Practice Problems : Limit, Continuity, Differentiability

1. If $f(x)=\begin{cases}\sin x+x\sin\left(\frac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$, then $f(x)$ is :

(a) continuous at everywhere (b) discontinuous at $x=0$

(c) $\lim\limits_{x\to 0} f(x)$ does not exist (d) none of the above

2. Find the value of $\lim\limits_{x\to 0}\frac{e^{[|\sin x|]}}{[x+1]}$, (where [ ] greatest integer) is :

(a) 1 (b) ∞ (c) does not exist (d) none of these

3. If $f(x)=\frac{\sin(\pi[x^2])}{[x]^2+[x]+1}$, then $f(x)$ is :

(a) discontinuous at $x=0$ (b) non-differentiable at $x=0$

(c) an odd as well as an even function (d) none of the above

4. If $f(x)=\begin{cases}x, & \text{if } x \text{ is rational}\\ -x, & \text{if } x \text{ is irrational}\end{cases}$, then :

(a) $f(x)$ is an odd function (b) $f(x)$ is continuous at $x=\frac{1}{2}$

(c) $f(x)$ is continuous at $x=0$ (d) $f(x)$ is a periodic function

5. The number of points of non-differentiability for the function $f(x)=|x|+|\cos x|+\tan\left(x+\frac{\pi}{4}\right)$ in (-1, 2) is :

(a) 1 (b) 2 (c) 3 (d) 4

6. If $f(x)=x+|x|+\cos[\pi^2]x$ and $g(x)=\sin x$, then :

(a) $f(x)+g(x)$ is continuous everywhere (b) $f(x)+g(x)$ is differentiable everywhere

(c) $f(x)-g(x)$ is differentiable everywhere (d) $f(x)\times g(x)$ is continuous but not differentiable at

7. Let $f(x)=\begin{cases}-1, & x<0\\ 0, & x=0\\ 1, & x>0\end{cases}$ and $g(x)=\sin x+\cos x$. Then points of discontinuity of $f(g(x))$ in $(0, 2\pi)$ are:

(a) $\frac{\pi}{4},\frac{5\pi}{4}$ (b) $\frac{\pi}{4},\frac{3\pi}{4}$ (c) $\frac{\pi}{4},\frac{7\pi}{4}$ (d) $\frac{3\pi}{4},\frac{7\pi}{4}$

8. $\lim\limits_{x\to 0}\frac{\sin[\cos x]}{1+[\cos x]}$ ([ ] denotes the greatest integer function) :

(a) equal to 1 (b) equal to 0 (c) does not exist (d) none of these

9. If α and β be the roots of $ax^2+bx+c=0$, then $\lim\limits_{x\to\alpha}(1+ax^2+bx+c)^{1/x-\alpha}$ is :

(a) $a(\alpha-\beta)$ (b) $\log|a(\alpha-\beta)|$ (c) $e^{a(\alpha-\beta)}$ (d) $e^{a(\alpha+\beta)}$

10. The value of $\lim\limits_{x\to 0}\left(1-\frac{1}{2^x}\right)\left(\frac{1}{\sqrt{\tan x+4}-2}\right)$ is :

(a) log 16 (b) cannot exist (c) 3 log 2 (d) 6 log 2

11. The function $f(x)=[x]\cos\left(\frac{2x-1}{2}\right)\pi$, where $[\cdot]$ denotes the greatest integer function, is discontinuous at :

(a) all (b) all integer points (c) no point (d) x which is not an integer

12. Let $f(x)$ be a continuous function defined for $1\le x\le 3$. If $f(x)$ take rational values for all x and $f(2)=10$, then $f(1.5)$ is equal to :

(a) 10 (b) 7.5 (c) 5 (d) none of these

13. Let $h(x)=\min\{x, x^2\}$, for every real number of x. Then :

(a) h is continuous for all x except 0 (b) h is differentiable for all x

(c) $h'(x) = 1$, for all real x (d) h is not differentiable at two values of x

14. The function $f(x) = [x]^2 - [x^2]$ (where $[y]$ is the greatest integer less than or equal to y), is discontinuous:

(a) all integers (b) all integers except 0 and 1

(c) all integers except 0 (d) all integers except 1

15. The function $f(x) = (x^2 - 1)\left|x^2 - 3x + 2\right| + \cos(|x|)$ is not differentiable at :

(a) – 1 (b) 0 (c) 1 (d) 2

16. The left hand derivative of $f(x) = [x]\ \sin(\pi x)$ at $x = k$, k an integer, is :

(a) $(-1)^k (k-1)\pi$ (b) $(-1)^{k-1}(k-1)\pi$ (c) $(-1)^k k\pi$ (d) $(-1)^{k-1} k\pi$

17. Which of the following functions is differentiable at $x = 0$?

(a) $\cos(|x|) + |x|$ (b) $\cos(|x|) - |x|$ (c) $\sin(|x|) + |x|$ (d) $\sin(|x|) - |x|$

18. Let $f : R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$. Then $\lim\limits_{x \to 0}\left(\dfrac{f(1+x)}{f(1)}\right)^{1/x}$ is equal to :

(a) 1 (b) $e^{1/2}$ (c) e^2 (d) e^3

19. If $\lim\limits_{x \to 0} \dfrac{((a-n)nx - \tan x)\sin nx}{x^2} = 0$, where n is non-zero real number, then a is equal to :

(a) 0 (b) $\dfrac{n+1}{n}$ (c) n (d) $n + \dfrac{1}{n}$

20. Let $f(x) = \dfrac{4^x}{4^x + 2}$. Then $f(x) + f(1-x)$ is equal to:

(a) 0 (b) 1 (c) – 1 (d) none of these

21. If $\lim\limits_{x \to a} \dfrac{a^x - x^a}{x^x - a^a} = -1$, then the value of a is:

(a) 1 (b) 0 (c) c (d) none of these

22. The value of $\lim\limits_{x \to 0} \dfrac{(1+x)^{1/x} - e}{x}$ is:

(a) (b) $\dfrac{e}{2}$ (c) $-\dfrac{e}{2}$ (d) $\dfrac{2}{e}$

23. $\lim\limits_{x \to 0} \dfrac{\sin x^n}{(\sin x)^m}, n > m > 0$ is equal to:

(a) 1 (b) 0 (c) $\dfrac{n}{m}$ (d) $\dfrac{m}{n}$

24. $\lim\limits_{x \to 0} \dfrac{\sin(\pi \cos^2 x)}{x^2}$ equals:

(a) $-\pi$ (b) π (c) $\dfrac{\pi}{2}$ (d) 1

25. If $\lim\limits_{x \to 0} \phi(x) = a^3, a \neq 0$, then $\lim\limits_{x \to 0} \phi\left(\dfrac{x}{a}\right)$ is:

(a) a^2 (b) $\dfrac{1}{a^3}$ (c) $\dfrac{1}{a^2}$ (d) a^3

Q	1	2	3	4	5	6	7	8	9	10
A	A	C	C	C	C	A	D	B	C	A
Q	11	12	13	14	15	16	17	18	19	20
A	C	A	D	D	D	A	D	C	D	B
Q	21	22	23	24	25	Ans Key				
	A	C	B	B	D					

MATHS ASSIGNMENT : LIMIT & CNTD..

1. The value of $f(0)$ so that the function $f(x)=\frac{1-\cos(1-\cos x)}{x^4}$ is continuous everywhere is

(a) 1/8 (b) 1/2 (c) 1/4 (d) none of these

2. If $f(x)=\frac{1+\sin x-\cos x}{1-\sin x-\cos x}$ is not defined at $x=0$. The value of $f(0)$ so that $f(x)$ is continuous at $x=0$, is

(a) 1 **(b)** -1 (c) 0 (d) none of these

3. The number of points at which the function $f(x)=1/(x-[x])$ is not continuous is

(a) 1 (b) 2 (c) 3 **(d)** none of these

4. $\lim\limits_{x\to 0}\frac{x\cos x-\log(1+x)}{x^2}$ is equal to

(a) 1/2 (b) 0 (b) 1 (d) none of these

5. For $m, n\in I^+$, $\lim\limits_{x\to 0}\frac{\sin x^n}{(\sin x)^m}$ is equal to

(a) 1 **(b)** 0 (c) n/m **(d)** ∞

6. $\lim\limits_{x\to 0}\frac{\sin[\cos x]}{1+[\cos x]}$ ([ ] denotes the greatest integer function) :

(a) equal to 1 **(b)** equal to 0 (c) does not exist (d) none of these

7. Let $f(x)=\begin{cases}-2, & x<0\\ x-2, & x\geq 0\end{cases}$ and $g(x)=|f(x)|$, then :

(a) $g(x)$ is continuous for all values of x (b) $g(x)$ is differentiable for all values of x

(c) $g(x)$ is differentiable in R (d) none of the above

8. $\lim\limits_{x\to\infty}\frac{\log x-[x]}{[x]}$, ([·] denotes the greatest integer function) :

(a) has value -1 (b) has value 0 (c) has value 1 (d) does not exist

9. The function $f(x)=[x]\cos\left(\frac{2x-1}{2}\right)\pi$, where [·] denotes the greatest integer function, is discontinuous at :

(a) all (b) all integer points **(c)** no point (d) x which is not an integer

10. $\lim\limits_{x\to\infty}\frac{2(x)^{1/2}+3(x)^{1/3}+4(x)^{1/4}+.....+n(x)^{1/n}}{(2x-3)^{1/2}+(2x-3)^{1/3}+.....+(2x-3)^{1/n}}$ is

(a) 2 **(b)** $\sqrt{2}$ (c) 3 (d) none of these

11. For $f(x)=\frac{1}{1-x}$. Then no of points of discontinuity of function $y=f^{3n}(x)$, if $f^n(x)=fofo.....$ of (n times)

(a) 1 **(b)** 2 (c) 0 (d) none of these

12. Let $f(x)=\lim\limits_{x\to\infty}(\sin x)^{2n}$. Then f is

(a) continuous at $x=\frac{\pi}{2}$ **(b)** discontinuous at $x=\frac{\pi}{2}$

(c) discontinuous at an infinite no of points **(d)** discontinuous at $x=-\frac{\pi}{2}$.

13. With usual notation for the greatest integer function $f(x)=[x]^3-[x^3]$ is discontinuous at all

(a) integers n **(b)** integers $n\neq 1$

(c) integers $n\neq 1$, since $f(n^-)\neq f(n)$ (d) integers $n\neq 1$, since $f(n^+)\neq f(n)$.

14. If $f(x)=\frac{\cos(\sin x)-\cos x}{x^2}$, $x\neq 0$ & $f(0)=a$ If $f(x)$ is continuous at $x=0$, then $a=.....$

(a) 0 (b) 4 (c) 5 (d) 6.

15. If $f(x)=px^2-q, x\in[0,1]$

$=x+1$ if $x\in[1,2]$ & $f(1)=2$, then the pair (p, q) for which $f(x)$ cannot be continuous at $x=1$ is

(a) (2, 0) (b) (1, - 1) (c) (4, 2) **(d)** (1, 1).

16. Let $f(x)$ be a continuous function defined on [1, 3]. If $f(x)$ takes rational values for all x and $f(2) = 10$, then the value of $(1 \cdot 5)$ is .....

(a) $7 \cdot 5$ **(b)** 10 (c) 5 (d) none of these

17. $\lim_{x \to 2^+} \left(\frac{|x|^3}{3} - \left[\frac{x}{3} \right]^3 \right)$, where $[x]$ is the greatest integer less than or equal to x is

(a) $\frac{5}{3}$ **(b)** $\frac{8}{3}$ (c) $\frac{7}{9}$ (d) none of these

18. Total number of points of discontinuity of $f(x) = [\sin x + \cos x]$, where [.] denotes the greatest integer function in $(\pi, 2\pi)$ is equal to

(a) 9 (b) 6 (c) 8 **(d)** 5

19. If $\lim_{x \to 0} \frac{1 - 3^x - 4^x + 12^x}{\sqrt{2\cos x + 7} - 3} = K$, then which of the following is true

(a) $-6, \log_e 12$ **(b)** $-6 \log_e 4 . \log_e 3$ (c) $-12 \log_e 4$ (d) $-3 \log_e \left(\frac{4}{3} \right)$.

20. If $f(x) = \lim_{x \to \infty} \frac{(1 + \sin \pi x)^n - 1}{(1 + \sin \pi x)^n + 1}$ then which of the following is false?

(a) f is cntd at $x = 1$ (b) f is not cntd at $x = 1$ (c) $\lim_{x \to 1^-} f(x) = 1$ (d) $\lim_{x \to 1^+} f(x) = -1$.

Find the correct ans:

21. $\lim_{n \to \infty} \sum_{x=1}^{10} \cos^{2n}(x - 5)$ is equal to

(a) 0 (b) 1 (c) 9 (d) 10

22. If $\lim_{x \to 0} \frac{x^{2n} \sin^n x}{x^{2n} - \sin^{2n} x}$ is a non zero finite number, then n must be equal to

(a) 1 (b) 2 (c) 3 (d) none of these

23. $\lim_{x \to \infty} \left(\frac{x + 2h}{x - h} \right)^x = 2$ then the value of h is

(a) $\log_3^2$ (b) $\ln 2^3$ (c) $\ln 2^{1/3}$ (d) none of these

24. $\lim_{n \to \infty} \frac{1}{n} (n^2 - 1) \sin \frac{\pi}{n}$ is equal to

(a) -1 (b) 1 (c) $-\pi$ (d) π

25. $\lim_{n \to \infty} \left(\frac{1^3}{n^4} + \frac{3^3}{n^4} + \frac{5^3}{n^4} + ... + \frac{(2n-1)^3}{n^4} \right)$ equals

(a) 0 (b) 2 (c) 4 (d) none of these

26. $\lim_{x \to 0} \frac{\int_0^x (\tan^{-1} x)^2 \, dx}{\sqrt{x^2 + 1}}$ is

(a) -1 (b) 1 (c) 0 (d) none of these

27. If a is an integer then $\lim_{x \to a} (a - x) + [x - a] + [a - x]$ is

(a) $\frac{a}{2}$ (b) $2a$ (c) independent of a (d) does not exist

28. If $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{k/x}$ $(a, b, c, k > 0)$

(a) equals 1 if $k = 1$ (b) equals abc if $k = 1$ (c) equals abc if $k = 3$ (d) equals $(a^2 b^2 c^2)^{1/3}$ if $k = 2$