FORCE AND FRICTION

GUPTA CLASSES

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FORCE & FRICTION

Force:

A force is something that changes the state of rest or motion of a body. It causes a body to start moving, if it is at rest or stops it, if it is in motion or to deflect it from its initial path of motion.

Concurrent forces: When many forces act at a point on a body, they are called concurrent forces. In the system of concurrent forces, the forces may be collinear, i.e., along the same straight line or coplanar, i.e., in the same plane.

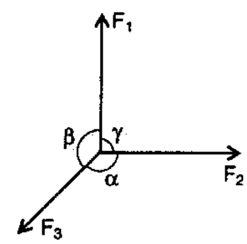
Superposition of forces: When many forces are acting on a single body, the resultant force is obtained by using the laws of vector addition. $\vec{F} = \vec{F_1} + \vec{F_2} + \ldots\ldots\ldots\ldots + \vec{F_n}$

Condition for equilibrium of concurrent forces:

1. For equilibrium; the vector sum of all the forces must be zero.
2. If the object is at rest and is in equilibrium, then it is called static equilibrium. If the body is in motion and in equilibrium ($\sum F = 0$), then it is called dynamic equilibrium.
3. The static equilibrium may be any one of the three types, viz, (1) stable, (2) unstable, (3) neutral.
4. For an object in equilibrium, acceleration is zero.
5. For an object in equilibrium under the action of conservative forces, F = -(dU/dr) where U represents potential energy.

Lami's theorem: If three forces F_1, F_2 and F_3 are acting simultaneously on a body and the body is in equilibrium, then according to Lami's theorem,

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$



where α, β, and γ are the angles opposite to the forces F_1, F_2 and F_3 respectively.

Basic forces in nature:

1. There are four types of basic forces that exist in nature:
   (a) Gravitational forces (b) Electromagnetic forces (c) Nuclear forces (d) Weak forces
2. Many other well known forces like frictional force, elastic force, viscous force, spring force, intermolecular force are example of electromagnetic forces.
3. The gravitational and the electromagnetic forces are long-range forces having infinite range while nuclear and weak forces are very short-range forces.
4. The nuclear force is the strongest force while the gravitational force is the weakest force of nature. The relative magnitudes of different forces can be expressed as $F_G : F_W : F_E : F_N :: 1 : 10^{25} : 10^{36} : 10^{38}$ i.e; $F_N > F_E > F_W > F_G$
5. Gravitational and electromagnetic forces are conservative forces and obey inverse square law, while nuclear force is a non-conservative force and varies inversely with some higher power of distance.

First law of motion:

1. Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by a resultant force to change that state.
2. This law is also known as law of inertia. Inertia is the property of inability of a body to change its position of rest or uniform motion in a straight line unless some external force acts on it. Mass is a measure of inertia of a body.
3. A frame of reference in which Newton's first law is valid is called inertial frame, i.e., if a frame of reference is at rest or in uniform motion it is called inertial, otherwise non-inertial.

Second law of motion:

1. According to 2nd law of motion, rate of change of momentum of a body is proportional to the resultant force acting on the body, i.e., $\vec{F} \propto (d\vec{p}/dt)$. Here, the change in momentum takes place in the direction of the applied resultant force.

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Momentum, $\vec{p} = m\vec{v}$ is a measure of sum of the motion contained in the body. According to this, $\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$

2. If the mass of the system is finite and remains constant w.r.t. time, then (dm/dt) = 0 and $\vec{F} = m(d\vec{v}/dt) = m\vec{a} = (\overrightarrow{p_2} - \overrightarrow{p_1})/t$
3. If the force is parallel or anti-parallel to the motion, it changes only the magnitude of $\vec{v}$ but not the direction and the path followed by the body is a straight line.
4. If the force is acting perpendicular to the motion of body, it changes only the direction but not the magnitude of $\vec{v}$ and the path followed by the body is a circle (uniform circular motion).
5. If the force acts at an angle to the motion of a body, it changes both the magnitude and direction of $\vec{v}$ In this case path followed by the body may be elliptical, non-uniform circular, parabolic or hyperbolic.

Third law of motion:

1. According to this law, for every action there is an equal and opposite reaction. When two bodies A and B exert force on each other, the force of A on B (i.e., action represented by F_{AB}) is always equal and opposite to the force of B on A (i.e., reaction represented by F_{BA}). Thus, $F_{AB} = -F_{BA}$.
2. The two forces involved in any interaction between two bodies are called action and reaction. But we cannot say that this particular force is action and the other one is reaction. Action and Reaction always act on different bodies.

Reaction: Reaction of a body on another body is always normal to surface of contact.

Pseudo Force: If an object is placed in an accelerated frame then for the convenience of calculation an imaginary force called as Pseudo Force acting on object is taken in account. This force is given by F = ma, where 'm' is mass of object and 'a' is acceleration of frame. Direction of this force is always opposite to acceleration of frame.

Strings: While dealing with problems involving string, remember the following:

1. String is assumed to be perfectly elastic (or inextensible) unless stated.
2. String is assumed to be massless unless stated.
3. On applying two equal and opposite force, tension is produced in string. The tension is directed inward opposite to applied force and is equal in magnitude throughout the length of string.

Springs: Regarding spring note the following;

1. If a spring is stretched or compressed by 'x' then restoring force is give by F = - k x , where k is called as spring constant or force constant of spring.
2. Spring constant $k \propto 1/l$ where l is normal length of spring.
3. If few springs of force constant k_1, k_2, k_3,……….are in combination then effective force constant of (k) of combination is given by (i) $k = k_1 + k_2 + k_3 + \ldots\ldots$. for parallel combination and (ii) $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots$..for series combination.

Impulse:

1. Impulse is defined as the product of the force and the time for which that force acts, i.e., I = Ft. If the force is time varying then I = ∫ F dt.
2. Impulse is measure of change of momentum produced in a body, i.e., impulse = $\Delta\vec{p} = m\vec{v} = \vec{F}\Delta t$
3. Area under force-time graph gives the magnitude of impulse.
4. To catch a ball, the player lower his hand so that Δt (time required for change of momentum) increases. This makes F small because F = (impulse/time).
5. Bogies of a train are provided with buffers since they increase the time interval of jerks during shunting and hence reduce the force with which the bogies pull each other.

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Some important points concerning Newton's laws of motion:

1. The forces of interaction between bodies composing a system are called internal forces. The forces exerted on bodies of a given system by bodies situated outside are called external forces.
2. Whenever one force acts on a body it gives rise to another force called reaction, i.e., a single isolated force is physically impossible. This is why total internal force on an isolated system is always zero.

Apparent weight of a body in a lift:

1. When the lift is at rest or moving with uniform velocity, i.e., $a = 0$: $W_{app.} = W_0$
2. When the lift moves upwards with an acceleration 'a': $W_{app.} = W_0\left(1+\frac{a}{g}\right) = mg\left(1+\frac{a}{g}\right)$
3. When the lift moves downwards with an acceleration 'a': $W_{app.} = W_0\left(1-\frac{a}{g}\right) = mg\left(1-\frac{a}{g}\right)$ Here, if $a > g$, W_{app} will be negative. Negative apparent weight will mean that the body is pressed against the roof of the lift instead of floor.
4. When the lift falls freely, i.e., $a = g$: $W_{app.} = 0$

Systems of variable mass:

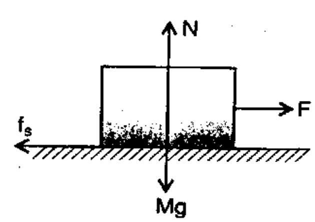
1. According to Newton's 2nd law: $\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$ If the mass of the system changes and the velocity of escaping mass is constant, then $(d\vec{v}/dt) = 0$ and $\vec{F} = \vec{v}\left(\frac{dm}{dt}\right)$.
2. If 'n' bullets each of mass m are fired per unit time from a machine gun, then the force required to hold the gun = $v\,dm/dt = v(mn) = mnv$.
3. If sand is dropped on to a conveyor belt at a constant rate, then the extra force required to maintain the speed of the belt constant = $v\,(dm/dt)$.
4. If a jet of liquid coming out from a pipe of uniform area of cross-section A hits a wall normally with a velocity v, then the force applied by the jet on the wall = $Av^2\rho$ (ρ = density of liquid).

Friction:

1. If we slide or try to slide one body over another body, there is a force that opposes the motion. This opposing force is called the force of friction. The origin of friction is adhesive and cohesive forces.
2. The force of friction is always tangent to the surfaces of two bodies and is always in a direction opposite to that of intended motion.
3. When a force is applied on a body and the body tends to move (but does not move), then the frictional force is called force of static friction. When the body is in motion, then the frictional force is called force of kinetic friction. Generally, the force of kinetic friction is slightly less than force of static friction.

Static friction:

1. The force of static friction, which develops in a direction opposite to the applied force, is a self adjusting force. Consider a block of weight $W = Mg$ placed on a rough horizontal surface. It is found that
   (a) When $F = 0$, then f_S,=0.
   (b) When $F \neq 0$ and small, then $f_S = F$ till f_S becomes equal to some $(f_S)_{max.}$.
   Once $f_S = (f_S)_{max.}$ Then f_S does not increase further.
   It is found that $(f_S)_{max.} = \mu_S N$ (μ_S = coefficient of static friction)



2. Maximum static friction $(f_S)_{max.}$ is also called the limiting friction. (f_S) is a self-adjusting force in the sense that till $(f_S)_{max.}$ force of friction $(f_S) = F$. But once $F > (f_S)_{max.}$

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then force of friction $(f_S) = (f_S)_{max.} = \mu_S N$

3. The coefficient of friction (μ_S) is a dimensionless constant which depends on the nature of surfaces and the cohesive or adhesive forces. It does not depend on the area of surfaces in contact.

2. The force of static friction can be expressed as: $(f_S) \le \mu_S N$. Hence, $\mu_S = \dfrac{\text{limiting friction } [(f_S)_{max.}]}{\text{normal reaction (N)}}$

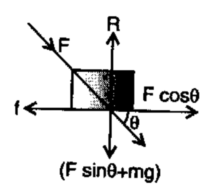
Kinetic friction:

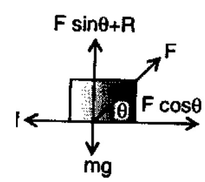
1. When applied force F exceeds the limiting friction, i.e., $(f_S)_{max.}$ then the body begins to move. The frictional force during motion is called the force of kinetic friction ((f_k)). It can be expressed as $(f_k) = \mu_k N$. where μ_k is called the coefficient of kinetic friction,
2. μ_k is a dimensionless constant. Usually $\mu_k < \mu_S$. Also, μ_k is independent of the relative velocities of the two objects.

Some other important points regarding friction:

1. Because limiting force is higher than kinetic friction, hence we require more force to start a motion than to maintain it against friction.
2. When a body rolls on a surface, the resistance offered by the surface is called rolling friction. In rolling the surfaces in contact do not rub each other. The velocity of the point of contact with respect to surface remains zero all the time, although the centre of the body moves forward. Rolling friction is negligible as compared to static or kinetic friction, i.e., $\mu_R < \mu_k < \mu_S$
3. If the normal force remains constant, then the force of friction does not depend on the area of surfaces in contact and their relative velocity.
4. If lubricants are used then force of friction generally decreases.
5. If a rubber wheel is rolling on a road, then lesser is the deformation in it, lesser is the friction,
6. Friction is a non-conservative force, i.e., work done against friction is path dependent. In its presence, mechanical energy is not conserved. Due to frictional forces, heat is produced.
7. Frictional forces are essential in many walks of life. For example, the walking process can only take place because there is friction between the shoes and ground. In the process of walking, in order to step forward, you must press your foot backward on the ground. A friction force tends to oppose this movement of the foot and the ground pushes you forward which allows you to walk. When there is no friction, as on ice or polished granite or oily surface, walking is difficult.
8. When a person is pedalling a bicycle then direction of frictional force on the front wheel is backward and is forward on the rear wheel. However, if pedalling is stopped, both wheels move themselves and so experience a force of friction in backward direction.

Pushing and Pulling of a body on a rough horizontal surface:





While pushing frictional force towards left, $f = \mu R = \mu\,(F \sin\theta + mg)$

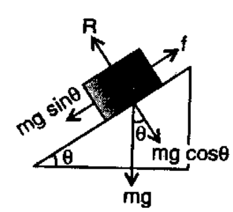
Force towards right = $F \cos\theta$

While pulling frictional force towards left, $f = \mu R = \mu\,(mg - F\sin\theta)$

Force towards right = $F\cos\theta$

Hence, It is easy to pull a lawn roller than to push it, since frictional force is more in the latter case.

Motion of a body on rough inclined plane:

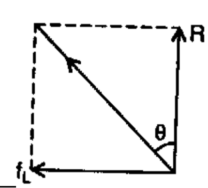


1. If we consider a body on an inclined plane which is just at the verge of sliding as shown in the figure. Consider the equilibrium of forces along and perpendicular to the plane.

   $f = mg \sin\theta$ and $R = mg \cos\theta$

   Now, $f = \mu_S R$ or $mg\sin\theta = \mu_S\,.mg\cos\theta$

   and $\mu_S = \tan\theta$, where θ is called as angle of friction or angle of repose.



2. Angle of friction may also be defined as the angle made by the resultant of maximum force of static friction and the reaction force, with the reaction, ie., $f/R = \mu_S = \tan\theta$.

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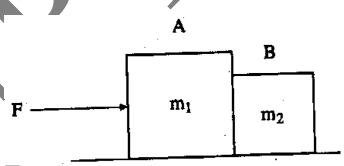
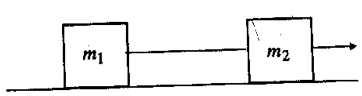
3. Angle of repose is defined as the minimum angle given a rough plane with horizontal such that a body placed on it is on the verge sliding. If angle of repose is θ then μ_S = tan θ. The angle of friction and angle of repose are equal.

FORCE & FRICTION Assignment

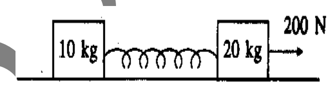
1. A body of mass 2 kg, moving on a horizontal surface with an initial velocity of 4 m/s, comes to rest after 2 seconds. If one wants to keep this body moving on the same surface with a velocity of 4 m/s, the force required is
(a) zero (b) 2 N (c) 4 N (d) 8 N
2. A weight of 290 N and another of 200 N are suspended by a rope on either side of a frictionless pulley. The acceleration of each weight (in m/s^2) is
(a) 1.5 (b) 1.8 (c) 2.2 (d) 2.5
3. What force should be applied on a 5 kg body so that it has a downward acceleration of 4 m/s^2?
(a) 69 N upwards (b) 69 N downwards
(b) 29 N upwards (d) 29 N downwards
4. A certain force gives a 2 kg object an acceleration of 0.5 m/s^2. What acceleration would the same force give a 10 kg object ?
(a) 0.1 (b) 0.2 (c) 0.5 (d) 1.0
5. An 80 kg man stands on a spring balance in an elevator. When it starts to move, the scale reads 700 N., What is the acceleration of the elevator ?
(a) 1.25 m/s^2 upwards (b) 2.0 m/s^2 downwards
(c) 2.0 m/s^2 upwards (d) 1.25 m/s^2 downwards
6. The mass of an elevator is 4000 kg. When the tension in the supporting cable is 48000 N, the acceleration of the elevator is (g = 10 m/s^2)
(a) 2 m/s^2 upwards (b) 2 m/s^2 downwards
(c) 20 m/s^2 upwards (d) 20 m/s^2 downwards
7. Two blocks A and B, having masses m_1 and m_2 respectively, are placed .in contact on a smooth horizontal surface as shown in fig. A force F is applied horizontally on A. The contact force between A and B is



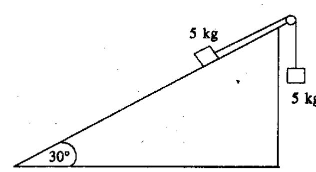
(a) $\frac{m_1}{m_2}F$ (b) $\frac{m_2}{m_1}F$ (c) $\frac{m_1}{m_2+m_1}F$ (d) $\frac{m_2}{m_2+m_1}F$
8. Two masses m_1 and m_2 placed on a smooth horizontal surface are connected by a massless inextensible string. A horizontal force F is applied on m_2 as shown. The tension in the string is



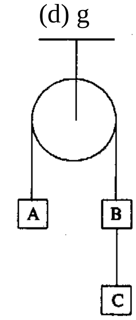
(a) $\frac{m_1}{m_2}F$ (b) $\frac{m_2}{m_1}F$
(c) $\frac{m_1}{m_2+m_1}F$ (d) $\frac{m_2}{m_2+m_1}F$
9. Two masses of 10 kg and 20 kg are connected by a massless spring as shown. A force of 200 N acts on the 20 kg mass. At a certain instant the acceleration of 10 kg mass is 12 m/s^2. The acceleration of the 20 kg mass at that instant is



(a) 4 m/s^2
(b) 10 m/s^2
(c) 12 m/s^2
(d) none of these.
10. Two masses, each of 5 kg, are connected by a massless, inextensible string passing over a smooth peg. One of the masses is on a frictionless 30° incline and the other hangs vertically. When the masses are released, the tension in the string and the common acceleration of the masses are (g = 10m/s^2)



(a) 37.5 N; 2.5 m/s^2 (b) 37.5 N ; 5 m/s^2
(c) 25 N ; 2.5 m/s^2. (d) 25 N ; 5 m/s^2
11. The minimum acceleration with which a fireman can slide down a rope of breaking strength two-third of his weight is
(a) zero (b) g/3 (c) 2g/3 (d) g
12. Three blocks A, B and C. each of mass 2 kg are hanging over a fixed pulley as shown. The tension in the string connecting B and C is



(a) zero
(b) 3.3 N
(c) 13.3 N
(d) 19.6 N
13. A body kept on a smooth inclined plane having inclination 1 in x will remain stationary relative to the inclined plane if the plane is given a horizontal acceleration equal to
(a) $\frac{g}{\sqrt{x^2-1}}$ (b) $\frac{gx}{\sqrt{x^2-1}}$

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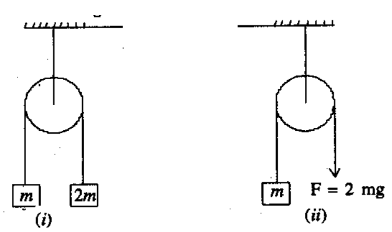
(c) $\frac{\sqrt{x^2-1}}{g}$ (d) $g\sqrt{x^2-1}$

14. A cricket ball of mass 150 g is moving with a velocity of 12 m/s and is hit by a bat so that it is turned back with a velocity of 20 m/s. The force of blow acts for 0.01 s. The average force exerted by the bat on the ball is
(a) 120 N (b) 240 N (c) 480 N (d) 960 N

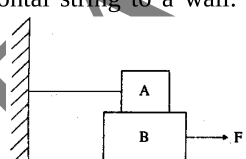
15. A vehicle, having a mass of 1000 kg is moving with a uniform velocity of 20 m/s. Sand is dropped into it at the rate of 5 kg/min. The force needed to keep the vehicle moving with uniform velocity is
(a) 3 N (b) 5 N (c) 3/5 N (d) 5/3 N

16.



(a) the same in both cases
(b) less in (i) than in (ii) by a factor 1/3.
(c) more in (i) than in (ii) by a factor 3.
(d) less in (i) than in (ii) by a factor 1/2.

17. A block A of mass 200 kg rests on a block B of mass 300 kg. A is tied with a horizontal string to a wall. Coefficient of friction between A and B is 0.25 and that between B and floor is 0.2. The horizontal force F needed to move the block B is

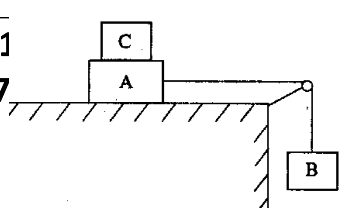


(a) 550 N (b) 1100 N (c) 1500 N (d) 2200 N

18. A block of mass 2 kg rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on block is
(a) 9.8 N (b) 0.7 x 9.8 N
(c) 9.8 x √3 N (d) 0.7 x 9.8 x √3 N

19. A uniform chain of length L lies on a table. If the coefficient of friction is μ, then the maximum length of the chain which can hang from the edge of the table without the chain sliding down is
(a) $\frac{L}{\mu}$ (b) $\frac{L}{\mu-1}$ (c) $\frac{\mu L}{\mu+1}$ (d) $\frac{\mu L}{\mu-1}$

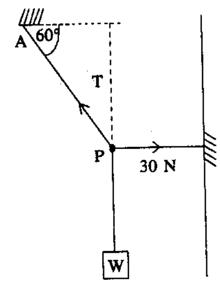
20. Two masses A and B of 10 kg and 5 kg respectively are connected with a string passing over a frictionless pulley fixed at the comer of a table as shown. The coefficient of static friction between A and the table is 0.2. The minimum mass C that should be placed on A to prevent it from moving is equal to



(a) 12 kg (b) 5 kg (c) 10 kg {d) 15 kg

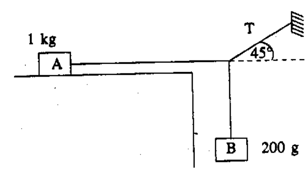
21. A block of mass 5 kg is placed on a rough inclined plane. The inclination of the plane is gradually increased till the block just begins to slide down. The inclination of the plane is then 3 in 5. The coefficient of friction between the block and the plane is (g = 10 m/s^2)
(a) 3/5 (b) 3/4 (c) 4/5 (d) 2/3

22. Three light strings are connected at the point P. A weight W is suspended from one of the strings. End A of string AP and end B of string PB are fixed as shown. In equilibrium PB is horizontal and PA makes an angle of 60° with the horizontal. If the tension in PB is 30 N then the tension in PA and weight W are respectively



(a) 60 N ; 30 N (b) 60/√3 N; 30/√3 N
(c) 60 N, 30√3 N (d) 60√3 N; 30√3 N

23. The mass-string system shown in the figure is in equilibrium. If the coefficient of friction between A and the table is 0.3, the frictional force on A is



(a) 9.8 N (b) 2.04 N
(c)1.96N (d) 0.59N

24. A plumb bob is hanging from the ceiling of a car. If the car moves with an acceleration a, the angle made by the string with the vertical is
(a) $\sin^{-1}$ (a/g) (b) $\sin^{-1}$ (g/a)
(c) $\tan^{-1}$ (a/g) (d) $\tan^{-1}$ (g/a)

25. A balloon has 2g of air. If a small hole is pierced into it, the air comes out with a speed of 2 m/s. If the balloon shrinks completely in 5 s. the average thrust experienced by the balloon is
(a) 8 x 10^{-1}N (b) 8 x 10^{-2}N
(c) 8 x 10^{-3}N (d) 8 x 10^{-4}N

26. A certain force applied to mass m_1 gives it an acceleration of 10 m/s^2. The same force applied to mass m_2 gives it an acceleration of 15 m/s^2. If the two

masses are joined together and the same force is applied to the combination, the acceleration will be

(a) 6 m/s^2 (b) 3 m/s^2 (c) 9 m/s^2 (d) 12 m/s^2

27. An impulse is supplied to a moving object with the force at an angle of 120° with the velocity vector. The angle between the impulse vector and the change in momentum vector is

(a) 120° (b) 0° (c) 60° (d) 240°

28. A machine gun is mounted on a 5 quintal vehicle on a smooth horizontal road. The gun fires 10 bullets per second, each of mass 10 g, with a speed of 500 m/s. The acceleration produced in the vehicle is

(a) 10 cm/s (b) 20 cm/s
(c) 50 cm/s (d) 1 m/s

29. In a rocket of mass 1000 kg fuel is consumed at the rate of 40 kg/s. The velocity of the gases ejected from the rocket is 5 x 10^4 m/s. The thrust on the rocket is

(a) 2 x 10^3 N (b) 5 x 10^4 N
(c) 2 x 10^6 N (d) 2 x l0^9 N

Answers: 1-c, 2-b, 3-c, 4-a, 5-d, 6-a, 7-d, 8-c, 9-a, 10-a, 11-b, -12-c, 13-a, 14-c, 15-d, 16-b, 17-c, 18-a, 19-c, 20-b, 21-b, 22-c, 23-c, 24-c, 25-d, 26-a, 27-b, 28-a, 29-c,

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