A

3021

Register Number | | | | | |

Part III — MATHEMATICS

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 200

SECTION - A

N. B. : i) *All* questions are compulsory.

ii) Each question carries *one* mark.

iii) Choose the most suitable answer from the given four alternatives. $40 \times 1 = 40$

1. The projection of $\vec{OP}$ on a unit vector $\vec{OQ}$ equals thrice the area of parallelogram *OPRQ*. Then $\angle POQ$ is

a) $\tan^{-1}\left(\frac{1}{3}\right)$ b) $\cos^{-1}\left(\frac{3}{10}\right)$

c) $\sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$ d) $\sin^{-1}\left(\frac{1}{3}\right)$.

2. The value of $\left[\, \vec{i}+\vec{j} \quad \vec{j}+\vec{k} \quad \vec{k}+\vec{i}\, \right]$ is

a) 0 b) 1

c) 2 d) 4.

[Turn over

3. If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ for non-coplanar vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ then

a) $\vec{a}$ is parallel to $\vec{b}$

b) $\vec{b}$ is parallel to $\vec{c}$

c) $\vec{c}$ is parallel to $\vec{a}$

d) $\vec{a} + \vec{b} + \vec{c} = \vec{0}$.

4. The vector equation of a plane passing through a point whose position vector is $\vec{a}$ and perpendicular to a vector $\vec{n}$ is

a) $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

b) $\vec{r} \times \vec{n} = \vec{a} \times \vec{n}$

c) $\vec{r} + \vec{n} = \vec{a} + \vec{n}$

d) $\vec{r} - \vec{n} = \vec{a} - \vec{n}$.

5. The centre and radius of the sphere $\left| \vec{r} - (2\vec{i} - \vec{j} + 4\vec{k}) \right| = 5$ are

a) (2, – 1, 4) and 5

b) (2, 1, 4) and 5

c) (– 2, 1, 4) and 6

d) (2, 1, – 4) and 5.

6. The locus of foot of perpendicular from the focus to a tangent of the curve

$16x^2 + 25y^2 = 400$ is

a) $x^2 + y^2 = 4$

b) $x^2 + y^2 = 25$

c) $x^2 + y^2 = 16$

d) $x^2 + y^2 = 9$.

7. One of the foci of the rectangular hyperbola $xy = 18$ is

a) (6, 6)

b) (3, 3)

c) (4, 4)

d) (5, 5).

8. The chord of contact of tangents from any point on the directrix of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, passes through its

a) vertex

b) focus

c) directrix

d) latus rectum.

9. The velocity v of a particle moving along a straight line when at a distance x from the origin is given by $a + bv^2 = x^2$, where a and b are constants. Then the acceleration is

a) $\frac{b}{x}$

b) $\frac{a}{x}$

c) $\frac{x}{b}$

d) $\frac{x}{a}$.

10. The angle between the curves $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and $\frac{x^2}{8} - \frac{y^2}{8} = 1$ is

a) $\frac{\pi}{4}$

b) $\frac{\pi}{3}$

c) $\frac{\pi}{6}$

d) $\frac{\pi}{2}$.

11. The area bounded by the line $y = x$, the x-axis, the ordinates $x = 1$ and $x = 2$ is

a) $\frac{3}{2}$

b) $\frac{5}{2}$

c) $\frac{1}{2}$

d) $\frac{7}{2}$.

12. The volumes of the solid obtained by revolving the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about major and minor axes are in the ratio

a) $b^2 : a^2$

b) $a^2 : b^2$

c) $a : b$

d) $b : a$.

13. $\int_0^\infty x^6 e^{-x/2}\, dx =$

a) $\frac{\lfloor 6}{2^7}$

b) $\frac{\lfloor 6}{2^6}$

c) $2^6 \lfloor 6$

d) $2^7 \lfloor 6$.

14. If $\cos x$ is an integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$, then $P =$

a) $-\cot x$

b) $\cot x$

c) $\tan x$

d) $-\tan x$.

15. The differential equation of all circles with centre at the origin is

a) $x\,dy + y\,dx = 0$

b) $x\,dy - y\,dx = 0$

c) $x\,dx + y\,dy = 0$

d) $x\,dx - y\,dy = 0$.

16. Which of the following is a tautology ?

a) $p \vee q$

b) $p \wedge q$

c) $p \vee \sim p$

d) $p \wedge \sim p$.

17. X is a discrete random variable which takes the values 0, 1, 2 and

$P(X = 0) = \frac{144}{169}$, $P(X = 1) = \frac{1}{169}$, then the value of $P(X = 2)$ is

a) $\frac{145}{169}$

b) $\frac{24}{169}$

c) $\frac{2}{169}$

d) $\frac{143}{169}$.

18. Variance of a random variable X is 4 and mean is 2. Then $E\left(X^2\right)$ is

a) 2

b) 4

c) 6

d) 8.

19. If in a Poisson distribution $P(X=0)=k$, then the variance is

a) $\log\left(\frac{1}{k}\right)$

b) $\log k$

c) e^k

d) $\frac{1}{k}$.

20. For a standard normal distribution, the mean and variance are

a) μ, σ^2

b) μ, σ

c) 0, 1

d) 1, 1.

21. If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then the rank of AA^T is

a) 3

b) 0

c) 1

d) 2.

22. If A and B are any two matrices such that $AB = 0$ and A is non-singular, then

a) $B = 0$

b) B is singular

c) B is non-singular

d) $B = A$.

23. The system of equations $ax + y + z = 0$, $x + by + z = 0$, $x + y + cz = 0$ has a non-trivial solution, then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} =$

a) 1 b) 2

c) -1 d) 0.

24. If $\rho(A) = \rho[A, B]$ then the system is

a) consistent and has infinitely many solutions

b) consistent and has a unique solution

c) consistent

d) inconsistent.

25. If $\vec{p}$, $\vec{q}$ and $\vec{p} + \vec{q}$ are vectors of magnitude λ, then $|\vec{p} - \vec{q}|$ is

a) 2λ b) $\sqrt{3}\,\lambda$

c) $\sqrt{2}\,\lambda$ d) 1.

26. If z represents a non-zero complex number then $arg(z) + arg(\bar{z})$ is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$

c) 0 d) $-\frac{\pi}{4}$.

27. If $x = \cos\theta + i\sin\theta$ then $x^n + \frac{1}{x^n}$ is

a) $2\cos n\theta$ b) $2i\sin n\theta$

c) $2\sin n\theta$ d) $2i\cos n\theta$.

28. If ω is a complex cube root of unity then the value of

$(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8)$ is

a) 9 b) -9

c) 16 d) 32.

29. Which of the following is incorrect ?

a) $Re(z) \leq |z|$

b) $Im(z) \leq |z|$

c) $z\bar{z} = |z|^2$

d) $Re(z) \geq |z|$.

30. The length of the latus rectum of the parabola $y^2 - 4x + 4y + 8 = 0$ is

a) 8 b) 6

c) 4 d) 2.

31. The C of Lagrange's Mean value theorem for the function $f(x) = x^2 + 2x - 1$, $a = 0$, $b = 1$ is

a) -1 b) 1

c) 0 d) $\frac{1}{2}$.

32. The statement : "If f has a local extremum (minimum or maximum) at c and if $f'(c)$ exists then $f'(c) = 0$" is

a) the extreme value theorem

b) Fermat's theorem

c) Law of Mean

d) Rolle's theorem.

33. If $u = f\left(\frac{y}{x}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

a) 0 b) 1

c) $2u$ d) u.

34. In which region does the curve $y^2(a + x) = x^2(3a - x)$ not lie ?

a) $x > 0$

b) $0 < x < 3a$

c) $x \leq -a$ and $x > 3a$

d) $-a < x < 3a$.

35. The value of $\int_0^1 x(1 - x)^4 \, dx$ is

a) $\frac{1}{12}$ b) $\frac{1}{30}$

c) $\frac{1}{24}$ d) $\frac{1}{20}$.

36. The differential equation obtained by eliminating a and b from $y = ae^{3x} + be^{-3x}$ is

a) $\frac{d^2y}{dx^2} + ay = 0$

b) $\frac{d^2y}{dx^2} - 9y = 0$

c) $\frac{d^2y}{dx^2} - 9\frac{dy}{dx} = 0$

d) $\frac{d^2y}{dx^2} + 9x = 0$.

37. The order and degree of the differential equation

$\sin x (dx + dy) = \cos x (dx - dy)$ are

a) 1, 1

b) 0, 0

c) 1, 2

d) 2, 1.

38. The conditional statement $p \rightarrow q$ is equivalent to

a) $p \vee q$

b) $p \vee \sim q$

c) $\sim p \vee q$

d) $p \wedge q$.

39. ' – ' is a binary operation on

a) N

b) $Q - \{ 0 \}$

c) $R - \{ 0 \}$

d) Z.

40. The order of [7] in $\left(Z_9 , +_9 \right)$ is

a) 9

b) 6

c) 3

d) 1.

SECTION - B

N. B. : i) Answer any *ten* questions.

ii) Question No. **55** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *six* marks. $10 \times 6 = 60$

41. Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and verify that

$A\,(\,adj\,A\,) = (\,adj\,A\,)\,A = |\,A\,|\,I.$

42. Solve the following equations by determinant method :

$$2x + 2y + z = 5$$

$$x - y + z = 1$$

$$3x + y + 2z = 4.$$

43. 'Diagonals of a rhombus are at right angles.' Prove by vector method.

44. a) For any vector $\vec{a}$, prove that

$$\vec{i} \times (\vec{a} \times \vec{i}) + \vec{j} \times (\vec{a} \times \vec{j}) + \vec{k} \times (\vec{a} \times \vec{k}) = 2\vec{a}.$$

b) Find the angle between the lines $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-4}{6}$ and

$x + 1 = \dfrac{y+2}{2} = \dfrac{z-4}{2}$.

45. P represents the variable complex number Z. Find the locus of P, if

$$| Z - 3i | = | Z + 3i |.$$

46. Solve the equation $x^4 - 8x^3 + 24x^2 - 32x + 20 = 0$ if $3 + i$ is a root.

47. a) Verify Rolle's theorem for the function $f(x) = x^3 - 3x + 3, \ 0 \leq x \leq 1$.

b) Evaluate $\lim_{x \to \infty} \frac{x^2}{e^x}$.

48. Find the intervals of concavity and the points of inflexion of the function

$$f(x) = 2x^3 + 5x^2 - 4x.$$

49. If $W = x + 2y + z^2$ and $x = \cos t, \ y = \sin t, \ z = t$, find $\frac{dW}{dt}$.

50. Evaluate $\int_0^{\pi/2} \log(\tan x)\, dx$.

51. Solve : $\frac{dy}{dx} + xy = x$.

52. Construct the truth table for $(p \wedge q) \vee r$.

53. In a continuous distribution the p.d.f. of X is

$$f(x) = \begin{cases} \frac{3}{4} x(2 - x) \ ; & 0 < x < 2 \\ 0 \ ; & \text{otherwise.} \end{cases}$$

Find the mean and variance of the distribution.

54. In a Poisson distribution if $P(X = 2) = P(X = 3)$, find $P(X = 5)$.

[Given $e^{-3} = 0{\cdot}050$].

55. a) Show that $p \to q$ and $q \to p$ are not equivalent.

OR

b) Prove that the tangent at any point to the rectangular hyperbola forms with the asymptotes a triangle of constant area.

SECTION – C

N. B. : i) Answer any *ten* questions.

ii) Question No. **70** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *ten* marks. 10 × 10 = 100

56. For what values of μ the equations $x + y + 3z = 0$, $4x + 3y + \mu z = 0$, $2x + y + 2z = 0$ have a

i) trivial solution

ii) non-trivial solution (use rank method).

57. Verify $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\,\vec{a}\ \vec{b}\ \vec{d}\,]\,\vec{c} - [\,\vec{a}\ \vec{b}\ \vec{c}\,]\,\vec{d}$ where

$\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} + \vec{k}$, $\vec{c} = 2\vec{i} + \vec{j} + \vec{k}$ and $\vec{d} = \vec{i} + \vec{j} + 2\vec{k}$.

58. Find the vector and Cartesian equations of the plane passing through the points $(-1, 1, 1)$ and $(1, -1, 1)$ and perpendicular to the plane $x + 2y + 2z = 5$.

59. If α and β are the roots of $x^2 - 2x + 4 = 0$, prove that

$\alpha^n - \beta^n = i\, 2^{n+1} \sin \frac{n\pi}{3}$ and deduce $\alpha^9 - \beta^9$, $n \in N$.

60. The girder of a railway bridge is in the parabolic form with span 100 ft and the highest point on the arch is 10 ft above the bridge. Find the height of the arch of the bridge at 10 ft to the left or right from the mid-point of the bridge.

61. Find the eccentricity, centre, vertices and foci of the ellipse

$9x^2 + 25y^2 - 18x - 100y - 116 = 0$ and draw its curve.

62. Find the equation of the hyperbola if its asymptotes are parallel to $x + 2y - 12 = 0$ and $x - 2y + 8 = 0$, $(2, 4)$ is the centre of the hyperbola and it passes through $(2, 0)$.

63. If the curves $y^2 = x$ and $xy = k$ intersect each other orthogonally then prove that $8k^2 = 1$.

64. Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.

65. Trace the curve $y^2 = 2x^3$.

66. Find the length of the curve $x = a(t - \sin t)$, $y = a(1 - \cos t)$ between $t = 0$ and $t = \pi$.

67. The number of bacteria in microbial culture grows at a rate which is proportional to the number of bacteria present in it. If the population of a colony of bacteria triples in 1 hour, show that the number of bacteria at the end of five hours will be 3^5 times of the population at initial time.

68. Solve : $\left(D^2 - 6D + 9\right) y = x + e^{2x}$.

69. Show that the set G of all rational numbers except '-1' forms an Abelian group with respect to the operation $*$ given by $a * b = a + b + ab$ for all $a, b \in G$.

70. a) The mean weight of 500 male students in a certain college is 151 pounds and the standard deviation is 15 pounds. Assuming the weights are normally distributed, find how many students weigh

i) between 120 and 155 pounds

ii) more than 185 pounds.

Z	2·067	0·2667	2·2667
Area	0·4803	0·1026	0·4881

OR

b) Find the area of the region bounded by the curve $y = 3x^2 - x$ and x-axis between $x = -1$ and $x = 1$.