Multiple Choice Questions with ONE Correct Answer

1. The smallest +ve integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$ is :
(a) 8 (b) 16
(c) 12 (d) None of these.
(1980 - 2 Marks)

2. The complex numbers $z = x + iy$ which satisfy the equation $\left|\frac{z-5i}{z+5i}\right| = 1$ lie on
(a) the x-axis
(b) the straight line $y = 5$
(c) the circle passing through the origin
(d) none of these.
(1981 - 2 Marks)

3. If $z = \left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$ then
(a) $R_e(z) = 0$ (b) $I_m(z) = 0$
(c) $R_e(z) = 0,\ I_m(z) > 0$ (d) $R_e(z) > 0, I_m(z) < 0$.
(1982 - 2 Marks)

4. The inequality $|z-4| < |z-2|$ represents the region given by
(a) $R_e(z) \geq 0$ (b) $R_e(z) < 0$
(c) $R_e(z) > 0$ (d) none of these.
(1982 - 2 Marks)

5. If $z = x + iy$ and $W = \frac{1-iz}{z-i}$ then $|W| = 1$ implies that, in the complex plane :
(a) z lies on the imaginary axis
(b) z lies on the real axis
(c) z lies on the unit circle
(d) none of these.
(1983 - 2 Marks)

6. The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram if and only if
(a) $z_1 + z_2 = z_2 + z_3$ (b) $z_1 + z_3 = z_2 + z_4$
(c) $z_1 + z_2 = z_3 + z_4$ (d) none of these.
(1983 - 2 Marks)

7. If $a, b, c; u, v, w$ are complex numbers representing the vertices of two triangles such that $c = (1-r)\,a + rb$, $w = (1-r)\,u + rv$ where r is a complex number, the two triangles :
(a) have the same area (b) are similar
(c) are congruent (d) none of these.
(1985 - 2 Marks)

8. If z_1 and z_2 are two non - zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then Arg z_1 - Arg z_2 is equal to
(a) $-\pi$ (b) $-\frac{\pi}{2}$ (c) 0
(d) $\frac{\pi}{2}$ (e) π
(1987 - 2 Marks)

9. The value of
$$\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right) \text{ is}$$
(a) -1 (b) 0 (c) $-i$
(d) i (e) none of these
(1987 - 2 Marks)

10. The complex number $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other, for
(a) $x = n\pi$ (b) $x = 0$
(b) $x = (n+1/2)\,\pi$ (d) no value of x
(1988 - 2 Marks)

11. If $\omega\,(\neq 1)$ is a cube root of unity and $(1+\omega)^7 = A + B\omega$ then A and B are respectively
(a) 0, 1 (b) 1, 1 (c) 1, 0 (d) -1, 1.
(1995 - 2 Marks)

12. Let z and ω be two non zero complex numbers such that $|z| = |\omega|$ and Arg z + Arg $\omega = \pi$, then z equals
(a) ω (b) $-\omega$ (c) $\bar{\omega}$ (d) $-\bar{\omega}$.
(1995 - 2 Marks)

13. Let z and ω be two complex numbers such that $|z| \leq 1$, $|\omega| \leq 1$ and $|z + i\omega| = |z - i\bar{\omega}| = 2$ then z equals
(a) 1 or i (b) i or $-i$
(c) 1 or -1 (d) i or -1.
(1995 - 2 Marks)

14. For positive integers n_1 and n_2 the values of the expression $(1+i)^{n_1} + (1+i^3)^{n_1} + (1+i^5)^{n_2} + (1+i^7)^{n_2}$ when $i = \sqrt{-1}$ is real if and only if
(a) $n_1 = n_2 + 1$ (b) $n_1 = n_2 - 1$
(c) $n_1 = n_2$ (d) $n_1 > 0, n_2 > 0$.
(1996 - 2 Marks)

15. If ω is an imaginary cube roots of unity, then $(1 + \omega - \omega^2)^7$ equals

(a) $128\ \omega$ (b) $-128\ \omega$
(c) $128\ \omega^2$ (d) $-128\ \omega^2$
(1998 - 2 Marks)

16. The value of the sum $\sum_{n=1}^{13}(i^n + i^{n+1})$, where $i = \sqrt{-1}$ equals
(a) i (b) $i - 1$ (c) $-i$ (d) 0
(1998 - 2 Marks)

17. If $i = \sqrt{-1}$, then $4 + 5\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$ is equal to
(a) $1 - i\sqrt{3}$ (b) $-1 + i\sqrt{3}$ (c) $i\sqrt{3}$ (d) $-i\sqrt{3}$
(1999 - 2 Marks)

18. If arg (z) < 0, then arg (–z) –arg (z) =
(a) π (b) $-\pi$ (c) $-\pi/2$ (d) $\pi/2$.
(2000 - 2 Marks)

19. If z_1, z_2, z_3 are complex numbers such that

$|z_1| = |z_2| = |z_3| = 1, \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is

(a) equal to 1 (b) less than 1
(c) greater than 3 (d) equal to 3
(2000 - 2 Marks)

20. Let z_1 and z_2 be n^{th} roots of unity which subtend a right angle at the origin, then 'n' must be of the form
(a) $4k + 1$ (b) $4k + 2$ (c) $4k + 3$ (d) $4k$
(2001 - 1 Mark)

21. The complex number z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of the triangle which is
(a) of area zero
(b) right angled isosceles
(c) equilateral
(d) obtuse angled isosceles. (2001 - 1 Mark)

22. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of

$$\begin{vmatrix} 1 & 1 & -1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$$

(a) 3ω (b) $3\omega(\omega - 1)$
(c) $3\omega^2$ (d) $3\omega(1 - \omega)$
(2002 - 1 Mark)

23. For all complex numbers Z_1, Z_2 satisfying $|Z_1| = 12$ and $|Z_2 - 3 - 4i| = 5$, the minumum value of $|Z_1 - Z_2|$ is
(a) 0 (b) 2 (c) 7 (d) 17
(2002 - 1 Mark)

24. If $|z| = 1$ and $\omega = \frac{z-1}{z+1}$ (where $z \neq -1$), then Re (ω) is

(a) 0 (b) $-\frac{1}{|z+1|^2}$

(c) $\left|\frac{z}{z+1}\right| \cdot \frac{1}{|z+1|^2}$ (d) $\frac{\sqrt{2}}{|z+1|^2}$.
(2003 - 1 Mark)

25. If $\omega\ (\neq 1)$ be a cube root of unity and $(1 + \omega^2)^n = (1 + \omega^4)^n$, then the least positive value of n is
(a) 2 (b) 3 (c) 5 (d) 6
(2004 - 1 Mark)

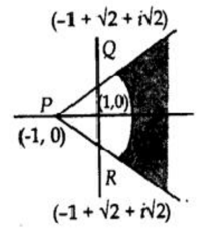
26. PQ and PR are two infinite rays, QAR is an arc. Point lying in the shaded region excluding the boundary satisfies

(a) $|z+1| > 2 : |\arg(z+1)| < \frac{\pi}{4}$

(b) $|z+1| < 2 : |\arg(z+1)| < \frac{\pi}{2}$

(c) $|z-1| > 2 : |\arg(z-1)| < \frac{\pi}{4}$

(d) $|z-1| < 2 : |\arg(z-1)| < \frac{\pi}{2}$



(2005 - 1 Mark)

27. The minimum value of $|a + b\omega + c\omega^2|$, where a, b and c are all not equal integers and $(\omega \neq 1)$ is a cube of root of unity, is
(a) 1 (b) 0
(c) $\sqrt{3}$ (d) $\frac{1}{2}$.
(2005 - 1 Mark)

28. If x, y, z are real and distinct then
$u = x^2 + 4y^2 + 9z^2 - 6yz - 32x - 2xy$ is always
(a) non negative (b) non positive
(c) zero (d) none of these.

29. If $\frac{\omega - \bar{\omega}z}{1 - z}$ is purely real where $\omega = \alpha + i\beta$, $\beta \neq 0$ and $z \neq 1$, then the set of the values of z is
(a) $\{z : |z| = 1\}$ (b) $\{z : z = \bar{z}\}$
(c) $\{z : z \neq 1\}$ (d) $\{z : |z| = 1, z \neq 1\}$.
(2006 - 3 Marks)

30. A man walks a distance of 3 units from the origin towards the north-east (N 45°E) direction. From there, he walks a distance of 4 units towards the north-west (N 45°W) direction to reach a point P. Then the position of P in the Argand plane is
(a) $3e^{i\pi/4} + 4i$ (b) $(3 - 4i)e^{i\pi/4}$
(c) $(4 + 3i)e^{i\pi/4}$ (d) $(3 + 4i)e^{i\pi/4}$.
(2007 - 3 Marks)

31. If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1 - z^2}$ lie on
(a) a line not passing through the origin
(b) $|z| = \sqrt{2}$ (c) the x-axis
(d) the y-axis (2007 - 3 Marks)

Multiple Choice Questions with ONE or MORE THAN ONE Correct Answer

32. If $z_1 = a + ib$ and $z_2 = c + id$ are complex number such that $|z_1| = |z_2| = 1$ and $Re(z_1\bar{z}_2) = 0$, then the pair of complex numbers $w_1 = a + ic$ and $w_2 = b + id$ satisfies :

(a) $|w_1| = 1$ (b) $|w_2| = 1$

(c) $\text{Re}(w_1\bar{w}_2) = 0$ (d) none of these.

(1985 - 2 Marks)

33. Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\dfrac{z_1+z_2}{z_1-z_2}$ may be

(a) zero (b) real and positive

(c) real and negative (d) none of these.

(1986 - 2 Marks)

Subjective Problems

34. If $x+iy=\sqrt{\dfrac{a+ib}{c+id}}$ prove that $(x^2+y^2)^2=\dfrac{a^2+b^2}{c^2+d^2}$

(1978 - 2 Marks)

35. If $x = a + b$, $y = a\alpha + b\beta$, $z = a\beta + b\alpha$ where α, β are complex cube roots of unity show that $xyz = a^3 + b^3$

(1979 - 3 Marks)

36. Express $\dfrac{1}{(1-\cos\theta)+2i\sin\theta}$ in the form $A + iB$.

(1979 - 3 Marks)

37. Find the real values of x and y for which the following equation is satisfied:

$$\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$$

(1980 - 2 Marks)

38. Let the complex numbers z_1, z_2 and z_3 be the vertices of an equilateral triangle. If z_0 be the circumcentre of the triangle, then prove that $z_1^2+z_2^2+z_3^2=3z_0^2$.

(1981 - 2 Marks)

39. A relation R on the set of complex numbers is defined by z_1Rz_2 if and only if $\dfrac{z_1-z_2}{z_1+z_2}$ is real show that R is an equivalence relation

(1982 - 2 Marks)

40. Prove that the complex numbers z_1, z_2 and the origin form an equilateral triangle only if $z_1^2+z_2^2-z_1z_2=0$

(1983 - 2 Marks)

41. If $1, a_1, a_2, \ldots\ldots, a_{n-1}$ are the n roots of unity then show that $(1-a_1)(1-a_2)\ldots(1-a_{n-1}) = n$.

(1984 - 2 Marks)

42. It is given that n is an odd integer greater than three, but n is not a multiple of 3. Prove that $x^3 + x^2 + x$ is a factor of $(x+1)^n - x^n - 1$

(1985 - 2 Marks)

43. Show that the area of the triangle on the argand diagram formed by the complex numbers, z, iz and $z + iz$ is $\frac{1}{2}|z|^2$

(1986 - $2\frac{1}{2}$ Marks)

44. Complex numbers z_1, z_2, z_3 are the vertices A, B, C respectively of an isosceles right angled triangle with right angle at C show that

$$(z_1-z_2)^2 = 2(z_1-z_3)(z_3-z_2)$$

(1986 - $2\frac{1}{2}$ Marks)

45. Let $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$. If z is any complex number such that the argument of $(z-z_1)/(z-z_2)$ is $\frac{\pi}{4}$, then prove that $|z-7-9i| = 3\sqrt{2}$

(1990 - 4 Marks)

46. If $iz^3 + z^2 - z + i = 0$ then show that $|z| = 1$

(1995 - 5 Marks)

47. Let z and ω be two complex numbers such that $|z|\le 1$, $|\omega|\le 1$ then show that

$$|z-\omega|^2 \le (|z|-|\omega|)^2 + (\text{Arg } z - \text{Arg } \omega)^2$$

(1995 - 5 Marks)

48. Find all non zero complex number z satisfying $\bar{z}=iz^2$

(1996 - 3 Marks)

49. Let z_1 and z_2 be roots of the equation, $z^2+pz+q=0$, where the co-efficients p and q may be complex numbers. Let A and B represent z_1 and z_2 in the complex plane. If $\angle AOB=\alpha\neq 0$ and $OA = OB$, where O is the origin, prove that $p^2=4q\cos^2(\alpha/2)$.

(1997 - 5 Marks)

50. Prove that $\sum_{k=1}^{n-1}(n-k)\cos\dfrac{2k\pi}{n}=-\dfrac{n}{2}$, where $n\ge 3$ is an integer.

(1997 - 5 Marks)

51. Let $\bar{b}z+b\bar{z}=c$, $b\neq 0$, be a line in the complex plane, where $\bar{b}$ is the complex conjugate of b, if a point z_1 is the deflection of a point z_2 through the line, then show that, $c=\bar{z}_1b+z_2\bar{b}$

(1997 - 3 Marks)

52. For complex numbers z and w, prove that $|z^2|w-|w|^2z=z-w$ if and only if $z=w$ or $z\bar{w}=1$.

(1999 - 10 Marks)

53. Let a complex number α, $\alpha\neq 1$ be a root of the equation $z^{p+q}-z^p-z^q+1=0$, when p, q are distinct primes. Show that either $1+\alpha+\alpha^2+\ldots\alpha^{p-1}=0$ or $1+\alpha+\alpha^2+\ldots+\alpha q=0$, but not both together.

(2002 - 5 Marks)

54. If z_1 & z_2 are two complex numbers such that $|z_1|<1<|z_2|$ then prove that $\left|\dfrac{1-z_1\bar{z}_2}{z_1-z_2}\right|<1$

(2003 - 5 Marks)

55. Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r z^r = 1$ where $|a_r| < 2$.
(2003 - 5 Marks)

56. $|z-1| = \sqrt{2}$ is a circle inscribed in a square whose one vertex is $2 + i\sqrt{3}$. Find the remaining vertex.
(2005 - 4 Marks)

True / False

57. For complex number $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, we write $z_1 \cap z_2$, if $x_1 \le x_2$ and $y_1 \le y_2$. Then for all complex numbers z with $1 \cap z$, we have $\frac{1-z}{1+z} \cap 0$.
(1981 - 2 Marks)

58. If the complex numbers, Z_1, Z_2 and Z_3 represent the vertices of an equilateral triangle such that $|Z_1| = |Z_2| = |Z_3|$ then $Z_1 + Z_2 + Z_3 = 0$.
(1984 - 1 Mark)

59. If three complex numbers are in A.P. then they lie on a circle in the complex plane.
(1985 - 1 Mark)

60. The cube roots of unity when represented on Argand diagram form the vertices of an equilateral triangle
(1988 - 1 Mark)

Fill in the blanks

61. If the expression

$$\frac{\left[\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right) + i\tan(x)\right]}{1 + 2i\sin\left(\frac{x}{2}\right)}$$

is real, then the set of all possible values of x is ....
(1987 - 3 Marks)

62. For any two complex numbers z_1, z_2 and any real numbers a and b, $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 =$ ....
(1988 - 2 Marks)

63. If a and b are real numbers between 0 and 1 such that the points $z_1 = a + i, z_2 = 1 + bi, z_3 = 0$ form an equilateral triangle, then a = ... and b = ...
(1989 - 2 Marks)

64. $ABCD$ is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy $BD = 2AC$. If the points D and M represents the complex numbers $1 + i$ and $2 - i$ respectively, then A represents the complex number .... or ....
(1993 - 3 Marks)

65. Suppose z_1, z_2, z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$ then z_2 = ... and z_3 = ...
(1994 - 2 Marks)

66. The value of the expression $1\cdot(2-\omega)(2-\omega^2) + 2\cdot(3-\omega)(3-\omega^2) + \ldots + (n-1)\cdot(n-\omega)(n-\omega^2)$, where ω is an imaginary cube root of unity, is .....
(1996 - 2 Marks)

Match the following

67. $z \ne 0$ is a complex number

COLUMN 1	COLUMN 2
(i) Re $z = 0$	(A) Re $z^2 = 0$
(ii) Arg $z = \frac{\pi}{4}$	(B) Im $z^2 = 0$
	(C) Re z^2 = Im z^2

(1992 - 2 Marks)

ANSWER KEY

1. (d)	2. (a)	3. (b)	4. (c)	5. (b)	6. (b)	7. (b)
8. (b)	9. (d)	10. (d)	11. (b)	12. (d)	13. (c)	14. (d)
15. (d)	16. (b)	17. (c)	18. (a)	20. (d)	21. (c)	22. (b)
23. (b)	24. (a)	25. (b)	26. (a)	27. (a)	28. (a)	29. (d)

30. (d) 31. (d) 32. (a, b, c) 33. (a, d) 36. $A = \frac{1}{2(1+3\cos^2\frac{\theta}{2})}, B = \frac{-\cot\frac{\theta}{2}}{1+3\cos^2\frac{\theta}{2}}$

37. 3, -1 48. $0+i, \frac{\sqrt{3}}{4} - \frac{1}{2}i, \frac{-\sqrt{3}}{4} - \frac{1}{2}i$ 56. $(1+i\sqrt{3})i+1, i\sqrt{3}, (1+\sqrt{3})-i$ 57. T 58. T

59. F 60. T 61. $2n\pi$ or $2n\pi + \frac{\pi}{4}$ 62. $(a^2+b^2)(|z_1|^2 + |z_2|^2$

63. $2-\sqrt{3}, 2-\sqrt{3}$ 64. $3 - \frac{1}{2}i$ or $1 - \frac{3}{2}i$ 65. $z_2 = -2, z_3 = 1 - i\sqrt{3}$

66. $\frac{1}{2}(n-1)n(n^2+3n+4)$ 67. (i) → (B), (ii) → (A).