Maths Model Practice Paper For AIEEE 2010

Q. 1. If the roots of the equation $bx^2 + cx + a = 0$ If the roots of the equation be imaginary, then for all real values of *x*, the expression is

1. less than –4*ab*
2. greater than 4*ab*
3. less than 4*ab*
4. greater than –4*ab*

Sol.:

$$Given\ c^2 < 4ab$$
$$3b^2x^2 \_ 6bcx + 2c^2 = (bx+c)^2 - c^2$$
$$Now,\ 3\,(bx+c)^2\ c^2 \geq -c^2 > -4ab$$

Answer: (4)

Q. 2. For real *x*, let , f (x) $x^3 + 5x + 1$ then

1. *f* is neither one-one nor onto **R**
2. *f* is one-one but not onto **R**
3. *f* is onto **R** but not one-one
4. *f* is one-one and onto **R**

Sol:

$$f`\ (x) = 3x2 + 5,\ which\ is\ positive.$$
$$\Rightarrow f(x)\ is\ strictly\ increa\sin\ g\ hence\ it\ is\ one-one.$$
$$Also,\ f\ (\infty) \to \infty\ and\ f\ (-\infty) \to -\infty$$
$$Therefore\ range\ of\ f\ (x)\ is\ R$$

Answer: (4)

Q. 3. Let a, b, c be such that

$$b\,(a+c) \neq 0.\ If \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0$$

, then the value of n is

1. any integer
2. zero
3. any even integer
4. any odd integer

Sol: The given equation can be written as

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n} \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$\Rightarrow n\ has\ to\ be\ any\ odd\ \text{int}\ eger.$

Answer: (4)

Q. 4. The lines p (p^2 + 1) x – y + q = 0and (p^2 + 1) x+ (p^2 + 1) y + 2q = 0 are perpendicular to a common line for

1. more than two values of p
2. no value of p
3. exactly one value of p
4. exactly two values of p

Sol:

$$p\ (p^2+1) = \frac{(p^2+1)}{(p^2+1)}\ as\ the\ given\ lines\ will\ be\ parallel.$$

$$\Rightarrow p = -1$$

Answer: (3)

Q. 5. In a binomial distribution $B\left(n, p=\frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then *n* is greater than

1. $\frac{4}{\log_{10}^{4} - \log_{10}^{3}}$

2. $\frac{1}{\log_{10}^{4} - \log_{10}^{3}}$

3. $\frac{4}{\log_{10}^{4} + \log_{10}^{3}}$

4. $\frac{9}{\log_{10}^{4} - \log_{10}^{3}}$

Sol:

$$p\ (at\ least\ 1 sycess) = 1 - p\ (all\ \ failures)$$

$$Given\ that\ 1-\left(\frac{3}{4}\right)^{n} \geq \frac{9}{10} \qquad \left[p(at\ least\ 1\ sucess) = 1-\left(\frac{3}{4}\right)^{n}\right]$$

$$\Rightarrow \frac{1}{10} \geq \left(\frac{3}{4}\right)^{n} \Rightarrow -1 \geq \left(\log_{10}^{3} - \log_{10}^{4}\right) \Rightarrow n \geq \frac{1}{\log_{10}^{4} - \log_{10}^{3}}$$

Answer: (2)

Q. 6. The differential equation which represents the family of curves $y = c_1 e^{c2x}$, $where\ c_1\ and\ c_2$, where and are arbitrary constants, is

1. $yy` = (y`)^2$

2. $y` = y^2$

3. $yy` = y`y$

4. $yy` = y`$

Sol:

$$y = c_1 e^{c2x} \Rightarrow y` = c_1 e^{c2x} = c_2 y \Rightarrow \frac{y`}{|y} = c_2$$

Differentiating again we get $y``y = (y`)^2$

Answer: (1)

Q. 7. If the mean deviation of the numbers 1, 1 + d, 1 + 2d,……. ……, 1 + 100d from their mean is 255, then the *d* is equal to

1. 20.2

2. 10.0

3. 20.0

4. 10.1

Sol:

$$\textit{Mean of given numbers, } \bar{x} = \frac{1 + (1+d) + \ldots\ldots \quad \ldots\ldots (1+100d)}{101} = 1 + 50d$$

$$\textit{Mean deviation} = \frac{\sum |x_i - \bar{x}|}{n} = \frac{\sum_{r=0}^{100} |(1 + |rd - (1+50d))|}{101}$$

$$\Rightarrow \frac{\sum_{r=0}^{100} |(r-50)d|}{101} = \frac{d\,50 \times 51}{101}$$

Given that mean deviation is 255

$$\Rightarrow \frac{d\,50 \times 51}{101} = 255 \Rightarrow d = 10.1$$

Answer: (4)

Q. 8. Let A and B denote the statements

$$A : \cos\alpha + \cos\beta + \cos\gamma = 0,\ B : \sin\alpha + \sin\beta + \sin\gamma = 0.$$

$$\textit{If } \cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}, \textit{ then}$$

1. both *A* and *B* are false

2. *A* is true and *B* is false

3. *A* is false and *B* is true

4. both *A* and *B* are true

Sol:

$$os\ (\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)=-\frac{3}{2}$$
$$\Rightarrow 3+2\left(\cos((\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)\right)=0$$
$$\Rightarrow (\cos\alpha+\cos\beta+\cos\gamma)^2+(\sin\alpha+\sin\beta+\sin\gamma)^2=0$$

Answer: (4)

Q. 9.

$Statement-1:\sim(p\leftrightarrow\sim q)$ *is equivalent to* $p\leftrightarrow q$.
$Statement-2\sim(p\leftrightarrow\sim q)$ *is a tautology*.

1. Statement-1 is false, Statement-2 is true.

2. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

3. Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

4. Statement-1 is true, Statement-2 is false.

Sol.: Since, nothing has been stated about statements ***p*** and ***q*** we need to consider them as two independent statements i.e. they can be simultaneously true as well as false. Now, this can also be seen through the truth table:

P	Q	$p\leftrightarrow q$	~q	$p\leftrightarrow\sim q$	$\sim(p\leftrightarrow\sim q)$
T	T	F	F	F	T
T	F	T	T	T	F
F	T	F	F	T	F
F	F	T	T	F	T

Clearly column of $p\leftrightarrow q$ *and* $(\sim p\leftrightarrow\sim q)$ *in truth table show equivalence.*
Also $\sim(p\leftrightarrow\sim q)$ *does not show tauology in this case.*

Answer: (4)

However if we consider ‘*p*’ and ‘*q*’ as two statements which may be dependent on each other then the equivalence becomes meaningless. Nothing can be said about tautology in general.

Q. 10.

Let A be a 2×2 matrix

Statement 1: adj (adj A) = A.

Statement 2: $|adj\ A| = |A|$

1. Statement-1 is false, Statement-2 is true.

2. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

3. Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

4. Statement-1 is true, Statement-2 is false.

Sol:

$$Consider\ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} adj\ A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \Rightarrow \quad adj\ (adj\ A) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$Also\ |adj\ A| = |A|\ but\ this\ does\ not\ \exp lain\ the\ statement - 1.$$

Answer: (3)

Q. 11. Statement-1: The variance of first *n* even natural numbers is $\frac{n^2 - 1}{4}$.

Statement-2: The sum of first *n* natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first *n* natural numbers is $\frac{n(n+1)(2n+1)}{6}$.

1. Statement-1 is false, Statement-2 is true.

2. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

3. Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

4. Statement-1 is true, Statement-2 is false.

$$Variance,\ \sigma = \frac{\sum (xi - \bar{x})^2}{n},\ where\ \bar{x}\ is\ the\ mean.\ Let\ 2,4,6,...2n\ be\ the\ numbers.$$

$$\Rightarrow \bar{x} = \frac{2\,(n)\,(n+1)}{2n} = n+1$$

$$\sigma = \frac{\sum 4r^2 - 4r\,(n+1) + (n+1)^2}{n} = \frac{4n\,(n+1)\,(2n+1)}{6n} - \frac{4n\,(n+1)^2}{2n} + (n+1)2$$

$$= \frac{2\,(2n^2+3n+1)}{3} - (n+1)^2 = \frac{4n^2+6n+2-3n^2-6n-3}{3} = \frac{n^2-1}{3}$$

Sol:

Answer: (1)

Q. 12.

$Let\ f\ (x) = (x+1)^2 - 1,\ x \geq -1.$

$Statement\ 1: The\ set\ \{x : f\ (x) = f^{-1}\ (x)\} = \{0, -1\}.$

Statement-2: *f* is a bijection.

1. Statement-1 is false, Statement-2 is true.

2. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

3. Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

4. Statement-1 is true, Statement-2 is false.

Sol:

$$f\ (x) = y = (x+1)^2 - 1,\ x \geq -1,\ y \geq -1$$

$$f^{-1}\ (x) = -1 + \sqrt{1+x},\ x \geq -1 \quad [f^{-1}\ (x)\ exist\ only\ if\ f\ (x)\ is\ bijective]$$

$$Also,\ f^{-1}\ (x) = f\ (x)$$

$$\Rightarrow (x+1)^2 - 1 = -1 + \sqrt{1+x} \Rightarrow x = 0, -1$$

Answer: (3)

Q. 13. Let f (x) = x |x| and g (x) = sinx.

Statement-1: is differentiable at *x* = 0 and its derivative is continuous at that point.

Statement-2: is twice differentiable at *x* = 0.

1. Statement-1 is false, Statement-2 is true.

2. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

3. Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

4. Statement-1 is true, Statement-2 is false.

Sol:

$$h\,(x) = g\,(f\,(x)) = \sin\,x^2,\ x \geq 0$$
$$= -\sin\,x^2,\ x\ \ 0$$
$$h`\,(x) = 2x\cos x^2,\ x < 0$$
$$\Rightarrow h`\,(0^+)\ h`\,(0-) = 0$$
$$h``\,(x) = -4x^2\ \sin\,x^2 + 2\cos x^2,\ x \geq 0$$
$$= -\left[-4x^2\ \sin\,x^2 + 2\cos x^2,\ x \geq 0\right],\ x < 0$$
$$\Rightarrow h``\,(0+) = 2$$
$$h``\,(0-) = -2$$

Answer: (4)

Q. 14. Given p(x) = $x^4 + ax^3 + bx^2 + cx + d$ such that x = 0 is the only real root of p` (x) = 0. If p(-1) < p(1), then in the interval [-1, 1]

1. neither p (-1)is the minimum nor p(1)is the maximum of p

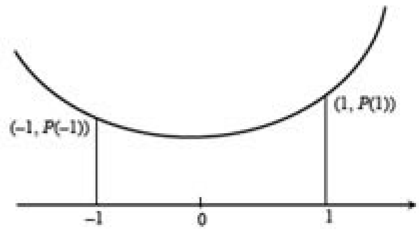
2. p (-1)is the minimum and p(1) is the maximum of p

3. p (-1) is not minimum but p(1) is the maximum of p

4. p (-1) is the minimum but p(1) is not the maximum of p

Sol:

$$p\,(x) = x^4 + ax^3 + bx^2 + cx + cx + d$$
$$Now\ p`\,(0) = 0 \Rightarrow c = 0$$
$$\Rightarrow p(x) = x^4 + ax^3 + bx^2 + d$$
$$Clearly\ \ p(1)\ is\ \max imum\ but\ p(-1)\ is\ not\ \min imum.$$



Answer: (3)

Q. 15. The shortest distance between the line y – x = 1 and curve $x = y^2$ is

1. $\frac{\sqrt{3}}{4}$

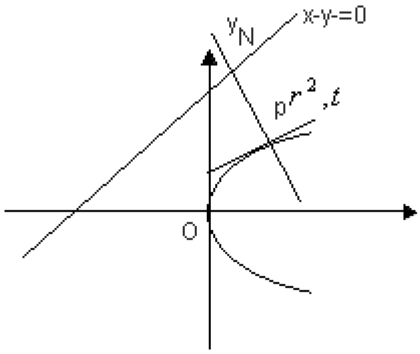
2. $\frac{3\sqrt{2}}{8}$

3. $\frac{2\sqrt{3}}{8}$

4. $\frac{3\sqrt{2}}{5}$

Sol:

$$x = y2$$
$$\Rightarrow y` = \frac{2}{2y}$$
$$m = \frac{1}{2t} = 1 \Rightarrow t = \frac{1}{2};\ p\left(\frac{1}{4}, \frac{1}{2}\right)$$
$$so,\ PN = \frac{\left|\frac{1}{4} - \frac{1}{2} + 1\right|}{\sqrt{2}} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$$



Answer: (2)

Q. 16. The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at the point (2, 3) and the *x*-axis is

1. 12

2. 3

3. 6

4. 9

Sol:

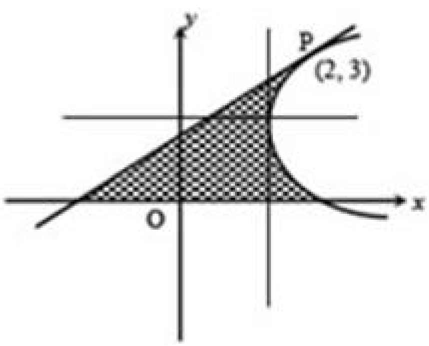
$$(y-2)^2 = x-1$$

$$2(y-2)y' = 1 \Rightarrow y' = \frac{1}{2(y-2)} \Rightarrow y' = \frac{1}{2} \text{ at } y = 3$$

$$\text{Equation of tangent } y - 3 = \frac{1}{2}(x-2)$$

$$\Rightarrow 2y - 6 = x - 2 \Rightarrow x = 2y - 4$$

$$A = \int_0^3 \left((y-2)2 + 1 - (2y-4)\right) dy = \int_0^3 \left(y2 - 6y + 9\right) dy = \frac{27}{3} = 9$$



Answer: (4)

Q. 17. The sum to infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots\ldots\ldots\ \ \ldots\ldots\ is$

1. 6

2. 2

3. 3

4. 4

Sol:

$$S=1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots\ldots\ldots \quad (i)$$
$$\frac{S}{3}=1+\frac{1}{3}+\frac{2}{3^2}+\frac{6}{3^3}+\frac{10}{3^4}+\ldots\ldots\ldots \quad (ii)$$
$$\text{Subtracting } (ii) \text{ from } (i) \text{ we get}, S\frac{2}{3}=1+\frac{1}{3}+\frac{4}{3^2}+\frac{4}{3^3}\ \ldots\ \ \ldots.$$
$$=\frac{4}{3}+\frac{4}{3^2}\left(\frac{1}{1-\frac{1}{3}}\right)=\frac{4}{3}+\frac{4}{3^2}\times\frac{3}{2}=2$$
$$S=3$$

Answer: (3)

Q. 18. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is

1. $4x^2 + 64y^2 = 48$

2. $x^2 + 16y^2 = 16$

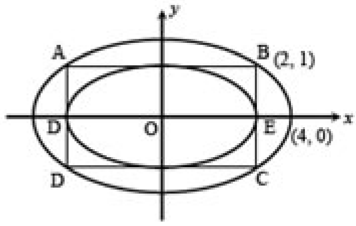
3. $x^2 + 12y^2 = 16$

4. $4x^2 + 48y^2 = 48$

Sol:

$\text{Let the new ellipse be } \frac{x^2}{16} + \frac{y^2}{b^2} = 1$

$\text{Satisfying } (2, 1) \text{ we get } b^2 = \frac{16}{12}$



Answer: (3)

Q. 19. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are

1. $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$

2. 6, -3, 2

3. $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$

4. $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$

Sol.: Direction ratios are: 6, –3, 2

Direction cosines are: $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$

Answer: (4)

Q. 20. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangements is

1. at least 1000

2. less than 500

3. at least 500 but less than 750

4. at least 750 but less than 1000

Sol:

$\text{Required number of arrangement} = {}^6C_4 \times {}^3C_1 \times {}^4C_2\,(2!)^2$
$= 15 \times 3 \times 6 \times 4 = 1080$

Answer: (1)

Q. 21. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and ***p, q*** are real numbers, then the equality
$[3\vec{u}\ p\vec{v}\ p\vec{w}] - [p\vec{v}\ \vec{w}\ q\vec{u}] - [2\vec{w}\ q\vec{v}\ q\vec{u}] = 0$ holds for

1. all values of (p, q)

2. exactly one value of (p, q)

3. exactly two values of (p, q)

4. more than two but not all values of (p, q)

Sol:

$(3p^2 - pq + 2q^2)[\vec{u}, \vec{v}, \vec{w}] = 0$
$\Rightarrow 3p^2 - pq + 2q^2 = 0$ for real p
$D \geq 0$
$\Rightarrow q^2 - 4 \times 2 \times 3q^2 \geq 0 \Rightarrow -23q^2 \geq 0 \Rightarrow q = 0$
$\Rightarrow p = 0$
So exactly one value of (p, q)

Answer: (2)

Q. 22.
Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$.
Then (α, β) equals

1. (-5, 5)

2. (6, -17)

3. (-6, 7)

4. (5, -15)

Sol:

$(2,1,-2)$ *lies on* $x+3y-\alpha z+\beta=0$

$\Rightarrow 2\alpha+\beta+-5$

Also $3-15-2\alpha=0 \Rightarrow \alpha=-6,\ \beta=7$

$(\alpha,\ \beta)\equiv(-6,7)$

Answer: (3)

Q. 23.

If $\left|Z-\frac{4}{z}\right|=2$, *then the* max *imum value of* $|Z|$ *is equal to*

1. $2+\sqrt{2}$

2. $\sqrt{2}+1$

3. $\sqrt{5}+1$

4. 2

Sol:

$\left||Z_1|-|Z_2|\right|\le|Z_1-Z_2|$

$\Rightarrow |Z|-\frac{4}{|Z|}\le 2$

$\Rightarrow |Z|^2-2|Z|-4\le 0 \Rightarrow |Z|_{max}=\sqrt{5}+1$

Answer: (3)

Q. 24. The remainder left out when $8^{2n}-(62)^{2n+1}$ is divided by 9 is

1. 8

2. 0

3. 2

4. 7

Sol:

$8^{2n} - (62)^{2n+1}$
$\Rightarrow (9-1)^{2n} - (63-1)^{2n+1} \Rightarrow \left({}^{2n}C_0 9^{2n} - {}^{2n}C_1 9^{2n-1} + ... + 1\right) - \left({}^{2n+1}C_0 63^{2n+1} - {}^{2n+1}C_1 63^{2n} + ... - 1\right)$
$\Rightarrow 9K + 2$
So the remainder is 2

Answer: (3)

Q. 25. If *A*, *B* and *C* are three sets such that $A \cap B = A \cap C$ *and* $A \cup B = A \cup C$, *then*

1. $A \cap B = \phi$
2. A = B
3. A = C
4. B = C

Sol:

$A \cap B = A \cap C$ *and* $A \cup B = A \cup C$
$\Rightarrow B = C$

Answer: (4)

Q. 26. $\int_0^{\pi} [\cot x] dx$, *where* $[.]$ denotes the greatest integer function, is equal to

1. $-\frac{\pi}{2}$
2. $\frac{\pi}{2}$
3. 1
4. -1

Sol:

$$I\int_0^{\pi}[\cot x]\,dx$$

$$I=\int_0^{\pi}[-\cot x]\,dx \Rightarrow dx\ 2I=\int_0^{\pi}([\cot x]+[-\cot x])dx=-\pi \Rightarrow I=\frac{\pi}{2}$$

Answer: (1)

Q. 27. If *P* and *Q* are the points of intersection of the circles $x^2 + y^2 + 3x - 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through *P*, *Q* and (1, 1) for

1. exactly one value of *p*
2. all values of *p*
3. all except one value of *p*
4. all except two values of *p*

Sol:

$$\text{Let the circle be } S_1+\lambda S_2=0$$

$$x^2+y^2+3x+7y+2p-5+\lambda\left(x^2+y^2+2x+2y-p^2\right)=0 \text{ passes through } (1,1)$$

$$7+2p+\lambda\left(6-p^2\right)=0, \text{ where } p=\pm\sqrt{6} \text{ required circle become } s_2=0$$

Answer: (2)

Q. 28. Three distinct points *A*, *B* and *C* are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0)to the distance from the point (-1, 0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle *ABC* is at the point

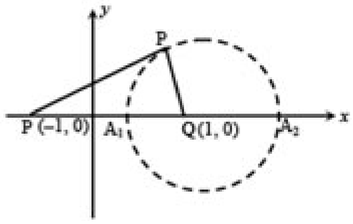
1. $\left(\frac{5}{3}, 0\right)$
2. (0, 0)
3. $\left(\frac{5}{4}, 0\right)$
4.

$\left(\frac{5}{2}, 0\right)$

Sol:

$A_1 \equiv \left(\frac{1}{2}, 0\right); A_2 \equiv (2, 0)$

$Circumcentre \equiv \left(\frac{5}{4}, 0\right)$



Answer: (3)

Q. 29. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ……., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals

1. $\frac{1}{50}$

2. $\frac{1}{14}$

3. $\frac{1}{7}$

4. $\frac{5}{14}$

Sol:

00 10 203040

01 11 21 31 40

: : : : :

09 19 29 39 49

$n(S) = 14, n(A) = 1$

$\Rightarrow P(A) = \frac{1}{14}$

Answer: (2)

Q. 30. Let be an implicit function of x defined $x^{2x} - 2x^{x} \cot y - 1 = 0$. Then y`(1) equals

1. -log2

2. -1

3. 1

4. log2

Sol:

$At\ x = 1 \Rightarrow \cot y = 0$

$y` = \frac{2(\cot y - 1)}{2\ cosec^2 y} = \frac{-2}{2} = -1$

Answer: (2)