MORE PRACTICE PAPERS FOR IIT-JEE

ALGEBRA

Complex Numbers

MULTIPLE CHOICE QUESTIONS

Type – I

Questions having one Correct Answer Only

Note: *Indicate your choice of correct answer for each question by writing one of the letters a, b, c, d whichever is appropriate.*

1. If $z = x + iy$, $z^{1/3} = a - ib$, then

$$\frac{x}{a} - \frac{y}{b} = k\,(a^2 - b^2), \text{ where } k =$$

(*a*) 0 (*b*) 1 (*c*) 4 (*d*) 6

2. For any two complex numbers z_1, z_2 and $a, b \in R$,

$$|az_1 + bz_2|^2 + |bz_1 - az_2|^2 =$$

(*a*) $|z_1|^2 + |z_2|^2$ (*b*) $(a^2 + b^2)(|z_1|^2 + |z_2|^2)$
(*c*) $(a^2 - b^2)(|z_1|^2 + |z_2|^2)$ (*d*) $|z_1| + |z_2|$

3. If $\alpha \neq 1$ is any nth root of unity, then

$$S = 1 + 3\alpha + 5\alpha^2 + \ldots\ldots \text{ upto } n \text{ terms is equal to}$$

(*a*) $2n/(1 - \alpha)$ (*b*) $-2n/(1 - \alpha)$ (*c*) $2n/(1 + \alpha)$ (*d*) $-2n/(1 + \alpha)$

4. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then

(*a*) $x = 3, y = 1$ (*b*) $x = 1, y = 3$ (*c*) $x = 0, y = 3$ (*d*) $x = 0, y = 0$

5. If $x = a + b$, $y = a\alpha + b\beta$, $z = a\beta + b\alpha$, where α, β are complex cube roots of unity, then $x^3 + y^3 + z^3 =$

(*a*) $(a^3 + b^3)$ (*b*) $a^3 - b^3$ (*c*) $3(a^3 + b^3)$ (*d*) $3(a^3 - b^3)$

6. If three complex numbers are in arithmetic progression, then they lie on a

(*a*) circle (*b*) straight line (*c*) parabola (*d*) ellipse

7. The value of the sum $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals

(*a*) i (*b*) $i - 1$ (*c*) $-i$ (*d*) 0

8. If $|z - 4/z| = 2$, then the greatest value of $|z|$ is

(*a*) $\sqrt{5} + 1$ (*b*) $\sqrt{3}$ (*c*) $\sqrt{2}$ (*d*) 1

9. If arg $(z) < 0$ then arg $(-z)$ – arg (z) is equal to

(*a*) π (*b*) $-\pi$ (*c*) $-\pi/2$ (*d*) $\pi/2$

10. If $x_n = \cos(\pi/2^n) + i\sin(\pi/2^n)$, then $x_1 x_2 x_3$ .... to ∞ is

(*a*) 0 (*b*) –1 (*c*) –2 (*d*) –3

11. The real value of θ for which the expression $\dfrac{1+i\cos\theta}{1-2i\cos\theta}$ is a real number is

(*a*) $2n\pi \pm \dfrac{\pi}{2}$ (*b*) $2n\pi + \dfrac{\pi}{3}$ (*c*) $2n\pi \pm \pi/4$ (*d*) $(2n + 1)\pi + \pi/2$

12. The maximum value of $|z|$ when z satisfies the condition $\left|z + \dfrac{2}{z}\right| = 2$ is

(*a*) $\sqrt{2} + \sqrt{3}$ (*b*) $\sqrt{3}$ (*c*) $\sqrt{3} + 1$ (*d*) $\sqrt{3} - 1$

13. If $1, \alpha_1, \alpha_2, ... , \alpha_{n-1}$ are the n roots of unity, then value of $(1 - \alpha_1)(1 - \alpha_2) .... (1 - \alpha_{n-1})$ is equal to

(*a*) 1 (*b*) n (*c*) -1 (*d*) $n - 1$

14. Let z_1 and z_2 be nth roots of unity which subtend a right angle at the origin. The n must be of the form:

(*a*) $4k + 1$ (*b*) $4k + 2$ (*c*) $4k + 3$ (*d*) $4k$

15. The curve represented by $\left|\dfrac{z - z_1}{z - z_2}\right| = c$, where $|c| \neq 1$, is

(*a*) straight line (*b*) circle (*c*) ellipse (*d*) None of these

16. If $x + iy = \sqrt{(a + ib)/(c + id)}$, then the value of $x^2 + y^2$ is

(*a*) $\sqrt{(a^2 + b^2)}\Big/\sqrt{(c^2 + d^2)}$ (*b*) $\sqrt{(c^2 + d^2)}\Big/\sqrt{(a^2 + b^2)}$

(*c*) $(a^2 + b^2)\big/(c^2 + d^2)$ (*d*) ab/cd

17. If $z + \sqrt{2}\ |z + 1| + i = 0$, then value of z is

(*a*) $2 + i$ (*b*) $2 - i$ (*c*) $-2 + i$ (*d*) $-2 - i$

18. For all complex numbers z_1, z_2 satisfying $|z_1 = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is

(*a*) 0 (*b*) 2 (*c*) 7 (*d*) 17

19. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals

(*a*) $128\,\omega$ (*b*) $-128\,\omega$ (*c*) $128\,\omega^2$ (*d*) $-128\,\omega^2$

20. If z_1 and z_2 are two complex numbers such that $|z_1| = |z_2| + |z_1 - z_2|$, then the value of arg z_1 – arg z_2 is

(*a*) 0 (*b*) 1 (*c*) -1 (*d*) 2

21. If z lies on the circle $|z| = 1$, then $2/z$ lies on

(*a*) a plane (*b*) a straight line (*c*) a circle (*d*) None of these

22. The points z_1, z_2, z_3 and z_4 in the complex plane are the vertices of a parallelogram taken in order if and only if

(*a*) $z_1 + z_2 + z_3 + z_4 = 0$ (*b*) $z_1/z_2 = z_3/z_4$

(*c*) $z_1 + z_3 = z_2 + z_4$ (*d*) $z_1 + z_2 = z_3 + z_4$

23. If z_1, z_2 and z_3 are in harmonic progression, then they lie on a

(*a*) circle (*b*) straight line (*c*) plane (*d*) ellipse

24. If z is purely imaginary, then

(*a*) $z = \bar{z}$ (*b*) $z \neq \bar{z}$

(*c*) $z = \dfrac{1}{\bar{z}}$ (*d*) $z\,\bar{z}$ is a negative quantity

25. For positive integers n_1, n_2 the value of the expression $(1 + i)^{n_1} + (1 + i^3)^{n_1} + (1 + i^5)^{n_2} + (1 + i^7)^{n_2}$, where $i = \sqrt{-1}$, is a real number only if

(*a*) $n_1 = n_2 + 1$ (*b*) $n_1 = n_2 - 1$ (*c*) $n_1 = n_2$ (*d*) $n_1 > 0,\ n_2 > 0$

26. If α_1 and α_2 are the two roots of the equation $z^2 + az + b = 0$, then the origin, α_1 and α_2 form an equilateral triangle if

(*a*) $a = b$ (*b*) $a^2 = 2b$ (*c*) $a^2 = 3b$ (*d*) $a = 3b^2$

27. The complex numbers z_1, z_2 and z_3 satisfying $\dfrac{z_1 - z_3}{z_2 - z_3} = \dfrac{1 - i\sqrt{3}}{2}$ are the vertices of a triangle which is:

(*a*) of area zero (*b*) right-angled isosceles

(*c*) equilateral (*d*) obtuse – angled isosceles

28. The area of the triangle formed by the complex numbers z, iz, $z + iz$ in the argand diagram is

(*a*) $\dfrac{1}{2}|z|^2$ (*b*) $|z|^2$ (*c*) $2|z|^2$ (*d*) None of these

29. If the complex numbers z_1, z_2, z_3 are in A.P., then they lie on a

(*a*) ellipse (*b*) straight line (*c*) parabola (*d*) circle

30. Let α be a complex number such that $|a| < 1$ and z_1, z_2, ... be vertices of a polygon such that $z_R = 1 + \alpha + \alpha^2 + ... + \alpha^{k-1}$. Then the vertices of the polygon lie within a circle

(*a*) $\left|z - \dfrac{1}{1-a}\right| = |1 - a|$ (*b*) $|z - (1 - a)| = |1 - a|$

(*c*) $\left|z - \dfrac{1}{1-a}\right| = \dfrac{1}{|1-a|}$ (*d*) $|z - a| = a$

31. For any two complex numbers z_1, z_2 the value of $|z_1 + z_2|^2 + |z_1 - z_2|^2$ is

(*a*) $(|z_1| + |z_2|)^2$ (*b*) $|z_1|^2 + |z_2|^2$ (*c*) $2\,(|z_1|^2 + |z_2|^2)$ (*d*) $\dfrac{1}{2}\,(|z_1|^2 + |z_2|)^2$

32. If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, then the value of $\cos 3\alpha + \cos 3\beta + \cos 3\gamma$ is

(*a*) 0 (*b*) $3\cos(\alpha + \beta + \gamma)$

(*c*) $3\sin(\alpha + \beta + \gamma)$ (*d*) $\cos(\alpha + \beta + \gamma)$

33. Value of the expression $\dfrac{(1+i)^n}{(1-i)^{n-2}}$ is:

(*a*) i^{n-1} (*b*) $2i^{n-1}$ (*c*) i^{n+1} (*d*) $2i^{n+1}$

34. The value of $\left[\sqrt{2}\,\{\cos(56^\circ\,15') + i\sin(56^\circ\,15')\}\right]^8$ is

(*a*) $4i$ (*b*) $8i$ (*c*) $-16\,i$ (*d*) $16\,i$

35. Let z_1 and z_2 be two non-zero complex numbers such that $|z_1| = |z_2|$ and arg (z_1) + arg $(z_2) = \pi$. Then z_1 equals

(*a*) z_2 (*b*) $-z_2$ (*c*) $\bar{z}_2$ (*d*) $-\bar{z}_2$

36. For $x_1, x_2, y_1, y_2 \in R$. If $0 < x_1 < x_2$, $y_1 = y_2$ and $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ and $z_3 = \frac{1}{2}(z_1 + z_2)$, then z_1, z_2 and z_3 satisfy:

(*a*) $|z_1| < |z_3| < |z_2|$ (*b*) $|z_1| > |z_2| > |z_3|$ (*c*) $|z_1| < |z_2| < |z_3|$ (*d*) $|z_1| = |z_2| = |z_3|$

37. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is

(*a*) equal to 1 (*b*) less than 1 (*c*) greater than 3 (*d*) equal to 3

38. If $z = \left(\frac{1+i}{2}\right)$, then the expression $2z^4 - 2z^2 + z + 3$ equals

(*a*) $(3 - i) / 2$ (*b*) $(3 + i) / 2$ (*c*) $3 + (i/2)$ (*d*) $3 - (i/2)$

39. If 1, ω, ω^2 are the three cube roots of unity then for $a, b, c, d, \in, R$, the expression $\left(\frac{a + b\omega + c\omega^2 + d\omega^2}{b + a\omega^2 + c\omega + d\omega}\right)$ is equal to

(*a*) ω^{-1} (*b*) $-\omega$ (*c*) ω (*d*) 1

40. $1, \alpha, \alpha^2, \ldots, \alpha^{n-1}$ are n, nth roots of unity, the value of $(g - \alpha)(g - \alpha^2) \ldots (g - \alpha^{n-1})$ will be

(*a*) n (*b*) 0 (*c*) $\frac{g^n - 1}{8}$ (*d*) $\frac{g^n + 1}{8}$

41. If $i = \sqrt{-1}$, then $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$ is equal to

(*a*) $1 - i\sqrt{3}$ (*b*) $-1 + i\sqrt{3}$ (*c*) $i\sqrt{3}$ (*d*) $-i\sqrt{3}$

42. If 1, ω and ω^2 are three cube roots of unity, then the roots of the equation $(x - 1)^3 - 8 = 0$ are

(*a*) $-1, -1 - 2\omega, -1 + 2\omega^2$ (*b*) $3, 1 + 2\omega, 1 + 2\omega^2$

(*c*) $3, 2\omega, 2\omega^2$ (*d*) None of these

43. If n is a positive integer but not a multiple of 3 and $z = -1 + i\sqrt{3}$, then $(z^{2n} + 2^n z^n + 2^{2n})$ is equal to

(*a*) 3×2^n (*b*) 1 (*c*) -1 (*d*) 0

Type – II

Questions having more than one correct answers.

Note: *Each question in this part, has one or more than one correct answers. For each question, write the letters a, b, c, d corresponding to the correct answers.*

44. If z satisfies $|z + 1| < |z - 2|$ then $\omega = 3z + 2 + i$ satisfies

(*a*) $|\omega + 5| < |\omega - 4|$ (*b*) $Re\left(\frac{1}{2\omega - 7}\right) > 0$

(*c*) $|\omega + 1 + i| < |\omega - 8 + i|$ (*d*) $|\omega + 1| < |\omega - 8|$

45. If $\frac{\tan\alpha - i(\sin\alpha/2 + \cos\alpha/2)}{1 + 2i\sin\alpha/2}$ is purely imaginary, then α is given by

(*a*) $n\pi + \pi/4$ (*b*) $n\pi - \pi/4$ (*c*) $2n\pi$ (*d*) $2n\pi + \pi/4$

46. If z_1 and z_2 are non-zero complex numbers such that $|z_1 - z_2| = |z_1| + |z_2|$ then

(*a*) $z_1 + kz_2 = 0$ for some positive number *k*

(*b*) $\arg z_1 = \arg z_2$ (*c*) $z_1\bar{z}_2 + \bar{z}_1 z_2 < 0$ (*d*) $|\arg z_1 - \arg z_2| = \pi$

47. If *z* satisfies $|z - 1| < |z + 3|$ then $\omega = 2z + 3 - i$ satisfies

(*a*) $|\omega - 5 - i| < |\omega + 3 + i|$ (*b*) $\arg(\omega - 1) < \pi/2$

(*c*) $|\omega - 5| < |\omega + 3|$ (*d*) $I_m(i\omega) > 1$

48. ω is a cube root of unity and *n* is a positive integer satisfying $1 + \omega^n + \omega^{2n} = 0$; then *n* is of the type

(*a*) $3m + 2$ (*b*) $3m + 1$ (*c*) $3m$ (*d*) None of these

49. The equation whose roots are *n*th powers of the roots of the equation, $x^2 - 2x\cos\theta + 1 = 0$, is given by

(*a*) $x^2 - 2x\cos n\theta + 1 = 0$ (*b*) $x^2 + 2x\cos n\theta + 1 = 0$

(*c*) $(x + \cos n\theta)^2 + \sin^2 n\theta = 0$ (*d*) $(x - \cos n\theta)^2 + \sin^2 n\theta = 0$

50. If $|z| = 1$, then $\left(\frac{1+z}{1+\bar{z}}\right)^n + \left(\frac{1+\bar{z}}{1+z}\right)^n$ is equal to

(*a*) $2\cos n(\arg(z/2))$ (*b*) $2\cos n(\arg(z))$

(*c*) $2\sin n(\arg(z/2))$ (*d*) $2\sin(\arg(z))$

51. The cube roots of $-i$ ($i = \sqrt{-1}$) in terms of ω (cube root of unity) are

(*a*) $i\omega^3$ (*b*) $i\omega^2$ (*c*) $i\omega$ (*d*) i

52. If $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$, then

(*a*) $z_1\bar{z}_2 + z_2\bar{z}_1 = 0$ (*b*) amp $\frac{z_1}{z_2} = \frac{\pi}{2}$

(*c*) $\frac{z_1}{z_2}$ is purely real (*d*) $\frac{z_1}{z_2}$ is purely imaginary

53. Let z_1, z_2 be two complex numbers represented by points on the circle $|z| = 1$ and $|z| = 2$ respectively, then

(*a*) $\min|z_1 - z_2| = 1$ (*b*) $\max.|2z_1 + z_2| = 4$

(*c*) $\left|z_2 + \frac{1}{z_1}\right| \le 3$ (*d*) None of these

54. If *z* is a complex number and $a_1, a_2, a_3, b_1, b_2, b_3$ all are real then, the value of

$$\begin{vmatrix} a_1 z + b_1\bar{z} & a_2 z + b_2\bar{z} & a_3 z + b_3\bar{z} \\ b_1 z + a_1\bar{z} & b_2 z + a_2\bar{z} & b_3 z + a_3\bar{z} \\ b_1 z + a_1 & b_2 z + a_2 & b_3 z + a_3 \end{vmatrix} \text{ is}$$

(*a*) $|z|^2$ (*b*) 3

(*c*) $(a_1 a_2 a_3 + b_1 b_2 b_3)^2 |z|^2$ (*d*) 0

55. If α is a complex constant such that $\alpha z^2 + z + \bar{\alpha} = 0$ has a real root then

(*a*) $\alpha + \bar{\alpha} = 0$ (*b*) $\alpha + \bar{\alpha} = 1$

(*c*) $\alpha + \bar{\alpha} = -1$ (*d*) The absolute value of the real root is 1

56. If $z = x + iy$ and $\omega = \frac{1-iz}{z-i}$, then $|\omega| = 1$ implies that in the complex plane

(*a*) z lies on real axis (*b*) z lies on imaginary axis
(*c*) z lies on unit circle (*d*) z lies inside a unit circle

57. If one root of the quadratic equation $(1 + i)\ x^2 - (7 + 3i)\ x + (6 + 8i) = 0$ is $4 - 3i$ then the other root must be

(*a*) $4 + 3i$ (*b*) $1 - i$ (*c*) $1 + i$ (*d*) $i\ (1 - i)$

58. The argument of the principal value of the complex number $\frac{2+i}{4i+(1+i)^2}$ are

(*a*) $\tan^{-1}(-2)$ (*b*) $-\tan^{-1}(2)$ (*c*) $\tan^{-1}\left(\frac{1}{2}\right)$ (*d*) $-\tan^{-1}\left(\frac{1}{2}\right)$

59. If z_1, z_2 are two complex numbers satisfying the equation $\left|\frac{z_1+z_2}{z_1-z_2}\right| = 1$, then z_1/z_2 is a number which is

(*a*) 0 (*b*) positive real (*c*) negative real (*d*) purely imaginary

60. If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, then

(*a*) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$ (*b*) $\sin 2\alpha - \sin 2\beta - \sin 2\gamma = 0$
(*c*) $\cos(\beta + \gamma) + \cos(\gamma + \alpha) + \cos(\alpha + \beta) = 0$
(*d*) $\sin(\beta - \gamma) + \sin(\gamma - \alpha) + \sin(\alpha - \beta) = 0$

61. If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$, then

(*a*) $\frac{x}{y}+\frac{y}{x} = 2\cos(\theta-\phi)$ (*b*) $xy + \frac{1}{xy} = 2\sin(\theta + \phi)$

(*c*) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\theta + n\phi)$ (*d*) $\frac{x^m}{y^n} + \frac{y^n}{x^m} = 2\cos(n\theta - m\phi)$

62. The points representing the complex number z for which

$$\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3} \text{ lie on}$$

(*a*) a parabola (*b*) an ellipse (*c*) a straight line (*d*) a circle

63. If $1, \alpha_1, \alpha_2, ..., \alpha_{n-1}$ are the n roots of unity, then the value of $(1 + \alpha_1)(1 + \alpha_2) ...... (1 + \alpha_{n-1})$ is

(*a*) always zero (*b*) –1 always (*c*) 0 if n is odd (*d*) – 1 if n is even

64. If $(1 + x)^n = c_0 + c_1 x + ... + c_n x^n$, where n is a positive integer, then

(*a*) $c_1 - c_3 + c_5 - ..... = 2^{n/2} \sin\frac{n\pi}{4}$ (*b*) $c_0 + c_4 + c_8 + ... = 2^{n-2} + 2^{(n-2)/2}\cos\frac{n\pi}{4}$

(*c*) $c_0 - c_2 + c_4 - ..... = 2^{n/2} \sin\frac{n\pi}{4}$ (*d*) None of these

65. If $\frac{z+1}{z+i}$ is a purely imaginary number, then z lies on a

(*a*) circle (*b*) circle passing through origin
(*c*) straight line (*d*) imaginary axis

66. $\sin^{-1}\left\{\frac{1}{i}(z-1)\right\}$, where z is non real, can be the angle of a triangle if

(*a*) $Re(z) = 1, I_m(z) = 1$ (*b*) $Re(z) = 1, I_m(z) \le 1$
(*c*) $Re(z) = 1, I_m(z) \ge -1$ (*d*) $Re(z) = 1, I_m(z) = 2$

ANSWERS

Type – I

1. (*c*)	**2.** (*b*)	**3.** (*b*)	**4.** (*d*)	**5.** (*c*)
6. (*b*)	**7.** (*b*)	**8.** (*a*)	**9.** (*a*)	**10.** (*b*)
11. (*a*)	**12.** (*c*)	**13.** (*b*)	**14.** (*d*)	**15.** (*b*)
16. (*a*)	**17.** (*d*)	**18.** (*b*)	**19.** (*d*)	**20.** (*a*)
21. (*c*)	**22.** (*c*)	**23.** (*b*)	**24.** (*a*)	**25.** (*d*)
26. (*c*)	**27.** (*c*)	**28.** (*a*)	**29.** (*b*)	**30.** (*c*)
31. (*c*)	**32.** (*b*)	**33.** (*b*)	**34.** (*d*)	**35.** (*d*)
36. (*a*)	**37.** (*a*)	**38.** (*d*)	**39.** (*c*)	**40.** (*c*)
41. (*c*)	**42.** (*b*)	**43.** (*d*)	**44.** (*c*), (*d*)	**45.** (*a*), (*c*), (*d*)
46. (*a*), (*c*), (*d*)	**47.** (*b*), (*c*), (*d*)	**48.** (*a*), (*b*)	**49.** (*a*), (*d*)	**50.** (*b*)
51. (*a*),(*b*),(*c*),(*d*)	**52.** (*a*), (*b*), (*d*)	**53.** (*a*), (*b*), (*c*)	**54.** (*d*)	**55.** (*b*), (*c*), (*d*)
56. (*a*)	**57.** (*c*), (*d*)	**58.** (*a*), (*b*)	**59.** (*a*), (*d*)	**60.** (*a*), (*c*)
61. (*a*), (*c*)	**62.** (*d*)	**63.** (*c*), (*d*)	**64.** (*a*), (*b*), (*c*)	**65.** (*a*), (*b*)
66. (*b*), (*c*)				

HINTS AND SOLUTIONS OF SELECTED QUESTIONS

1. $(x + iy)^{1/3} = a - ib \Rightarrow (x + iy) = (a - ib)^3$

$= (a^3 - 3ab^2) + i\,(b^3 - 3a^2 b)$

$\therefore\quad x = a^3 - 3ab^2,\ y = b^3 - 3a^2b$

$\Rightarrow\quad \dfrac{x}{a} - \dfrac{y}{b} = a^2 - 3b^2 - b^2 + 3a^2 = 4(a^2 - b^2)$

$\therefore\quad k = 4$

Hence (*c*) *is the correct answer.*

2. $|az_1 + bz_2|^2 + |bz_1 - az_2|^2 = |az_1|^2 + |bz_2|^2 + 2\,Re\,(az_1\,\overline{bz_2}) + |bz_1|^2 + |az_2|^2 - 2\,Re\,(bz_1\,\overline{az_2})$

$= a^2\,|z_1|^2 + b^2\,|z_2|^2 + 2\,ab\,Re\,(z_1\,\bar{z}_2) + b^2\,|z_1|^2 + a^2\,|z_1|^2$

$- 2ab\,Re\,(z_1\,\bar{z}_2)$ [$\because$ *a* and *b* are real]

$= (a^2 + b^2)\,(|z_1|^2 + |z_2|^2)$

Hence (*b*) *is the correct answer.*

3. Let $z = x + iy$. Then

$|z^2 - 1| = |z|^2 + 1 \Rightarrow |x^2 - y^2 + 2xy - 1| = x^2 + y^2 + 1$

$\Rightarrow\quad (x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)^2$

$\Rightarrow\quad (x^2 - y^2)^2 - 2(x^2 - y^2) + 1 + 4x^2\,y^2 = (x^2 + y^2)^2 + 2(x^2 + y^2) + 1$

$\Rightarrow\quad 4x^2 = 0 \Rightarrow x = 0.$

Thus *z* lies on imaginary axis.

Hence (*b*) *is the correct answer*.

5. We have

$x^3 + y^3 + z^3 = (a + b)^3 + (a\alpha + b\beta)^3 + (\alpha\beta + b\alpha)^3$

$= a^3 + b^3 + 3a^2b + 3ab^2 + a^3\alpha^3 + b^3\beta^3 + 3a^2b\alpha^2\beta + 3ab^2\alpha\beta^2 + a^3\beta^3$
$+ b^3\alpha^3 + 3a^2\,b\alpha\,\beta^2 + 3ab^2\,\alpha^2\beta$

$= 3a^3 + 3b^3 + 3\,(a^2b + ab^2)\,(1 + \alpha^2\beta + \alpha\beta^2)$ [$\because$ $\alpha^3 = \beta^3 = 1$]

$= 3a^3 + 3b^3 + 3(a^2b + ab^2)\,\{1 + \alpha\beta\,(\alpha + \beta)\}$

$= 3\,(a^3 + b^3)$ [$\because$ $\alpha\beta = 1$ and $1 + \alpha + \beta = 0$]

Hence (*c*) *is the correct answer*.

6. If z_1, z_2 and z_3 are in A.P., then $z_2 = \frac{z_1 + z_3}{2}$ i.e., z_2 is the mid point of the line joining z_1 and z_3.

Hence z_1, z_2, z_3 lie on a straight line.
Hence (*b*) *is the correct answer.*

7. $$\sum_{i=1}^{13} (i^n + i^{n+1}) = \sum_{i=1}^{13} i^n (1+i)$$

$$= (1+i) \left[\frac{i(1-i)^{13}}{1-i}\right] = (1+i) \left[\frac{i(1-i)}{1-i}\right]$$

$$= (1+i)\, i = -1 + i.$$

Hence (*b*) *is the correct answer.*

8. $$|z| = \left|z - \frac{4}{z} + \frac{4}{z}\right| \le \left|z - \frac{4}{z}\right| + \left|\frac{4}{z}\right| = 2 + \frac{4}{|z|}$$

$\Rightarrow \quad |z|^2 \le 2|z| + 4$
$\Rightarrow \quad (|z| - 1)^2 \le 5$
$\Rightarrow \quad |z| \le \sqrt{5} + 1.$

Hence (*a*) *is the correct answer.*

9. Let $\arg z = -\theta$, where θ is positive.
then $\arg(-z) = \pi + \theta = 2\pi - (\pi + \theta)$
$\Rightarrow \quad \arg(-z) = \pi - \theta = \pi + z$
$\Rightarrow \quad \arg(-z) - \arg z = \pi.$

Hence (*a*) *is the correct answer.*

13. Since 1, α_1, α_2, ... α_{n-1} are the roots of $x^n - 1 = 0$, hence
$x^n - 1 = (x - 1)(x - \alpha_1)(x - \alpha_2) \ldots\ldots (x - \alpha_{n-1})$

$$\Rightarrow \quad (x - \alpha_1)(x - \alpha_2) \ldots\ldots (x - \alpha_{n-1}) = \frac{x^n - 1}{x - 1} = x^{n-1} + x^{n-2} + \ldots\ldots x + 1$$

Putting $x = 1$,
$\Rightarrow \quad (1 - \alpha_1)(1 - \alpha_2) \ldots\ldots (1 - \alpha_{n-1}) = 1 + 1 + \ldots\ldots + 1 + 1 = n.$

Hence (*b*) *is the correct answer*.

14. Let $z = (1)^{1/n} = (\cos 0 + i \sin 0)^{1/n} = \cos \frac{2r\pi}{n} + i \sin \frac{2r\pi}{n}$

$$= e^{i\frac{2r\pi}{n}}$$

where r varies from 0 to $(n-1)$ and each root is unimodular as $|e^{i\theta}| = 1$.

Let $z_1 = 1$ and $z_2 = e^{i\frac{2k\pi}{n}}$ where $(z_2 - 0) = (z_1 - 0)\, e^{\frac{\pi}{2} i}$ (given condition)

or $e^{i\frac{2k\pi}{n}} = 1.\ e^{\frac{\pi}{2} i}$

$\therefore \quad n = 4k$

Hence (*d*) *is the correct answer.*

17. Putting $z = x + iy$, we have

$$x + iy + \sqrt{2}\ |(x+1) + iy| + i = 0$$

$$\Rightarrow \quad x + iy + \sqrt{2}\ .\ \sqrt{\{(x+1)^2 + y^2\}} + i = 0$$

$\Rightarrow \quad x + \sqrt{2}\ [(x+1)^2 + y^2]^{1/2} = 0$ and $y + 1 = 0$.

$\therefore \quad y = -1$ and

$2\,[(x+1)^2 + 1] = x^2 \quad \Rightarrow \quad x^2 + 4x + 4 = 0$

$\Rightarrow \quad (x+2)^2 = 0$, *i.e.,* $x = -2$

$\therefore \quad z = x + iy = -2 - i$.

Hence (*d*) *is the correct answer.*

19. Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

Then $|z_1| = \sqrt{x_1^2 + y_1^2}$, $|z_2| = \sqrt{x_2^2 + y_2^2}$,

$|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Then $|z_1| = |z_2| + |z_1 - z_2|$

$\Rightarrow \quad \sqrt{(x_1^2 + y_1^2)} - \sqrt{(x_2^2 + y_2^2)} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow \quad x_1^2 + y_1^2 + x_2^2 + y_2^2 - 2\sqrt{(x_1^2 + y_1^2)(x_2^2 + y_2^2)} = x_1^2 + x_2^2 - 2x_1x_2 + y_1^2 + y_2^2 - 2y_1y_2$

$\Rightarrow \quad x_1 x_2 + y_1 y_2 = \sqrt{(x_1^2 + y_1^2)\,(x_2^2 + y_2^2)}$

$\Rightarrow \quad x_1^2 x_2^2 + y_1^2 y_2^2 + 2x_1 x_2 y_1 y_2 = x_1^2 x_2^2 + x_1^2 y_2^2 + x_2^2 y_1^2 + y_1^2 y_2^2$

$x_1^2 y_2^2 + x_2^2 y_1^2 - 2x_1 x_2 y_1 y_2 = 0$

$\Rightarrow \quad (x_1 y_2 - x_2 y_1)^2 = 0 \quad \Rightarrow \quad x_1 y_2 - x_2 y_1 = 0$

$\Rightarrow \quad \dfrac{y_1}{x_1} = \dfrac{y_2}{x_2} \quad \Rightarrow \quad \tan^{-1}\left(\dfrac{y_1}{x_1}\right) = \tan^{-1}\left(\dfrac{y_2}{x_2}\right)$

$\Rightarrow \quad \arg z_1 = \arg z_2$.

Hence (*a*) *is the correct answer.*

21. The given points will form a parallelogram if and only if the midpoint of $z_1 z_3$ is the same as the midpoint of $z_2 z_4$, *i.e.,* $\dfrac{1}{2}(z_1 + z_3) = \dfrac{1}{2}(z_2 + z_4) \quad \Rightarrow \quad z_1 + z_3 = z_2 + z_4$

Hence (*c*) *is the correct answer.*

24. $(1+i)^{n_1} + (1-i)^{n_1} + (1+i)^{n_2} + (1-i)^{n_2}$

$= 2\left[{}^{n_1}c_0 + {}^{n_1}c_2\, i^2 + {}^{n_1}c_4\, i^4 + \ldots..\right] + 2\left[{}^{n_2}c_0 + {}^{n_2}c_2\, i^2 + {}^{n_2}c_4\, i^4 + \ldots..\right]$

As $i^2, i^4, i^6, \ldots..$ are all real. Hence the given expression is real if $n_1 > 0$ and $n_2 > 0$.

Hence (*d*) *is the correct answer.*

25. From the given condition

$z_1 + z_2 = -a$ and $z_1 z_2 = b$.

z_1, z_2 and origin will form an equilateral triangle if

$|z_1| = |z_2| = |z_1 - z_2|$ ...(1)

Let $\alpha = z_1 - 0, \quad \beta = z_2 - z_1$ and $\gamma = 0 - z_2$,

Then $\alpha + \beta + \gamma = 0$ ...(2)

so that $\bar{\alpha} + \bar{\beta} + \bar{\gamma} = 0$. Substituting in (1),

$|\alpha| = |\beta| = |\gamma| = k$ (say)

i.e., $\alpha\bar{\alpha} = \beta\bar{\beta} = \gamma\bar{\gamma} = k$.

Therefore,

$$\frac{k}{\alpha} + \frac{k}{\beta} + \frac{k}{\gamma} = 0 \quad \Rightarrow \quad \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = 0$$

$\Rightarrow \quad \dfrac{1}{z_1} + \dfrac{1}{z_2 - z_1} - \dfrac{1}{z_2} = 0$

$\Rightarrow \quad (z_2 - z_1)(z_1 - z_2) = z_1 z_2 \quad \Rightarrow \quad (z_1 - z_2)^2 = -z_1 z_2$

$\Rightarrow \quad (z_1 + z_2)^2 = 4 z_1 z_2 - z_1 z_2 = 3 z_1 z_2$

$\Rightarrow \quad (-a)^2 = a^2 = 3b$

Hence (*c*) *is the correct answer.*

26. Let z_1, z_2, z_3 are the points *A, B* and *C*. By taking modulus of the given relation

$$\frac{|z_1 - z_3|}{|z_2 - z_3|} = \frac{1}{4} + \frac{3}{4} = 1$$

$\therefore \quad AC = BC.$

Hence triangle is isosceles. Also,

$$\frac{z_1 - z_3}{z_2 - z_3} = \cos\frac{\pi}{3} - i\sin\frac{\pi}{3} = e^{-\frac{\pi}{3}i}$$

$\Rightarrow \quad (z_2 - z_3) = (z_1 - z_3)\ e^{i\pi/3}$

Anticlockwise rotation implies that $\angle ACB = \pi/3$. Hence isosceles triangle is equilateral.

Hence (*c*) *is the correct answer.*

29. $z_k = 1 + a + a^2 + ... + a^{k-1} = \dfrac{1 - a^k}{1 - a}$

$\Rightarrow \quad z_k - \dfrac{1}{1-a} = \dfrac{a^k}{1-a}$

$\Rightarrow \quad \left| z_k - \dfrac{1}{1-a} \right| = \dfrac{|a^k|}{|1-a|} = \dfrac{|a|^k}{|1-a|} < \dfrac{1}{|1-a|}$

$\Rightarrow \quad z_k$ lies within $\left| z - \dfrac{1}{1-a} \right| = \dfrac{1}{|1-a|}$.

Hence (*c*) *is the correct answer.*

35. Since $x_1 < x_2 \quad \Rightarrow \quad \sqrt{x_1^2 + y_1^2} < \sqrt{x_2^2 + y_2^2}$ $\quad [\because\ y_1 = y_2]$

$\therefore \quad |z_1| < |z_2|$ ...(1)

$$|z_3| = \frac{1}{2}|z_1 + z_2| < \frac{1}{2}[|z_1| + |z_2|] < \frac{1}{2}|z_2| + \frac{1}{2}|z_2|$$

$\Rightarrow \quad |z_3| < |z_2|$ ...(2)

From (1) and (2)

$|z_1| < |z_3| < |z_2|$

Hence (*a*) *is the correct answer.*

38. Given expression

$$= \frac{a\omega^3 + b\omega + c\omega^2 + d\omega^2}{b + a\omega^2 + c\omega + d\omega} = \frac{\omega(b + a\omega^2 + c\omega + d\omega)}{(b + a\omega^2 + c\omega + d\omega)}$$

$= \omega$

Hence (*c*) *is the correct answer.*

39. $1, \alpha, \alpha^2, ... \alpha^{n-1}$ are the roots of $x^n - 1 = 0$

$\Rightarrow \quad x^n - 1 = (x - 1)(x - \alpha)(x - \alpha^2) ...... (x - \alpha^{n-1})$

$$\Rightarrow \quad \frac{x^n - 1}{x - 1} = (x - \alpha)(x - \alpha^2) \ldots\ldots (x - \alpha^{n-1})$$

Putting $x = g$, we get

$$(g - \alpha)(g - \alpha^2) \ldots\ldots. (g - \alpha^{n-1}) = \frac{g^n - 1}{8}$$

Hence (*c*) *is the correct answer.*

42. $z = -1 + i\sqrt{3} = 2\left(\frac{-1 + i\sqrt{3}}{2}\right) = 2\omega,$

where ω is the cube root of unity.

$$\therefore \quad z^{2n} + 2^n z^n + 2^{2n} = 2^{2n}\,\omega^{2n} + 2^n \,.\, 2^n\,\omega^n + 2^{2n}$$

$$= 2^{2n}\,(\omega^{2n} + \omega^n + 1)$$

Let $n = 3m + 1, \quad m \in I$

$$= 2^{2n}\left\{(\omega)^{6m+2} + (\omega)^{3m+1} + 1\right\}$$

$$= 2^{2n}\,\{\omega^2 + \omega + 1\} = 0.$$

Hence (*d*) *is the correct answer.*

43. Given $|z + 1| < |z - 2|$ and $\omega = 3z + 2 + i$

$$\therefore \quad \omega + \overline{\omega} = 3z + 2 + i + 3\bar{z} + 2 - i$$

$$= 3(z + \bar{z}) + 4 \qquad \ldots(1)$$

$$|z + 1|^2 < |z - 2|^2$$

$$\Rightarrow \quad (z + 1)(\bar{z} + 1) < (z - 2)(\bar{z} - 2)$$

$$\Rightarrow \quad z + \bar{z} < 1 \qquad \ldots(2)$$

From (1) & (2)

$$\frac{\omega + \overline{\omega} - 4}{3} < 1$$

$$\Rightarrow \quad \omega + \overline{\omega} < 7 \qquad \ldots(3)$$

Option (*d*) $|\omega + 1| < |\omega - 8|$

$\Rightarrow$ $|\omega + 1|^2 < |\omega - 8|^2$

$\Rightarrow$ $(\omega + 1)\,(\overline{\omega} + 1) < (\omega - 8)\,(\overline{\omega} - 8)$

$\Rightarrow$ $\omega + \overline{\omega} < 7$ which is true from (3)

Option (*c*) $|\omega + 1 + i| < |\omega - 8 + i|$

$\Rightarrow$ $|\omega + 1 + i|^2 < |\omega - 8 + i|^2$

$\Rightarrow$ $(\omega + 1 + i)\,(\overline{\omega} + 1 - i) < (\omega - 8 + i)\,(\overline{\omega} - 8 - i)$

$\Rightarrow$ $\omega + \overline{\omega} < 7$ which is true from (3)

Option (*b*) $Re\left(\frac{1}{2\omega - 7}\right) > 0$

$$\Rightarrow \quad \frac{\frac{1}{2\omega - 7} + \frac{1}{2\overline{\omega} - 7}}{2} > 0$$

$\Rightarrow$ $\omega + \overline{\omega} > 7$ which is not true from (3)

Option (*a*) $|\omega + 5| < |\omega - 4|$

$\Rightarrow$ $|\omega + 5|^2 < |\omega - 4|^2$

$\Rightarrow$ $(\omega + 5)\,(\overline{\omega} + 5) < (\omega - 4)\,(\overline{\omega} - 4)$

$\Rightarrow \quad \omega + \overline{\omega} < -1$ which is not true from (3)

Hence (*c*), (*d*) *are correct answers.*

48. $x^2 - 2x \cos\theta + 1 = 0$

$\Rightarrow \quad x = \cos\theta \pm i \sin\theta$

If $\alpha = \cos\theta + i\sin\theta$ then $\beta = \cos\theta - i\sin\theta$

Required equation is

$$x^2 - (\alpha^n + \beta^n)\, x + \alpha^n \beta^n = 0$$

$\Rightarrow \quad x^2 - 2x \cos n\theta + 1 = 0$

$\Rightarrow \quad (x - \cos n\theta)^2 + \sin^2 n\theta = 0$

Hence (*a*) and (*d*) *are correct answers.*

49. If $|z| = 1$, $\left(\dfrac{1+z}{1+\bar{z}}\right)^n + \left(\dfrac{1+\bar{z}}{1+z}\right)^n$

$$= \left(\frac{z(1+z)}{z+z\bar{z}}\right)^n + \left(\frac{z+z\bar{z}}{z(1+z)}\right)^n$$

$$= \left(\frac{z(1+z)}{z+|z|^2}\right)^n + \left(\frac{z+|z|^2}{z(1+z)}\right)^2$$

$$= \left(\frac{z(1+z)}{z+1}\right)^n + \left(\frac{z+1}{z(1+z)}\right)^2$$

$$= z^n + \frac{1}{z^n} = 2\cos n\,(\arg z)$$

Hence (*b*) *is the correct answer.*

53. $$\begin{vmatrix} a_1 z + b_1 \bar{z} & a_2 z + b_2 \bar{z} & a_3 z + b_3 \bar{z} \\ b_1 z + a_1 \bar{z} & b_2 z + a_2 \bar{z} & b_3 z + a_3 \bar{z} \\ b_1 z + a_1 & b_2 z + a_2 & b_3 z + a_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 z + b_1 \bar{z} & a_2 z + b_2 \bar{z} & a_3 z + b_3 \bar{z} \\ b_1 (z - |z|^2) & b_2 (z - |z|^2) & b_3 (z - |z|^2) \\ b_1 z + a_1 & b_2 z + a_2 & b_3 z + a_3 \end{vmatrix} \quad \text{operating } R_2 \to R_2 - \bar{z}\, R_3$$

$$= (z - |z|^2) \left\{ \begin{vmatrix} a_1 z + b_1 \bar{z} & a_2 z + b_2 \bar{z} & a_3 z + b_3 \bar{z} \\ b_1 & b_2 & b_3 \\ b_1 z & b_2 z & b_3 z \end{vmatrix} + \begin{vmatrix} a_1 z + b_1 \bar{z} & a_2 z + b_2 \bar{z} & a_3 z + b_3 \bar{z} \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix} \right\}$$

$= 0 + 0$ [Operating $R_1 \to R_1\ (z R_3 + \bar{z}\ R_2)$ in second determinant]

$= 0$

Hence (*d*) *is the correct answer.*

61. Let $z = x + iy$. Then

$$\frac{z-2}{z+2} = \frac{(x-2)+iy}{(x+2)+iy} = \frac{[(x-2)+iy]\,[(x+2)-iy]}{[(x+2)+iy][(x+2)-iy]}$$

$$= \frac{(x-2)(x+2)+y^2+i[(x+2)y-y(x-2)]}{(x+2)^2+y^2}$$

$$= \frac{x^2+y^2-4+4iy}{(x+2)^2+y^2}$$

$$\arg\left(\frac{z-2}{z+2}\right) = \tan^{-1}\left(\frac{4y/\{(x+2)^2+y^2\}}{(x^2+y^2-4)/\{(x+2)^2+y^2\}}\right)$$

$$= \tan^{-1}\frac{4y}{x^2+y^2-4} = \frac{\pi}{3} \text{ (given)}$$

$$\Rightarrow \quad \frac{4y}{x^2+y^2-4} = \tan\frac{\pi}{3} = \sqrt{3}$$

$$\Rightarrow \quad x^2 + y^2 - \frac{4}{\sqrt{3}}\, y - 4 = 0$$

This is the equation of a circle.

Hence (d) *is the correct answer.*

63. Putting $x = i$ in the given expression.

$$c_0 + c_1 i + c_2 i^2 + c_3 i^3 + ... = (1 + i)^n$$

$$\Rightarrow \quad (c_0 - c_2 + c_4 .....) + i\,(c_1 - c_3 + c_5 + ...) = (1 + i)^n$$

But $\quad 1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$$\therefore \quad (1+i)^n = 2^{n/2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^n$$

$$= 2^{n/2}\left(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}\right)$$

$$\therefore \quad c_0 - c_2 + c_4 + ... = 2^{n/2}\cos\frac{n\pi}{4} \qquad ...(1)$$

and $\quad c_1 - c_3 + c_5 + ... = 2^{n/2}\sin\frac{n\pi}{4} \qquad ...(2)$

which means (a) and (c) are correct Again, adding (1) to the identity

$$c_0 + c_2 + c_4 + ... = 2^{n-1}, \text{ we get}$$

$$2\,(c_0 + c_4 + c_8 + ...) = 2^{n-1} + 2^{n/2}\cos\frac{n\pi}{4}$$

$$c_0 + c_4 + c_8 + ... = 2^{n-2} + 2^{(n-2)/2}\cos\frac{n\pi}{4}$$

which is option (b)

Hence (a), (b), (c) *are correct answers.*

65. $\quad \frac{z-1}{i} = \text{real} \Rightarrow \frac{x+iy-1}{i} = \text{real},$

$$\Rightarrow \quad \frac{x-1}{i} + y = \text{real}$$

$$\therefore \quad x - 1 = 0 \quad \Rightarrow \quad x = 1$$

Then $\quad \sin^{-1}\left(\frac{z-1}{i}\right) = \sin^{-1}(y)$

$$\Rightarrow \quad -1 \le y \le 1$$

$\therefore \quad Re\,|z| = x = 1, \; -1 \le I_m(z) \le 1$

Hence (b) and (c) *are correct answers.*

PRACTICE TEST – I

1. Let $z_1 = 6 + i$ and $z_2 = 4 - 3i$. Let z be a complex number such that

$$\arg\left(\frac{z-z_1}{z_2-z}\right) = \frac{\pi}{2}; \text{ then } z \text{ satisfies}$$

(*a*) $|z-(5-i)| = \sqrt{5}$ (*b*) $|z-(5+i)| = \sqrt{5}$
(*c*) $|z - (5 - i)| = 5$ (*d*) $|z - (5 + i)| = 5$

2. If n_1, n_2 are positive integers, then $(1 + i)^{n_1} + (1 + i^3)^{n_1} + (1 + i^5)^{n_2} + (1 + i^7)^{n_2}$ is a real number if and only if
(*a*) $n_1 = n_2 + 1$ (*b*) $n_1 + 1 = n_2$
(*c*) $n_1 = n_2$ (*d*) n_1, n_2 are any two positive integers.

3. The number of solutions of the equation $z^2 + |z|^2 = 0$, where $z \in$ c is
(*a*) one (*b*) two (*c*) three (*d*) infinitely many

4. The smallest positive integer for which $\left(\frac{1+i}{1-i}\right)^n = -1$, is
(*a*) 4 (*b*) 3 (*c*) 2 (*d*) 1

5. If a complex number lies in the third quadrant than its conjugate lies in
(*a*) first quadrant (*b*) 2nd quadrant (*c*) 3rd quadrant (*d*) 4th quadrant

6. If $(a + ib)^5 = \alpha + i\beta$ then $(b + ia)^5$ is equal to
(*a*) $\alpha - i\beta$ (*b*) $\beta - i\alpha$ (*c*) $\beta + i\alpha$ (*d*) $-\alpha - i\beta$

7. If z is any non-zero complex number then arg (z) + arg $(\bar{z})$ is equal to
(*a*) 0 (*b*) $\pi/2$ (*c*) π (*d*) $3\pi/2$

8. If ω is a complex cube root of unity and $(1 + \omega)^7 = A + B\,\omega$ then A and B are respectively equal to
(*a*) ω (*b*) $-\omega$ (*c*) ω^{-1} (*d*) 1

9. $\sum_{m=1}^{4n+3} i^m$ equal to
(*a*) i (*b*) $-i$ (*c*) 1 (*d*) –1

10. If $(1 + i)\, z = (1 - i)\, \bar{z}$ then z is
(*a*) $p\,(1 + i), p \in R$ (*b*) $p\,(1 - i), p \in R$ (*c*) $\frac{p}{1+i}, p \in R^+$ (*d*) $\frac{p}{1-i}, p \in R^+$

11. The value of $\sqrt{i} + \sqrt{(-i)}$ is
(*a*) i (*b*) $-i$ (*c*) $\sqrt{2}$ (*d*) 0

12. If the vertices of a triangle are $-3 + i$, $8 + 5i$, $-2 - 3i$, the modulus of the complex number representing the centroid of this triangle is:
(*a*) 2 (*b*) $\sqrt{2}$ (*c*) $2\sqrt{2}$ (*d*) 4

13. If the points represented by complex numbers $z = \alpha + i\beta$, $z_2 = \gamma + i\delta$ and $z_1 - z_2$ are collinear then
(*a*) $\alpha\delta - \beta\gamma = 0$ (*b*) $\alpha\delta + \beta\gamma = 0$ (*c*) $\alpha\beta - \gamma\delta = 0$ (*d*) $\alpha\beta - \gamma\delta = 0$

14. If z_1, z_2, z_3, z_4 represent the vertices of a rhombus taken in the anticlockwise order then
(*a*) $z_1 - z_2 + z_3 - z_4 = 0$ (*b*) $z_1 + z_2 + z_3 + z_4 = 0$
(*c*) amp. $\frac{z_1-z_2}{z_3-z_4} = \frac{\pi}{2}$ (*d*) amp. $\frac{z_2-z_4}{z_1-z_3} = 0$

15. The value of the expression $2(1+\omega)(1+\omega^2) + 3(2\omega+1)(2\omega^2+1) + 4(3\omega+1)(3\omega^2+1) + \ldots + (n+1)(n\omega+1)(n\omega^2+1)$, where ω is the cube root of unity is:

(*a*) $\left[\frac{n(n+1)}{2}\right]^2 + n$ (*b*) $\left[\frac{n(n+1)}{2}\right]^2 - n$ (*c*) $\left[\frac{n(n+1)}{2}\right]^2$ (*d*) None of these

16. If $z = x + iy$ satisfies amp $(z-1)$ = amp $(z+3i)$ then the value of $(x-1):y$ is equal to

(*a*) 2 : 1 (*b*) 1 : 3 (*c*) – 1 : 3 (*d*) None of these

17. Let $A = \frac{2}{\sqrt{3}} e^{-i5\pi/6}$, $B = \frac{2}{\sqrt{3}} e^{-i\pi/6}$, $C = \frac{2}{\sqrt{3}} e^{-i5\pi/6}$ be three points forming a triangle *ABC* in the argand plane. Then ΔABC is:

(*a*) scalene (*b*) isosceles (*c*) equilateral (*d*) None of these

18. If z_1, z_2 be two complex numbers representing the points on the circles $|z| = 1$ and $|z| = 2$ respectively, then:

(*a*) max $|2z_1 + z_2| = 4$ (*b*) min $|z_1 - z_2| = 2$

(*c*) $\left|z_2 + \frac{1}{z_1}\right| \geq 3$ (*d*) None of these

19. The distances of the roots of the equation $|\sin\theta_1| z^3 + |\sin\theta_2| z^2 + |\sin\theta_3| z + |\sin\theta_4| = 3$, from $z = 0$, are

(*a*) greater than $|\sin\theta_1| + |\sin\theta_2| + |\sin\theta_3| + |\sin\theta_4|$

(*b*) less than $|\sin\theta_1| + |\sin\theta_2| + |\sin\theta_3| + |\sin\theta_4|$

(*c*) less than 2/3 (*d*) greater than 2/3

20. If $x^2 - x + 1 = 0$ then the value of $\sum_{n=1}^{5}\left(x^n + \frac{1}{x^n}\right)^2$ is

(*a*) 8 (*b*) 12 (*c*) 10 (*d*) 15

21. If $f_n(\alpha) = e^{i\alpha/n^2} \cdot e^{2i\alpha/n^2} \cdot e^{3i\alpha/n^2} \ldots\ldots e^{i\alpha/n}$ then value of $\lim_{n\to\infty} f_n(\pi)$ is

(*a*) – 1 (*b*) 1 (*c*) $-i$ (*d*) i

22. The equation $z^5 + z^4 + z^3 + z^2 + z + 1 = 0$ is satisfied by

(*a*) $z = \pm 1$ (*b*) $z = -1$ (*c*) $z = \pm\frac{1}{2} + i\frac{\sqrt{3}}{2}$ (*d*) $z = \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

23. If z is a non real root of $(-1)^{1/7}$ then $z^{86} + z^{175} + z^{289}$ is equal to

(*a*) 1 (*b*) –1 (*c*) 3 (*d*) 0

24. $z\bar{z} + a\bar{z} + \bar{a}z + b = 0$ where $b \in R$ represents a real circle of non zero radius if

(*a*) $|\bar{a}|^2 > b$ (*b*) $|a|^2 < b$ (*c*) $|a|^2 \geq b$ (*d*) $|a|^2 \leq b$

25. If z is a complex number, then

(*a*) $z\bar{z}$ is purely real but $z + \bar{z}$ is not (*b*) $z + \bar{z}$ is purely real but $z\bar{z}$ is not

(*c*) $z + \bar{z}$ and $z\bar{z}$ are both purely real (*d*) neither $z + \bar{z}$ nor $z\bar{z}$ need be purely real

ANSWERS

Type – I

1. (*a*)	**2.** (*d*)	**3.** (*d*)	**4.** (*c*)	**5.** (*b*)
6. (*c*)	**7.** (*a*)	**8.** (*a*)	**9.** (*d*)	**10.** (*b*)
11. (*c*)	**12.** (*b*)	**13.** (*a*)	**14.** (*a*)	**15.** (*a*)

16. (*b*)	**17.** (*c*)	**18.** (*a*)	**19.** (*d*)	**20.** (*a*)
21. (*d*)	**22.** (*b*), (*c*), (*d*),	**23.** (*b*)	**24.** (*a*)	**25.** (*c*)

SOME HINTS

1. $(1+i)^{n_1} + (1-i)^{n_1} + (1+i)^{n_2} + (1-i)^{n_2}$

$$= 2^{n_1/2}\left[\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{n_1} + \left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^{n_1}\right]$$

$$+ 2^{n_2/2}\left[\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{n_2} + \left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^{n_2}\right]$$

$$= 2^{n_1/2}\,.\,2\cos\frac{n_1\pi}{4} + 2^{n_2/2}\,.\,2\cos\frac{n_2\pi}{4} = \text{real}$$

6. $(b + ia)^5 = i^5\,(a - ib)^5 = i\,(\alpha - i\beta)$

15. $S = 2(1 + \omega)(1 + \omega^2) + 3(2\omega + 1)(2\omega^2 + 1) + 4(3\omega + 1)(3\omega^2 + 1) + \ldots + (n + 1)(n\omega + 1)(n\omega^2 + 1)$

$T_n = (n + 1)(n\omega + 1)(n\omega^2 + 1)$

$= (n + 1)(n^2 - n + 1) = n^3 + 1$

$$\therefore \quad S = \sum n^3 + \sum 1 = \left(\frac{n(n+1)}{2}\right)^2 + n.$$

17. $A = \dfrac{2}{\sqrt{3}}\, e^{i\pi/2} = \dfrac{2}{\sqrt{3}}\, i,\ B = \dfrac{2}{\sqrt{3}}\, e^{-i\pi/6}$

$$= \frac{2}{\sqrt{3}}\left(\frac{\sqrt{3} - i}{2}\right),\quad c = \frac{2}{\sqrt{3}}\, e^{-5\pi/6} = \frac{2}{\sqrt{3}}\left(\frac{-\sqrt{3} - i}{2}\right)$$

$\therefore \quad |A - B| = 2, \quad |B - C| = 2, \quad |C - A| = 2.$

19. We know that

$|\sin\theta_1| \le 1, \quad |\sin\theta_2| \le 1$ etc., we have

$|\sin\theta_1|\, z^3 + |\sin\theta_2|\, z^2 + |\sin\theta_3|\, z^3 + |\sin\theta_4| = 3$

$\therefore \quad |z| = \big|\, |\sin\theta_1|\, z^3 + |\sin\theta_2|\, z^2 + |\sin\theta_3|\, z + |\sin\theta_4|$

$\Rightarrow \quad z \le ||\sin\theta_1||\,|z|^3 + ||\sin\theta_2||\,|z|^2 + ||\sin\theta_3||\,|z| + ||\sin\theta_4||$

$\Rightarrow \quad 3 \le |z|^3 + |z|^2 + |z| + 1$

$\le 1 + |z| + |z|^2 + |z|^3 + \ldots\infty$

$$\Rightarrow \quad 3 < \frac{1}{1-|z|} \quad \Rightarrow \quad 1 - |z| < \frac{1}{3}$$

$$\Rightarrow \quad |z| > 2/3. \quad \Rightarrow \quad |z - 0| > \frac{2}{3}.$$

23. Given expression $= (z^7)^{12}\,.\,z^2 + (z^7)^{25} + (z^7)^{41}.\,z^2$

$= (-1)^{12}\,.\,z^2 + (-1)^{25} + (-1)^{41}.\,z^2$

$= z^2 - 1 - z^2 = -1.$

PRACTICE TEST – II

1. If $\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)^{50} = 3^{25}(x+iy)$ where $x, y \in R$, then the values of x and y are:

(*a*) 0, 3 (*b*) 0, – 3 (*c*) – 3, 0 (*d*) 1/2, $\sqrt{3}/2$

2. If $z(1+a) = b+ic$ and $a^2+b^2+c^2 = 1$, then value of $\frac{1+iz}{1-iz}$ is

(*a*) $\frac{a+ib}{1+c}$ (*b*) $\frac{a-ib}{1+c}$ (*c*) $\frac{a+ib}{1-c}$ (*d*) $\frac{a-ib}{1-c}$

3. If ω is a non real cube root of unity then the value of
$1.\ (2-\omega)(2-\omega^2) + 2.\ (3-\omega)(3-\omega^2) + ... + (n-1)(n-\omega)(n-\omega^2)$ is

(*a*) $\frac{n^2(n-1)^2}{4}-n+1$ (*b*) $\left(\frac{n(n+1)}{2}\right)^2-n$ (*c*) real (*d*) non real

4. The continued product of the four values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{3/4}$ is

(*a*) 1 (*b*) $1+i\sqrt{3}$ (*c*) $1-i\sqrt{3}$ (*d*) None of these

5. If α, β be two complex numbers then $|\alpha|^2 + |\beta|^2$ is equal to

(*a*) $|\alpha+\beta|^2 + |\alpha-\beta|^2$ (*b*) $\frac{1}{2}(|\alpha+\beta|^2 + |\alpha-\beta|^2)$

(*c*) $\frac{1}{2}(|\alpha+\beta|^2 - |\alpha-\beta|^2|)$ (*d*) None of these

6. Two of the three values of $(-1)^{1/3}$ are $\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$ and $\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}$ · The third value is:

(*a*) $\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}$ (*b*) $\cos\frac{5\pi}{3}-i\sin\frac{5\pi}{3}$

(*c*) –1 (*d*) 1

7. Value of $\left(\frac{1+\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}}{1+\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}}\right)^8$ is

(*a*) $1+i$ (*b*) $1-i$ (*c*) 1 (*d*) – 1

8. If z be a complex number satisfying $z^4+z^3+2z^2+z+1=0$ then $|z|$ is equal to

(*a*) 1 (*b*) $\frac{1}{2}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{3}{4}$

9. If $x_r = \cos\frac{\pi}{2^r}+i\sin\frac{\pi}{2^r}$, then value of $x_1.\ x_2.\ x_3\ .....\ \infty$ is

(*a*) 0 (*b*) 1 (*c*) π (*d*) – 1

10. If $i = \sqrt{-1}$ and n is a positive integer, value of
$i^n + i^{n+4} + i^{n+2} + i^{n+3}$ is

(*a*) 1 (*b*) i (*c*) i^n (*d*) 0

11. If $z^4 = i$, then value of z will be

(*a*) 1 (*b*) i (*c*) $\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}$ (*d*) $\cos\frac{\pi}{8} - i\sin\frac{\pi}{8}$

12. If α is a non real fourth root of unity and $n \in N$, then the value of $\alpha^{4n-1} + \alpha^{4n-2} + \alpha^{4n-3}$ is:

(*a*) 0 (*b*) –1 (*c*) 1 (*d*) 3

13. *ABCD* is a square, vertices being taken in the anticlockwise sense. If *A* represents the complex number z and the intersection of the diagonals is the origin then

(*a*) *B* represents the complex number iz (*b*) *B* represents the complex number $i\bar{z}$

(*c*) *D* represents the complex number $i\bar{z}$ (*d*) *D* represents the complex number iz

14. The locus of z which satisfies the inequality

$\log_{0.3}|z-1| > \log_{0.3}|z-i|$ is given by

(*a*) $x + y < 0$ (*b*) $x + y > 0$ (*c*) $x - y > 0$ (*d*) $x - y < 0$

15. If the roots of $z^3 + iz^2 + 2i = 0$ represent the vertices of a ΔABC in the Argand plane then the area of the triangle is

(*a*) $\frac{3\sqrt{7}}{4}$ (*b*) $\frac{3\sqrt{7}}{2}$ (*c*) 2 (*d*) 1

16. If z_1 and z_2 are any two complex numbers, then

$$\left|z_1 + \sqrt{z_1^2 - z_2^2}\right| + \left|z_1 - \sqrt{z_1^2 - z_2^2}\right| \text{ is equal to}$$

(*a*) $|z_1 + z_2|$ (*b*) $|z_1 - z_2|$ (*c*) $|z_1 + z_2| - |z_1 - z_2|$ (*d*) $|z_1 + z_2| + |z_1 - z_2|$

17. If $f(n) = i^n + i^{-n}$, where $i = \sqrt{-1}$ and n is an integer, then the total number of distinct values of $f(n)$ is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

18. The number whose multiplicative inverse is $(\sqrt{3} + 4i)/19$, will be

(*a*) $4 - i\sqrt{3}$ (*b*) $4 + i\sqrt{3}$ (*c*) $\sqrt{3} - 4i$ (*d*) $\sqrt{3} + 4i$

19. The value of $\sum_{k=1}^{10}\left(\sin\frac{2\pi k}{11} - i\cos\frac{2\pi k}{11}\right)$ is

(*a*) 0 (*b*) i (*c*) $-i$ (*d*) -1

20. If α, β, γ are the roots of $x^3 - 3x^2 + 3x + 7 = 0$ and ω is cube roots of unity, then value of

$\frac{\alpha-1}{\beta-1} + \frac{\beta-1}{\gamma-1} + \frac{\gamma-1}{\alpha-1}$ is

(*a*) ω^2 (*b*) $2\omega^2$ (*c*) $3\omega^2$ (*d*) $\frac{3}{\omega}$

21. The equation $z\bar{z} + (4-3i)z + (4+3i)\bar{z} + 5 = 0$ represents a circle whose radius is

(*a*) $\sqrt{5}$ (*b*) $2\sqrt{5}$ (*c*) 5 (*d*) $\frac{5}{2}$

22. Let α and β be two distinct complex numbers such that $|\alpha| = |\beta|$. If real part of α is positive and imaginary part of β is negative, then the complex number $(\alpha + \beta)/(\alpha - \beta)$ may be

(*a*) real and positive (*b*) real and negative

(*c*) purely imaginary (*d*) zero

23. If $\alpha = \cos\left(\frac{8\pi}{11}\right) + i\sin\left(\frac{8\pi}{11}\right)$ then $Re\,(\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5)$ is

(*a*) 0 (*b*) 1/2 (*c*) –1/2 (*d*) 1

24. If $|z_1 - 1| < 1$, $|z_2 - 2| < 2$, $|z_3 - 3| < 3$ then $|z_1 + z_2 + z_3|$

(*a*) < 6 (*b*) < 12 (*c*) > 3 (*d*) lies between 6 and 12

25. Let S be the set of all complex numbers z such that $|z| = 1$ and define relation R on S by $z_1 \, R \, z_2$ is $|\arg z_1 - \arg z_2| = \frac{2\pi}{3}$ then R is

(*a*) Symmetric (*b*) Antisymmetric (*c*) Reflexive (*d*) Transitive

ANSWERS

1. (*d*)	**2.** (*a*)	**3.** (*a*), (*c*)	**4.** (*a*)	**5.** (*b*)
6. (*c*)	**7.** (*d*)	**8.** (*a*)	**9.** (*d*)	**10.** (*d*)
11. (*c*)	**12.** (*b*)	**13.** (*a*)	**14.** (*c*)	**15.** (*c*)
16. (*d*)	**17.** (*c*)	**18.** (*c*)	**19.** (*b*)	**20.** (*c*)
21. (*b*)	**22.** (*c*)	**23.** (*c*)	**24.** (*b*)	**25.** (*a*)

SOME HINTS

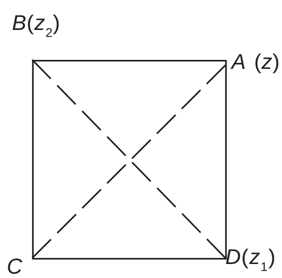
3.
$$T_n = (n-1)(n-\omega)(n-\omega^2)$$
$$= (n-1)\{n^2 - (\omega + \omega^2)\, n + \omega^3\}$$
$$= (n-1)(n^2 + n + 1) = n^3 - 1$$
$$\therefore \quad S = \sum n^3 - \sum 1 \text{ etc.}$$

8. $z^4 + z^3 + z^2 + z^2 + z + 1 = 0$

$\Rightarrow$ $(z^2 + z + 1)(z^2 + 1) = 0$

$\therefore$ $z = i, -i, \omega, \omega^2$ and for each $|z| = 1$.

13. $OB = OD = OA = |z|$



Let amp. $z = \theta$ then amp. $z_1 = \theta - \frac{\pi}{2}$,

amp. $z_2 = \theta + \frac{\pi}{2}$ and $z = |z|(\cos\theta + i\sin\theta)$

$$\therefore \quad z_1 = |z| \left\{\cos\left(\theta - \frac{\pi}{2}\right) + i \sin\left(\theta - \frac{\pi}{2}\right)\right\}$$
$$= |z| (\sin\theta - i\cos\theta)$$
$$= |z| (-i)(\cos\theta + i\sin\theta) = -iz$$
$$z_2 = |z| \left\{\cos\left(\theta + \frac{\pi}{2}\right) + i \sin\left(\theta + \frac{\pi}{2}\right)\right\} = iz.$$

14. $\log_{0.3} |z - 1| > \log_{0.3} |z - i|$

$\Rightarrow$ $|z - 1| < |z - i|$

$\Rightarrow$ $(z - 1)(\bar{z} - 1) < (z - i)(\bar{z} + i)$

$\Rightarrow$ $z\bar{z} - z - \bar{z} + 1 < z\bar{z} + iz - i\bar{z} + 1$

$\Rightarrow$ $(1 + i)\, z + (1 - i)\, \bar{z} > 0$

$\Rightarrow$ $(z + \bar{z}) + i\,(z - \bar{z}) > 0$

$\Rightarrow$ $\left(\frac{z+\bar{z}}{2}\right) - \left(\frac{z-\bar{z}}{2i}\right) > 0$

$\Rightarrow$ $x - y > 0.$

22. Let $\alpha = x + iy$ and $\beta = x - iy$ and $a, b > 0$.

Since $|\alpha| = |\beta|$

$$\therefore \quad \frac{\alpha+\beta}{\alpha-\beta} = \frac{2x}{2iy} = -\frac{ix}{y}$$

Again, if $\alpha = x - iy$ and $\beta = -x - iy$

then $$\frac{\alpha+\beta}{\alpha-\beta} = \frac{-2iy}{2x} = -\frac{iy}{x}.$$

25. $|z| = 1 \Rightarrow z = \cos\theta + i\sin\theta$

Let $z_1 = \cos\theta_1 + i\sin\theta_1$ and $z_2 = \cos\theta_2 + i\sin\theta_2$

$$\therefore \quad z_1 R z_2 \Leftrightarrow |\arg z_1 - \arg z_2| = \frac{2\pi}{3}$$

$$\Leftrightarrow |\theta_1 - \theta_2| = 2\pi/3 \Leftrightarrow |\theta_2 - \theta_1| = 2\pi/3$$

Hence R is symmetric.

SOME INTELLIGENT PROBLEMS

1. Complex numbers z_1, z_2, z_3 are the vertices A, B, C respectively of an isosceles right angled triangle with right angle at C. The value of $(z_1 - z_2)^2$ is

(*a*) $(z_1 - z_3)(z_3 - z_2)$ (*b*) $2(z_1 - z_3)(z_3 - z_2)$

(*c*) $(z_1 + z_3)(z_3 + z_2)$ (*d*) $(z_1 - z_2)(z_3 - z_2)$

2. The triangle whose vertices are the points represented by the complex numbers z_1, z_2, z_3 on the Argand diagram is equilateral if and only if the value of $\frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} + \frac{1}{z_1 - z_2}$ is

(*a*) 0 (*b*) i (*c*) 1 (*d*) indeterminate

3. If z_1 and z_2 both satisfy the relation $z + \bar{z} = 2|z - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{4}$, then the imaginary part of $(z_1 + z_2)$ is

(*a*) 1 (*b*) –1 (*c*) 2 (*d*) –2

4. If $|z - 1| = 1$, where z is a point on the Argand plane, then the value of $\frac{z-2}{z}$ is

(*a*) arg z (*b*) tan (arg z) (*c*) i tan (arg z) (*d*) None of these

5. The roots z_1, z_2, z_3 of the equation

$$x^3 + 3ax^2 + 3bx + c = 0$$

in which a, b, c are complex numbers, correspond to the points A, B, C on the Gaussian plane. Then centroid of the triangle ABC will be

(*a*) $-a$ (*b*) a (*c*) $-b$ (*d*) $-c$

6. Complex numbers z_1, z_2 and the origin form an isosceles triangle with vertical angle $\frac{2\pi}{3}$ if

(*a*) $z_1^2 + z_2^2 - z_1 z_2 = 0$ (*b*) $z_1^2 + z_2^2 + z_1 z_2 = 0$

(*c*) $z_1^2 + z_2^2 = 1$ (*d*) $z_1^2 - z_2^2 + z_1 z_2 = 1$

7. a is a complex number such that $|a| = 1$. Value of a, so that equation $az^2 + z + 1 = 0$ has one purely imaginary root, will be $\cos\alpha + i\sin\alpha$ where α is

(*a*) $\cos^{-1}\frac{\sqrt{5}+1}{2}$ (*b*) $\cos^{-1}\frac{\sqrt{5}-1}{2}$ (*c*) $\sin^{-1}\frac{\sqrt{5}-1}{2}$ (*d*) can not be determined

8. If ω is the nth root of unity and z_1 and z_2 any two complex numbers, then value of

$$\sum_{p=0}^{n-1} |z_1 + \omega^p z_2|^2 \text{ is } (n \in N)$$

(a) $n(|z_1| + |z_2|)^2$ (b) $n(|z_1| - |z_2|)^2$ (c) $n(|z_1|^2 + |z_2|^2)$ (d) $n(|z_1|^2 - |z_2|^2)$

9. Let z_1, z_2, z_3 be three non-zero complex numbers and $z_1 \neq z_2$. If

$$\begin{vmatrix} |z_1| & |z_2| & |z_3| \\ |z_2| & |z_3| & |z_1| \\ |z_3| & |z_1| & |z_2| \end{vmatrix} = 0$$

then z_1, z_2, z_3

(a) are coplanar (b) are collinear

(c) lie on a cirle with centre at origin (d) lie on a parabola with focus at origin

10. Let $\Delta = \begin{vmatrix} 1 & \omega & 2\omega^2 \\ 2 & 2\omega^2 & 4\omega^3 \\ 3 & 3\omega^3 & 6\omega^4 \end{vmatrix}$, where ω is the cube root of unity, then

(a) $\Delta = 1$ (b) $\Delta = 2$ (c) $\Delta = 3$ (d) $\Delta = 0$

11. If z is a complex number such that $\left|\dfrac{z-5i}{z+5i}\right| = 1$, then the locus of z is

(a) x – axis (b) y – axis

(c) straight line $y = 5$ (d) a circle passing through origin

12. If z is a complex number such that $z \neq 0$ and $Re(z) = 0$, then

(a) $Re(z^2) = Im(z^2)$ (b) $Re(z^2) = 0$ (c) $Im(z^2) = 0$ (d) None of these

13. The locus of z satisfying the inequality

$\log_{(1/3)} |z + 1| > \log_{(1/3)} |z - 1|$ is

(a) $R(z) > 0$ (b) $R_z < 0$ (c) $Im(z) > 0$ (d) $Im(z) < 0$

14. If $z = (\alpha + 3) + i\sqrt{(5-\alpha^2)}$, then the locus of z is

(a) real axis (b) imaginary axis (c) a circle (d) a hyperbola

15. The equation $|z - i| + |z + i| = k$, $a > 0$, can represent an ellipse if k is

(a) 1 (b) 2 (c) 3 (d) 4

16. If $1 + x^2 = \sqrt{3}\, x$, then value of $\sum_{n=1}^{24} (x^n - x^{-n})^2$ is equal to

(a) 48 (b) – 48 (c) $\pm 48(\omega - \omega^2)$ (d) $\pm 48(\omega + \omega^2)$

17. If $e^{i\theta} = \cos\theta + i\sin\theta$ then for the ΔABC, value of $e^{i(A+B+C)}$ is

(a) 1 (b) –1 (c) $-i$ (d) i

18. If α is non real and $\alpha = (1)^{1/5}$ then the value of $2^{|1+\alpha+\alpha^2+\alpha^{-2}-\alpha^{-1}|}$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

19. Let $z_1 = a + ib$, $z_2 = c + id$ be two unimodular complex numbers such that $Im(z_1 \bar{z}_2) = 1$. If $\omega_1 = a + ic$, $\omega_2 = b + id$ then

(a) $Re(\omega_1 \omega_2) = 0$ (b) $Re(\omega_1 \omega_2) = 1$ (c) $Im(\omega_1 \omega_2) = 1$ (d) $Im(\omega_1 \bar{\omega}_2) = 1$

20. If α, β and γ are the cube roots of p ($p < 0$), then for any a, b and c the value of

$\dfrac{a\alpha + b\beta + c\gamma}{a\beta + b\gamma + c\alpha}$ is (*a*) p (*b*) $-p$ (*c*) ω^2 (*d*) 0

21. If $z = \cos\theta + i\sin\theta$, the value of $\dfrac{z^{2n}-1}{z^{2n}+1}$ is

(*a*) $i\tan n\theta$ (*b*) $2\cos n\theta$ (*c*) $2i\sin n\theta$ (*d*) None of these

22. If $|z| < 1$ and $|(z_1 - z_2)/(1 - \bar{z}z_2)| < 1$ then

(*a*) $|z_2| = 1$ (*b*) $|z_2| < 1$ (*c*) $|z_2| > 1$ (*d*) $|z_2| \geq 1$

23. Let $z = 1 - p + i\sqrt{p^2 + p + 2}$, where p is a real parameter. The locus of z in the Argand plane is

(*a*) a straight line (*b*) a parabola (*c*) an ellipse (*d*) a hyperbola

24. The centre of square $ABCD$ is at $z = 0$. A is z_1. Then the centroid of ΔABC is

(*a*) $\dfrac{z_1}{3}(\cos\pi \pm i\sin\pi)$ (*b*) $\dfrac{z_1}{3}(\cos\pi/2 \pm i\sin\pi/2)$

(*c*) $z_1(\cos\pi \pm i\sin\pi)$ (*d*) $z_1(\cos\pi/2 \pm i\sin\pi/2)$

25. The system of equations $|z + 1 + i| = \sqrt{2}$ and $|z| = 3$ has

(*a*) one solution (*b*) two solutions

(*c*) Infinitely many solutions (*d*) no solution

26. Area of the triangle on the Argand diagram formed by the complex numbers z, iz and $z + iz$ is

(*a*) $|z|^2$ (*b*) $\dfrac{1}{2}|z|^2$ (*c*) $1/2\,|z|^2$ (*d*) None of these

27. Value of $\left(\dfrac{1+\sqrt{3}\,i}{2}\right)^n + \left(\dfrac{-1-\sqrt{3}\,i}{2}\right)^n$, when n is a multiple of 3 is

(*a*) 2 (*b*) -1 (*c*) 1 (*d*) -2

28. Let z_1 and z_2 be two non real complex cube roots of unity and $|z - z_1|^2 + |z - z_2|^2 = \lambda$ be the equation of a circle with z_1, z_2 as ends of a diameter then the value of λ is

(*a*) 2 (*b*) $\sqrt{2}$ (*c*) 3 (*d*) 4

29. The number of points in the complex plane that satisfying the conditions $|z - 2| = 2$, $z(1 - i) + \bar{z}(1 + i) = 4$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) more than 2

30. Let S be the set of complex number z which satisfy

$\log_{1/3}\{\log_{1/2}(|z|^2 + 4|z| + 3)\} < 0$. Then value of S is

(*a*) Singleton (*b*) Empty (*c*) Finite (*d*) Infinite

ANSWERS

1. (*b*)	**2.** (*a*)	**3.** (*c*)	**4.** (*c*)	**5.** (*a*)
6. (*b*)	**7.** (*b*)	**8.** (*c*)	**9.** (*c*)	**10.** (*d*)
11. (*a*)	**12.** (*c*)	**13.** (*a*)	**14.** (*c*)	**15.** (*d*)
16. (*b*)	**17.** (*b*)	**18.** (*d*)	**19.** (*d*)	**20.** (*c*)
21. (*a*)	**22.** (*b*)	**23.** (*d*)	**24.** (*b*)	**25.** (*d*)
26. (*b*)	**27.** (*a*)	**28.** (*c*)	**29.** (*c*)	**30.** (*b*)