. <u>Elastic collision in one dimension</u>

(a) Collision in one dimension involves the collision of two particles when their motion is along a straight line joining the centres of the two particles. In such case, the motion of the particles both before and after collision is in a straight line and hence is called collision in one dimension. aThis is also known as head on collision.

(b) Consider two particles as shown below



Let this be an elastic collision in one dimension.

(c) Then we can write

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{(By conservation of linear momentum)}$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad \text{(By conservation of kinetic energy)}$$

(d) If m_1, m_2, u_1 and u_2 are known, using the above two equations, we can determine the two unknown variables v_1 and v_2.

On solving the above equations, we get

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2u_2}{m_1 + m_2} \qquad \text{and} \qquad v_2 = \frac{2m_1u_1}{m_1 + m_2} + \frac{m_2 - m_1}{m_1 + m_2}u_2$$

(e) We shall now consider a few special cases of elastic collision in one dimension

(i) **Case-I : When the collision is elastic and the mass of both particles is same.** Here, $m_1 = m_2$

Substituting this in above equations, we get $v_1 = u_2$ and $v_2 = u_1$

Hence, we conclude that if collision is elastic in one dimension and the mass of both particles is same, the velocities of the particles after collision interchange.

(ii) **Case- II: When the collision is elastic, the second particle has initial velocity zero and also has large mass**

i.e, $u_2 = 0$ **and** $m_2 >> m_1$.

Substituting this in above equations, we get $v_1 = - u_1$ and $v_2 = 0$

Hence, we conclude that when a particle makes an elastic collision in one dimension with a large mass at rest, the colliding particle moves with the same speed and reverses its direction.

(iii) **Case - III - When the collision is elastic, the second particle has initial velocity zero and also has large mass**

i.e., $u_2 = 0$ and $m_2 >> m_1$.

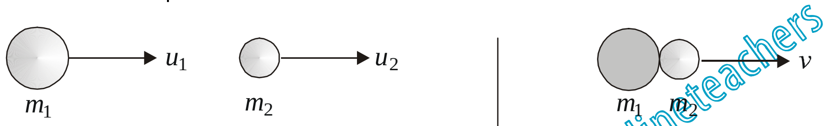
Substituting this in above equations, we get $v_1 = - eu_1$ and $v_2 = 0$

Hence, we conclude that when a particle makes an elastic collision in one dimension with a large mass at rest, the colliding particle reverses its direction and moves with a speed equal to e **times its initial speed.**

13. Perfectly inelastic collision in one dimension

(a) In this case, the two particles stick together after collision and move together with same velocity say 'v'.

(b) Consider the two particles as shown



Before collision **After collision**

(c) Since, the collision is perfectly inelastic, $v_1 = v_2 = v$

Then, by conservation of linear momentum, we can write $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

(d) If m_1, m_2, u_1 and u_2 are known, we have only one unknown variable v. Hence, $v = \dfrac{m_1u_1 + m_2u_2}{m_1 + m_2}$

(e) Clearly since, $v_1 = v_2 = v$, we have $e = \dfrac{v_2 - v_1}{u_1 - u_2} = 0$.

(f) If a body is dropped from a height h_o and rises to a height h_1 after collision with ground,

then the coefficient of restitution is given by $e = \sqrt{h_1/h_o}$ or $h_1 = h_o e^2$.

(g) If after n collisions, the body rises to a height h_n, then $h_n = h_o e^{2n}$.

14. Summary of collisions

	Elastic	Inelastic	Perfectly Inelastic
Linear Momentum is conserved	**yes**	**yes**	**yes**
K.E. is conserved	**yes**	**no**	**no**
Coeff. of restitution (e)	$e = 1$	$0 < e < 1$	$e = 0$
Deformation during collision is relieved	**completely**	**partly**	**not at all**
Particles stick together after collision	**no**	**no**	**yes**