IIT JEE PHYSICS FREE NOTES Module : 1 Vectors & Scalars

Physical quantities are divided into vectors and scalars. A scalar quantity is denoted by a magnitude or real number-Eg.Temperature of a room,volume of a jug,density of steel or presssure of air in a tyre.In the case of vector quantities,a specific direction related to some underlying reference frame is needed to define it,in addition to a magnitude...Eg.displacement of a car and velocity of a body.The length of a vector is called it's magnitude.It is indicated by using vertical bars or by using italics - | A| or A .If the length of a vector is one unit, then it's a unit vector.

A component of a vector is the projection of the vector on an axis.The find this ,we draw perpendicular lines from from the two ends of the vector to the axis.The process of finding the components is called resolving the vector.The component of a vector has the same direction [along an axis] as the vector.

A unit vector lacks both dimension and unit,it only specifies a direction.Regarding the relations among vectors,we have great freedom in choosing a co-ordinate system,as the relations does not depend on the location of the origin of the co-ordinate system or on the orientation of the axis.

Signage of unit vectors : Multiplication of two unit vectors in anti-clockwise direction gives the third vector + ve. Whereas, multiplication of any two unit vectors in clockwise direction gives the third unit vector -ve sign.

Basic Set : Consider two non zero vectors, a and b,where the direction of b is neither the same or the opposite to that of a .Let OA and OB be representations of a and b and P is the plane of the triangle, OAB.Now,any vector v whose representation OV lies in the plane P can be written as $v = \lambda a + \mu b$. Here the co-efficients, λ and μ are unique.As the vectors have their directions parellel to the same plane,they are coplanar.Any vector coplanar with a and b can be expanded uniquely in the above form.Also, the expansion set cannot be reduced in number[say,to a single vector].Hence the pair of vectors (a,b) is said to be a basic set for vectors lying in the plane P.If we are dealing with three co-planar vectrors,a, b and c , then it's $v = \lambda a + \mu b + v c$, again a,b,c is a basic set..Eventhough any set of three non-coplanar vectors form a basic,the basic vectors are considered as orthogonal unit vectors. Here, the basic set is denoted by (i,j,k).

Position vector : Vectors which are used to specify the positions of points in space.

A vector does not necessarily have location,eventhough a vector may refer to a quantity defined at a particular point.Two vectors can be compared ,eventhough they measure a physical quantity defined at different points of space & time.Note- the applicability of vectors is largely based on euclidean geometry.... that space is flat [for huge distances].In such a scanario, we can compare two vectors at differtent points.

A Vector must a) satisfy the parallelogram law of addition b) have a magnitude and direction independent of choice of co-ordinate system.

Vector addition : Usually denoted by a ' tip to tail ' method,where a second vector is joined at it's tail to the head or tip of the first vector. Now, a third vector is drawn from the tail of the first vector to the head of the second one. This third vector is the resultant vector.In case one needs to add a vector acting in the opposite direction,you have to just add the second vector with a -ve sign i.e, $| r| = |a| + | -b|$.

Triangle law of Vector Addition : When two vectors are represented both in magnitude and direction by the two sides of a triangle,taken in the same order, thei

r resultant is represented by the third side of the triangle (both in magnitud
e & direction) taken in the reverse direction.
Parallelogram law of Vector Addition : If two vectors can be represented both in
magnitude and direction by two adjacent sides of a parallelogram, then the resu
ltant is represented both in direction and magnitude by the corresponding diagon
al of the parallelogram.
here, A + B = B + A ,hence it is commutative, A+ (B+C) = (A+B)+C,he
nce associative. If k is a scalar,then k(A+B) = kA + kB, hence distributive.

vector addition

parallelogram/ vector addition

All quantities having magnitude and direction need not be vectors - For eg, in t
he case of the rotation of a rigid body about a particular axis fixed in space,
eventhough it has both magnitude (angle of rotation) and direction (the direc
tion of the axis), but two such rotations do not combine according to the vecto
r law of addition, unless the angles of rotation are infinitesimally small. Eg -
when the two axes are perpendicular to each other and the rotations are by π/2 ra
d or 90°. Therefore,the commutative law of addition is not satisfied.

Product of two vectors : A x (B + C) = A x B + A x C - here one product is a s
calar and the other is mostly a vector...... why is A x B not useful ? If D = B
+ C then, AD ≠ AB + AC .... there is no distributive property.

Scalar product (Dot product) a.b or a dot b = [a] [b] cos θ . Now, a . b =
b . a i.e, commuitative law is followed. In addition, a . (b+c) = a.b + a.c [
distributive law] and λ a.b = λ (a.b) [associative with scalar multiplication
].

scalar product
Other properties are : -

a.a = $[a]^2$
a.b = 0 , only if a and b are perpendicular or if one of them is zero.
If [i,j,k] is on an orthogonal basis , then i.i = j.j = k.k = 1 and i.j = j.k
= k.i = 0
If a1 = λ1 i + μ1j + v1k and a2 = λ2i + μ2j + v2k then a1.a2 = λ1λ2 + μ1μ2 + v1v2 .
A scalar product is a scalar, not a vector and it may be + ve , -ve or ze
ro. Note :

If θ is between 90° and 180° ---------> cos θ < 0 , hence - ve
If θ is between 0° and 90° ---------> cos θ > 0 , hence + ve
When θ = 90° (vectors are perpendicular) = 0
Vector Product : of two parallel or antiparallel vectors are always zero.T
he vector product of any vector with itself is zero.

vector product
Component of a vector : Let n be a unit vector. The component of vector v in the
direction of n is defined as vn . If it's a general vector a , then the component
of v is v.â.

if v is a sum of vectors, v = v1 +v2 + v3, then the component of v in the direction of n i
s v.n = (v1+v2+v3) and n = (v1.n)+(v2.n)+(v3.n)and this isbased on the distributive law

a scalar product. Therefore, the component of the sum of a no of vectors in a given direction is equal to the sum of the components of the individual vectors in that direction.

if a vector v is expanded in terms of a general basis set (a,b,c) in the form λa + μb + vc, the co-efficients λ,μ and v are not components of vector v in the direction of ,b and c. But , if v is expanded in terms of an orthonormal basis set [i,j,k] in the form v = λi +μj + vk, then the component of v in the i direction is v. i = (λi +μj i = λ(i.i) +μ(j. i) + v(k.i) = λ + 0+0 = λ . Likewise, μ and v are the components o in the j and k directions. Therefore, when a vector v is expanded in terms of an orthonormal basis set (i ,j , k) in the form v =λi +μj + vk , the co-efficients λ , μ an are the components of v in the i , j and k directions.

Note : In vector addition, |C| = | A | + | B| isn't always true. The magnitude of | A| + | B| depends on the magnitudes of | A| and | B| and on the angle between them.Only in the case where, they are parellel , is the magnitude of |C| = | A | + | B| . If they are anti-parallel,|C| = | A | - | B|. As vectors are not ordinary numbers,ordinary multiplication is not applicable to them.

Method of Components : Adding vectors by measuring a scale diagram offers limited accuracy and calculations with right triangles work only when the two vectors are perpendicular.So.another method is to add the components of a vector i.e, | A | = | Ax | + | Ay |.Components of vectors are not vectors themselves,they are just numbers.The components of a vector may be +ve or -ve numbers.

Note : Relating a vector's magnitude and direction to it's components are correct only when the angle θ is measured from the +ve x axis .When finding the direction of a vector from it's components, check to which quadrant the angle belongs to.Eg,if tan θ = -1 , the angle could be either 135° or 315°........ hence,only by checking the components, the angle can be found out.

VECTOR CLASSIFICATION : Based on the character of their magnitude / direction or both,vectors can be broadly classified as :-

Polar vectors (true vectors) : Vectors having a starting point or point of application.

Axial vectors (pseudo vectors) : Vectors whose directions are along the axis of rotation. Called pseudo vector because it's sign changes when the orientation in space changes . Such a sign change doesn't happen in a polar vector.Pseudo vectors usually occur as the cross - product of two normal vectors. Eg : Angular velocity, Torque, Magnetic feild.

Collinear vectors : Two or more vectors acting along the same line or along the parallel lines. They may act either in the same direction or in the opposite direction.

Parallel vectors : Collinear vectors having the same direction . Hence, angle between them is 0°.

Anti-parallel vectors : Collinear vectors in opposite direction .Hence,angle between them is 180° or Π radian.

Equal vectors : Two vectors having equal magnitude and same direction.

Coplanar vectors : Vectors lying in the same plane.

Position vector : Vector which provides an idea about the direction and distance of a point from origin in space.

Null vector (zero vector) : A vector whose length is zero. In co-ordinates , the vector is (0,0,0), and it is commonly denoted , or 0.It doesn't have a direction and cannot be normalized.. i.e, a unit vector is not possible,which is a multiple of a null vector. The sum of the null vector with any vector a is a (that is, 0+a=a).

axial vector

polar vector

Scalar triple product (box product or mixed triple product) : Refers to a method of applying the vector product or scalar product to three vectors . It could be denoted as (a b c)

(a b c) = a . (b x c)

Points to consider :

Used in finding the volume of the parallelepiped which has edges defined by three vectors .
It's product could be zero, if all the vectors lie in the same plane (linearily dependent),and hence can't make a volume.
It's value is +ve , if all the three vectors are right handed.

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