ALTERNATING CURRENT

GUPTA CLASSES

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Alternating Current

Transient Current

When a DC circuit is switched on or off the current change instantaneously. The current flowing in circuit during this short interval are called transient current

Charging of a capacitor

When a voltage is applied across the terminals of a capacitor, the potential cannot rise to its final value instantaneously.

1. The manner in which charge q on plates builds up and the potential across the capacitor V increases, is given by the equations: $q = q_0 (1 - e^{-t/RC})$ and $V = V_0(1 - e^{-t/RC})$. Here R is the resistance of circuit.
2. The current in the circuit (Transient) during charging is given by: $I = I_0 e^{-t/RC}$ where $I_0 = (V_0/R)$
3. Time constant of the circuit is τ = RC. It is that time in which the charge on the plates grows to 63.2% of the final value or it is also the time in which current falls to 36.8% of the initial value.

Discharging of a capacitor

When a charged capacitor (at t = 0) is allowed to discharge through a resistance R, then the voltage across the capacitor and the current in the circuit falls exponentially as follows:

$q = q_0 e^{-t/RC}$ and $V = V_0 e^{-t/RC}$ Also $I = I_0 e^{-t/RC}$

The rate of discharging depends on the time constant again given by: τ = RC. It is that time in which the charge or the voltage or the current decays to reach 36.8% of their initial value.

Growth and decay of current in LR circuit

When a switch in an LR circuit is closed, the current does not become maximum immediately but it takes some time, i.e., there is a time lag.

1 The instantaneous current in the circuit during its growth is given by: $I = \left(\frac{E}{R}\right)(1 - e^{-\frac{R}{L}t})$

Here, (L/R) = time constant of LR circuit. The time constant is the time in which current rises to 0.6321 times the maximum current which is equal to (E/R).

2 When the switch in an LR circuit is opened, the instantaneous current I is given by: $I = \left(\frac{E}{R}\right)e^{-\frac{R}{L}t}$

Hence, the time constant of an LR circuit may also be defined as the time in which the current falls to 0.3679 times of its initial current.

3 Decay or growth of current in LR circuit is fast when L/R is small and slow when (L/R) is large.

Alternating current and voltage

The alternating current changes in magnitude continuously and periodically in direction. It is represented by a sine or cosine curve.

The instantaneous value of an AC is given by: $I = I_0 \sin\omega t$ and the alternating voltage is given by: $E = E_0 \sin\omega t$. Here, ω is the angular frequency of AC and $(\omega/2\pi)$ is the frequency of AC. $(2\pi/\omega)$ represents the time period of AC.

Peak value: Peak value of AC is the maximum value of current in either direction of the cycle, In $I = I_0 \sin\omega t$, I_0 is the peak value of AC.

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Mean value: The mean value of AC represented by the equation, $I = I_0 \sin \omega t$ is zero over one complete cycle and is meaningless. In practice, mean value of alternating current refers to its average value over either the first half cycle or the second half cycle. Over first half cycle, $I_{mean} = 2I_0/\pi$ Over second half cycle, $I_{mean} = -2I_0/\pi$

Numerically, $I_{mean} = 2I_0/\pi$

A moving coil galvanometer, connected to an AC source of 50 Hz AC, shows a steady zero reading of the pointer. If the frequency is 2 Hz or 3 Hz, the pointer oscillates with equal amplitude on either side of zero position.

RMS or Virtual value: The RMS value is defined as the square root of the mean of square of the instantaneous value of current over the complete cycle. It may also be defined as the direct current which produces the same heating effect in a resistor as the actual AC $I_V = I_0/\sqrt{2} = I_{rms}$

(b) AC ammeter or voltmeter measures virtual current or virtual voltage.

(c) For an AC mains of 220 volt, the peak value of the voltage is given by: $E_0 = \sqrt{2}E_V = \sqrt{2} \times 220 = 311$ volt

(d) Form Factor = $\dfrac{\text{Virtual Value}}{\text{Mean Value}} = \dfrac{e_0/\sqrt{2}}{2e_0/\pi} = \dfrac{\pi}{2\sqrt{2}} = 1.1$

AC through pure resistor R

1 Alternating e.m.f. of the source: $E = E_0 \sin \omega t$

2 A resistance opposes current but does not oppose a change in current. Hence, current is in phase with e.m.f.

3 The instantaneous value of the current is given by: $I = \dfrac{E_0 \sin \omega t}{R}$

4 The virtual value of current I_V is given by: $I_V = \dfrac{E_V}{R}$

AC through pure inductor of inductance L

1 Alternating e.m.f. of the source: $E = E_0 \sin \omega t$

2 An inductor does not oppose current but opposes a change in current.

3 Since the voltage changes continuously, hence the current reacts to the change and inductive reactance $X_L = \omega L$

4 The current lags behind the voltage by $\pi/2$.

5 The instantaneous current is given by: $I = \dfrac{E_0 \sin\left(\omega t - \frac{\pi}{2}\right)}{X_L}$

6 The virtual value of the current is given by: $I_V = E_V / X_L$.

AC through a pure capacitor of capacitance C

1 Alternating e.m.f. of the source: $E = E_0 \sin \omega t$

2 A capacitor has infinite resistance for a DC source. With an AC source, voltage changes, hence charge on the plates of the capacitor changes with time, i.e., there is a current. The current leads the voltage by $\pi/2$.

3 The capacitor has a capacitive reactance X_C in AC circuit, given by: $X_C = (1/\omega C)$.

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4 The instantaneous value of current is: $I = \dfrac{E_0 \sin\left(\omega t + \frac{\pi}{2}\right)}{X_C}$

5 The virtual value of current I_V is given by: $I_V = \dfrac{E_V}{X_C}$

AC source connected to resistor R, an inductor L and a capacitor C in series

1 The virtual current I_V is given by: $I_V = \dfrac{E_V}{\sqrt{R^2 + (X_L - X_C)^2}}$

2 The current and voltage have a phase difference ϕ given by: $\tan\phi = \dfrac{|X_L - X_C|}{R}$

3 The impedance which represents the total opposition of the circuit to AC source is represented by Z. The impedance Z is the vector sum of resistance and reactances (X_L & X_C) in AC series circuit $Z = \sqrt{R^2 + (X_L - X_C)^2}$

4 In AC series combination, virtual voltages are added vectorically, i.e; $E_V = \sqrt{V_R^2 + (V_L - V_C)^2}$

where $V_R = I_V R$, $V_C = I_V X_c$ and $V_L = I_V X_L$

Power in an AC circuit

The power in an electric circuit is the rate at which electric energy is consumed in the circuit . The average power of an AC circuit is given by $P = E_{rms} \times I_{rms} \times \cos\phi$, where $\cos\phi$ is known as power factor.

For a LCR series circuit: $\cos\phi = \dfrac{R}{Z}$

Wattless current: Consider the case of a circuit which contains either inductance only or the capacitance only, i.e., ohmic resistance is zero. In such cases, phase difference between voltage and current is 90°, i.e., ϕ = 90°. Now, we know that the average power in a circuit is given by: $P = E_{rms} \times I_{rms} \times \cos\phi$ So,

$$P = E_{rms} \times I_{rms} \times \cos 90^0 = 0$$

This shows that when ohmic resistance of an AC circuit is zero then average power remains zero though current flows in the circuit. In this way, there is no dissipation of energy in the circuit. The current in such a circuit is called as wattless current.

Series resonant circuit

1 A particular frequency of AC at which impedance of a series LCR circuit becomes minimum or the current becomes maximum, is called the resonant frequency and the circuit is called as series resonance circuit.

2 At resonance frequency $\omega_0 L = \dfrac{1}{\omega_0 C}$ and $\omega_0 = \dfrac{1}{\sqrt{LC}}$ or $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

3 At resonance $\tan\phi = \dfrac{|X_L - X_C|}{R} = \text{zero}$, i.e., $\phi = 0$ i.e., phase difference is zero. So, $\cos\phi = +1$, i.e., power factor is maximum.

4 At resonance, $I_{max.} = \dfrac{E_V}{Z_{min.}} = \dfrac{E_V}{R}$ In this case, $V_L = I_V \omega_0 L$, $V_C = I_V(1/\omega_0 C)$

$V_L = V_C$ and $E_V = \sqrt{V_R^2 + (V_L - V_C)^2}$ Hence, $E_V = V_R$

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i.e., at resonance with L and C in series, the current is maximum through the L and C combination but potential difference across the combination is zero. Hence, reactance of the LC combination at resonance is zero. At resonance, the whole of the source voltage appears across the resistor.

Quality factor: The quality factor is a measure of the efficiency of energy stored in an inductor or capacitor when an alternating current is applied. $Q = 2\pi\left[\dfrac{\text{energy stored}}{\text{energy lost per period}}\right]$

Q = $\omega L/R$ or $1/\omega CR$, i.e., the quality factor may also be defined as the ratio of reactance of either inductance or capacitance at resonance frequency to the resistance of the circuit.

Choke coil

We know that in purely resistive circuit, the power loss is maximum because power factor cos $\phi = (R/Z) = 1$ (because Z = R). Hence, the use of resistance is avoided in AC circuits.

A choke coil is a coil which has high inductance and negligible resistance. Thus the power factor is almost zero. So a choke coil controls the alternating current without an appreciable energy loss. This is used in fluorescent tubes to control the current.

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Alternating Current Assignment

1 3Hot wire ammeters can be used for measuring
(a) alternating current only
(b) direct current only
(c) both alternating and direct current
(d) neither alternating nor direct current.

2 3A steady p.d. of 10 V produces heat at a rate x in a resistor. The peak value of the alternating voltage which will produce heat at a rate x/2 in the same resistor is
(a) 5V (b) $5\sqrt{2}$ V (c) 10 V (d) $10\sqrt{2}$ V

3 4If a current $I = I_o \sin(\omega t - \pi/2)$ flows in a circuit across which an alternating potential $E = E_o \sin \omega t$ has been applied, then the power consumed in the circuit is
(a) $E_oI_o/\sqrt{2}$ (b) $E_oI_o/2$ (c) $EI/\sqrt{2}$ (d) zero.

4 1In a circuit containing an inductance of zero resistance, the current lags behind the applied ac voltage by a phase angle equal to
(a) 90° (b) 45° (c) 30° (d) 0°

5 2An alternating voltage $V = 200\sqrt{2} \sin 100t$ where V is in volt and t seconds, is connected to a series combination of 1 μF capacitor and 10 kΩ resistor through an ac ammeter. The reading of the ammeter will be
(a) $\sqrt{2}$ mA (b) $10\sqrt{2}$ mA (c) 2 mA (d) 20 mA

6 3An inductor of 1 henry is connected across a 220 V, 50 Hz supply. The peak value of the current is approximately
(a) 0.5 A (b) 0.7 A (c) 1 A (d) 1.4 A

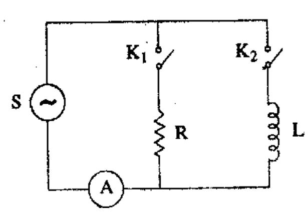
7 2An inductive coil has a resistance of 100 Ω. When an ac signal of frequency 1000 Hz is applied to the coil, the voltage leads the current by 45°. The inductance of the coil is
(a) $1/10\pi$ (b) $1/20\pi$ (c) $1/40\pi$ (d) $1/60\pi$

8 4A resistor R and a capacitor C are connected in series across an ac source of rms voltage 5V. If the rms voltage across C is 3V then that across R is
(a) IV (b) 2V (c) 3V (d) 4V

9 3An LCR series circuit contains L = 8H, C = 0.5μF and R= 100Ω. The resonant frequency of circuit is
(a) $1000/\pi$ (b) $500/\pi$ (c) $250/\pi$ (d) $125/\pi$

10 2A resistance R, an inductance L and a capacitance C are connected in series across an ac source of angular frequency ω. If the resonant frequency is ω_o then the current will lag behind the voltage if
(a) $\omega < \omega_o$ (b) $\omega > \omega_o$ (c) $\omega = \omega_o$ (d) $\omega = 0$

11 1In the given circuit, R is a pure resistor, L is a pure inductor, S is a 100 V, 50 Hz ac source, and A is an ac ammeter. With either K_1 or K_2 alone closed, the ammeter reading is I If the source is changed to 100 V, 100 Hz, the ammeter reading with K_1, alone closed and with K_2 alone closed will be respectively,



(a) I, $I/2$ (b) I, $2I$ (c) $2I$, I (d) $2I$, $I/2$.

12 1A resistor, an inductor and a capacitor are connected in series to an ac source. An ac voltmeter measures the voltages across them as 80 V, 30 V and 90 V respectively. The rms value of the supply voltage is
(a) 100 V (b) 100 $\sqrt{2}$2V (c) 200V (d) 200 $\sqrt{2}$2 V

13 1An LCR series circuit consists of a resistance of 10 Ω, a capacitance of reactance 60 Ω and an inductor coil. The circuit is found to resonate when put across a 300 V, 100 Hz supply. The inductance of the coil is (take π =3).
(a) 0.1H (b) 0.01 H (c) 0.2 H (d) 0.02 H

14 2A sinusoidal alternating current flows through a resistor R. If the peak current is I_p, then the power dissipated is
(a) $I_p^2 R$ (b) ½ $I_p^2 R$ (c) $(4/\pi) I_p^2 R$ (d) $(1/\pi) I_p^2 R$

15 2Power delivered by an ac source of frequency ω to an LCR series circuit is maximum when
(a) $\omega L = \omega C$ (b) $\omega L = 1/\omega C$
(c) $\omega L = R - (1/\omega C)$ (d) $\omega C = R - (1/\omega L)$

16 2An electric bulb which runs at 80 V dc and consumes 10 A current is connected across a 100 V, 50 Hz ac supply. The inductance of the choke required is (take π = 3)
(a) 0.01 H (b) 0.02 H (c) 0.04H (d) 0.08H

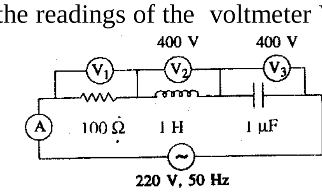
17 2A 1 μF capacitor is connected across an ac source whose voltage amplitude is 50 V and angular frequency is 100 rad/s. The current amplitude will be
(a) 2.5 mA (b) 5 mA (c) 10mA (d) 15 mA

18 1A 60 μF capacitor, 0.3 H inductor and a 50 Ω, resistor are connected in series with a 120 V, 60 Hz source. The current in the circuit is
(a) 1.5 A (b) 2 A (c) 3 A (d) 4 A

19 3Two coils A and B are connected in series across a 240 V, 50 Hz supply. The resistance of A is 5 Ω and the inductance of B is 0 02 H. The power consumed is 3 kW and the power factor is 0.75. The impedance of the circuit is
(a) 0.144Ω (b) 1.44Ω (c) 14.4Ω (d) 144 Ω

20 2The reactance of a capacitor at 50 Hz is 10Ω. Its reactance at 100 Hz is
(a) 2.5 Ω (b) 5 Ω (c) 10Ω (d) 20Ω

21 1In the given circuit the readings of the voltmeter V_1 and the ammeter A are



(a) 220 V : 2.2A
(b) 110 V ; 1.1 A
(c) 220V ; 1.1 A
(d) 110 V; 2.2A.

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22 1An LCR series circuit containing a resistance of 120 Ω has angular resonance frequency 4×10^5 rad s^{-1}, At resonance the voltage across resistance and inductance are 60 V and 40 V respectively. The values of L and C are

(a) 0.2mH, 1/32 μF (b) 0.4 mH, 1/16 μF

(c) 0.2 mH, 1/16 μF (d) 0.4 mH, 1/32 μF

Answers

1c ,2c ,3d ,4a ,5b ,6c ,7b ,8d ,9c ,10b ,11a
12a ,13a , 14b ,15b ,16b ,17b ,18a ,19c ,20b
21a ,22a

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