SIMPLE HARMONIC MOTION

GUPTA CLASSES

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Simple Harmonic Motion

Periodic motion

A motion is said to be periodic when it repeats itself. A periodic motion can be synthesized from or analysed into a number of simple harmonic motions. In a periodic motion, force is always directed towards a fixed point which may or may not be on the path of motion.

Oscillatory motion

A body is said to be in oscillatory motion if it moves back and forth repeatedly about a mean position. Oscillatory motion is basically a constrained periodic motion between two fixed limits. Though every oscillatory motion is definitely periodic but every periodic motion is not always oscillatory.

Simple harmonic motion

SHM is the simplest kind of periodic motion. It may be defined as the periodic motion of a body such that its acceleration at any instant is always directed towards a fixed point, and the magnitude of its acceleration at that instant is directly proportional to its displacement from a fixed point.

The fixed point is known as mean position and the force obeying above two conditions is called as restoring force. In SHM body oscillates under the action of a constant restoring force given by:$F = -Ky$.

1. All SHMs are periodic but reverse may or may not be true.
2. In every SHM, the time period is independent of amplitude,

Displacement in SHM

It is given by the equation $y = A \sin (\omega t + \phi_o)$

where ϕ_o = epoch or initial phase and $\omega t = 2\pi/T$.

Phase

The argument $(\omega t + \phi_o)$ of the sine function is called the phase of the SHM. The constant ϕ_o in the equation for SHM is called epoch or phase constant or initial phase. This enables us to find the position from where time is calculated in SHM.

The phase of an oscillating system at any instant represents its state regarding its position and direction of motion at that instant.

If the phase is zero at a certain instant then from the equation of SHM, i.e., $y = A \sin (\omega t + \phi_o)$, $y = 0$ and $v = (dy/dt) = A\omega$, i.e., particle is crossing the equilibrium position; and if $(\omega t + \phi_o) = \pi/2$, $y = A$ and $v = 0$, i.e., particle is at extreme position.

Velocity in SHM

In case of SHM, when motion is considered from the equilibrium position, velocity at any instant t is given by:

$$v = A\omega \cos \omega t = \omega\sqrt{A^2 - y^2}$$

At the mean position, i.e., when $y = 0$, $v = \mathbf{v_{max.} = \omega A}$, At the extreme positions, i.e; when $y = \pm A$, $v = \mathbf{v_{min.} = 0}$.

Acceleration in SHM

In case of SHM, when time is considered from equilibrium position, acceleration at any instant is:

$$a = -A\omega^2 \sin \omega t = -\omega^2 y$$

1. At the mean position, i.e., when $y = 0$, | acceleration | will be minimum (= 0). Hence, In SHM acceleration is zero in equilibrium position where | velocity | is maximum (= ωA).
2. At the extreme positions, i.e., when $y = \pm A$, | acceleration | will be maximum (= $\omega^2 A$). Hence, in SHM acceleration is maximum at the extreme positions where velocity is minimum (= 0).

Energy associated with SHM

1. Kinetic energy = KE = $\frac{1}{2} mv^2 = \frac{1}{2} m\omega^2(A^2 - y^2)$
2. Potential energy = PE = $\frac{1}{2} m\omega^2 y^2$.

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3. At the mean position, $y = 0$, hence KE = $(KE)_{max.} = \frac{1}{2} m\omega^2 A^2$ and PE = $(PE)_{min.} = 0$
4. At the extreme positions, $y = \pm A$; hence KE = $(KE)_{min.} = 0$ and PE = $(PE)_{max.} = \frac{1}{2} m\omega^2 A^2$.
5. Total energy = (KE) + (PE) = $\frac{1}{2} m\omega^2 A^2$. Also, Total energy = $(KE)_{max} = (PE)_{max.}$ i.e., energy is conserved in SHM.

Time period

The time taken to complete simple harmonic motion once is called the time period. It is given by:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\left(\frac{\text{displacment}}{\text{acceleration}}\right)} = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$$

Simple pendulum

A simple pendulum consists of a heavy point mass suspended by a weightless, inextensible and perfectly flexible string from a rigid support. Such an ideal pendulum is not possible in practice but a heavy bob suspended by a light inextensible thread works as simple pendulum.

1. Time period $T = 2\pi\sqrt{(L/g)}$.
2. When the time period of a simple pendulum is 2 second, it is called a second pendulum.
3. Time period of simple pendulum is independent of amplitude as long as θ is small.
4. Time period of simple pendulum is independent of the mass of bob.
5. Time period of simple pendulum depends on L as $T \propto \sqrt{L}$; hence the graph between T and L will be a parabola while that between T^2 and L will be a straight line.
6. As $T \propto (1/\sqrt{g})$, hence with change in g, T will change.
7. If the length of a pendulum is comparable to the radius of earth (R). Time period is given by: $T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{L}+\frac{1}{R}\right)}}$

    (a) If L >> R, then $T = 2\pi\sqrt{(R/g)} \cong 84.6$ minutes.

    (b) If L = R, then $T' = 2\pi\sqrt{(R/2g)} = (T/\sqrt{2}) \cong 1$ hour.

Spring Mass Combination

A point mass attached with a massless spring constitutes a spring mass combination. Such an ideal combination is not possible in practice. So, a small heavy mass suspended from a light spring is nearly ideal combination.

1. In the limit of small displacement, restoring force developed on a spring is given by: $F = -ky$.
2. Time period: $T = 2\pi\sqrt{\frac{m}{k}}$ and frequency $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
3. If a spring of force constant k is divided into n equal parts and one such part is attached to a mass m, then the time period is given by: $T' = 2\pi\sqrt{\frac{m}{nk}} = \frac{T}{\sqrt{n}}$
4. If two masses m_1 and m_2 are connected by a spring, then the time period is given by: $T = 2\pi\sqrt{\frac{\mu}{k}}$

    where μ = reduced mass = $\frac{m_1 m_2}{m_1 + m_2}$
5. If two springs of force constants k_1 and k_2 are connected in parallel and a mass m is attached to them, then the time period is given by: $T = 2\pi\sqrt{m/(k_1 + k_2)}$ (with effective spring constant $k = k_1 + k_2$)

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6. If two springs of force constants k_1 and k_2 are connected in series and a mass m is attached to them, then the time period is given by: $T = 2\pi\sqrt{m\left(\frac{k_1 + k_2}{k_1 k_2}\right)}$ (with effective spring constant $k = k_1k_2/k_1 + k_2$)

7. If M be the mass of the spring and mass m is suspended from it, then the time period is: $T = 2\pi\sqrt{\frac{m + (M/3)}{k}}$

Some other points

1. The simple harmonic oscillations may also be expressed as $y = A \sin \omega t + B \cos \omega t$, where A and B are constants related to the amplitude. We can write

   $y = A \sin \omega t + B \cos \omega t = A_R \sin (\omega t + \phi)$ where $A_R = (A^2 + B^2)^{1/2}$ and $\tan \phi = (B/A)$.

2. Under weightlessness or in the freely falling lift $T= 2\pi(L/g)^{1/2} = \infty$. This means, the pendulum does not oscillate at all.
3. The *y*, *v*, *A* of SHM vary simple harmonically with the same time period and frequency.
4. The KE as well as PE vary periodically but not simple harmonically in SHM. The frequency of KE or the PE is just two times that of *y*, *v* or *a*.

Different Type of SHM

1. If a wire of length L, area of cross-section A, Young's modulus Y is stretched by suspending a mass m, then the mass can oscillate with time period: $T = 2\pi\sqrt{\frac{mL}{YA}}$

2. If the lower surface of a cube of side L and modulus of rigidity η is fixed while fixing a particle of mass m on the upper face, a force parallel to upper face is applied to mass m and then withdrawn, the mass m can oscillate with a time period: $T = 2\pi\sqrt{\frac{m}{\eta L}}$

3. If a gas is enclosed in a cylinder of volume V_0 fitted with a piston of cross-section A and mass M and the piston is slightly depressed and released, the piston can oscillate with a frequency: $f = \frac{1}{2\pi}\sqrt{\frac{BA^2}{MV_0}}$, where B represents bulk modulus of elasticity of the gas.

4. If a simple pendulum is suspended from the roof of a compartment of a train moving down an inclined plane of inclination θ, then the time period of oscillation is: $T = 2\pi\sqrt{\frac{L}{g\cos\theta}}$

5. If a ball of radius r oscillates in a bowl of radius R, then its time period of oscillation is: $T = 2\pi\sqrt{\frac{R - r}{g}}$

6. If a disc of radius r oscillates about a point at its rim, then its time period is given by: $T = 2\pi\sqrt{\frac{r}{g}}$

Undamped Oscillation

The oscillations whose amplitude remain constant with time are called undamped oscillation. A pendulum oscillating in vacuum is example of these oscillation.

Damped Oscillation

The oscillations whose amplitude goes on decreasing with time are called damped oscillation. A pendulum oscillating in air is example of these oscillation. The frequency of damped oscillation is lesser than natural frequency. The equation of damped oscillation is $m\left(\frac{d^2y}{dt^2}\right) + b\left(\frac{dy}{dt}\right) + ky = 0$ and the amplitude of oscillation decreases exponently.

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Free Oscillation

When a body oscillate with its natural frequency, it is said to execute free oscillation.

Forced Oscillation

In this body is maintained in oscillation by an external periodic force of frequency other than natural frequency of oscillator. Initially oscillator tend to oscillate with natural frequency but external periodic force superpose its frequency and oscillator finally oscillate with frequency of force.

Resonant Oscillation

It is a special case of forced oscillation, in which, frequency of periodic force equals the natural frequency of oscillator. In resonant oscillation amplitude is large, which become infinite in absence of damping.

Maintained Oscillation

If in damped oscillation the loss of energy is provided by some external source then amplitude of oscillation become constant and such oscillations are called maintained oscillation.

Simple Harmonic Motion Assignment

1. The total energy of a particle executing S.H.M. of amplitude A is proportional to
(a) A^2 (b) A^{-2} (c) A (d) 1/A
2. A metallic sphere is filled with water and hung by a long thread. It is made to oscillate. If there is a small hole in the bottom through which water slowly flows out, the time period will
(a) go on increasing till the sphere is empty
(b) go on decreasing till the sphere is empty
(c) remain unchanged throughout
(d) first increase, then it will decrease till the sphere is empty and the period will now be the same as when the sphere was full of water.
3. A body falling freely on a planet covers 8 m in 2 s. The time period of a one metre long simple pendulum on this planet will be
(a) 1.57 s (b) 3.14s (c) 6.28 s (d) none of these.
4. The bob of a simple pendulum of period T is given a negative charge. If it is allowed to oscillate above a positively charged plate, the new time period will be
(a) equal to T
(b) more than T
(c) less than T
(d) infinite.
5. A simple pendulum suspended from the ceiling of a train has a period T when the train is at rest. If the train starts moving with a constant acceleration, the time period of the pendulum will
(a) increase
(b) decrease
(c) remain unaffected
(d) become infinite.
6. A body of mass 5 g is executing S.H.M. with amplitude 10 cm. Its maximum velocity is 100 cm/s. Its velocity will be 50 cm/s at a displacement from the mean position equal to
(a) 5 cm (b) $5\sqrt{3}$cm (c) 10cm (d) $10\sqrt{3}$cm
7. A simple harmonic oscillator has amplitude A and time period T. Its maximum speed is
(a) 4A/T (b) 2A/T (c) 4πA/T (d) 2πA/T
8. A weakly damped harmonic oscillator of frequency n_1 is driven by an external periodic force of frequency n_2. When the steady state is reached, the frequency of the oscillator will be
(a) n_1 (b) n_2 (c) $(n_1 + n_2)/2$ (d) $\sqrt{n_1 + n_2}$
9. The frequency of a vibrating body situated in air
(a) is the same as its natural frequency
(b) is higher than its natural frequency
(c) is lower than its natural frequency
(d) can have any value
10. The equation $\frac{d^2y}{dt^2} + b\frac{dy}{dt} + \omega^2 y = 0$ represents the equation of motion for a
(a) free vibration
(b) damped vibration
(c) forced vibration
(d) resonant vibration
11. The displacement equation of an oscillator is y = 5 sin ($0.2\pi t + 0.5\pi$) in SI units. The time period of oscillation is
(a) 10 s (b) 1 s (c) 0.2 s (d) 0.5 s.
12. The amplitude at resonance of a vibrating body situated in air becomes :
(a) infinite
(b) zero
(c) large but finite

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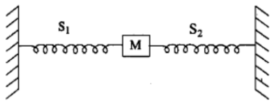
(d) small but non-zero.

13. A simple pendulum has a time period T. The pendulum is completely immersed in a non-viscous liquid whose density is $1/10_{th}$ of that of the material of the bob. The time period of the pendulum immersed in the liquid is

(a) $T\sqrt{10/9}$ (b) $T\sqrt{9/10}$ (c) T (d) T/10

14. In the given diagram S_1 and S_2 are identical springs. The frequency of oscillation of the mass M is f. If one of the springs is removed the frequency will be



(a) f/2 (b) 2f (c) f√2 (d) f/√2

15. The vertical extension in a light spring by a weight of 1 kg, in equilibrium, is 9.8 cm. The period of oscillation of the spring, in seconds, will be

(a) 2π/10 (b) 2π/100 (c) 20π (d) 200π

16. A particle of mass 0.5 kg executes S.H.M. Its energy is 0.04 J. If its time period is π seconds, its amplitude is

(a) 10 cm (b) 15 cm (c) 20 cm (d) 40 cm

17. A body executes S.H.M. with an amplitude A. Its energy is half kinetic and half potential when the displacement is

(a) A/3 (b) A/2 (c) A/√2 (a) A/2√2

18. A particle is executing S.H.M. of period 4 s. Then the time taken by it to move from the extreme position to half the amplitude is

(a) 1/3 s (b) 2/3 s (c) 3/4 s (d) 4/3 s

19. A simple pendulum has a length '*l*'. The mass of the bob is 'm'. The bob is given a charge +q. The pendulum is suspended between the plates of a charged parallel plate capacitor which are placed vertically. If E is the electric field intensity between the plates, then the time period of oscillation will be :

(a) $2\pi\sqrt{\frac{l}{g}}$ (b) $2\pi\sqrt{\frac{l}{g+\frac{qE}{m}}}$

(c) $2\pi\sqrt{\frac{l}{g-\frac{qE}{m}}}$ (d) $2\pi\sqrt{\frac{l}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$

20. Three masses of 0.1 kg, 0.3 kg and 0.4 kg are suspended at the end of a spring. When the 0.4 kg mass is removed, the system oscillates with a period 2 s. When the 0.3 kg mass is also removed, the system will oscillate with a period

(a) 1 s (b) 2 s (c) 3 s (d) 4 s

21. The equation of S.H.M. of a particle is $\frac{d^2y}{dt^2}+ky=0$, where k is a positive constant. The time period of motion is given by

(a) $\frac{2\pi}{\sqrt{k}}$ (b) $\frac{2\pi}{k}$ (c) $\frac{k}{2\pi}$ (d) $\frac{\sqrt{k}}{2\pi}$

22. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant K. A mass m hangs freely from the free end of the spring. The area of cross-section and Young's modulus of the wire are A and Y, respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to

(a) $2\pi\sqrt{\frac{m}{K}}$ (b) $2\pi\sqrt{\frac{m(YA+KL)}{YAK}}$

(c) $2\pi\sqrt{\frac{mYA}{KL}}$ (d) $2\pi\sqrt{\frac{mL}{YA}}$

23. A uniform cylinder of mass M and cross-sectional area A is suspended from a fixed point by a light spring of force constant k. The cylinder is partially submerged in a liquid of density ρ. If it is given a small downward push and released, it will oscillate with time period

(a) $2\pi\sqrt{\frac{M}{k+A\rho g}}$ (b) $2\pi\sqrt{\frac{M}{k-A\rho g}}$

(c) $2\pi\sqrt{\frac{M}{k}}$ (d) $2\pi\sqrt{\frac{M}{A\rho g}}$

24. A particle is vibrating in S.H.M. If its velocities are v_1 and v_2 when the displacements from the mean position are y_1 and y_2 respectively, then its time period is

(a) $2\pi\sqrt{\frac{y_1^2+y_2^2}{v_1^2+v_2^2}}$ (b) $2\pi\sqrt{\frac{v_1^2+v_2^2}{y_1^2+y_2^2}}$

(c) $2\pi\sqrt{\frac{v_2^2-v_1^2}{y_1^2+y_2^2}}$ (d) $2\pi\sqrt{\frac{y_1^2-y_2^2}{v_2^2-v_1^2}}$

25. A person normally weighing 60 kg stands on a platform which oscillates up and down simple harmonically with a frequency 2 Hz and an amplitude 5 cm. If a machine on the platform gives, the person's weight, then; ($g = 10\ m/s^2$, $\pi^2 = 10$)

(a) the maximum reading of the machine will be 108 kg
(b) the maximum reading machine of the will be 90 kg
(c) the minimum reading machine of the will be zero.
(d) None of the above is correct

26. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that it executes simple harmonic oscillations with period T. If the mass is increased by m, the period becomes 5T/4. The ratio (m/M) is

(a) 4/5 (b) 5/4 (c) 9/16 (d) 25/16

27. A pole is floating in a liquid with 80 cm of its length immersed. It is pushed down a certain distance and then released. The time period of vertical oscillation is
(a) $4\pi/7$ s (b) $3\pi/7$ s (c) $2\pi/7$ s (d) $\pi/7$ s
28. When a particle oscillates simple harmonically, its potential energy varies periodically. If the frequency of oscillation of the particle is n, the frequency of potential energy variation is
(a) n/2 (b) n (c) 2n (d) 4n
29. A particle, moving along the x-axis, executes simple harmonic motion when the force acting on it is given by (A and k are positive constants)
(a) – Akx (b) A cos kx
(c) A exp (- kx) (d) Akx
30. The length of a simple pendulum is increased by 1%. Its time period will
(a) increase by 1% (b) increase by 0.5%
(c) decrease by 0.5% (d) increase by 2%
31. A pendulum clock is set to give correct time at the sea level. This clock is moved to a hill station at an altitude of 2500 m above the sea level. In order to keep correct time on the hill station, the length of the pendulum
(a) has to be reduced
(b) has to be increased
(c) needs no adjustment
(d) needs no adjustment but its mass has to be increased
32. A simple harmonic oscillator has time period T. The time taken by it to travel from the extreme position to half the amplitude is
(a) T/6 (b) T/4 (c) T/3 (d) T/2

ANSWERS:

1a ,2d ,3b ,4c ,5b ,6b ,7d ,8b ,9c ,10b ,11a ,12c ,13a ,14d ,15a ,16c ,17c ,18b ,19d ,20a ,21a ,22b ,23a ,24d ,25a ,26c ,27a ,28c ,29a ,30b ,31a ,32a

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