CLASS : XII (ABCD) **DPP ON PROBABILITY** **DPP. NO.- 1**

After 1st Lecture

Q.1_{46} 100 cards are numbered from 1 to 100. The probability that the randomly chosen card has a digit 5 is:
(A) 0.01 (B) 0.09 (C*) 0.19 (D) 0.18

[Hint: from 50 to 60, 10 fives and 9 other fives $\Rightarrow$ total 19 $\Rightarrow$ (C)]

Q.2_{4} A quadratic equation is chosen from the set of all the quadratic equations which are unchanged by squaring their roots. The chance that the chosen equation has equal roots is :
(A*) 1/2 (B) 1/3 (C) 1/4 (D) 2/3

Q.3_{10} If the letters of the word "MISSISSIPPI" are written down at random in a row, the probability that no two S's occur together is :
(A) 1/3 (B*) 7/33 (C) 6/13 (D) 5/7

Q.4_{29} A sample space consists of 3 sample points with associated probabilities given as 2p, p^2, 4p – 1 then

(A*) p = $\sqrt{11}-3$ (B) $\sqrt{10}-3$ (C) $\frac{1}{4} < p < \frac{1}{2}$ (D) none

[Hint: $p^2 + 2p + 4p - 1 = 1$ (exhaustive)
$p^2 + 6p - 2 = 0$ $\Rightarrow$ (A)]

Q.5_{39} A committee of 5 is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all is :
(A) 1/2 (B) 5/9 (C*) 4/9 (D) 2/3

[Hint : $\frac{{}^7C_3 + {}^7C_5}{{}^9C_5}$]

Q.6_{53} There are only two women among 20 persons taking part in a pleasure trip. The 20 persons are divided into two groups, each group consisting of 10 persons. Then the probability that the two women will be in the same group is :
(A*) 9/19 (B) 9/38 (C) 9/35 (D) none

[Hint: n(S) = number of ways in which 20 people can be divided into two equal groups

$$= \frac{20!}{10!\,10!\,2!}$$

n(A) = 18 people can be divided into groups of 10 and 8

$$= \frac{18!}{10!\,8!}$$

$$P(E) = \frac{18!}{10!\,8!} \cdot \frac{10!\,10!\,2}{20!} = \frac{10.9.2}{20.19} = \frac{9}{19} \text{ Ans]}$$

Q.7 A bag contain 5 white, 7 black, and 4 red balls, find the chance that three balls drawn at random are all white. [Ans. 1/56]

Q.8 If four coins are tossed, find the chance that there should be two heads and two tails. [Ans. 3/8]

Q.9 Thirteen persons take their places at a round table, show that it is five to one against two particular persons sitting together.

Q.10 In shuffling a pack of cards, four are accidentally dropped, find the chance that the missing cards should be one from each suit. [Ans. $\frac{(13)^2}{^{52}C_4} = \frac{2197}{20825}$]

Q.11 A has 3 shares in a lottery containing 3 prizes and 9 blanks, B has 2 shares in a lottery containing 2 prizes and 6 blanks. Compare their chances of success. [Ans. 952 to 715]

Q.12 There are three works, one consisting of 3 volumes, one of 4 and the other of one volume. They are placed on a shelf at random, prove that the chance that volumes of the same works are all together is $\frac{3}{140}$.

Q.13 The letter forming the word *Clifton* are placed at random in a row. What is the chance that the two vowels come together? [Ans. 2/7]

Q.14 Three bolts and three nuts are put in a box. If two parts are chosen at random, find the probability that one is a bolt and one is a nut. [Ans. 3/5]

Q.15 There are '*m*' rupees and '*n*' ten nP's, placed at random in a line. Find the chance of the extreme coins being both ten nP's. [Ans. $\frac{n(n-1)}{(m+n)(m+n-1)}$]

Q.16 A fair die is tossed. If the number is odd, find the probability that it is prime. [Ans. 2/3]

Q.17 Three fair coins are tossed. If both heads and tails appear, determine the probability that exactly one head appears. [Ans. 1/2]

Q.18 3 boys and 3 girls sit in a row. Find the probability that (i) the 3 girls sit together. (ii) the boys are girls sit in alternative seats. [Ans. 1/5, 1/10]

Q.19 A coin is biased so that heads is three times as likely to appear as tails. Find P (H) and P (T). [Ans. 3/4, 1/4]

Q.20 In a hand at "whist" what is the chance that the 4 kings are held by a specified player? [Ans. $\frac{^4C_4 \cdot {}^{48}C_9}{^{52}C_{13}}$]

After 2nd Lecture

Q.1 Given two independent events A, B such that P (A) = 0.3, P (B) = 0.6. Determine
(i) P (A and B) (ii) P (A and not B) (iii) P (not A and B)
(iv) P (neither A nor B) (v) P (A or B)
[**Ans.** (i) 0.18, (ii) 0.12, (iii) 0.42, (iv) 0.28, (v) 0.72]

Q.2 A card is drawn at random from a well shuffled deck of cards. Find the probability that the card is a
(i) king or a red card (ii) club or a diamond (iii) king or a queen
(iv) king or an ace (v) spade or a club (vi) neither a heart nor a king.
[**Ans.** (i) $\frac{7}{13}$, (ii) $\frac{1}{2}$, (iii) $\frac{2}{13}$, (iv) $\frac{2}{13}$, (v) $\frac{1}{2}$, (vi) $\frac{9}{13}$]

Q.3 A coin is tossed and a die is thrown. Find the probability that the outcome will be a head or a number greater than 4. [**Ans.** $\frac{2}{3}$]

Q.4 Let A and B be events such that $P(\tilde{A})$ = 4/5, P(B) = 1/3, P(A/B) = 1/6, then
(a) $P(A \cap B)$; (b) $P(A \cup B)$; (c) P(B/A) ; (d) Are A and B independent?
[Ans. (a) 1/18, (b) 43/90, (c) 5/18, (d) NO]

[Sol. (a) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{6} \Rightarrow P(A \cap B) = \frac{P(B)}{6} = \frac{1}{18}$ Ans.]

(b) $P(A \cup B) = \frac{1}{5} + \frac{1}{3} - \frac{1}{18} = \frac{18+30-5}{90} = \frac{43}{90}$ Ans.

(c) $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{1}{18} \cdot \frac{5}{1} = \frac{5}{18}$ Ans.

(d) $P(A) \cdot P(B) = \frac{1}{5} \cdot \frac{1}{3} = 15 \neq P(A \cap B)$. A & B are not independent]

Q.5 If A and B are two events such that P (A) = $\frac{1}{4}$, P (B) = $\frac{1}{2}$ and P (A and B) = $\frac{1}{8}$, find
(i) P (A or B), (ii) P (not A and not B) [**Ans.** (i) $\frac{5}{8}$, (ii) $\frac{3}{8}$]

Q.6_{168} A 5 digit number is formed by using the digits 0, 1, 2, 3, 4 & 5 without repetition. The probability that the number is divisible by 6 is :
(A) 8 % (B) 17 % (C*) 18 % (D) 36 %

[Hint: Number should be divisible by 2 and 3.
$n(S) = 5 \cdot 5!$; n (A) : reject '0' = $2 \cdot 4!$
reject 3, $4! + 2 \cdot 3 \cdot 3!$
Total $n(A) = 3 \cdot 4! + 6 \cdot 3! = 18 \cdot 3!$

$\therefore \quad p = \frac{18 \cdot 3!}{5 \cdot 5!}$ = 18%]

Q.7$_{54}$ An experiment results in four possible out comes S_1, S_2, S_3 & S_4 with probabilities p_1, p_2, p_3 & p_4 respectively. Which one of the following probability assignment is possbile.
[Assume $S_1\ S_2\ S_3\ S_4$ are mutually exclusive]
(A) $p_1 = 0.25$, $p_2 = 0.35$, $p_3 = 0.10$, $p_4 = 0.05$
(B) $p_1 = 0.40$, $p_2 = -0.20$, $p_3 = 0.60$, $p_4 = 0.20$
(C) $p_1 = 0.30$, $p_2 = 0.60$, $p_3 = 0.10$, $p_4 = 0.10$
(D*) $p_1 = 0.20$, $p_2 = 0.30$, $p_3 = 0.40$, $p_4 = 0.10$

Q.8$_{88}$ In throwing 3 dice, the probability that atleast 2 of the three numbers obtained are same is
(A) 1/2 (B) 1/3 (C*) 4/9 (D) none
[Hint : $P(E) = 1 - P(\text{all different}) = 1 - (6/6) \cdot (5/6) \cdot (4/6) = 1 - (120/216) = 4/9$]

Q.9$_{173}$ There are 4 defective items in a lot consisting of 10 items. From this lot we select 5 items at random. The probability that there will be 2 defective items among them is
(A) $\frac{1}{2}$ (B) $\frac{2}{5}$ (C) $\frac{5}{21}$ (D*) $\frac{10}{21}$

[Hint: $10 \begin{cases} 4D \\ 6G \end{cases} \xrightarrow[5]{} \begin{cases} 2D \\ 3G \end{cases}$ **[12th (26-12-2004)]**

$$p = \frac{{}^4C_2 \cdot {}^6C_3}{{}^{10}C_5} = \frac{10}{21} \Rightarrow (D)\ ]$$

Q.10$_{134}$ From a pack of 52 playing cards, face cards and tens are removed and kept aside then a card is drawn at random from the ramaining cards. If
A : The event that the card drawn is an ace
H : The event that the card drawn is a heart
S : The event that the card drawn is a spade
then which of the following holds ?
(A*) $9\,P(A) = 4\,P(H)$ (B) $P(S) = 4P(A \cap H)$
(C) $3\,P(H) = 4\,P(A \cup S)$ (D) $P(H) = 12\,P(A \cap S)$

[Hint: $52 \xrightarrow[\text{10's}]{\text{face cards}} 36 \begin{cases} 9\text{ H} \\ 9\text{ S} \\ 9\text{ D} \\ 9\text{ C} \end{cases}$;

$$P(A) = \frac{1}{9}\ ;\ P(H) = \frac{1}{4}\ ;\ P(S) = \frac{1}{4}\ ;\ P(A \cap H) = \frac{1}{36}\ ;\ P(A \cap S) = \frac{1}{36}\ ;\ P(A \cup S) = \frac{1}{3}\ ]$$

Q.11$_{220}$ 6 married couples are standing in a room. If 4 people are chosen at random, then the chance that exactly one married couple is among the 4 is :
(A*) $\frac{16}{33}$ (B) $\frac{8}{33}$ (C) $\frac{17}{33}$ (D) $\frac{24}{33}$
[Hint: $n(S) = {}^{12}C_4 = 55 \times 9$
$n(A) = {}^6C_1 . {}^5C_2 . 2^2 = 6 \times 10 \times 4$

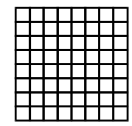
$$P(E) = \frac{6 \times 10 \times 4}{55 \times 9} = \frac{2.2.4}{11.3} = \frac{16}{33} \text{ Ans]}$$

Q.12_{45} The chance that a 13 card combination from a pack of 52 playing cards is dealt to a player in a game of bridge, in which 9 cards are of the same suit, is

(A*) $\dfrac{4\cdot{}^{13}C_9\cdot{}^{39}C_4}{{}^{52}C_{13}}$ (B) $\dfrac{4!\cdot{}^{13}C_9\cdot{}^{39}C_4}{{}^{52}C_{13}}$ (C) $\dfrac{{}^{13}C_9\cdot{}^{39}C_4}{{}^{52}C_{13}}$ (D) none

Q.13_{64} If two of the 64 squares are chosen at random on a chess board, the probability that they have a side in common is :

(A) 1/9 (B*) 1/18 (C) 2/7 (D) none



[Hint: $n(S) = {}^{64}C_2\cdot 2$; $n(A) = \dfrac{4\cdot 2+6\cdot 4\cdot 3+36\cdot 4}{64\cdot 63}$.

Alternatively : $n(A) = 7\cdot 8 + 7\cdot 8 = 112$

Ask: Prob that they have a corner in common]

Q.14_{182} Two red counters, three green counters and 4 blue counters are placed in a row in random order. The probability that no two blue counters are adjacent is

(A) $\dfrac{7}{99}$ (B) $\dfrac{7}{198}$ (C*) $\dfrac{5}{42}$ (D) none

[Sol. R R G G G B B B B when counters are alike **[14-8-2005, 13th]**

$n(S) = \dfrac{9!}{2!3!4!}$

$n(A) = \dfrac{5!}{3!2!}\cdot{}^6C_4$ | R | R | G | G | G |

$\therefore$ $P(A) = \dfrac{5!\cdot 15}{3!2!}\cdot\dfrac{2!3!4!}{9!} = \dfrac{6!\cdot 60}{9\cdot 8\cdot 7\cdot 6!} = \dfrac{60}{7\cdot 8\cdot 9} = \dfrac{15}{7\cdot 2\cdot 9} = \dfrac{5}{42}$

Alternatively : $n(S) = 9!$ $R_1R_1G_1G_2G_3B_1B_2B_3B_4$

$n(A) = 5!\cdot{}^6C_4\cdot 4!$ when counters are different

$p = \dfrac{5!\cdot 6\cdot 5\cdot 4\cdot 3}{9!} = \dfrac{5\cdot 4\cdot 3}{9\cdot 8\cdot 7} = \dfrac{5}{42}$]

Q.15 The probabilities that a student will receive A, B, C or D grade are 0.40, 0.35, 0.15 and 0.10 respectively. Find the probability that a student will receive

(i) not an A grade (ii) B or C grade (iii) at most C grade

[**Ans.** (i) 0.6, (ii) 0.5, (iii) 0.25]

Q.16 In a single throw of three dice, determine the probability of getting

(i) a total of 5 (ii) a total of at most 5 (iii) a total of at least 5.

[**Ans.** (i) $\dfrac{1}{36}$, (ii) $\dfrac{5}{108}$, (iii) $\dfrac{53}{54}$]

Q.17 A die is thrown once. If E is the event "the number appearing is a multiple of 3" and F is the event "the number appearing is even", find the probability of the event "E and F". Are the events E and F independent?

[**Ans.** $P(E) = \dfrac{1}{3}$, $P(F) = \dfrac{1}{2}$, $P(E \text{ and } F) = \dfrac{1}{6}$; Yes]

Q.18 In the two dice experiment, if E is the event of getting the sum of number on dice as 11 and F is the event of getting a number other than 5 on the first die, find P (E and F). Are E and F independent events?

[**Ans.** $P(E) = \dfrac{2}{36}$, $P(F) = \dfrac{30}{36}$, $P(E\cap F) = \dfrac{1}{36}$; Not independent]

Q.19 A natural number x is randomly selected from the set of first 100 natural numbers. Find the probability that it satisfies the inequality. $x + \frac{100}{x} > 50$ [Ans: $\frac{55}{100} = \frac{11}{20}$]

Note: {1, 2, 48, 49, 50, ... ,100 } $\left[\text{wrong Ans given by students } \frac{1}{50}, \frac{27}{50}, \frac{53}{100}\right]$

Q.20 3 students A and B and C are in a swimming race. A and B have the same probability of winning and each is twice as likely to win as C. Find the probability that B or C wins. Assume no two reach the winning point simultaneously.

[Sol. P(C) = p ; P(A) = 2p ; P(B) = 2p

$\therefore$ 5p = 1 $\Rightarrow$ p = 1/5

$$P(B \text{ or } C) = P(B) + P(C) = \frac{2}{5} + \frac{1}{5} = \frac{3}{5} \quad]$$

Q.21 A box contains 7 tickets, numbered from 1 to 7 inclusive. If 3 tickets are drawn from the box, one at a time, determine the probability that they are alternatively either odd-even-odd or even-odd-even.

$$[\text{Ans. } p = \frac{4 \cdot 3 \cdot 3 + 3 \cdot 4 \cdot 2}{7 \cdot 6 \cdot 5} = \frac{6}{210} = \frac{2}{7}\ ]$$

Q.22 5 different marbles are placed in 5 different boxes randomly. Find the probability that exactly two boxes remain empty. Given each box can hold any number of marbles.

[Sol. n(S) = 5^5 ; For computing favourable outcomes.

2 boxes which are to remain empty, can be selected in ${}^5C_2$ ways and 5 marbles can be placed in the remaining 3 boxes in groups of 221 or 311 in

$$3!\left[\frac{5!}{2!2!2!} + \frac{5!}{3!2!}\right] = 150 \text{ ways} \Rightarrow n(A) = {}^5C_2 \cdot 150$$

Hence $P(E) = {}^5C_2 \cdot \frac{150}{5^5} = \frac{60}{125} = \frac{12}{25}$ Ans.]

Q.23$_{132}$ South African cricket captain lost the toss of a coin 13 times out of 14. The chance of this happening was

(A*) $\frac{7}{2^{13}}$ (B) $\frac{1}{2^{13}}$ (C) $\frac{13}{2^{14}}$ (D) $\frac{13}{2^{13}}$

[Hint: L and W can be filled at 14 places in 2^{14} ways.

$\therefore$ n(S) = 2^{14}.

Now 13 L's and 1W can be arranged at 14 places in 14 ways.

Hence n(A) = 14

$\therefore$ $p = \frac{14}{2^{14}} = \frac{7}{2^{13}}$] **[14-8-2005, 13th]**

Q.24 There are ten prizes, five A's, three B's and two C's, placed in identical sealed envelopes for the top ten contestants in a mathematics contest. The prizes are awarded by allowing winners to select an envelope at random from those remaining. When the 8^{th} contestant goes to select the prize, the probability that the remaining three prizes are one A, one B and one C, is

(A*) 1/4 (B) 1/3 (C) 1/12 (D) 1/10

[Hint: n(S) = ${}^{10}C_7$ = 120 **[08-01-2006, 12 & 13]**

n(A) = ${}^5C_4 \cdot {}^3C_2 \cdot {}^2C_1$

$P(E) = \frac{5 \cdot 3 \cdot 2}{120} = \frac{1}{4}$ Ans.]

CLASS : XII (ABCD)	DPP ON PROBABILITY	DPP. NO.- 3

After 3rd Lecture

Q.1$_{33}$ Whenever horses a, b, c race together, their respective probabilities of winning the race are 0.3, 0.5 and 0.2 respectively. If they race three times the probability that "the same horse wins all the three races" and the probablity that a, b, c each wins one race, are respectively

(A*) $\frac{8}{50}$; $\frac{9}{50}$ (B) $\frac{16}{100}$, $\frac{3}{100}$ (C) $\frac{12}{50}$; $\frac{15}{50}$ (D) $\frac{10}{50}$; $\frac{8}{50}$

[Sol. P(a) = 0.3 ; P(b) = 0.5 ; P(c) = 0.2 $\Rightarrow$ a, b, c are exhaustive
P(same horse wins all the three races) = P(aaa or bbb or ccc)

$$= (0.3)^3 + (0.5)^3 + (0.2)^3 = \frac{27+125+8}{1000} = \frac{160}{1000} = \frac{4}{25}$$

P(each horse wins exactly one race)

$$= \text{P(abc or acb or bca or bac or cab or cba)} = 0.3\times 0.5 \times 0.2 \times 6 = 0.18 = \frac{9}{50}$$]

Q.2$_{63}$ Let A & B be two events. Suppose P(A) = 0.4 , P(B) = p & $P(A \cup B) = 0.7$. The value of p for which A & B are independent is :

(A) 1/3 (B) 1/4 (C*) 1/2 (D) 1/5

[Sol. $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$0.7 = 0.4 + p - 0.4p$

$\therefore \quad 0.6p = 0.3 \Rightarrow p = \frac{1}{2}$]

Q.3$_{78}$ A & B are two independent events such that $P(\overline{A}) = 0.7$, $P(\overline{B}) = a$ & $P(A \cup B) = 0.8$, then, a =

(A) 5/7 (B*) 2/7 (C) 1 (D) none

Q.4$_{72}$ A pair of numbers is picked up randomly (without replacement) from the set $\{1, 2, 3, 5, 7, 11, 12, 13, 17, 19\}$. The probability that the number 11 was picked given that the sum of the numbers was even, is nearly :

(A) 0.1 (B) 0.125 (C*) 0.24 (D) 0.18

[Hint : $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{{}^7C_1}{{}^8C_2 + 1} = \frac{7}{29}$; A : 11 is picked , B : sum is even]

Q.5$_{42}$ For a biased die the probabilities for the diffferent faces to turn up are given below :

Faces :	1	2	3	4	5	6
Probabilities :	0.10	0.32	0.21	0.15	0.05	0.17

The die is tossed & you are told that either face one or face two has turned up. Then the probability that it is face one is :

(A) 1/6 (B) 1/10 (C) 5/49 (D*) 5/21

[Hint : $P(A/A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A) + P(B)} = \frac{0.10}{0.10 + 0.32}$]

Q.6$_{170}$ A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the determinant chosen has the value non negative is :

(A) 3/16 (B) 6/16 (C) 10/16 (D*) 13/16

[Hint : 1 – P (Determinant has negative value)

$$1 - \frac{3}{16} = \frac{13}{16}\left(\begin{vmatrix}1 & 1\\1 & 0\end{vmatrix} ; \begin{vmatrix}0 & 1\\1 & 1\end{vmatrix} ; \begin{vmatrix}0 & 1\\1 & 0\end{vmatrix}\right) \Big]$$

Q.7$_{99}$ 15 coupons are numbered 1, 2, 3,..... , 15 respectively. 7 coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9 is :

(A) $\left(\frac{9}{16}\right)^6$ (B) $\left(\frac{8}{15}\right)^7$ (C) $\left(\frac{3}{5}\right)^7$ (D*) $\frac{9^7 - 8^7}{15^7}$

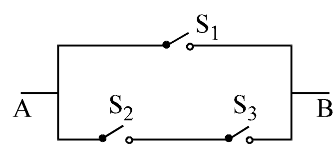
[Hint: n(S) = $\times\ \times\ \times\ \times\ \times\ \times\ \times = 15^7$; came of the number must be 9]

Q.8$_{229}$ A card is drawn & replaced in an ordinary pack of 52 playing cards. Minimum number of times must a card be drawn so that there is atleast an even chance of drawing a heart, is

(A) 2 (B*) 3 (C) 4 (D) more than four

[Hint: $\frac{1}{4} + \frac{3}{4} \cdot \frac{1}{4} + \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4}$]

Q.9 An electrical system has open-closed switches S_1, S_2 and S_3 as shown.



The switches operate independently of one another and the current will flow from A to B either if S_1 is closed or if both S_2 and S_3 are closed. If $P(S_1) = P(S_2) = P(S_3) = \frac{1}{2}$, find the probability that the circuit will work.

[Sol. $P(S_1) = P(S_2) = P(S_3) = \frac{1}{2}$ [**Ans.** $\frac{5}{8}$]

E : event that the current will flow.

$$P(E) = P\big((S_2 \cap S_3) \text{ or } S_1\big) = P(S_2 \cap S_3) + P(S_3) - P(S_1 \cap S_2 \cap S_3) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}\Big]$$

Q.10 A certain team wins with probability 0.7, loses with probability 0.2 and ties with probability 0.1. The team plays three games. Find the probability

(i) that the team wins at least two of the games, but lose none.

(ii) that the team wins at least one game. [**Ans.** (i) 0.49 ; (ii) 0.973]

[Sol. P (W) = 0.7 ; P (L) = 0.2 ; P (T) = 0.1

E : winning at least 2 games but lose none

P (E) = P (W W T or W T W or T W W or W W W)

$= 3 \times 0.7 \times 0.7 \times 0.1 + (0.7)^3 = 0.7 \times 0.7\,[0.3 + 0.7] = 0.49$

F : wining at least 1 game

A = L or T $\Rightarrow$ P (A) = 0.3 ; P (F) = 1 – P (A A A) = $1 - (0.3)^3 = 1 - 0.027 = 0.973$]

Q.11 An integer is chosen at random from the first 200 positive integers. Find the probability that the integer is divisible by 6 or 8.

[Sol. n (S) = 200 [Ans. $\frac{1}{4}$]

n (A) = divisible by 6 or 8

n (6) = 6 + 12 + ... + 198 = 33

n (8) = 8 + 16 + ...+ 200 = 25

n $(6 \cap 8)$ = 24 + 48 + ... + 192 = 8

n (A) = 33 + 25 – 8 = 50

$P(A) = \frac{1}{4}$]

Q.12 A clerk was asked to mail four report cards to four students. He addresses four envelops that unfortunately paid no attention to which report card be put in which envelope. What is the probability that exactly one of the students received his (or her) own card?

[**Ans.** $\frac{8}{24} = \frac{1}{3}$]

Q.13 Find the probability of at most two tails or at least two heads in a toss of three coins.

[Sol. A = at most two tails n (s) – {T T T} [**Ans.** $\frac{7}{8}$]

B = at least two heads H H H, T H H, H T H, H H T

$P(A) = \frac{7}{8}$; $P(B) = \frac{4}{8}$; $P(A \cap B) = \frac{4}{8}$

$P(A \cup B) = \frac{7}{8} + \frac{4}{8} - \frac{4}{8} = \frac{7}{8}$ Ans.]

Q.14 What is the probability that in a group of

(i) 2 people, both will have the same date of birth.

(ii) 3 people, at least 2 will have the same date of birth.

Assume the year to be ordinarry consisting of 365 days.

[Sol.**(i)** A : both have the same date of birth

$P(A) = \frac{1}{365}\cdot\frac{1}{365} + \frac{1}{365}\cdot\frac{1}{365} + \text{.......365 times} = \frac{1}{365}$ Ans.

(ii) B : at least 2 in a group of 3 will have the same birthday

P (B) = 1 – P (all 3 have different birthday)

$= 1 - 1 \times \frac{364}{365}\cdot\frac{363}{365} = 1 - \frac{364 \times 363}{(365)^2}$ Ans.]

Q.15 The probability that a person will get an electric contract is $\frac{2}{5}$ and the probability that he will not get plumbing contract is $\frac{4}{7}$. If the probability of getting at least one contract is $\frac{2}{3}$, what is the probability that he will get both ?

[Sol. $P(E) = \frac{2}{5}$; $P(F) = P(\text{plumbing}) = 1 - \frac{4}{7} = \frac{3}{7}$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\frac{2}{3} = \frac{2}{5} + \frac{3}{7} - x \quad \Rightarrow \quad x = \frac{17}{105}$ Ans.]

Q.16 Five horses compete in a race. John picks two horses at random and bets on them. Find the probability that John picked the winner. Assume dead heal. [Ans. $\frac{4}{10}$]

[Sol. $\left.\begin{array}{l} n(S) = {}^5C_2 = 10 \\ n(A) = 1 \cdot {}^4C_1 = 4 \end{array}\right] \Rightarrow p = \frac{2}{5}$]

Q.17 Two cubes have their faces painted either red or blue. The first cube has five red faces and one blue face. When the two cubes are rolled simultaneously, the probability that the two top faces show the same colour is 1/2. Number of red faces on the second cube, is

(A) 1 (B) 2 (C*) 3 (D) 4

[Sol. Let the number of red faces on the 2^{nd} cube = x **[08-01-2006, 12 & 13]**

number of blue faces = (6 – x)

P (R R or B B) = 1/2

$$\frac{5}{6}\cdot\frac{x}{6} + \frac{1}{6}\cdot\frac{6-x}{6} = \frac{1}{2}$$

5x + 6 – x = 18

4x = 12 $\Rightarrow$ x = 3 Ans.]

Q.18 A H and W appear for an interview for two vaccancies for the same post.
P(H) = 1/7 ; P(W) = 1/5. Find the probability of the events
(a) Both are selected (b) only one of them is selected (c) none is selected.

[Ans. $\frac{1}{35}, \frac{2}{7}, \frac{24}{35}$]

Q.19 A bag contains 6R, 4W and 8B balls. If 3 balls are drawn at random determine the probability of the event
(a) all 3 are red ; (b) all 3 are black ; (c) 2 are white and 1 is red ;
(d) at least 1 is red ; (e) 1 of each colour are drawn
(f) the balls are drawn in the order of red, white, blue.

[Ans. (a) $\frac{5}{204}$, (b) $\frac{7}{102}$, (c) $\frac{3}{68}$, (d) $\frac{149}{204}$, (e) $\frac{4}{17}$, $\frac{2}{51}$]

Q.20 The odds that a book will be favourably reviewed by three independent critics are 5 to 2, 4 to 3, and 3 to 4 respectively. What is the probability that of the three reviews a majority will be favourable?

[Ans. $\frac{209}{343}$]

Q.21 In a purse are 10 coins, all five nP's except one which is a rupee, in another are ten coins all five nP's. Nine coins are taken from the former purse and put into the latter, and then nine coins are taken from the latter and put into the former. Find the chance that the rupee is still in the first purse.

[Ans. $\frac{10}{19}$]

Q.22 A, B, C in order cut a pack of cards, replacing them after each cut, on condition that the first who cuts a spade shall win a prize. Find their respective chances. [Ans. $\frac{16}{37}, \frac{12}{37}, \frac{9}{37}$]

Q.23 A and B in order draw from a purse containing 3 rupees and 4 nP's, find their respective chances of first drawing a rupee, the coins once drawn not being replaced. [Ans. $\frac{22}{35}, \frac{13}{35}$]

CLASS : XII (ABCD) **DPP ON PROBABILITY** **DPP. NO.- 4**

After 4th Lecture

Q.1$_{37}$ There are n different gift coupons, each of which can occupy $N(N > n)$ different envelopes, with the same probability 1/N

P_1: The probability that there will be one gift coupon in each of n definite envelopes out of N given envelopes

P_2: The probability that there will be one gift coupon in each of n arbitrary envelopes out of N given envelopes

Consider the following statements

(i) $P_1 = P_2$ (ii) $P_1 = \frac{n!}{N^n}$ (iii) $P_2 = \frac{N!}{N^n(N-n)!}$

(iv) $P_2 = \frac{n!}{N^n(N-n)!}$ (v) $P_1 = \frac{N!}{N^n}$

Now, which of the following is true

(A) Only (i) (B*) (ii) and (iii) (C) (ii) and (iv) (D) (iii) and (v)

[Sol. From the given data $n(S) = N^n$; $n(A) = n!$ $\Rightarrow$ $P_1 = \frac{n!}{N^n}$

$P_1 = \frac{n!}{N^n}$ Since the n different gift coupans can be placed in the n definite (Out of N) envelope in $^nP_n = n!$ ways

$P_2 = \frac{N!}{(N-n)!\,N^n}$ As n arbitrary envelopes out of N given envelopes can be chosen in ^NC_n ways and the n gift coupans can occupy these envelopes in n ! ways.]

$$= \frac{N!}{N^n(N-n)!}$$]

Q.2$_{61}$ The probability that an automobile will be stolen and found withing one week is 0.0006. The probability that an automobile will be stolen is 0.0015. The probability that a stolen automobile will be found in one week is

(A) 0.3 (B*) 0.4 (C) 0.5 (D) 0.6

[Hint: $P(S \cap F) = 0.0006$, where S : moter cycle is stolen ; F : moter cycle found

$P(S) = 0.0015$

$$P(F/S) = \frac{P(F \cap S)}{P(S)} = \frac{6 \times 10^{-4}}{15 \times 10^{-4}} = \frac{2}{5} \Rightarrow \text{(B)}$$]

Q.3$_{25}$ One bag contains 3 white & 2 black balls, and another contains 2 white & 3 black balls. A ball is drawn from the second bag & placed in the first, then a ball is drawn from the first bag & placed in the second. When the pair of the operations is repeated, the probability that the first bag will contain 5 white balls is:

(A) 1/25 (B) 1/125 (C*) 1/225 (D) 2/15

[Hint: Bag 1 < 3 W, 2 B   Bag 2 < 2 W, 3 B

$$P(E) = P[W\,B\,W\,B] = \frac{2}{5}\cdot\frac{2}{6}\cdot\frac{1}{5}\cdot\frac{1}{6} = \frac{1}{225}$$]

Q.4$_{75}$ A child throws 2 fair dice. If the numbers showing are unequal, he adds them together to get his final score. On the other hand, if the numbers showing are equal, he throws 2 more dice & adds all 4 numbers showing to get his final score. The probability that his final score is 6 is:

(A) $\frac{145}{1296}$ (B) $\frac{146}{1296}$ (C) $\frac{147}{1296}$ (D*) $\frac{148}{1296}$

[Hint: P (6) = (51, 15, 24, 42) or 11 & (22 or 13 or 31) or (22 & 11)]

Q.5$_{118}$ A person draws a card from a pack of 52 cards, replaces it & shuffles the pack. He continues doing this till he draws a spade. The probability that he will fail exactly the first two times is :

(A) 1/64 (B*) 9/64 (C) 36/64 (D) 60/64

[Hint: P(E) = P(FFS) = 3/4.3/4.1/4]

Q.6 Indicate the correct order sequence in respect of the following :

I. If the probability that a computer will fail during the first hour of operation is 0.01, then if we turn on 100 computers, exactly one will fail in the first hour of operation.

II. A man has ten keys only one of which fits the lock. He tries them in a door one by one discarding the one he has tried. The probability that fifth key fits the lock is 1/10.

III. Given the events A and B in a sample space. If P(A) = 1, then A and B are independent.

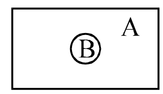
IV. When a fair six sided die is tossed on a table top, the bottom face can not be seen. The probability that the product of the numbers on the five faces that can be seen is divisible by 6 is one.

(A) FTFT (B*) FTTT (C) TFTF (D) TFFF

[Hint: **I.** $P(X = 1) = {}^{100}C_1\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{100}$ **[18-12-2005, 12 & 13]**

II. Every key that fits have the same probability = 1/10

III. Consider $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



but $P(A \cup B) = P(A) = 1$

$1 = 1 + P(B) - P(A \cap B)$

$P(A \cap B) = P(B) = P(B) \cdot P(A)$ (P(A) = 1)

IV. Each product 1 2 3 4 5 ; 1 2 3 4 6 ; 1 2 3 5 6 ; 1 2 4 5 6 ; 1 3 4 5 6 ; 2 3 4 5 6 is divisible by six.]

Q.7$_{156}$ An unbaised cubic die marked with 1, 2, 2, 3, 3, 3 is rolled 3 times. The probability of getting a total score of 4 or 6 is

(A) $\frac{16}{216}$ (B*) $\frac{50}{216}$ (C) $\frac{60}{216}$ (D) none

[Hint: 1 , 2 , 2 , 3 , 3 , 3 (thrown 3 times)

$P(1) = \frac{1}{6}$; $P(2) = \frac{2}{6}$; $P(3) = \frac{3}{6}$

P(S) = P(4 or 6) = P(112 (3 cases) or 123 (6 cases) or 222)

$$= 3.\frac{1}{6}.\frac{1}{6}.\frac{2}{1} + 6\frac{1}{6}.\frac{2}{6}.\frac{3}{1} + \frac{2}{6}.\frac{2}{6}.\frac{2}{6} = \frac{6+36+8}{216} = \frac{50}{216} = \frac{25}{108}$$]

Q.8$_{200}$ A bag contains 3 R & 3 G balls and a person draws out 3 at random. He then drops 3 blue balls into the bag & again draws out 3 at random. The chance that the 3 later balls being all of different colours is

(A) 15% (B) 20% (C*) 27% (D) 40%

[Sol. 3R 3G

$1R + 2G \to \boxed{2R + 1G + 3B}$

$2R + 1G \to \boxed{1R + 2G + 3B}$

$3R \to \boxed{3G + 3B} \xrightarrow{\text{zero}}$

$3G \to \boxed{3R + 3B} \xrightarrow{\text{zero}}$

; $$\frac{{}^3C_1 \cdot {}^3C_2}{{}^6C_2} \cdot \frac{{}^2C_1 \cdot {}^1C_1 \cdot {}^3C_1}{{}^6C_3} + \frac{{}^3C_2 \cdot {}^3C_1}{{}^6C_3} \cdot \frac{{}^1C_1 \cdot {}^2C_1 \cdot {}^3C_1}{{}^6C_3}$$]

Q.9$_{169}$ A biased coin with probability P, 0 < P < 1, of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5 then the value of P is

(A) 1/4 (B) 1/6 (C*) 1/3 (D) 1/2

Q.10$_{202}$ If $a, b \in N$ then the probability that $a^2 + b^2$ is divisible by 5 is

(A*) $\frac{9}{25}$ (B) $\frac{7}{18}$ (C) $\frac{11}{36}$ (D) $\frac{17}{81}$

[Hint : Square of a number ends in 0, 1, 4, 5, 6 and 9 favourable ordered pairs of (a^2, b^2) can be (0, 0) ; (5, 5) ; (1, 4) , (4, 1) ; (1, 9) , (9, 1) ; (4, 6) , (6, 4) ; (6, 9) , (9, 6) and P(0) = 1/10 = P(5) ; P(1) = P(4) = P(6) = P(9) = 2/10]

Q.11$_{52}$ In an examination, one hundred candidates took paper in Physics and Chemistry. Twenty five candidates failed in Physics only. Twenty candidates failed in chemistry only. Fifteen failed in both Physics and Chemistry. A candidate is selected at random. The probability that he failed either in Physics or in Chemistry but not in both is

(A*) $\frac{9}{20}$ (B) $\frac{3}{5}$ (C) $\frac{2}{5}$ (D) $\frac{11}{20}$

[13th Test (5-12-2004)]

Q.12 In a certain game A's skill is to be B's as 3 to 2, find the chance of A winning 3 games at least out of 5.

[Hint: odd in favour of A 3 : 2 $\Rightarrow$ P(A) = $\frac{3}{5}$ = p ; q = $\frac{2}{5}$ [Ans. $\frac{2133}{3125}$]

$$P(E) = {}^5C_3\left(\frac{3}{5}\right)^3\cdot\left(\frac{2}{5}\right)^2 + {}^5C_4\left(\frac{3}{5}\right)^4\cdot\left(\frac{2}{5}\right) + \left(\frac{3}{5}\right)^5 \;]$$

Q.13 In each of a set of games it is 2 to 1 in favour of the winner of the previous game. What is the chance that the player who wins the first game shall wins three at least of the next four? [Ans. 4/9]

[Hint: P(W/W) = $\frac{2}{3}$; P(L/W) = $\frac{1}{3}$; P(W/L) = $\frac{1}{3}$; P(L/L) = $\frac{2}{3}$]

Q.14 A coin is tossed n times, what is the chance that the head will present itself an odd number of times?

[Hint: $P(E) = \dfrac{{}^nC_1 + {}^nC_3 + {}^nC_5 + ...}{2^n} = \dfrac{2^{n-1}}{2^n} = \dfrac{1}{2}$] [Ans. $\frac{1}{2}$]

Q.15 A fair die is tossed repeatidly. A wins if it is 1 or 2 on two consecutive tosses and B wins if it is 3, 4, 5 or 6 on two consecutive tosses. The probability that A wins if the die is tossed indefinitely, is

(A) $\frac{1}{3}$ (B*) $\frac{5}{21}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$

[Sol. Let P(S) = P(1 or 2) = 1/3 **[29-10-2005, 12th Jaipur]**

P(F) = P(3 or 4 or 5 or 6) = 2/3

P(A wins) = P[(S S or S F S S or S F S F S S or ....) or (F S S or F S F S S or ....)]

$$= \frac{\frac{1}{9}}{1-\frac{2}{9}} + \frac{\frac{2}{27}}{1-\frac{2}{9}} = \frac{1}{9}\times\frac{9}{7} + \frac{2}{27}\times\frac{9}{7} = \frac{1}{7} + \frac{2}{21} = \frac{3+2}{21} = \frac{5}{21}$$

P (A winning) = $\frac{5}{21}$; P (B winning) = $\frac{16}{21}$ Ans.]

Q.16 Counters marked 1, 2, 3 are placed in a bag, and one is withdrawn and replaced. The operation being repeated three times, what is the chance of obtaining a total of 6? [Ans. 7 / 27]

[Hint: $P(1\;2\;\overset{(6\text{ cases})}{3} \text{ or } 2\;2\;2) = \frac{6+1}{3^3} = \frac{7}{27}$]

Q.17 A normal coin is continued tossing unless a head is obtained for the first time. Find the probability that
(a) number of tosses needed are at most 3.
(b) number of tosses are even. [Ans. (a) $\frac{7}{8}$, (b) $\frac{1}{3}$]

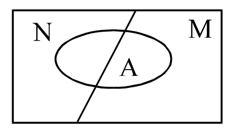
[Sol. (a) P (H or T H or T T H); $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=\frac{7}{8}$

(b) T H or T T T H or .... ; $P(E) = \frac{1/4}{1-1/4} = \frac{1}{4}\times\frac{4}{3} = \frac{1}{3}$]

Q.18 A purse contains 2 six sided dice. One is a normal fair die, while the other has 2 ones, 2 threes, and 2 fives. A die is picked up and rolled. Because of some secret magnetic attraction of the unfair die, there is 75% chance of picking the unfair die and a 25% chance of picking a fair die. The die is rolled and shows up the face 3. The probability that a fair die was picked up, is

(A*) $\frac{1}{7}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{24}$

[Sol. N = Normal die ; P(N) = 1/4 **[08-01-2006, 12 & 13]**
M = magnetic die ; P(M) = 3/4
A = die shows up 3



$P(A) = P(A \cap N) + P(A \cap M)$
$= P(N)\ P(A/N) + P(M) \cdot P(A/M)$

$$= \frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{2}{6} = \frac{7}{24}$$

$$P(N/A) = \frac{P(N \cap A)}{P(A)} = \frac{(1/4)\cdot(1/6)}{7/24} = \frac{1}{7} \text{ Ans.]}$$

Q.19 Before a race the chance of three runners, A, B, C were estimated to be proportional to 5, 3, 2, but during the race A meets with an accident which reduces his chance to 1/3. What are the respective chance of B and C now? [Ans. B = 2/5 ; C = 4/15]

Q.20 A fair coin is tossed a large number of times. Assuming the tosses are independent which one of the following statement, is True?
(A) Once the number of flips is large enough, the number of heads will always be exactly half of the total number of tosses. For example, after 10,000 tosses one should have exactly 5,000 heads.
(B*) The proportion of heads will be about 1/2 and this proportion will tend to get closer to 1/2 as the number of tosses inreases
(C) As the number of tosses increases, any long run of heads will be balanced by a corresponding run of tails so that the overall proportion of heads is exactly 1/2
(D) All of the above **[29-10-2005, 12^{th}]**

Q.21 A and B each throw simultaneously a pair of dice. Find the probability that they obtain the same score.

Hint: [P [(2&2) or (3&3) or (4&4) ...] [Ans: $\frac{73}{648}$]

Q.22 A is one of the 6 horses entered for a race, and is to be ridden by one of two jockeys B or C. It is 2 to 1 that B rides A, in which case all the horses are equally likely to win; if C rides A, his chance is trebled, what are the odds against his winning? [Ans. 13 to 5]

Direction for Q.23 to Q.25

Let S and T are two events defined on a sample space with probabilities
$P(S) = 0.5$, $P(T) = 0.69$, $P(S/T) = 0.5$

Q.23 Events S and T are:

(A) mutually exclusive
(B*) independent
(C) mutually exclusive and independent
(D) neither mutually exclusive nor independent

Q.24 The value of P(S and T)

(A*) 0.3450 (B) 0.2500 (C) 0.6900 (D) 0.350

Q.25 The value of P(S or T)

(A) 0.6900 (B) 1.19 (C*) 0.8450 (D) 0

[Sol. $P(S/T) = \dfrac{P(S \cap T)}{P(T)} \Rightarrow 0.5 = \dfrac{P(S \cap T)}{0.69} \Rightarrow P(S \cap T) = 0.5 \times 0.69 = P(S)P(T)$

$\Rightarrow$ S and T are independent **Ans.**

$\therefore$ $P(S \text{ and } T) = P(S) \cdot P(T) = 0.69 \times 0.5 = 0.345$ **Ans.**

$P(S \text{ or } T) = P(S) + P(T) - P(S \cap T) = 0.5 + 0.69 - 0.345 = 0.8450$ **Ans.**]

[29-10-2005, 12th]

After 5th Lecture

Q.1$_{113}$ The first 12 letters of the english alphabets are written down at random. The probability that there are 4 letters between A & B is :

(A) 7/33 (B) 12/33 (C) 14/33 (D*) 7/66

[Hint: n(S) = 12 ! ; n(A) = $\boxed{{}^{10}C_4 \cdot 4!\ 2!}$ 7!

$\therefore$ p = 7 / 66]

Q.2$_{223}$ Events A and C are independent. If the probabilities relating A, B and C are P (A) = 1/5; P (B) = 1/6 ; P (A $\cap$ C) = 1/20 ; P (B $\cup$ C) = 3/8 then

(A*) events B and C are independent

(B) events B and C are mutually exclusive

(C) events B and C are neither independent nor mutually exclusive

(D) events B and C are equiprobable

[Hint: P (A $\cap$ C) = P (A) $\cdot$ P (C)

$$\frac{1}{20} = \frac{1}{5} \cdot P(C) \quad \Rightarrow \quad P(C) = \frac{1}{4}$$

now $P(B \cup C) = \frac{1}{6} + \frac{1}{4} - P(B \cap C)$, hence $P(B \cap C) = \frac{3}{8} - \frac{1}{3} = \frac{1}{24} = P(B) \cdot P(C) \Rightarrow$ (A)]

Q.3 Assume that the birth of a boy or girl to a couple to be equally likely, mutually exclusive, exhaustive and independent of the other children in the family. For a couple having 6 children, the probability that their "three oldest are boys" is

(A) $\frac{20}{64}$ (B) $\frac{1}{64}$ (C) $\frac{2}{64}$ (D*) $\frac{8}{64}$

[Hint: E : $B_1\ B_2\ B_3 \times \times \times$ where $\times$ means B or G **[27-11-2005, 12th]**

$P(E) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} = \frac{8}{64}$ Ans.]

Q.4 A and B play a game. A is to throw a die first, and is to win if he throws 6, If he fails B is to throw, and to win if he throws 6 or 5. If he fails, A is to throw again and to win with 6 or 5 or 4, and so on, find the chance of each player. [Ans. A : $\frac{169}{324}$; B : $\frac{155}{324}$]

Q.5$_{69}$ Box A contains 3 red and 2 blue marbles while box B contains 2 red and 8 blue marbles. A fair coin is tossed. If the coin turns up heads, a marble is drawn from A, if it turns up tails, a marble is drawn from bag B. The probability that a red marble is chosen, is

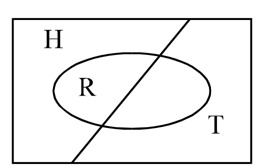
(A) $\frac{1}{5}$ (B*) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $\frac{1}{2}$

[Hint: A $<^{3\text{ R}}_{2\text{ B}}$; B $<^{2\text{ R}}_{8\text{ B}}$

R = event that a red marble is drawn

P (R) = P (R $\cap$ H) + P (R $\cap$ T)

= P (H) P (R / H) + P (T) $\cdot$ P (R / T)



$$= \frac{1}{2}\left[\frac{3}{5} + \frac{2}{10}\right] = \frac{8}{10} \cdot \frac{1}{2} = \frac{2}{5}$$]

Q.6_{74} A examination consists of 8 questions in each of which one of the 5 alternatives is the correct one. On the assumption that a candidate who has done no preparatory work chooses for each question any one of the five alternatives with equal probability, the probability that he gets more than one correct answer is equal to :

(A) $(0.8)^8$ (B) $3\,(0.8)^8$ (C) $1 - (0.8)^8$ (D*) $1 - 3\,(0.8)^8$

[Hint: $p = \frac{1}{5} = 0.2$; $q = 0.8$; $P(E) = 1 - P(0 \text{ or } 1)$]

Q.7_{181} The germination of seeds is estimated by a probability of 0.6. The probability that out of 11 sown seeds exactly 5 or 6 will spring is :

(A*) $\frac{^{11}C_5 \,.\, 6^5}{5^{10}}$ (B) $\frac{^{11}C_6 \left(3^5\, 2^5\right)}{5^{11}}$ (C) $^{11}C_5 \left(\frac{5}{6}\right)^{11}$ (D) none of these

Q.8_{152} The probability of obtaining more tails than heads in 6 tosses of a fair coins is :

(A) 2/64 (B*) 22/64 (C) 21/64 (D) none

[Hint : $P(4 \text{ or } 5 \text{ or } 6) = \frac{^6C_4 + {^6C_5} + {^6C_6}}{64} = \frac{22}{64}$]

Q.9_{192} An instrument consists of two units. Each unit must function for the instrument to operate. The reliability of the first unit is 0.9 & that of the second unit is 0.8. The instrument is tested & fails. The probability that "only the first unit failed & the second unit is sound" is :

(A) 1/7 (B*) 2/7 (C) 3/7 (D) 4/7

[Hint:
A : the instrument has failed
B_1 : first unit fails and second is healthy
B_2 : first unit healthy and second unit fails
B_3 : both fails
B_4 : both healthy

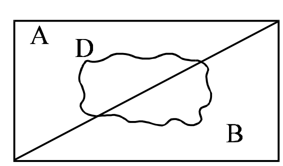
$$\left.\begin{array}{l} P(B_1) = 0.1 \times 0.8 = 0.08 \\ P(B_2) = 0.2 \times 0.9 = 0.18 \\ P(B_3) = 0.1 \times 0.2 = 0.02 \\ P(B_4) = 0.9 \times 0.8 = 0.72 \end{array}\right] \quad \begin{array}{l} P(A/B_1) = P(A/B_2) = P(A/B_3) = 1 \\ P(A/B_4) = 0 \end{array}$$

Now compute $P(B_1/A)$]

Q.10_{196} Lot A consists of 3G and 2D articles. Lot B consists of 4G and 1D article. A new lot C is formed by taking 3 articles from A and 2 from B. The probability that an article chosen at random from C is defective, is

(A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C*) $\frac{8}{25}$ (D) none

[Hint : A = event that the item came from lot A ; $P(A) = \frac{3}{3+2} = \frac{3}{5}$



B = item came from B ; P (B) = 2/5
D = item from mixed lot ' C ' is defective
$P(D) = P(D \cap A) + P(D \cap B)$
$= P(A).\, P(D/A) + P(B).\, P(D/A)$
$= \frac{3}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{1}{5} = \frac{8}{25}$ Ans.]

Q.11$_{28}$ A die is weighted so that the probability of different faces to turn up is as given :

Number	1	2	3	4	5	6
Probability	0.2	0.1	0.1	0.3	0.1	0.2

If $P(A/B) = p_1$ and $P(B/C) = p_2$ and $P(C/A) = p_3$ then the values of p_1, p_2, p_3 respectively are
Take the events A, B & C as A = {1, 2, 3}, B = {2, 3, 5} and C = {2, 4, 6}

(A) $\frac{2}{3}, \frac{1}{3}, \frac{1}{4}$ (B) $\frac{1}{3}, \frac{1}{3}, \frac{1}{6}$ (C) $\frac{1}{4}, \frac{1}{3}, \frac{1}{6}$ (D*) $\frac{2}{3}, \frac{1}{6}, \frac{1}{4}$

Q.12 If *mn* coins have been distributed into *m* purses, n into each find
(1) the chance that two specified coins will be found in the same purse, and
(2) what the chance becomes when r purses have been examined and found not to contain either of the specified coins.

[Ans. (1) $\frac{n-1}{mn-1}$, (2) $\frac{n-1}{mn-rn-1}$]

Q.13 A box has four dice in it. Three of them are fair dice but the fourth one has the number five on all of its faces. A die is chosen at random from the box and is rolled three times and shows up the face five on all the three occassions. The chance that the die chosen was a rigged die, is

(A) $\frac{216}{217}$ (B) $\frac{215}{219}$ (C*) $\frac{216}{219}$ (D) none

[Sol. $4 \begin{cases} 3 \ \textit{normal die} \\ 1 \ \textit{rigged die} \end{cases}$ **[27-11-2005, 12th]**

A : die shows up the face 5
B_1 : it is a rigged die ; $P(B_1) = 1/4$
B_2 : it is a normal die ; $P(B_2) = 3/4$

$P(A/B_1) = 1$; $P(A/B_2) = \frac{1}{216}$

$$P(B_1/A) = \frac{\frac{1}{4}\cdot 1}{\frac{1}{4}\cdot 1 + \frac{3}{4}\cdot\frac{1}{216}} = \frac{216}{219} \quad \text{Ans.]}$$

Q.14 On a Saturday night 20% of all drivers in U.S.A. are under the influence of alcohol. The probability that a driver under the influence of alcohol will have an accident is 0.001. The probability that a sober driver will have an accident is 0.0001. If a car on a saturday night smashed into a tree, the probability that the driver was under the influence of alcohol, is
(A) 3/7 (B) 4/7 (C*) 5/7 (D) 6/7

[Hint: A : car met with an accident **[29-10-2005, 12th Jaipur]**
B_1: driver was alcoholic, $P(B_1) = 1/5$
B_2: driver was sober, $P(B_2) = 4/5$
$P(A/B_1) = 0.001$; $P(A/B_2) = 0.0001$

$$P(B_1/A) = \frac{(.2)(.001)}{(.2)(.001)+(.8)(.0001)} = 5/7 \ \textbf{Ans.}]$$

Direction for Q.15 to Q.17 (3 Questions)

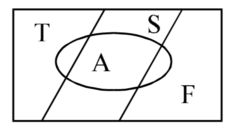
A JEE aspirant estimates that she will be successful with an 80 percent chance if she studies 10 hours per day, with a 60 percent chance if she studies 7 hours per day and with a 40 percent chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7, respectively

Q.15 The chance she will be successful, is
(A) 0.28 (B) 0.38 (C*) 0.48 (D) 0.58

Q.16 Given that she is successful, the chance she studied for 4 hours, is
(A) $\frac{6}{12}$ (B*) $\frac{7}{12}$ (C) $\frac{8}{12}$ (D) $\frac{9}{12}$

Q.17 Given that she does not achieve success, the chance she studied for 4 hour, is
(A) $\frac{18}{26}$ (B) $\frac{19}{26}$ (C) $\frac{20}{26}$ (D*) $\frac{21}{26}$

[Sol. A : She get a success **[18-12-2005, 12 & 13]**
T : She studies 10 hrs : P(T) = 0.1
S : She studies 7 hrs : P(S) = 0.2
F : She studies 4 hrs : P(F) = 0.7



P(A/T) = 0.8 ; P(A/S) = 0.6 ; P(A/F) = 0.4

$P(A) = P(A \cap T) + P(A \cap S) + P(A \cap F)$
$= P(T) \cdot P(A/T) + P(S) \cdot P(A/S) + P(F) \cdot P(A/F)$
$= (0.1)(0.8) + (0.2)(0.6) + (0.7)(0.4)$
$= 0.08 + 0.12 + 0.28 = 0.48$ **Ans.(15)**

$$P(F/A) = \frac{P(F \cap A)}{P(A)} = \frac{(0.7)(0.4)}{0.48} = \frac{0.28}{0.48} = \frac{7}{12}$$ **Ans.(16)**

$$P(F/\overline{A}) = \frac{P(F \cap \overline{A})}{P(\overline{A})} = \frac{P(F) - P(F \cap A)}{0.52} = \frac{(0.7) - 0.28}{0.52} = \frac{0.42}{0.52} = \frac{21}{26}$$ **Ans.(17)**]

Q.18 There are four balls in a bag, but it is not known of what colour they are ; one ball is drawn at random and found to be white. Find the chance that all the balls are white. Assume all number of white ball in the bag to be equally likely. [Ans. 2/5]

Q.19 A letter is known to have come either from London or Clifton. On the postmark only the two consecutive letters ON are legible. What is the chance that it came from London? [Ans. 12/17]

Q.20 A purse contains *n* coins of unknown value, a coin drawn at random is found to be a rupee, what is the chance that is it the only rupee in the purse? Assume all numbers of rupee coins in the purse is equally likely.

[Ans. $\frac{2}{n(n+1)}$]

Q.21 One of a pack of 52 cards has been lost, from the remainder of the pack two cards are drawn and are found to be spades, find the chance that the missing card is a spade. [Ans. 11/50]

Q.22 A, B are two inaccurate arithmeticians whose chance of solving a given question correctly are (1/8) and (1/12) respectively. They solve a problem and obtained the same result. If it is 1000 to 1 against their making the same mistake, find the chance that the result is correct. [Ans. 13/14]

[Hint: A : they obtained the same result

$$\left.\begin{array}{l} B_1 : A \cap \overline{B} \ ; P(B_1) = \dfrac{1}{18} \cdot \dfrac{11}{12} \\[2ex] B_2 : \overline{A} \cap B \ ; P(B_1) = \dfrac{7}{8} \cdot \dfrac{1}{12} \\[2ex] B_3 : A \cap B \ ; P(B_3) = \dfrac{1}{8} \cdot \dfrac{1}{12} \\[2ex] B_4 : \overline{A} \cap \overline{B} \ ; P(B_4) = \dfrac{7}{8} \cdot \dfrac{11}{12} \end{array}\right\} \qquad \begin{array}{l} \text{Now} \quad P(A/B_1) = 0 \\[2ex] P(A/B_2) = 0 \\[2ex] P(A/B_3) = 1 \\[2ex] P(A/B_4) = \dfrac{7}{8} \cdot \dfrac{11}{12} \cdot \dfrac{1}{1001} \end{array}$$

$$\text{Now} \quad P(B_3/A) = \frac{\frac{1}{8} \cdot \frac{1}{12}}{\frac{1}{8} \cdot \frac{1}{12} + \frac{7}{8} \cdot \frac{11}{12} \cdot \frac{1}{1001}} = \frac{13}{14} \ ]$$

Q.23 We conduct an experiment where we roll a die 5 times. The order in which the number read out is important.

(a) What is the total number of possible outcomes of this experiment?

(b) What is the probability that exactly 3 times a "2" appears in the sequence (say event E)?

(c) What is the probability that the face 2 appears at least twice (say event F)?

(d) Which of the following are true : $E \subset F$, $F \subset E$?

(e) Compute the probabilities : $P(E \cap F)$, $P(E/F)$, $P(F/E)$

(f) Are the events E and F independent?

[Hint: (a) 6^5 ; (b) $P(E) = \dfrac{{}^5C_3 \cdot 5^2}{6^5} = \dfrac{10 \cdot 5^2}{6^5}$

(c) $P(F) = 1 - P(\text{face two appears exact once or no two}) = 1 - \dfrac{{}^5C_1 \cdot 5^4 + 5^5}{6^5}$; $P(F) = 1 - 2 \cdot \left(\dfrac{5}{6}\right)^5$

(d) $E \subset F$; (e) $P(E \cap F) = P(E)$; $P(E/F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{P(E)}{P(F)}$; $P(F/E) = \dfrac{P(E \cap F)}{P(E)} = 1$

(f) No]

After 6th Lecture

Q.1 A bowl has 6 red marbles and 3 green marbles. The probability that a blind folded person will draw a red marble on the second draw from the bowl without replacing the marble from the first draw, is

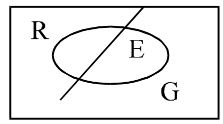
(A*) $\frac{2}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{5}{8}$

[Hint: 9 $<$ 6R, 3G **[27-11-2005, 12th]**

E : Event that the 2^{nd} drawn marble is red; R : 1^{st} drawn is red; G = 1^{st} drawn is green

$P(E) = P(E \cap R) + P(E \cap G)$

$= P(R) \cdot P(E/R) + P(E) \cdot P(E/G)$



$= \frac{6}{9} \cdot \frac{5}{8} + \frac{3}{9} \cdot \frac{6}{8} = \frac{48}{72} = \frac{2}{3}$]

Q.2_{114} 5 out of 6 persons who usually work in an office prefer coffee in the mid morning, the other always drink tea. This morning of the usual 6, only 3 are present. The probability that one of them drinks tea is :

(A*) 1/2 (B) 1/12 (C) 25/72 (D) 5/72

[Hint: 6 persons $<$ 5 coffee, 1 tea

Total number of ways in which 3 persons one of which drinks tea and 2 others can be selected

$= {}^1C_1 \cdot {}^5C_2$ ways

number of ways any 3 can be selected ${}^6C_3$

$\therefore \quad P(E) = \frac{{}^5C_2}{{}^6C_3} = \frac{10}{20} = \frac{1}{2}$]

Q.$3_{29/5}$ Pal's gardner is not dependable, the probability that he will forget to water the rose bush is 2/3. The rose bush is in questionable condition. Any how if watered, the probability of its withering is 1/2 & if not watered then the probability of its withering is 3/4. Pal went out of station & after returning he finds that rose bush has withered. What is the probability that the gardner did not water the rose bush.

[Ans: 3/4]

[Sol. A = Rose bush has withered

B_1 = Gardener did not water the rose bush $P(B_1) = 2/3$

B_2 = Gardener watered the rose bush $P(B_2) = 1/3$

$P(A/B_1) = \frac{3}{4}$; $P(A/B_2) = \frac{1}{2}$

$$P(B_1/A) = \frac{P(B_1).P(A/B_1)}{P(B_1).P(A/B_1) + P(B_2).P(A/B_2)} = \frac{\frac{2}{3}.\frac{3}{4}}{\frac{2}{3}.\frac{3}{4} + \frac{1}{3}.\frac{1}{2}} = \frac{6}{6+2} = \frac{3}{4} \quad \text{Ans]}$$

Q.4_{66} The probability that a radar will detect an object in one cycle is p. The probability that the object will be detected in n cycles is :

(A) $1 - p^n$ (B*) $1 - (1 - p)^n$ (C) p^n (D) $p(1 - p)^{n-1}$

[Hint: $P(A) = p$
p (object is not detected in one cycle) = $1 - p$
p (object is not detected in *n* cycle) = $(1 - p)^n$
p (object will be detected) = $1 - (1 - p)^n$]

Q.5$_{139}$ Nine cards are labelled 0, 1, 2, 3, 4, 5, 6, 7, 8. Two cards are drawn at random and put on a table in a successive order, and then the resulting number is read, say, 07 (seven), 14 (fourteen) and so on. The probability that the number is even, is

(A*) $\frac{5}{9}$ (B) $\frac{4}{9}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

[Sol. n (S) = number of ways in which two numbers are drawn in a definite order
$= 9 \times 8 = 72$
n (A) = any one number from 0, 2, 4, 6, 8 can be taken in $^5C_1$ ways and any one can be taken from the remaining 8 in $^8C_1$ ways.

Hence total ways $= 8 \times 5 = 40$; $p = \frac{40}{72} = \frac{5}{9}$]

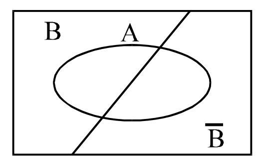
Q.6$_{140}$ Two cards are drawn from a well shuffled pack of 52 playing cards one by one. If
A : the event that the second card drawn is an ace and
B : the event that the first card drawn is an ace card.
then which of the following is true?

(A) $P(A) = \frac{4}{17}$; $P(B) = \frac{1}{13}$ (B*) $P(A) = \frac{1}{13}$; $P(B) = \frac{1}{13}$

(C) $P(A) = \frac{1}{13}$; $P(B) = \frac{1}{17}$ (D) $P(A) = \frac{16}{221}$; $P(B) = \frac{4}{51}$

[Sol. $P(A) = P\{(B \cap A) \cup (\overline{B} \cap A)\} = P(B \cap A) + P(\overline{B} \cap A)$

$= P(B)\,P(A/B) + P(\overline{B})\,P(A/\overline{B}) = \frac{4}{52} \cdot \frac{3}{51} + \frac{48}{52} \cdot \frac{4}{51} = \frac{1}{13}$



$P(B) = \frac{1}{13}$]

Q.7$_{213}$ If $\frac{(1+3p)}{3}, \frac{(1-p)}{4}$ & $\frac{(1-2p)}{2}$ are the probabilities of three mutually exclusive events defined on a sample space S, then the true set of all values of p is

(A*) $\left[\frac{1}{3}, \frac{1}{2}\right]$ (B) $\left[\frac{1}{3}, 1\right]$ (C) $\left[\frac{1}{4}, \frac{1}{3}\right]$ (D) $\left[\frac{1}{4}, \frac{1}{2}\right]$

[Hint: $P(A) \geq 0$; $P(B) \geq 0$; $P(C) \geq 0$; $P(A) + P(B) + P(C) \leq 1$]

Q.8$_{48/5}$ A lot contains 50 defective & 50 non defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as :
A = { the first bulb is defective}; B = { the second bulb is non defective}
C = { the two bulbs are both defective or both non defective}
Determine whether (i) A,B,C are pair wise independent (ii) A,B,C are independent
[Ans: **(i)** A,B,C are pairwise independent **(ii)** A,B,C are not independent.]

[Sol. Let $\begin{cases} 50 \text{ defective} \\ 50 \text{ good} \end{cases}$

A : first bulb is defective
B : second bulb is good
C : two bulbs are either both good or both defective

$P(A) = \frac{1}{2}$ $\quad$ $P(B) = \frac{1}{2}$ $\quad$ $P(C) = \frac{1}{4}$

$$\left.\begin{aligned} P(A \cap B) &= \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4} \\ P(B \cap C) &= \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4} \\ P(C \cap A) &= \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4} \end{aligned}\right\}$$

Note :
$P(A \cap B) = P(A)\cdot P(B)$
$P(B \cap C) = P(B)\cdot P(C)$
$P(C \cap A) = P(C)\cdot P(A)$
Hence the events are pairwise independent.
$P(A \cap B \cap C) = 0 \rightarrow$ Hence, A, B, C are non independent]

Q.9$_{150}$ An Urn contains 'm' white and 'n' black balls. All the balls except for one ball, are drawn from it. The probability that the last ball remaining in the Urn is white, is

(A*) $\frac{m}{m+n}$ $\quad$ (B) $\frac{n}{m+n}$ $\quad$ (C) $\frac{1}{(m+n)!}$ $\quad$ (D) $\frac{mn}{(m+n)!}$

[Hint: $n(S) = {}^{m}C_{m-1} \cdot {}^{n}C_{n}$ $\quad$ Urn $\begin{cases} m\ W \\ n\ W \end{cases}$
$n(A) = {}^{m+n}C_{m+n-1}$

$\Rightarrow \quad P = \frac{m}{m+n}$]

Q.10$_{151}$ A Urn contains 'm' white and 'n' black balls. Balls are drawn one by one till all the balls are drawn. Probability that the second drawn ball is white, is

(A*) $\frac{m}{m+n}$ $\quad$ (B) $\frac{n(m+n-1)}{(m+n)(m+n-1)}$

(C) $\frac{m(m-1)}{(m+n)(m+n-1)}$ $\quad$ (D) $\frac{mn}{(m+n)(m+n-1)}$

[Hint: E : event that 2^{nd} drawn is white

$$P(E) = P(B\,W \text{ or } W\,W) = \frac{n}{m+n}\cdot\frac{m}{m+n-1} + \frac{m}{m+n}\cdot\frac{m-1}{m+n-1}$$

$$= \frac{m(m+n-1)}{(m+n)(m+n-1)} = \frac{m}{m+n} \quad \Rightarrow \quad (A)\ ]$$

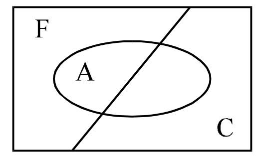
Q.11 Mr. Dupont is a professional wine taster. When given a French wine, he will identify it with probability 0.9 correctly as French, and will mistake it for a Californian wine with probability 0.1. When given a Californian wine, he will identify it with probability 0.8 correctly as Californian, and will mistake it for a French wine with probability 0.2. Suppose that Mr. Dupont is given ten unlabelled glasses of wine, three with French and seven with Californian wines. He randomly picks a glass, tries the wine, and solemnly says : "French". The probability that the wine he tasted was Californian, is nearly equal to

(A) 0.14 (B) 0.24 (C*) 0.34 (D) 0.44

[Sol. P(F/F) = 0.9 ; P(C/F) = 0.1 ; P(C/C) = 0.8 ; P(F/C) = 0.2

$P(F) = \frac{3}{10}$; $P(C) = \frac{7}{10}$

A : Wine tasted was French



B_1 : It is a Californian wine ; $P(B_1) = \frac{7}{10}$

B_2 : It is a French wine ; $P(B_2) = \frac{3}{10}$

$P(A/B_1) = 0.2$; $P(A/B_2) = 0.9$

$P(B_1/A) = \dfrac{0.7\times 0.2}{0.7\times 0.2 + 0.3\times 0.9} = \dfrac{0.14}{0.14+0.27} = \dfrac{14}{41}$ Ans.]

Q.12$_{120}$ Let A, B & C be 3 arbitrary events defined on a sample space 'S' and if, $P(A) + P(B) + P(C) = p_1$, $P(A \cap B) + P(B \cap C) + P(C \cap A) = p_2$ & $P(A \cap B \cap C) = p_3$, then the probability that exactly one of the three events occurs is given by :

(A) $p_1 - p_2 + p_3$ (B) $p_1 - p_2 + 2p_3$ (C) $p_1 - 2p_2 + p_3$ (D*) $p_1 - 2p_2 + 3p_3$

Q.13$_{226}$ Three numbers are chosen at random without replacement from {1, 2, 3,...... , 10}. The probability that the minimum of the chosen numbers is 3 or their maximum is 7 is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D*) $\frac{11}{40}$

[Hint: N = {1, 2,.......10} $\rightarrow$ 3 are drawn

A = minimum of the chosen number is 3

B = maximum number of the chosen number is 7.

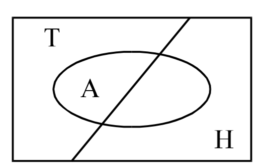
$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B) = \dfrac{{}^7C_2 + {}^6C_2 - {}^3C_1}{{}^{10}C_3}$]

Q.14 A biased coin which comes up heads three times as often as tails is tossed. If it shows heads, a chip is drawn from urn–I which contains 2 white chips and 5 red chips. If the coin comes up tails, a chip is drawn from urn–II which contains 7 white and 4 red chips. Given that a red chip was drawn, what is the probability that the coin came up heads? [Ans. 165/193]

[Sol. A = red chip was drawn

H = coin shows up head ; P(H) = 3/4

T = coin shows up tail ; P(T) = 1/4



Urn-I < 2W, 5R ; Urn-II < 7W, 4R

now $A = (A \cap H) + (A \cap T)$

$P(A) = P(H)\, P(A/T) + P(T)\, P(T/T) = \frac{3}{4}\cdot\frac{5}{7} + \frac{1}{4}\cdot\frac{4}{11} = \frac{15}{28} + \frac{1}{11} = \frac{193}{11\cdot 28}$

now $P(H/A) = \dfrac{P(H\cap A)}{P(A)} = \dfrac{15}{28}\times\dfrac{28\cdot 11}{193} = \dfrac{165}{193}$ Ans.]

Q.15$_{68/5}$ In a college, four percent of the men and one percent of the women are taller than 6 feet. Further 60 percent of the students are women. If a randomly selected person is taller than 6 feet, find the probability that the student is a women. [Ans. 3/11]

[Sol. Let A = person is taller than 6 feet

$P(A / M) = 0.04$; $P(A / W) = 0.01$

$P(M) = 0.4$; $P(W) = 0.6$

$$P(W / A) = \frac{P(W)\cdot P(A/W)}{P(W)\cdot P(A/W) + P(M)\cdot P(A/M)} = \frac{(0.60)\cdot(0.01)}{(0.60)\cdot(0.01)+(0.40)\cdot(0.04)}$$

$$= \frac{60}{60+160} = \frac{6}{22} = \frac{3}{11} \text{ Ans.]}$$

Q.16$_{81}$ If at least one child in a family with 3 children is a boy then the probability that 2 of the children are boys, is

(A*) $\frac{3}{7}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{3}{8}$

[Hint: n (S) = B G G (3) ; B B G (3) ; B B B (1) ; hence n(S) = 7

n (A) = B B G (3) $\Rightarrow$ $p = \frac{3}{7}$]

Q.17$_{116}$ The probabilities of events, $A \cap B$, A, B & $A \cup B$ are respectively in A.P. with probability of second term equal to the common difference. Therefore the events A and B are

(A) compatible (B) independent

(C) such that one of them must occur (D*) such that one is twice as likely as the other

[Hint: $P(A \cap B)$, $P(A)$, $P(B)$, $P(A \cup B)$ are in A.P. with d = P(A)

$\therefore$ $P(A) - P(A \cap B) = P(A)$ $\Rightarrow$ $P(A \cap B) = 0$ $\Rightarrow$ A & B are ME or incompatible

also $P(B) - P(A) = P(A)$ $\Rightarrow$ $2P(A) = P(B)$

$\therefore$ if $P(A) = p$; $P(B) = 2p$ $\Rightarrow$ (D) compatible means whcih can happen simultaneously]

Q.18$_{211}$ From an urn containing six balls, 3 white and 3 black ones, a person selects at random an even number of balls (all the different ways of drawing an even number of balls are considered equally probable, irrespective of their number). Then the probability that there will be the same number of black and white balls among them

(A) $\frac{4}{5}$ (B*) $\frac{11}{15}$ (C) $\frac{11}{30}$ (D) $\frac{2}{5}$

[Sol. Total number of possible cases = 3 (either 2 or 4 or 6 are drawn)

$$\text{Hence required probability} = \frac{1}{3}\left(\frac{{}^3C_1 \times {}^3C_1}{{}^6C_2} + \frac{{}^3C_2 \times {}^3C_2}{{}^6C_4} + \frac{{}^3C_3 \times {}^3C_3}{{}^6C_6}\right) = \frac{11}{15} \Rightarrow \text{(B)]}$$

Q.19$_{241}$ One purse contains 6 copper coins and 1 silver coin ; a second purse contains 4 copper coins. Five coins are drawn from the first purse and put into the second, and then 2 coins are drawn from the second and put into the first. The probability that the silver coin is in the second purse is

(A) $\frac{1}{2}$ (B) $\frac{4}{9}$ (C*) $\frac{5}{9}$ (D) $\frac{2}{3}$

[Sol. (4C + 1S) goes from A to B and 2C return from B to A.

$$\Rightarrow \quad P(E) = \frac{{}^6C_4 \cdot {}^1C_1}{{}^7C_5} \cdot \frac{{}^8C_2}{{}^7C_2} = \frac{15}{21} \cdot \frac{28}{36} = \frac{5}{9} \quad]$$

Q.20$_{31}$ 7 persons are stopped on the road at random and asked about their birthdays. If the probability that 3 of them are born on Wednesday, 2 on Thursday and the remaining 2 on Sunday is $\frac{K}{7^6}$, then K is equal to

(A) 15 (B*) 30 (C) 105 (D) 210

[Hint: $\frac{K}{7^6} = {}^7C_3 . \left(\frac{1}{7}\right)^3 . {}^4C_2 \left(\frac{1}{7}\right)^2 . \left(\frac{1}{7}\right)^2 \quad \Rightarrow \quad K = 30$]

Q.21$_{95}$ Two buses A and B are scheduled to arrive at a town central bus station at noon. The probability that bus A will be late is 1/5. The probability that bus B will be late is 7/25. The probability that the bus B is late given that bus A is late is 9/10. Then the probabilities

(i) neither bus will be late on a particular day and

(ii) bus A is late given that bus B is late, are respectively

(A) 2/25 and 12/28 (B) 18/25 and 22/28 (C*) 7/10 and 18/28 (D) 12/25 and 2/28

[Hint: (i) $P(A) = \frac{1}{5}$; $P(B) = \frac{7}{25}$; $P(B/A) = \frac{9}{10}$

$$P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$$
$$= 1 - [P(A) + P(B) - P(A \cap B)]$$
$$= 1 - \left[\frac{1}{5} + \frac{7}{25} - P(A) \cdot P(B/A)\right]$$
$$= 1 - \left[\frac{1}{5} + \frac{7}{25} - \frac{1}{5} \cdot \frac{7}{25}\right] = \frac{7}{10} \text{ Ans.]}$$

(ii) $$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B/A)}{P(B)}$$

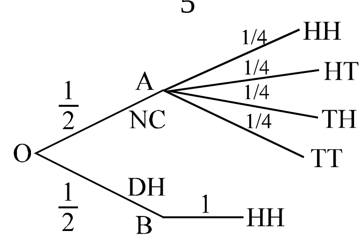
$$= \frac{\frac{1}{5} \cdot \frac{9}{10}}{\frac{7}{25}} = \frac{9}{50} \times \frac{25}{7} = \frac{9}{14} = \frac{18}{28} \text{ Ans.]}$$

Q.22$_{48}$ A box contains a normal coin and a doubly headed coin. A coin selected at random and tossed twice, fell headwise on both the occasions. The probability that the drawn coin is a doubly headed coin is

(A) $\frac{2}{3}$ (B) $\frac{5}{8}$ (C) $\frac{3}{4}$ (D*) $\frac{4}{5}$

[Sol. $A \rightarrow$ Normal coin

$B \rightarrow$ DH coin



$$P(B/HH) = \frac{P(B \cap HH)}{P(HH)}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot 1} = \frac{\frac{1}{2}}{\frac{1}{8} + \frac{4}{8}} = \frac{1}{2} \times \frac{8}{5} = \frac{4}{5} \Rightarrow \text{D]}$$

Q.23$_{56}$ A box contains 5 red and 4 white marbles. Two marbles are drawn successively from the box without replacement and the second drawn marble drawn is found to be white. Probability that the first marble is also while is

(A*) $\frac{3}{8}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

[Sol. Method-1 : since the 2nd is known to be W, there are only 3 ways of the remaining 8 in which the 1st can be white, so that probability = 3/8

Method-2 : A – 2nd drawen found to be white

B_1 - 1st drawn is W ; B_2 – 1st drawn is R

$$P(B_1) = \frac{4}{9} \quad ; \quad P(B_2) = \frac{5}{9}$$

$$P(A / B_1) = \frac{3}{8} \quad ; \quad P(A / B_2) = \frac{4}{8}$$

$$P(B_1 / A) = \frac{\frac{4}{9}\cdot\frac{3}{8}}{\frac{4}{9}\cdot\frac{3}{8}+\frac{5}{9}\cdot\frac{4}{8}} = \frac{12}{12+20} = \frac{12}{32} = \frac{3}{8}$$

Alternatively : 9 – 5R, 4W:
R R = 20/72
R W = 20/72
W W = 12/72
W R = 20/72

$P[(W W) / (R W \text{ or } W W)] = \frac{12}{32} = \frac{3}{8}$ Ans.]

Q.24 A and B in order draw a marble from bag containing 5 white and 1 red marbles with the condition that whosoever draws the red marble first, wins the game. Marble once drawn by them are not replaced into the bag. Then their respective chances of winning are

(A) $\frac{2}{3}$ & $\frac{1}{3}$ (B) $\frac{3}{5}$ & $\frac{2}{5}$ (C) $\frac{2}{5}$ & $\frac{3}{5}$ (D*) $\frac{1}{2}$ & $\frac{1}{2}$

[Sol. Bag – W W W W W ; R **[29-10-2005, 12th Jaipur]**

P(A) = P(R or W W R or W W W W R)

$$= \frac{1}{6} + \frac{5}{6}\cdot\frac{4}{5}\cdot\frac{1}{4} + \frac{5}{6}\cdot\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}; \qquad \therefore \qquad P(B) = \frac{1}{2}$$]

Q.25$_{112}$ In a maths paper there are 3 sections A, B & C. Section A is compulsory. Out of sections B & C a student has to attempt any one. Passing in the paper means passing in A & passing in B or C. The probability of the student passing in A, B & C are p, q & 1/2 respectively. If the probability that the student is successful is 1/2 then :

(A) $p = q = 1$ (B) $p = q = \frac{1}{2}$ (C) $p = 1, q = 0$ (D*) $p = 1, q = \frac{1}{2}$

[Hint: $p(S) = P\big(A \text{ and } (B \text{ or } C)\big) = p \cdot \frac{1}{2}\left(q + \frac{1}{2}\right)$

$$\frac{1}{2} = \frac{p}{2}\left(q+\frac{1}{2}\right); \qquad 1 = p\left(q+\frac{1}{2}\right) \Rightarrow \text{(D)}$$]

Q.26$_{210}$ A box contains 100 tickets numbered 1, 2, 3,.... ,100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The minimum number on them is 5, with probability

(A*) $\frac{1}{9}$ (B) $\frac{2}{11}$ (C) $\frac{3}{19}$ (D) none

[Hint: N = {1, 2, ...5, ...10, ...100}
two tickets are drawn
A : maximum number on the two chosen ticket is $\le 10 \Rightarrow$ n(S) = 10
B : minimum number on the two chosen ticket is 5

$$P(B / A) = \frac{P(A \cap B)}{P(A)} = \frac{{}^{5}C_1}{{}^{10}C_2} = \frac{5}{45} = \frac{1}{9}$$ [one of the ticket is 5 and one is frm 6, 7, 8, 9, 10]]

Q.27$_{240}$ Sixteen players $s_1, s_2, \ldots, s_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength. The probability that "exactly one of the two players s_1 & s_2 is among the eight winners" is

(A) $\frac{4}{15}$ (B) $\frac{7}{15}$ (C*) $\frac{8}{15}$ (D) $\frac{9}{15}$

[Hint: 7 players (leaving s_1 & s_2) out of 14 can be selected in ${}^{14}C_7$ and the 8th player can be chosen in two ways i.e. either s_1 or s_2. Hence the total ways = ${}^{14}C_7 . 2$

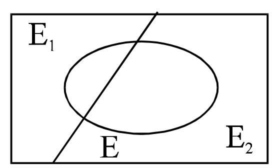
Therefore $$p = \frac{2 \cdot {}^{14}C_7}{{}^{16}C_8} = \frac{8}{15}$$]

[Alternatively : Let

E_1 : S_1 and S_2 are in the same group
E_2 : S_1 and S_2 are in the different group
E : exactly one of the two players S_1 & S_2 is among the eight winners.

$E = (E \cap E_1) + (E \cap E_2)$
$P(E) = P(E \cap E_1) + p(E \cap E_2)$
$P(E) = P(E_1).P(E/E_1) + p(E_2).P(E/E_2)$ ....(1)

Now $$P(E_1) = \frac{\frac{(14)!}{(2)^7 . 7!}}{\frac{16!}{2^8 . 8!}} = \frac{1}{15}$$



$$P(E_2) = 1 - \frac{1}{15} = \frac{14}{15}$$

$$P(E) = \frac{1}{15}.1 + \frac{14}{15} \cdot P(\text{ exactly one of either } S_1 \text{ \& } S_2 \text{ wins})$$

$$= \frac{1}{15} + \frac{14}{15} . \left(\frac{1}{2} . \frac{1}{2} + \frac{1}{2} . \frac{1}{2}\right) = \frac{1}{15} + \frac{1}{14} . \frac{1}{2} = \frac{1}{15} + \frac{7}{15} = \frac{8}{15}$$ Ans]

Q.28 The number 'a' is randomly selected from the set {0, 1, 2, 3, ... 98, 99}. The number 'b' is selected from the same set. Probability that the number $3^a + 7^b$ has a digit equal to 8 at the units place, is

(A) $\frac{1}{16}$ (B) $\frac{2}{16}$ (C) $\frac{4}{16}$ (D*) $\frac{3}{16}$

[Hint:

3^a ends in $\rightarrow$ 7^b ends in $\downarrow$	1	3	7	9
1			8	
3				
7	8			
9				8

[27-11-2005, 12th]

Out of 16 case 3 are favorable $\Rightarrow$ $p = \frac{3}{16}$]

Q.29 We are given two urns as follows :

Urn A contains 5 red marbles, 3 white marbles and 8 blue marbles.

Urn B contains 3 red marbles and 5 white marbles

A fair dice is tossed if 3 or 6 appears, a marble is chosen from B, otherwise a marble is chosen from A. Find the probability that (i) a red marble is chosen, (ii) a white marble is chosen, (iii) a blue marble is chosen. (Use Tree Diagram)

[Ans. (i) $\frac{1}{3}$; (ii) $\frac{1}{3}$; (iii) $\frac{1}{3}$]

Q.30 We are given two Urns as follows :

Urn A contains 5 red marbles, 3 white marbles.

Urn B contains 1 red marbles and 2 white marbles.

A fair die is tossed, if a 3 or 6 appears, a marble is drawn from B and put into A and then a marble is drawn from A ; otherwise, a marble is drawn from A and put into B and then a marble is drawn from B. (Use Tree Diagram)

(i) What is the probability that both marbles are red?

(ii) What is the probability that both marbles are white?

[Ans. (i) $\frac{61}{216}$; (ii) $\frac{371}{1296}$]

Q.31 Two boys A and B find the jumble of *n* ropes lying on the floor. Each takes hold of one loose end. If the probability that they are both holding the same rope is $\frac{1}{101}$ then the number of ropes is equal to

(A) 101 (B) 100 (C*) 51 (D) 50

[Sol. The n strings have a total of 2n ends. One boy picks up one end, this leaves (2n – 1) ends for the second boy to choose, of which only one is correct.

$\therefore \quad p = \frac{1}{2n-1} \Rightarrow \frac{1}{2n-1} = \frac{1}{101} \Rightarrow 2n - 1 = 101 \Rightarrow n = 51$] **[08-01-2006, 12 & 13]**

Direction for Q.32 to Q.35 (4 Questions)
Read the passage given below carefully before attempting these questions.

A standard deck of playing cards has 52 cards. There are four suit (clubs, diamonds, hearts and spades), each of which has thirteen numbered cards (2, ..., 9, 10, Jack, Queen, King, Ace)

In a game of card, each card is worth an amount of points. Each numbered card is worth its number (e.g. a 5 is worth 5 points) ; the Jack, Queen and King are each worth 10 points ; and the Ace is either worth your choice of either 1 point or 11 points. The object of the game is to have more points in your set of cards than your opponent without going over 21. Any set of cards with sum greater than 21 automatically loses.

Here's how the game played. You and your opponent are each dealt two cards. Usually the first card for each player is dealt face down, and the second card for each player is dealt face up. After the initial cards are dealt, the first player has the option of asking for another card or not taking any cards. The first player can keep asking for more cards until either he or she goes over 21, in which case the player loses, or stops at some number less than or equal to 21. When the first player stops at some number less than or equal to 21, the second player then can take more cards until matching or exceeding the first player's number without going over 21, in which case the second player wins, or until going over 21, in which case the first player wins.
We are going to simplify the game a little and assume that all cards are dealt face up, so that all cards are visible. Assume your opponent is dealt cards and plays first.

Q.32 The chance that the second card will be a heart and a Jack, is

(A) $\frac{4}{52}$ (B) $\frac{13}{52}$ (C) $\frac{17}{52}$ (D*) $\frac{1}{52}$

Q.33 The chance that the first card will be a heart or a Jack, is

(A) $\frac{13}{52}$ (B*) $\frac{16}{52}$ (C) $\frac{17}{52}$ (D) none

Q.34 Given that the first card is a Jack, the chance that it will be the heart, is

(A) $\frac{1}{13}$ (B) $\frac{4}{13}$ (C*) $\frac{1}{4}$ (D) $\frac{1}{3}$

Q.35 Your opponent is dealt a King and a 10, and you are dealt a Queen and a 9. Being smart, your opponent does not take any more cards and stays at 20. The chance that you will win if you are allowed to take as many cards as you need, is

(A) $\frac{97}{564}$ (B) $\frac{25}{282}$ (C) $\frac{15}{188}$ (D*) $\frac{1}{6}$

[Sol. P (2^{nd} card is J of H) = $\frac{51}{52}\cdot\frac{1}{51}=\frac{1}{52}$ **Ans.(32)** **[29-01-2006, 12th & 13th]**

There are 16 ways to get a Jack or a hearts : get one of the thirteen hearts (Ace through King of hearts), or get one of the Jack of clubs, Jack of spades, or Jack of diamonds.
Hence, probability (Jack or hearts) = 16/52. **Ans.(33)**

$$P(H / J) = \frac{P(H\cap J)}{P(J)} = \frac{1}{4}$$ **Ans.(34)**

P(win) = P(any 2's or Ace or Ace and Ace) = $\frac{4}{48}+\frac{4}{48}=\frac{1}{6}$ **Ans.(35)**]

More than one alternative are correct:

Q.36$_{230}$ If A & B are two events such that $P(B) \neq 1$, B^C denotes the event complementry to B, then

(A*) $P\left(A/B^C\right) = \dfrac{P(A) - P(A \cap B)}{1 - P(B)}$

(B*) $P(A \cap B) \geq P(A) + P(B) - 1$

(C*) $P(A) > < P\left(A/B\right)$ according as $P\left(A/B^C\right) > < P(A)$

(D*) $P\left(A/B^C\right) + P\left(A^C/B^C\right) = 1$

[Sol. (B) $1 \geq P(A) + P(B) - P(A \cap B)$ or $P(A \cap B) \geq P(A) + P(B) - 1$ Þ (B)

(C) Let $P(A) > P(A/B)$

or $P(A) > \dfrac{P(A \cap B)}{P(B)}$

$P(A).\,P(B) > P(A \cap B)$ ....(1)

TPT $P(A/B^C) > P(A)$

$\dfrac{P(A \cap B^c)}{P(B^c)} > P(A)$

$P(A) - P(A \cap B) > P(A)\,[\,1 - P(B)]$

$-P(A \cap B) > -P(A).\,P(B)$

or $P(A).\,P(B) > P(A \cap B)$ ....(2)

from (1) and (2) $P(A) > P(A/B) \Rightarrow P(A/B^c) > P(A)$]

Q.37$_{225}$ A bag initially contains one red & two blue balls. An experiment consisting of selecting a ball at random, noting its colour & replacing it together with an additional ball of the same colour. If three such trials are made, then :

(A*) probability that atleast one blue ball is drawn is 0.9

(B*) probability that exactly one blue ball is drawn is 0.2

(C*) probability that all the drawn balls are red given that all the drawn balls are of same colour is 0.2

(D*) probability that atleast one red ball is drawn is 0.6.

[Hint : (i) $P(E_1) = 1 - P(R\,R\,R)$

$= 1 - \left[\dfrac{1}{3} \cdot \dfrac{2}{4} \cdot \dfrac{3}{5}\right] = 0.9$ Bag $\begin{cases} R \\ B\,B \end{cases}$

(ii) $P(E_2) = 3\,P(B\,R\,R) = 3.\ \dfrac{2}{3} \cdot \dfrac{1}{4} \cdot \dfrac{2}{5} = 0.2$

(iii) $P(E_3) = P(R\,R\,R/R\,R\,R \cup B\,B\,B) = \dfrac{P(RRR)}{P(RRR) + P(BBB)}$

but $P(B\,B\,B) = \dfrac{2}{3} \cdot \dfrac{3}{4} \cdot \dfrac{4}{5} = \dfrac{8}{20} \Rightarrow P(E_3) = \dfrac{0.1}{0.1 + 0.4} = 0.2$

(iv) $P(E_4) = 1 - P(B\,B\,B) = 1 - \dfrac{2}{5} = 0.6$]

Q.38$_{227}$ Two real numbers, x & y are selected at random. Given that $0 \le x \le 1$; $0 \le y \le 1$. Let A be the event that $y^2 \le x$; B be the event that $x^2 \le y$, then :

(A*) $P(A \cap B) = \frac{1}{3}$ (B*) A & B are exhaustive events

(C) A & B are mutually exclusive (D) A & B are independent events.

Q.39$_{221}$ For any two events A & B defined on a sample space ,

(A*) $P(A/B) \ge \frac{P(A) + P(B) - 1}{P(B)}$, $P(B) \ne 0$ is always true

(B*) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$

(C*) $P(A \cup B) = 1 - P(A^c).\ P(B^c)$, if A & B are independent

(D) $P(A \cup B) = 1 - P(A^c).\ P(B^c)$, if A & B are disjoint

[Hint: For A $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow$ T.P.T. $P(A \cap B) \ge P(A) + P(B) - 1$

or T.P.T. , $1 \ge P(A) + P(B) - P(A \cap B)$

or T.P.T. , $1 \ge P(A \cap B)$

which is true $\Rightarrow$ (A) is correct

(B) and (C) are obvious]

Q.40$_{205}$ If E_1 and E_2 are two events such that $P(E_1) = 1/4$, $P(E_2/E_1) = 1/2$ and $P(E_1/E_2) = 1/4$

(A*) then E_1 and E_2 are independent

(B) E_1 and E_2 are exhaustive

(C*) E_2 is twice as likely to occur as E_1

(D*) Probabilities of the events $E_1 \cap E_2$, E_1 and E_2 are in G.P.

[Hint : $P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$

$$\frac{1}{2} = \frac{P(E_1 \cap E_2)}{1/4} \Rightarrow P(E_1 \cap E_2) = \frac{1}{8} = P(E_2).\ P(E_1/E_2)$$

$$= P(E_2).\ \frac{1}{4} \Rightarrow P(E_2) = \frac{1}{2}$$

Since $P(E_1 \cap E_2) = \frac{1}{8} = P(E_1).\ P(E_2)$ $\Rightarrow$ events are independent

Also $P(E_1 \cup E_2) = \frac{1}{2} + \frac{1}{4} - \frac{1}{8} = \frac{5}{8}$ $\Rightarrow$ E_1 & E_2 are non exhaustive]

Q.41$_{179}$ Let $0 < P(A) < 1$, $0 < P(B) < 1$ & $P(A \cup B) = P(A) + P(B) - P(A).\ P(B)$, then :

(A) $P(B/A) = P(B) - P(A)$ (B) $P(A^C \cup B^C) = P(A^C) + P(B^C)$

(C*) $P((A \cup B)^C) = P(A^C).\ P(B^C)$ (D*) $P(A/B) = P(A)$

Q.42$_{27}$ If M & N are independent events such that $0 < P(M) < 1$ & $0 < P(N) < 1$, then :

(A) M & N are mutually exclusive (B*) M & $\overline{N}$ are independent

(C*) $\overline{M}$ & $\overline{N}$ are independent (D*) $P(M/N) + P(\overline{M}/N) = 1$