Dear student following is a Moderate level [O ● O] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-12(+3, –1), (Single option correct)

Q.1 If there is a term containing x^{2r} in $(x + 1/x^2)^{n-3}$, then

(A) n – 2r is a positive integral multiple of 3

(B) n – 2r is even

(C) n – 2r is odd

(D) None of these

Q.2 If the sum of the coefficients in the expansion of $(\alpha x^2 - 2x + 1)^{35}$ is equal to the sum of the coefficients in the expansion of $(x - \alpha y)^{35}$, then α

(A) 0 (B) 1

(C) May be any real number

(D) No such value exist

Q.3 In the polynomial $(x - 1)(x - 2)(x - 3)....(x - 100)$, the coefficient of x^{99} is

(A) 5050 (B) –5050

(C) 100 (D) 99

Q.4 The coefficient of x^{100} in the expansion of $\sum_{j=0}^{200}(1+x)^j$ is

(A) $\binom{200}{100}$ (B) $\binom{201}{102}$

(C) $\binom{200}{101}$ (D) $\binom{201}{100}$

Q.5 Middle term in the expansion of $(1 + 3x + 3x^2 + x^3)^6$ is

(A) 4^{th} (B) 3^{rd}

(C) 10^{th} (D) None of these

Q.6 Which of the following expression is divisible by 1225

(A) $6^{2n} - 35n - 1$ (B) $6^{2n} - 35n + 1$

(C) $6^{2n} - 35n$ (D) $6^{2n} - 35n + 2$

Q.7 If 7^{103} is divided by 25, then the remainder is

(A) 20 (B) 16

(C) 18 (D) 15

Q.8 For all $n \in N$, $2^{4n} - 15n - 1$ is divisible by

(A) 225 (B) 125

(C) 325 (D) None

Q.9 $9^7 + 7^9$ is divisible by

(A) 6 (B) 24

(C) 64 (D) 72

Q.10 The coefficients of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is same if α equals :

(A) $\frac{-3}{10}$ (B) $\frac{10}{3}$

(C) $\frac{-5}{3}$ (D) $\frac{3}{5}$

Q.11 If $\frac{T_2}{T_3}$ in the expansion of $(a + b)^n$ and $\frac{T_3}{T_4}$ in the expansion of $(a + b)^{n+3}$ are equal, then n =

(A) 3 (B) 4

(C) 6 (D) 5

Q.12 If in the expansion of $(1+ x)^m (1 - x)^n$, the coefficients of x and x^2 are 3 and –6 respectively, then m is

(A) 6 (B) 9 (C) 12 (D) 24

MATHEMATICS FOUNDATION (CLASS TEST - 2/7) (BINOMIAL THEOREM) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1.	○	○	○	○	5.	○	○	○	○	9.	○	○	○	○
2.	○	○	○	○	6.	○	○	○	○	10.	○	○	○	○
3.	○	○	○	○	7.	○	○	○	○	11.	○	○	○	○
4.	○	○	○	○	8.	○	○	○	○	12.	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	A	B	B	A	C	A	C	A	C	A	D	C

SOLUTIONS

Sol.1 (A)

$T_{p+1} = {}^{n-3}C_p\, x^{n-3-p}.\left(\frac{1}{x^2}\right)^p = {}^{n-3}C_p\, x^{n-3-3p}$

$2r = n-3-3p \Rightarrow p = \frac{n-2r-3}{3} = \frac{n-2r}{3} - 1$

$\Rightarrow p + 1 = (n-2r)/3$

$\therefore$ (n – 2r) is a positive integer, multiple of 3.

Sol.2 (B)

Putting x = y = 1, we have

$(\alpha - 1)^{35} = (1 - \alpha)^{35}$

$\Rightarrow 2(\alpha - 1)^{35} = 0 \Rightarrow \alpha = 1$

Sol.3 (B)

$(x-1)(x-2)(x-3)\ldots(x-100)$

Numbers of terms = 100;

$\therefore$ Coefficient of x^{99}

$= (x-1)(x-2)(x-3)\ldots(x-100)$

$= (-1-2-3-\ldots\ldots-100)$

$= -(1+2+\ldots+100)$

$= -\frac{100\times101}{2} = -5050$

Sol.4 (A)

$T_{r+1} = {}^{200}C_r\,(1)^{200-r}\,(x)^r$

Hence coefficient of $x^{100} = {}^{200}C_{100} = \binom{200}{100}$

Sol.5 (C)

$(1 + 3x + 3x^2 + x^3)^6$

$= \{(1+x)^3\}^6 = (1+x)^{18}$

Hence the middle term is 10^{th}

Sol.6 (A)

Consider expression = $6^{2n} - 35n - 1$

$= (6^2)^n - 35n - 1 = 36^n - 35n - 1$

so $6^{2n} - 35n - 1 = (1+35)^n - 35n - 1$

$= 1 + 35n + {}^nC_2.35^2 + \ldots\ldots + 35^n - 35n - 1$

$= (35)^2[{}^nC_2 + {}^nC_3.35 + \ldots\ldots. + 35^{n-2}]$

$= 1225 \times$ a positive integer if $n \geq 2$

If n = 1, then $6^{2n} - 35n - 1 = 0$, which is divisible by 1225.

so, (B), (C), (D) are not divisible by 1225.

Sol.7 (C)

We have, $7^{103} = 7(49)^{51} = 7(50-1)^{51}$

$= 7((50)^{51} - {}^{51}C_1 50^{50} + {}^{51}C_2 50^{49} - \ldots\ldots. - 1))$

$= 7\{(50^{51} - {}^{51}C_1 50^{50} + {}^{51}C_2 50^{49} - \ldots\ldots.)$

$- (7 + 18 - 18)\}$

= k + 18 (say) $\quad \because$ k is divisible by 25.

$\therefore$ remainder is 18.

Sol.8 (A)

We have, $2^{4n} = (2^4)^n = (16)^n = (1+15)^n$

$\therefore \quad 2^{4n} = 1 + {}^nC_1.15 + {}^nC_2.15^2 + {}^nC_3.15^3 + \ldots$

$\Rightarrow \quad 2^{4n} - 1 - 15n = 15^2[{}^nC_2 + {}^nC_3.15 + \ldots.]$

= 225 K, where K is an integer.

Hence $2^{4n} - 15n - 1$ is divisible by 225.

Sol.9 (C)

We have, $9^7 + 7^9 = (1+8)^7 - (1-8)^9$

$= (1 + {}^7C_1.8^1 + {}^7C_2.8^2 + \ldots\ldots. + {}^7C_7.8^7)$

$- (1 - {}^9C_1.8^1 + {}^9C_2.8^2 - \ldots\ldots. - {}^9C_9.8^9)$

$= 16 \times 8 + 64[({}^7C_2 + \ldots.. + {}^7C_7.8^5)$

$-({}^9C_2 - \ldots\ldots. - {}^9C_9.8^7)]$

= 64 (an integer)

$\Rightarrow \quad 9^7 + 7^9$ is divisible by 64.

Sol.10 (A)

In $(1+\alpha x)^4$, the middle term is $T_{(4/2+1)} = T_3$ and in $(1-\alpha x)^6$, the middle term is $T_{(6/2+1)} = T_4$.

In $(1+\alpha x)^4$, $T_3 = {}^4C_2(1)^2(\alpha x)^2$

and in $(1-\alpha x)^6$, $T_4 = {}^6C_3(1)^3(-\alpha x)^3$

$\Rightarrow \quad {}^4C_2\alpha^2 = {}^6C_3(-\alpha)^3$ (As given)

$\Rightarrow \quad 6\alpha^2 = -20\alpha^3$

$\Rightarrow \quad \alpha = -\frac{6}{20} = -\frac{3}{10}$.

(Note that α ought to be non-zero)

Sol.11 (D)

$$\frac{{}^{n}C_1\, a^{n-1} b}{{}^{n}C_2\, a^{n-2} b^2} = \frac{{}^{n+3}C_2\, a^{n+1} b^2}{{}^{n+3}C_3\, a^n b^3}$$

$$\Rightarrow \frac{n}{\frac{n(n-1)}{2}} = \frac{\frac{(n+3)\,(n+2)}{2}}{\frac{(n+3)\,(n+2)\,(n+1)}{6}}$$

$$\Rightarrow \frac{2}{n-1} = \frac{3}{n+1} \Rightarrow n = 5$$

Sol.12 (C)

We have,

$(1 + x)^m (1-x)^n$

$$= \left(1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \ldots\right) \times$$

$$\left(1 - nx + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\right)$$

$$= 1 + (m-n)\,x + \left\{\frac{m(m-1)}{2!} + \frac{n(n-1)}{2!} - mn\right\} x^2 + \ldots..$$

Given, $m - n = 3$ ....(1)

and $\frac{m(m-1) + n(n-1) - 2mn}{2!} = -6$

$\Rightarrow m^2 - m + n^2 - n - 2mn = -12$

$\Rightarrow (m-n)^2 - (m+n) = -12$

$\Rightarrow m + n = 9 + 12 = 21$ ....(2)

(Using (1))

Solving (1) and (2), we get $m = 12$.