Kartik Academy
Destination of Success

ZONE

Inverse Trigonometric Functions

Trigonometry Phase – IV

Contents:

KEY CONCEPTS

General Definition(s) :

1. $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ etc. denote angles or real numbers whose sine is x, whose cosine is x and whose tangent is x, provided that the answers given are numerically smallest available. These are also written as arc sinx, arc cosx etc.

   If there are two angles one positive & the other negative having same numerical value, then positive angle should be taken.

2. **Principal Values And Domains Of Inverse Circular Functions :**

   (i) $y = \sin^{-1}x$ where $-1 \le x \le 1$; $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ and $\sin y = x$.

   (ii) $y = \cos^{-1}x$ where $-1 \le x \le 1$; $0 \le y \le \pi$ and $\cos y = x$.

   (iii) $y = \tan^{-1}x$ where $x \in R$; $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and $\tan y = x$.

   (iv) $y = \text{cosec}^{-1}x$ where $x \le -1$ or $x \ge 1$; $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, $y \ne 0$ and $\text{cosec}\, y = x$.

   (v) $y = \sec^{-1}x$ where $x \le -1$ or $x \ge 1$; $0 \le y \le \pi$; $y \ne \frac{\pi}{2}$ and $\sec y = x$.

   (vi) $y = \cot^{-1}x$ where $x \in R$, $0 < y < \pi$ and $\cot y = x$.

Note That : (a) 1st quadrant is common to all the inverse functions.
(b) 3rd quadrant is **not used** in inverse functions.
(c) 4th quadrant is used in the **clockwise direction** i.e. $-\frac{\pi}{2} \le y \le 0$.

3. **Properties Of Inverse Circular Functions :**

P–1 (i) $\sin(\sin^{-1}x) = x$, $-1 \le x \le 1$ (ii) $\cos(\cos^{-1}x) = x$, $-1 \le x \le 1$

(iii) $\tan(\tan^{-1}x) = x$, $x \in R$ (iv) $\sin^{-1}(\sin x) = x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

(v) $\cos^{-1}(\cos x) = x$; $0 \le x \le \pi$ (vi) $\tan^{-1}(\tan x) = x$; $-\frac{\pi}{2} < x < \frac{\pi}{2}$

P–2 (i) $\text{cosec}^{-1}x = \sin^{-1}\frac{1}{x}$; $x \le -1$, $x \ge 1$

(ii) $\sec^{-1}x = \cos^{-1}\frac{1}{x}$; $x \le -1$, $x \ge 1$

(iii) $\cot^{-1}x = \tan^{-1}\frac{1}{x}$; $x > 0$

$= \pi + \tan^{-1}\frac{1}{x}$; $x < 0$

P–3 (i) $\sin^{-1}(-x) = -\sin^{-1}x$, $-1 \le x \le 1$
(ii) $\tan^{-1}(-x) = -\tan^{-1}x$, $x \in R$
(iii) $\cos^{-1}(-x) = \pi - \cos^{-1}x$, $-1 \le x \le 1$
(iv) $\cot^{-1}(-x) = \pi - \cot^{-1}x$, $x \in R$

P–4 (i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ $-1 \le x \le 1$ (ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ $x \in R$

(iii) $\text{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$ $|x| \ge 1$

P–5 $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\dfrac{x+y}{1-xy}$ where $x > 0$, $y > 0$ & $xy < 1$

$$= \pi + \tan^{-1}\frac{x+y}{1-xy} \quad \text{where } x > 0\ ,\ y > 0\ \&\ xy > 1$$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\dfrac{x-y}{1+xy}$ where $x > 0$, $y > 0$

P–6 **(i)** $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$ where $x \ge 0$, $y \ge 0$ & $(x^2 + y^2) \le 1$

Note that : $x^2 + y^2 \le 1 \Rightarrow 0 \le \sin^{-1}x + \sin^{-1}y \le \dfrac{\pi}{2}$

(ii) $\sin^{-1}x + \sin^{-1}y = \pi - \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$ where $x \ge 0$, $y \ge 0$ & $x^2 + y^2 > 1$

Note that : $x^2 + y^2 > 1 \Rightarrow \dfrac{\pi}{2} < \sin^{-1}x + \sin^{-1}y < \pi$

(iii) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$ where $x \ge 0$, $y \ge 0$

(iv) $\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\left[xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right]$ where $x \ge 0$, $y \ge 0$

P–7 If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\dfrac{x+y+z-xyz}{1-xy-yz-zx}\right]$ if, $x > 0, y > 0, z > 0$ & $xy + yz + zx < 1$

Note : (i) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ then $x + y + z = xyz$

(ii) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \dfrac{\pi}{2}$ then $xy + yz + zx = 1$

P–8 $2\tan^{-1}x = \sin^{-1}\dfrac{2x}{1+x^2} = \cos^{-1}\dfrac{1-x^2}{1+x^2} = \tan^{-1}\dfrac{2x}{1-x^2}$

Note very carefully that :

$$\sin^{-1}\frac{2x}{1+x^2} = \begin{cases} 2\tan^{-1}x & \text{if } |x| \le 1 \\ \pi - 2\tan^{-1}x & \text{if } x > 1 \\ -(\pi + 2\tan^{-1}x) & \text{if } x < -1 \end{cases} \qquad \cos^{-1}\frac{1-x^2}{1+x^2} = \begin{cases} 2\tan^{-1}x & \text{if } x \ge 0 \\ -2\tan^{-1}x & \text{if } x < 0 \end{cases}$$

$$\tan^{-1}\frac{2x}{1-x^2} = \begin{cases} 2\tan^{-1}x & \text{if } |x| < 1 \\ \pi + 2\tan^{-1}x & \text{if } x < -1 \\ -(\pi - 2\tan^{-1}x) & \text{if } x > 1 \end{cases}$$

REMEMBER THAT :

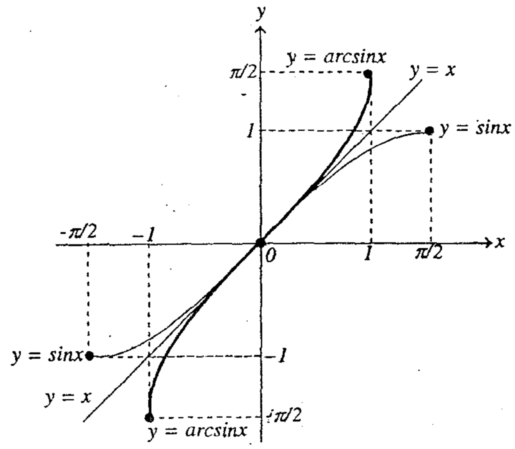
(i) $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \dfrac{3\pi}{2} \Rightarrow x = y = z = 1$

(ii) $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi \Rightarrow x = y = z = -1$

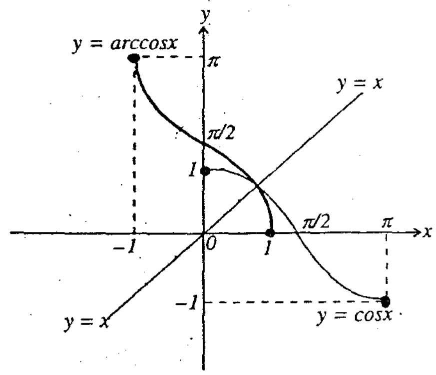
(iii) $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$ and $\tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{2}$

Inverse Trigonometric Functions
Some Useful Graphs

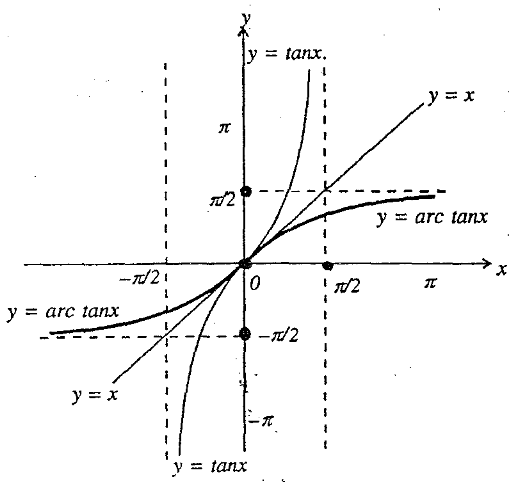
1. $y = \sin^{-1}x,\ |x| \le 1,\ y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



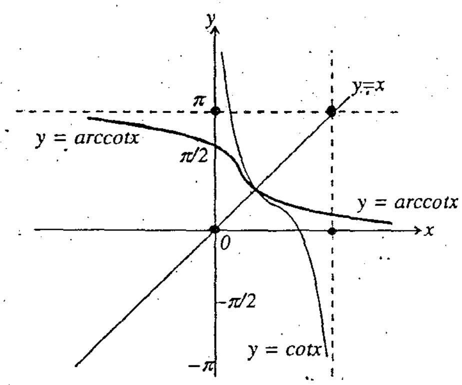
2. $y = \cos^{-1}x,\ |x| \le 1,\ y \in [0, \pi]$



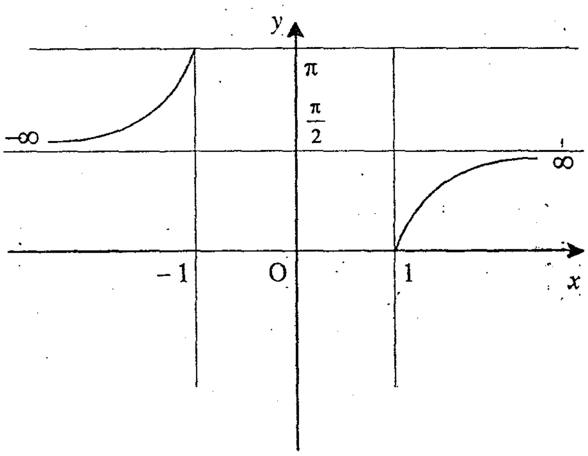
3. $y = \tan^{-1}x,\ x \in R,\ y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



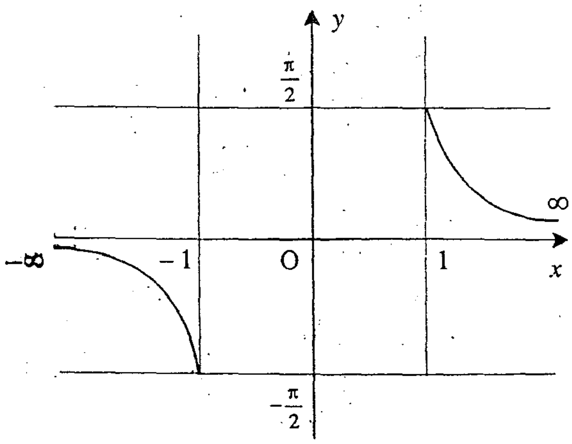
4. $y = \cot^{-1}x,\ x \in R,\ y \in (0, \pi)$



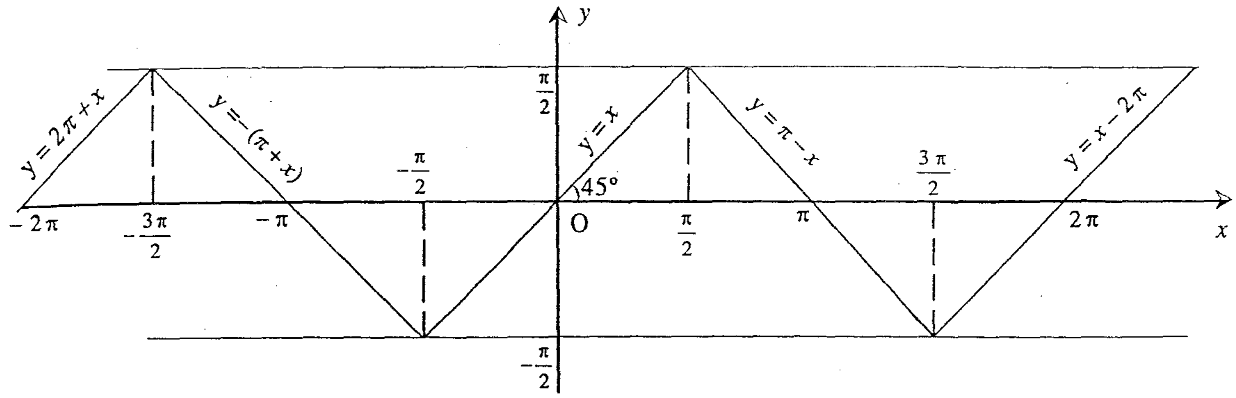
5. $y = \sec^{-1}x,\ |x| \ge 1,\ y \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



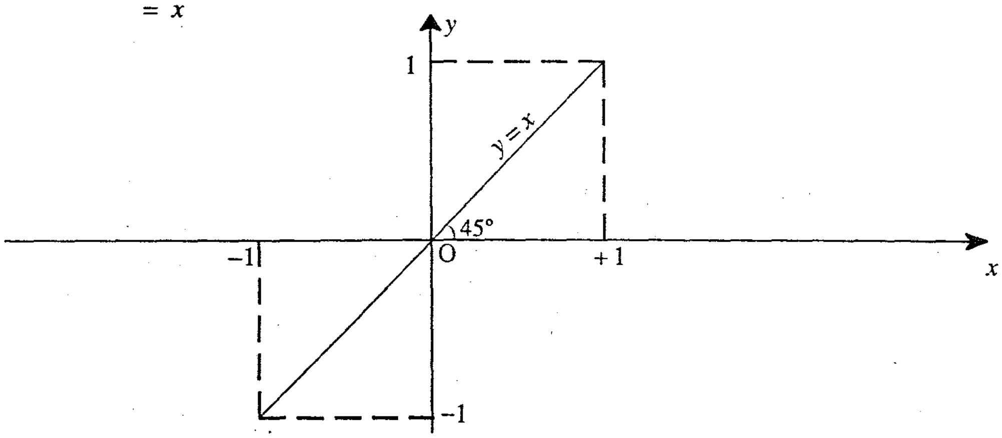
6. $y = \text{cosec}^{-1}x,\ |x| \ge 1,\ y \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$



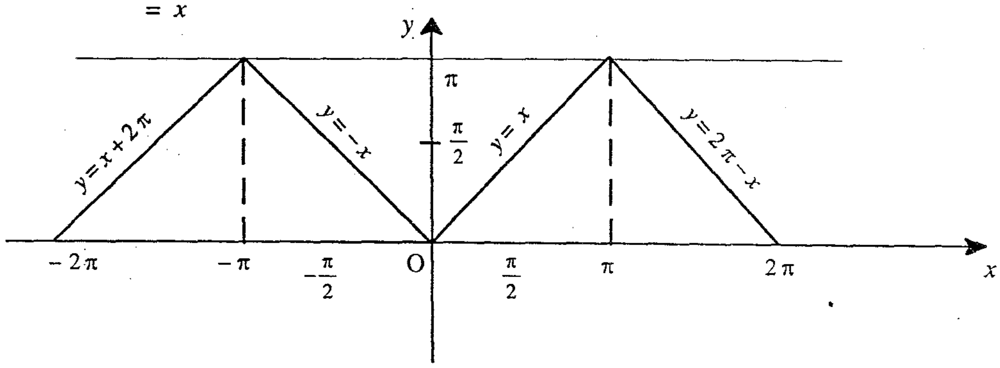
7. (a) $y = \sin^{-1}(\sin x)$, $x \in R$, $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, Periodic with period 2π



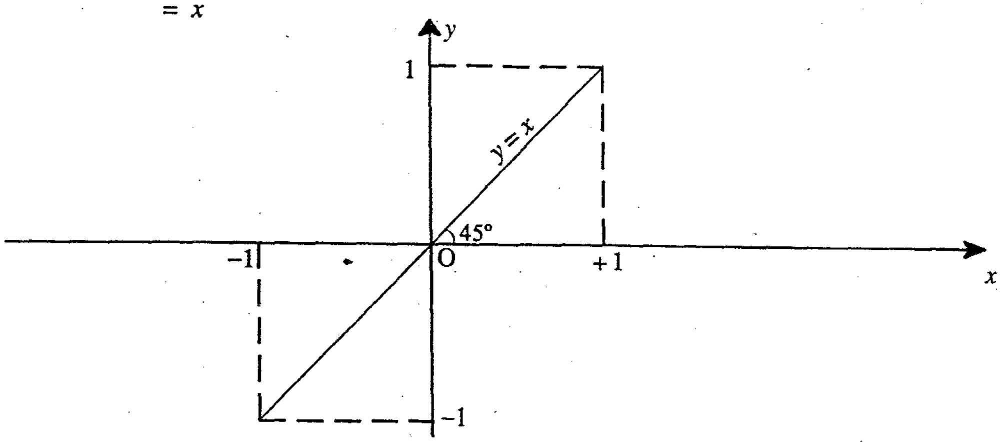
7.(b) $y = \sin(\sin^{-1}x)$, $x \in [-1, 1]$, $y \in [-1, 1]$, y is aperiodic
$= x$



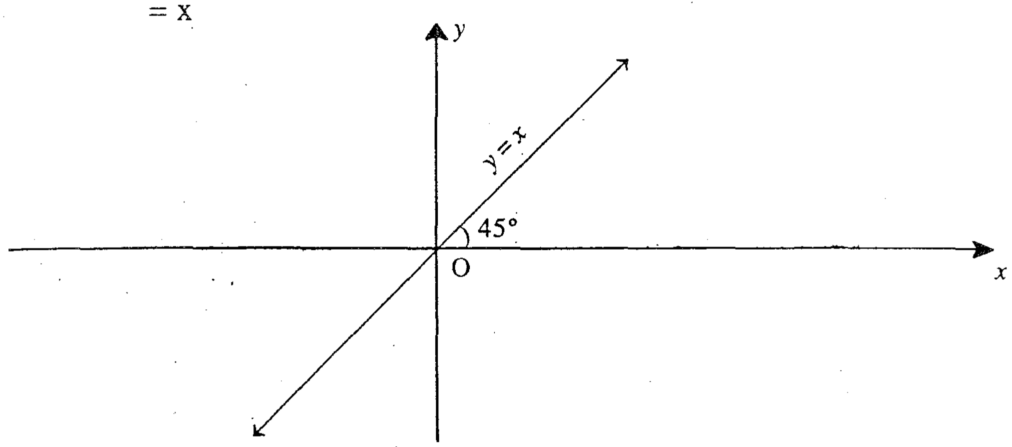
8. (a) $y = \cos^{-1}(\cos x)$, $x \in R$, $y \in [0, \pi]$, periodic with period 2π
$= x$



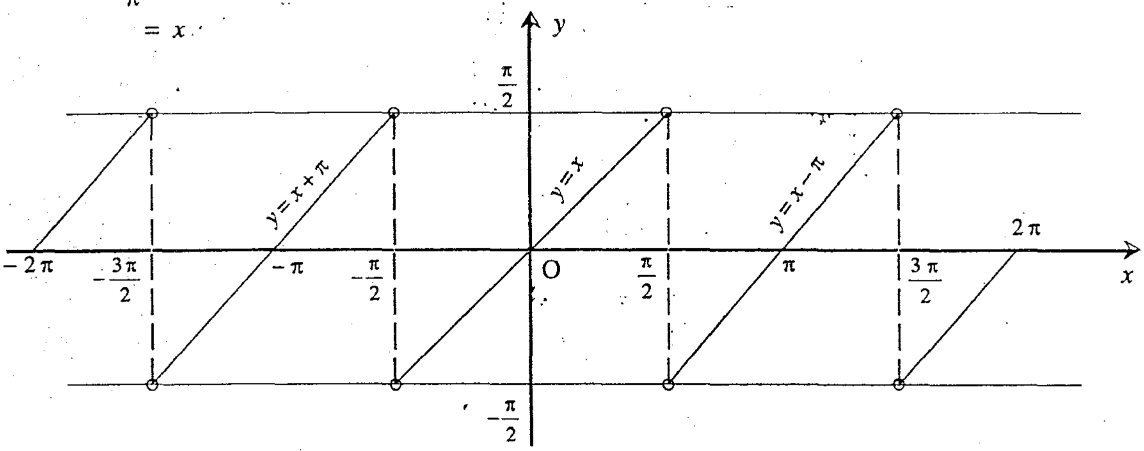
8. (b) $y = \cos(\cos^{-1}x)$, $x \in [-1, 1]$, $y \in [-1, 1]$, y is aperiodic
$= x$



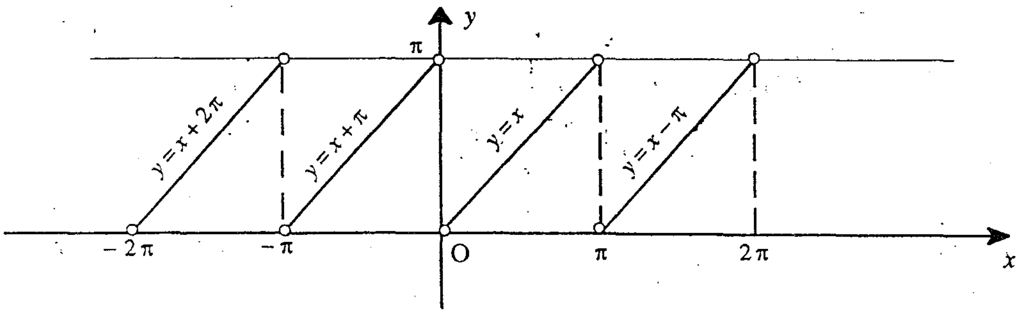
9. (a) $y = \tan(\tan^{-1}x)$, $x \in R$, $y \in R$, y is aperiodic
$= x$



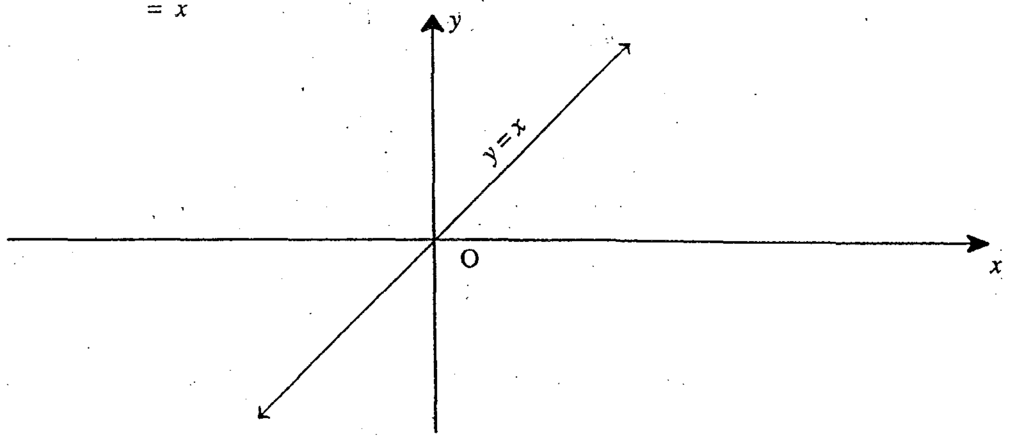
9. (b) $y = \tan^{-1}(\tan x)$, $x \in R - \left\{(2n-1)\frac{\pi}{2}\ n \in I\right\}$ $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, periodic with period π
$= x$



10. (a) $y = \cot^{-1}(\cot x)$, $x \in R - \{n\pi\}$, $y \in (0, \pi)$, periodic with π
$= x$



10. (b) $y = \cot(\cot^{-1}x)$, $x \in R$, $y \in R$, y is aperiodic
$= x$



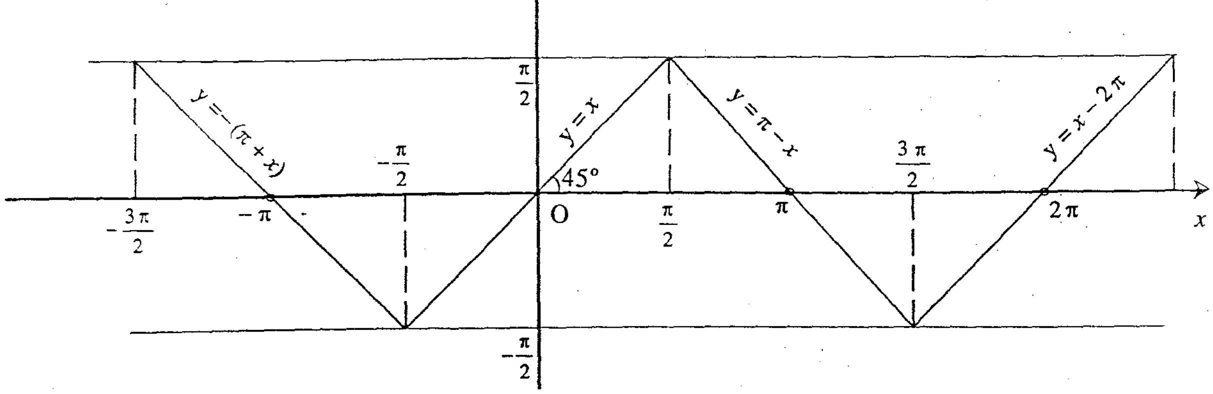
11.

12.

12. (

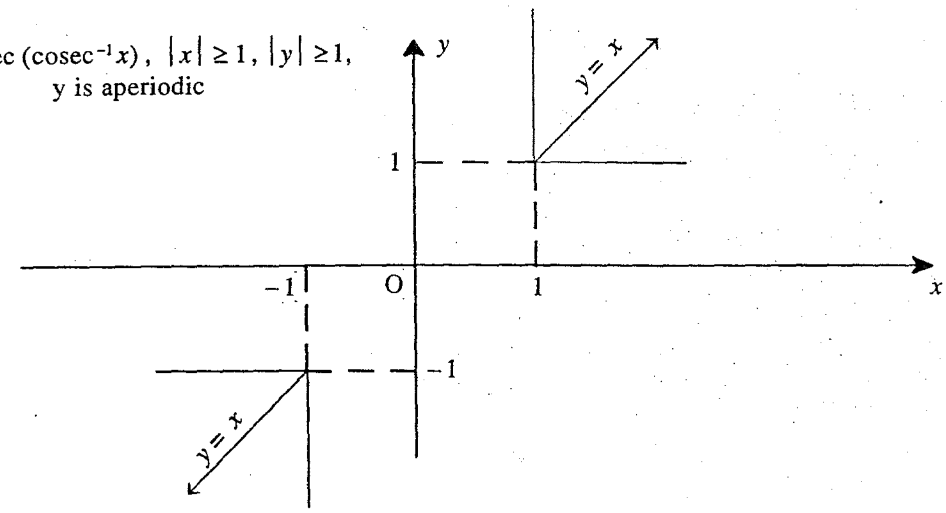
11. (a) $y = \text{cosec}^{-1}(\text{cosec}\,x)$, $x \in R - \{ n\pi , n \in I \}$ $y \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

$= x$, y is periodic with period 2π



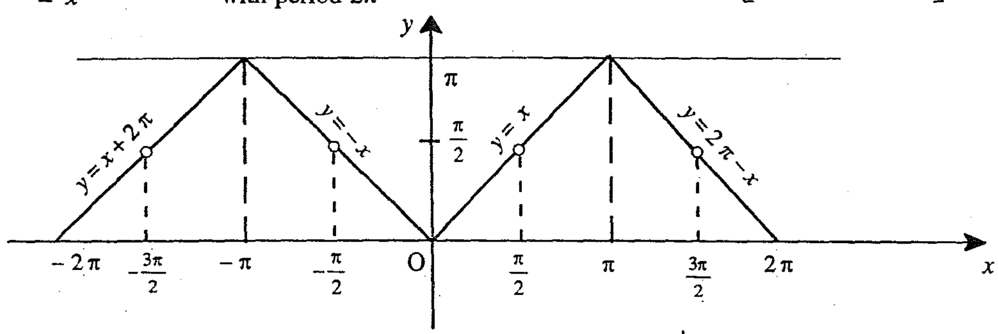
11. (b) $y = \text{cosec}(\text{cosec}^{-1}x)$, $|x| \geq 1$, $|y| \geq 1$,

$= x$ y is aperiodic



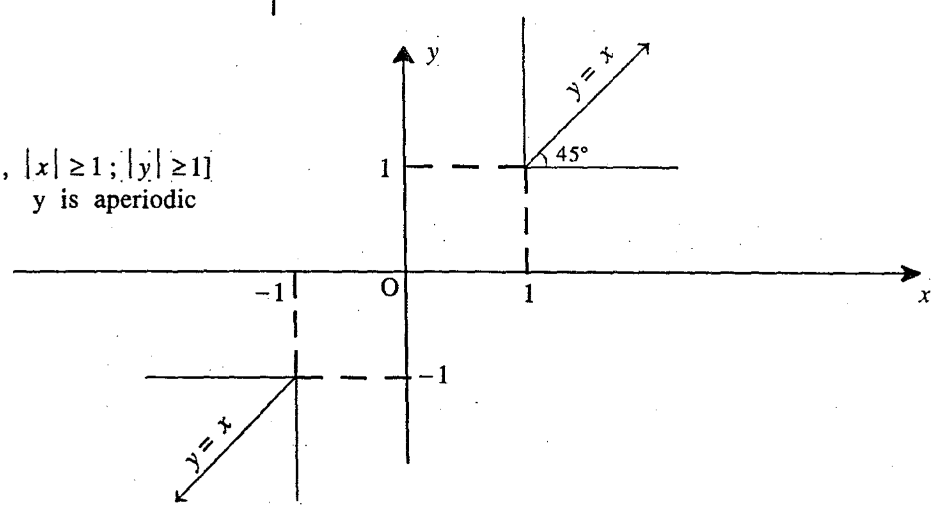
12. (a) $y = \sec^{-1}(\sec x)$, y is periodic with period 2π ; $x \in R - \left\{(2n-1)\frac{\pi}{2}\ n \in I\right\}$, $y \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

$= x$



12. (b) $y = \sec(\sec^{-1}x)$, $|x| \geq 1$; $|y| \geq 1]$

$= x$ y is aperiodic



EXERCISE I

Q 1. Find the following :

(i) $\tan\left[\cos^{-1}\frac{1}{2}+\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)\right]$ (ii) $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right]$ (iii) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

(iv) $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$ (v) $\cos\left(\tan^{-1}\frac{3}{4}\right)$ (vi) $\tan\left[\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right]$

Q 2. Find the following :

(i) $\sin\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right]$ (ii) $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$ (iii) $\tan^{-1}\left[\tan\frac{3\pi}{4}\right]$

(iv) $\cos^{-1}\left[\cos\frac{4\pi}{3}\right]$ (v) $\sin\left[\cos^{-1}\frac{3}{5}\right]$

(vi) $\tan^{-1}\left[\frac{3\sin 2\alpha}{5+3\cos 2\alpha}\right]+\tan^{-1}\left[\frac{\tan\alpha}{4}\right]$ where $-\frac{\pi}{2}<\alpha<\frac{\pi}{2}$

Q 3. Prove that :

(a) $2\cos^{-1}\frac{3}{\sqrt{13}}+\cot^{-1}\frac{16}{63}+\frac{1}{2}\cos^{-1}\frac{7}{25}=\pi$ (b) $\tan^{-1}2+\tan^{-1}3=\frac{3\pi}{4}$

(c) $\cot^{-1}9+\text{cosec}^{-1}\frac{\sqrt{41}}{4}=\frac{\pi}{4}$ (d) $\text{arc}\cos\sqrt{\frac{2}{3}}-\text{arc}\cos\frac{\sqrt{6}+1}{2\sqrt{3}}=\frac{\pi}{6}$

Q 4. Find the solution set of the equation, $3\cos^{-1}x=\sin^{-1}\left(\sqrt{1-x^2}\,(4x^2-1)\right)$.

Q 5. Prove that :

(a) $\sin^{-1}\cos(\sin^{-1}x)+\cos^{-1}\sin(\cos^{-1}x)=\frac{\pi}{2}$, $|x|\le 1$

(b) $2\tan^{-1}(\text{cosec}\tan^{-1}x-\tan\cot^{-1}x)=\tan^{-1}x$ $(x\neq 0)$

(c) $\tan^{-1}\left(\frac{2mn}{m^2-n^2}\right)+\tan^{-1}\left(\frac{2pq}{p^2-q^2}\right)=\tan^{-1}\left(\frac{2MN}{M^2-N^2}\right)$ where $M=mp-nq$, $N=np+mq$,

$\left|\frac{n}{m}\right|<1$; $\left|\frac{q}{p}\right|<1$ and $\left|\frac{N}{M}\right|<1$

(d) $\tan(\tan^{-1}x+\tan^{-1}y+\tan^{-1}z)=\cot(\cot^{-1}x+\cot^{-1}y+\cot^{-1}z)$

Q 6. Find the simplest value of, $\text{arc}\cos x+\text{arc}\cos\left(\frac{x}{2}+\frac{1}{2}\sqrt{3-3x^2}\right)$, $x\in\left[\frac{1}{2},1\right)$

Q 7. If $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$ then prove that $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha$.

Q 8. If $\text{arc}\sin x+\text{arc}\sin y+\text{arc}\sin z=\pi$ then prove that : $(x, y, z>0)$

(a) $x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}=2xyz$

(b) $x^4+y^4+z^4+4x^2y^2z^2=2(x^2y^2+y^2z^2+z^2x^2)$

Q 9. Find the greatest and the least values of the function, $f(x)=(\sin^{-1}x)^3+(\cos^{-1}x)^3$.

6, 10 c, e, f, g, 13

Q 10. Solve the following equations / system of equations :

(a) $\sin^{-1}x + \sin^{-1}2x = \frac{\pi}{3}$

(b) $\tan^{-1}\frac{1}{1+2x} + \tan^{-1}\frac{1}{1+4x} = \tan^{-1}\frac{2}{x^2}$

(c) $\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1}(3x)$

(d) $\sin^{-1}\frac{1}{\sqrt{5}} + \cos^{-1}x = \frac{\pi}{4}$

(e) $\cos^{-1}\frac{x^2-1}{x^2+1} + \tan^{-1}\frac{2x}{x^2-1} = \frac{2\pi}{3}$

(f) $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$ & $\cos^{-1}x - \cos^{-1}y = \frac{\pi}{3}$

(g) $2\tan^{-1}x = \cos^{-1}\frac{1-a^2}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2}$ $\quad a > 0,\ b > 0$.

Q 11. If $\tan^{-1}x, \tan^{-1}y, \tan^{-1}z$ are in A.P., then prove that, $y^2(x+z) + 2y(1-xz) = x+z$ where $y \in (-0, 1)$; $xz < 1$ & $x > 0,\ z > 0$.

Q12. Find the value of $\sin^{-1}(\sin 5) + \cos^{-1}(\cos 10) + \tan^{-1}[\tan(-6)] + \cot^{-1}[\cot(-10)]$.

Q13. Show that

$$\sin^{-1}\left(\sin\frac{33\pi}{7}\right) + \cos^{-1}\left(\cos\frac{46\pi}{7}\right) + \tan^{-1}\left(-\tan\frac{13\pi}{8}\right) + \cot^{-1}\left(\cot\left(-\frac{19\pi}{8}\right)\right) = \frac{13\pi}{7}$$

Q14. In a Δ ABC if $\angle A = 90°$, then prove that $\tan^{-1}\frac{b}{c+a} + \tan^{-1}\frac{c}{a+b} = \frac{\pi}{4}$

Q15. Prove that : $\sin\cot^{-1}\tan\cos^{-1}x = \sin\operatorname{cosec}^{-1}\cot\tan^{-1}x = x$ where $x \in (0,1)$

EXERCISE II

Q 1. Prove that :

(a) $\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right] = \frac{2b}{a}$

(b) $\cos^{-1}\frac{\cos x + \cos y}{1 + \cos x\cos y} = 2\tan^{-1}\left(\tan\frac{x}{2}\cdot\tan\frac{y}{2}\right)$ (c) $2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\cdot\tan\frac{x}{2}\right] = \cos^{-1}\left[\frac{b + a\cos x}{a + b\cos x}\right]$

Q 2. If $y = \tan^{-1}\left[\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right]$ prove that $x^2 = \sin 2y$.

Q 3. If $u = \cot^{-1}\sqrt{\cos 2\theta} - \tan^{-1}\sqrt{\cos 2\theta}$ then prove that $\sin u = \tan^2\theta$.

Q 4. If $\alpha = 2\ \text{arc}\tan\left(\frac{1+x}{1-x}\right)$ & $\beta = \text{arc}\sin\left(\frac{1-x^2}{1+x^2}\right)$ for $0 < x < 1$, then prove that $\alpha + \beta = \pi$, what the value of $\alpha + \beta$ will be if $x > 1$.

Q5. If $x \in \left[-1, -\frac{1}{2}\right]$ then express the function $f(x) = \sin^{-1}(3x - 4x^3) + \cos^{-1}(4x^3 - 3x)$ in the form of $a\cos^{-1}x + b\pi$, where a and b are rational numbers.

Q 6. Find the sum of the series :

(a) $\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\frac{\sqrt{2}-1}{\sqrt{6}} + \ldots.. + \sin^{-1}\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}} + \ldots\ldots \infty$

(b) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + \ldots.. + \tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}} + \ldots.. \infty$

(c) $\cot^{-1}7 + \cot^{-1}13 + \cot^{-1}21 + \cot^{-1}31 + \ldots\ldots$ to n terms .

(d) $\tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \tan^{-1}\frac{1}{x^2+7x+13}$ to n terms.

(e) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18} + \tan^{-1}\frac{1}{32} + \ldots.. \infty$

Q 7. Solve the following :

(a) $\cot^{-1}x + \cot^{-1}(n^2 - x + 1) = \cot^{-1}(n-1)$

(b) $\sec^{-1}\frac{x}{a} - \sec^{-1}\frac{x}{b} = \sec^{-1}b - \sec^{-1}a \quad a \geq 1 ;\ b \geq 1 ,\ a \neq b$.

(c) $\tan^{-1}\frac{x-1}{x+1} + \tan^{-1}\frac{2x-1}{2x+1} = \tan^{-1}\frac{23}{36}$

Q 8. Express $\frac{\beta^3}{2}\operatorname{cosec}^2\left[\frac{1}{2}\tan^{-1}\frac{\beta}{\alpha}\right] + \frac{\alpha^3}{2}\sec^2\left[\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}\right]$ as an integral polynomial in α & β . $\alpha>0, \beta>0$

Q 9. Find the integral values of K for which the system of equations ;

$$\begin{bmatrix} \arccos x + (\arcsin y)^2 = \frac{K\pi^2}{4} \\ (\arcsin y)^2 . (\arccos x) = \frac{\pi^4}{16} \end{bmatrix}$$

possesses solutions & find those solutions .

Q 10. Express the equation $\cot^{-1}\left[\frac{y}{\sqrt{1-x^2-y^2}}\right] = 2\tan^{-1}\sqrt{\frac{3-4x^2}{4x^2}} - \tan^{-1}\sqrt{\frac{3-4x^2}{x^2}}$ as a rational integral equation is x & y .

Q 11. If $X = \operatorname{cosec} . \tan^{-1} . \cos . \cot^{-1} . \sec . \sin^{-1} a$ & $Y = \sec \cot^{-1} \sin \tan^{-1} \operatorname{cosec} \cos^{-1} a$; where $0 \leq a \leq 1$. Find the relation between X & Y . Express them in terms of 'a' .

Q 12. If $A = \frac{1}{1}\cot^{-1}\left(\frac{1}{1}\right) + \frac{1}{2}\cot^{-1}\left(\frac{1}{2}\right) + \frac{1}{3}\cot^{-1}\left(\frac{1}{3}\right)$; $B = 1\cot^{-1}(1) + 2\cot^{-1}(2) + 3\cot^{-1}(3)$, then find the value of $(A^2 + B^2 - 2AB)^{1/2}$.

Q 13. Prove that the equation ,$(\sin^{-1}x)^3 + (\cos^{-1}x)^3 = \alpha\pi^3$ has no roots for $\alpha < \frac{1}{32}$ and $\alpha > \frac{7}{8}$

Q 14. Solve the following inequalities :

(a) $\operatorname{arc}\cot^2 x - 5 \operatorname{arc}\cot x + 6 > 0$ (b) $\operatorname{arc}\sin x > \operatorname{arc}\cos x$ (c) $\tan^2(\operatorname{arc}\sin x) > 1$

Q 15. Solve the following system of inequations

$4 \operatorname{arc}\tan^2 x - 8 \operatorname{arc}\tan x + 3 < 0$ & $4 \operatorname{arc}\cot x - \operatorname{arc}\cot^2 x - 3 \geq 0$

6(c) (8) (10) (12)

EXERCISE III

Q 1. Find all the positive integral solutions of, $\tan^{-1}x + \cos^{-1}\frac{y}{\sqrt{1+y^2}} = \sin^{-1}\frac{3}{\sqrt{10}}$.

[REE '93 , 6]

Q 2. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, then find the value of $x^2 + y^2 + z^2 + 2\,xyz$.

[REE '94 , 6]

Q 3. Convert the trigonometric function $\sin(2\cos^{-1}(\cot(2\tan^{-1}x)))$ into an algebraic function f(x). Then from the algebraic function, find all the values of x for which f(x) is zero. Express the values of x in the form $a \pm \sqrt{b}$ where a & b are rational numbers. [REE '95 , 6]

Q 4. If $\theta = \tan^{-1}(2\tan^2\theta) - \frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right)$ then find the general value of θ. [REE'97, 6]

Q 5. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is :

(A) zero (B) one (C) two (D) infinite [JEE '99, 2 (out of 200)]

Q 6. Using the principal values, express the following as a single angle :

$$3\tan^{-1}\left(\frac{1}{2}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) + \sin^{-1}\frac{142}{65\sqrt{5}}.$$

[REE '99, 6]

Q 7. Solve, $\sin^{-1}\frac{ax}{c} + \sin^{-1}\frac{bx}{c} = \sin^{-1}x$ where $a^2 + b^2 = c^2$, $c \neq 0$.

[REE 2000 (Mains) , 3 out of 100]

Q 8. Solve the equation:

$$\cos^{-1}\left(\sqrt{6}x\right) + \cos^{-1}\left(3\sqrt{3}x^2\right) = \frac{\pi}{2}$$

[REE 2001 (Mains) , 3 out of 100]

Q9. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \ldots\ldots\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \ldots\ldots\right) = \frac{\pi}{2}$ for $0 < |x| < \sqrt{2}$ then x equals to [JEE 2001 (screening)]

(A) 1/2 (B) 1 (C) – 1/2 (D) – 1

Q10. Prove that $\cos\tan^{-1}\sin\cot^{-1}x = \sqrt{\frac{x^2+1}{x^2+2}}$ [JEE 2002 (mains) 5]

6,7

ANSWER SHEET

EXERCISE I

Q 1. (i) $\frac{1}{\sqrt{3}}$ (ii) 1 (iii) $\frac{5\pi}{6}$ (iv) $-\frac{\pi}{3}$ (v) $\frac{4}{5}$ (vi) $\frac{17}{6}$

Q 2. (i) $\frac{1}{2}$ (ii) -1 (iii) $-\frac{\pi}{4}$ (iv) $\frac{2\pi}{3}$ (v) $\frac{4}{5}$ (vi) α

Q 4. $\left[\frac{\sqrt{3}}{2}, 1\right]$

Q 6. $\frac{\pi}{3}$

Q 9. $\frac{7\pi^3}{8}$ when $x = -1$ & $\frac{\pi^3}{32}$ when $x = \frac{1}{\sqrt{2}}$

Q 10. (a) $x = \frac{1}{2}\sqrt{\frac{3}{7}}$ (b) $x = 3$ (c) $x = 0, \frac{1}{2}, -\frac{1}{2}$ (d) $x = \frac{3}{\sqrt{10}}$ (e) $x = 2-\sqrt{3}$ or $\sqrt{3}$

(f) $x = \frac{1}{2}, y = 1$ (g) $x = \frac{a-b}{1+ab}$

Q 12. $8\pi - 21$

EXERCISE II

Q 4. $-\pi$

Q5. $6\cos^2 x - \frac{9\pi}{2}$, so $a = 6$, $b = -\frac{9}{2}$

Q 6. (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) arc cot$\left[\frac{2n+5}{n}\right]$ (d) arc tan $(x+n)$ – arc tan x (e) $\frac{\pi}{4}$

Q 7. (a) $x = n^2 - n + 1$ or $x = n$ (b) $x = ab$ (c) $x = \frac{4}{3}$

Q 8. $(\alpha^2 + \beta^2)(\alpha + \beta)$

Q 9. $K = 2$; $\cos\frac{\pi^2}{4}, 1$ & $\cos\frac{\pi^2}{4}, -1$

Q 10. $y^2 = \frac{x^2}{27}(9 - 8x^2)^2$

Q 11. $X = Y = \sqrt{3 - a^2}$

Q 12. $\frac{5\pi}{24} + \frac{5}{6}\cot^{-1}(3)$

Q 14. (a) $(\cot 2, \infty) \cup (-\infty, \cot 3)$ (b) $\left(\frac{\sqrt{2}}{2}, 1\right]$ (c) $\left(\frac{\sqrt{2}}{2}, 1\right) \cup \left(-1, -\frac{\sqrt{2}}{2}\right)$

Q15. $\left(\tan\frac{1}{2}, \cot 1\right]$

EXERCISE III

Q 1. $x = 1$; $y = 2$ & $x = 2$; $y = 7$

Q 2. 1

Q 3. $x = 0 \pm \sqrt{1}$; $x = 1 \pm \sqrt{2}$; $x = -1 \pm \sqrt{2}$

Q 4. 0, $\pi/4$ and $\tan^{-1}(2)$

Q 5. C

Q 6. π

Q 7. $x \in \{-1, 0, 1\}$

Q8. $x = \frac{1}{3}$

Q9. B