B

8021

Register Number

Part III — MATHEMATICS

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 200

SECTION – A

N. B. : i) *All* questions are compulsory.

ii) Each question carries *one* mark.

iii) Choose the most suitable answer from the given four alternatives. $40 \times 1 = 40$

1. If $\vec{PR} = 2\vec{i} + \vec{j} + \vec{k}$, $\vec{QS} = -\vec{i} + 3\vec{j} + 2\vec{k}$ then the area of the quadrilateral *PQRS* is

a) $5\sqrt{3}$ b) $10\sqrt{3}$

c) $\frac{5\sqrt{3}}{2}$ d) $\frac{3}{2}$.

2. The point of intersection of the lines $\frac{x-6}{-6} = \frac{y+4}{4} = \frac{z-4}{-8}$ and $\frac{x+1}{2} = \frac{y+2}{4} = \frac{z+3}{-2}$ is

a) (0, 0, – 4) b) (1, 0, 0)

c) (0, 2, 0) d) (1, 2, 0).

[Turn over

3. The shortest distance between the parallel lines $\frac{x-3}{4} = \frac{y-1}{2} = \frac{z-5}{-3}$ and $\frac{x-1}{4} = \frac{y-2}{2} = \frac{z-3}{-3}$ is

a) 3 b) 2

c) 1 d) 0.

4. Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{0}$. If $|\vec{u}| = 3$, $|\vec{v}| = 4$ and $|\vec{w}| = 5$ then $\vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{w} + \vec{w} \cdot \vec{u}$ is

a) 25 b) -25

c) 5 d) $\sqrt{5}$.

5. The centre and radius of the sphere $\left| 2\vec{r} + \left(3\vec{i} - \vec{j} + 4\vec{k} \right) \right| = 4$ are

a) $\left(-\frac{3}{2}, \frac{1}{2}, -2 \right)$ and 4

b) $\left(-\frac{3}{2}, \frac{1}{2}, -2 \right)$ and 2

c) $\left(-\frac{3}{2}, \frac{1}{2}, -2 \right)$ and 6

d) $\left(-\frac{3}{2}, \frac{1}{2}, -2 \right)$ and 5.

6. The tangent at any point P on the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ whose centre C meets the major axis at T and PN is the perpendicular to the major axis. Then $CN.CT =$

a) $\sqrt{6}$ b) 3

c) $\sqrt{3}$ d) 6.

7. The eccentricity of the hyperbola with asymptotes

$x + 2y - 5 = 0$, $2x - y + 5 = 0$ is

a) 3

b) $\sqrt{2}$

c) $\sqrt{3}$

d) 2.

8. The equation of the chord of contact of tangents from the point (− 3, 1) to the parabola $y^2 = 8x$ is

a) $4x - y - 12 = 0$

b) $4x + y + 12 = 0$

c) $4y - x - 12 = 0$

d) $4y - x + 12 = 0$.

9. Equation of the tangent to the curve $y = x^3$ at (1, 1) is

a) $y = 2x - 3$

b) $y = 2x + 3$

c) $y = 3x - 2$

d) $y = 3x + 2$.

10. If a normal makes an angle θ with positive x-axis, then the slope of the curve at the point, where the normal is drawn, is

a) $-\cot\theta$

b) $\tan\theta$

c) $-\tan\theta$

d) $\cot\theta$.

11. In the group $\left(Z_4 , +_4 \right)$, $O([3])$ is

a) 4 b) 3

c) 2 d) 1.

12. Given $E(X+C) = 8$ and $E(X-C) = 12$, then the value of C is

a) -2 b) 4

c) -4 d) 2.

13. If in a Poisson distribution $P(X=0) = k$ then the variance is

a) $\log \frac{1}{k}$

b) $\log k$

c) e^{λ}

d) $\frac{1}{k}$.

14. The marks secured by 400 students in a Mathematics test were normally distributed with mean 65. If 120 students got more marks above 85, the number of students securing marks between 45 and 65 is

a) 120

b) 20

c) 80

d) 160.

15. $F(x) = \frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1} x\right) \; -\infty < x < \infty$ is a distribution function of a continuous variable X, then $P(x \leq 1)$ is

a) $\frac{3}{4}$

b) $\frac{\pi}{4}$

c) $\frac{1}{2}$

d) $\frac{1}{4}$.

16. If A is a scalar matrix with scalar $k \neq 0$, of order 3, then A^{-1} is

a) $\frac{1}{k^2} I$

b) $\frac{1}{k^3} I$

c) $\frac{1}{k} I$

d) kI.

17. If A is a matrix of order 3, then det (kA) is

a) $k^3 \det(A)$

b) $k^2 \det(A)$

c) $k \det(A)$

d) $\det(A)$.

18. Inverse of $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ is

a) $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$

b) $\begin{pmatrix} -2 & 5 \\ 1 & -3 \end{pmatrix}$

c) $\begin{pmatrix} 3 & -1 \\ -5 & -3 \end{pmatrix}$

d) $\begin{pmatrix} -3 & 5 \\ 1 & -2 \end{pmatrix}$.

19. If A and B are any two matrices such that $AB = 0$ and A is non-singular, then

a) $B = 0$

b) B is singular

c) B is non-singular

d) $B = A$.

20. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, then $\left(\vec{a} + \vec{b}\right)$ is a unit vector if

a) $\theta = \frac{\pi}{3}$

b) $\theta = \frac{\pi}{4}$

c) $\theta = \frac{\pi}{2}$

d) $\theta = \frac{2\pi}{3}$.

21. If $f'(x) = \sqrt{x}$ and $f(1) = 2$ then $f(x)$ is

a) $-\frac{2}{3}\left(x\sqrt{x} + 2\right)$

b) $\frac{3}{2}\left(x\sqrt{x} + 2\right)$

c) $\frac{2}{3}\left(x\sqrt{x} + 2\right)$

d) $\frac{2}{3}\,x\left(\sqrt{x} + 2\right)$.

22. The order and degree of the differential equation

$$\frac{d^2y}{dx^2} - y + \left(\frac{dy}{dx} + \frac{d^3y}{dx^3}\right)^{3/2} = 0 \text{ are}$$

a) 2, 3

b) 3, 3

c) 3, 2

d) 2, 2.

23. If p is T, and q is F, then which of the following have the truth value T?

I. $p \vee q$

II. $\sim p \vee q$

III. $p \vee \sim q$

IV. $p \wedge \sim q$.

a) I, II, III

b) I, II, IV

c) I, III, IV

d) II, III, IV.

24. Which of the following is not a binary operation on R?

a) $a * b = ab$

b) $a * b = a - b$

c) $a * b = \sqrt{ab}$

d) $a * b = \sqrt{a^2 + b^2}$.

25. If p is true and q is false, then which of the following is not true?

a) $p \to q$ is false

b) $p \vee q$ is true

c) $p \wedge q$ is false

d) $p \leftrightarrow q$ is true.

26. The area of the region bounded by the graph of $y = \sin x$ and $y = \cos x$ between $x = 0$ and $x = \frac{\pi}{4}$ is

a) $\sqrt{2} + 1$

b) $\sqrt{2} - 1$

c) $2\sqrt{2} - 2$

d) $2\sqrt{2} + 2$.

27. The volume generated by rotating the triangle with vertices at (0, 0), (3, 0) and (3, 3) about x-axis is

a) 18π

b) 2π

c) 36π

d) 9π.

28. $\int_0^\infty x^6 e^{-x/2}\, dx =$

a) $\frac{\lfloor 6}{2^7}$

b) $\frac{\lfloor 6}{2^6}$

c) $2^6 \lfloor 6$

d) $2^7 \lfloor 6$.

29. The differential equation of the family of lines $y = mx$ is

a) $\frac{dy}{dx} = m$

b) $y dx - x dy = 0$

c) $\frac{d^2 y}{dx^2} = 0$

d) $y dx + x dy = 0$.

30. The amount present in a radioactive element disintegrates at a rate proportional to its amount (p). The differential equation corresponding to the above statement is (k is negative)

a) $\frac{dp}{dt} = \frac{k}{p}$

b) $\frac{dp}{dt} = kt$

c) $\frac{dp}{dt} = kp$

d) $\frac{dp}{dt} = -kt$.

31. $\lim_{x \to \infty} \frac{x^2}{e^x}$ is

a) 2

b) 0

c) ∞

d) 1.

32. The least possible perimeter of a rectangle of area 100 m^2 is

a) 10

b) 20

c) 40

d) 60.

33. If $u = \log\left(\frac{x^2 + y^2}{xy}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is

a) 0

b) u

c) $2u$

d) u^{-1}.

34. The curve $a^2 y^2 = x^2(a^2 - x^2)$ has

a) only one loop between $x = 0$ and $x = a$

b) two loops between $x = 0$ and $x = a$

c) two loops between $x = -a$ and $x = a$

d) no loop.

35. The value of $\int_{-\pi/2}^{\pi/2} \left(\frac{\sin x}{2 + \cos x}\right) dx$ is

a) 0

b) 2

c) log 2

d) log 4.

36. The polar form of the complex number $\left(i^{25}\right)^3$ is

a) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

b) $\cos\pi + i\sin\pi$

c) $\cos\pi - i\sin\pi$

d) $\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$.

37. If $z_1 = 4 + 5i$, $z_2 = -3 + 2i$ then $\frac{z_1}{z_2}$ is

a) $\frac{2}{13} - \frac{22}{13}i$

b) $\frac{-2}{13} + \frac{22}{13}i$

c) $\frac{-2}{13} - \frac{23}{13}i$

d) $\frac{2}{13} + \frac{22}{13}i$.

38. If $\frac{1-i}{1+i}$ is a root of the equation $ax^2 + bx + 1 = 0$, where a, b are real, then (a, b) is

a) (1, 1) b) (1, − 1)

c) (0, 1) d) (1, 0).

39. The arguments of n^{th} roots of a complex number differ by

a) $\frac{2\pi}{n}$ b) $\frac{\pi}{n}$

c) $\frac{3\pi}{n}$ d) $\frac{4\pi}{n}$.

40. The length of the latus rectum of the parabola whose vertex is (2, − 3) and the directrix $x = 4$ is

a) 2 b) 4

c) 6 d) 8.

SECTION - B

N. B. : i) Answer any *ten* questions.

ii) Question No. **55** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *six* marks. $10 \times 6 = 60$

41\. a) For any non-singular matrix A, show that $\left(A^{T}\right)^{-1} = \left(A^{-1}\right)^{T}$.

b) Find the inverse of the matrix : $\begin{pmatrix} -1 & 2 \\ 1 & -4 \end{pmatrix}$.

42\. Examine the consistency of the system of equations using rank method :

$$x + y + z = 7, \quad x + 2y + 3z = 18, \quad y + 2z = 6.$$

43\. a) A force given by $3\vec{i} + 2\vec{j} - 4\vec{k}$ is applied at the point (1, − 1, 2). Find the moment of the force about the point (2, − 1, 3).

b) If $\vec{x} \cdot \vec{a} = 0$, $\vec{x} \cdot \vec{b} = 0$, $\vec{x} \cdot \vec{c} = 0$ and $\vec{x} \neq \vec{0}$ then show that $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar.

44\. Find the Vector and Cartesian equation of the sphere on the join of the points A and B having position vectors $2\vec{i} + 6\vec{j} - 7\vec{k}$ and $-2\vec{i} + 4\vec{j} - 3\vec{k}$ respectively as a diameter. Find also the centre and radius of the sphere.

45\. For any two complex numbers z_1 and z_2 prove that

a) $|z_1 z_2| = |z_1| \cdot |z_2|$

b) $arg\left(z_1 \cdot z_2\right) = arg\, z_1 + arg\, z_2$.

46. If n is a positive integer, prove that $(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} \cos \frac{n\pi}{4}$.

47. Evaluate : $\lim_{x \to 1} x^{\frac{1}{x-1}}$.

48. Determine the points of inflexion if any, of the function $y = x^3 - 3x + 2$.

49. If u is a homogeneous function of x and y of degree n, prove that

$$x \frac{\partial^2 u}{\partial x \partial y} + y \frac{\partial^2 u}{\partial y^2} = (n-1) \frac{\partial u}{\partial y}.$$

50. Evaluate : $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$.

51. Solve : $(3D^2 + D - 14)\,y = 13e^{2x} + 10e^{x}$.

52. Use the truth table to determine whether the statement

$((\sim p) \vee q) \vee (p \wedge (\sim p))$ is a tautology.

53. Show that the set of all 2×2 non-singular matrices forms a non-Abelian infinite group under matrix multiplication (where the entries belong to R).

54. The overall percentage of passes in a certain examination is 80. If 6 candidates appear in the examination, what is the probability that at least 5 pass the examination ?

55. a) Prove that the tangent at any point to the rectangular hyperbola forms with the asymptotes a triangle of constant area.

OR

b) Find the mean and variance of the probability density function :

$$f(x) = \begin{cases} xe^{-x} & ; \text{ if } x > 0 \\ 0 & ; \text{ otherwise.} \end{cases}$$

SECTION - C

N. B. : i) Answer any *ten* questions.

ii) Question No. **70** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *ten* marks. $10 \times 10 = 100$

56. A small seminar hall can hold 100 chairs. Three different colours (red, blue and green) of chairs are available. The cost of a red chair is Rs. 240, cost of a blue chair is Rs. 260 and the cost of a green chair is Rs. 300. The total cost of chairs is Rs. 25,000. Find at least 3 different solutions of the number of chairs in each colour to be purchased.

57. Prove that $\sin(A - B) = \sin A \cos B - \cos A \sin B$ using vectors.

58. Find the Vector and Cartesian equations of the plane through the points (1, 2, 3) and (2, 3, 1) perpendicular to the plane $3x - 2y + 4z - 5 = 0$.

59. Find all the values of $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2} \right)^{3/4}$ and hence prove that the product of the values is 1.

60. A cable of a suspension bridge hangs in the form of a parabola when the load is uniformly distributed horizontally. The distance between two towers is 1500 ft, the points of support of the cable on the towers are 200 ft above the roadway and the lowest point on the cable is 70 ft above the roadway. Find the vertical distance to the cable from a pole whose height is 122 ft.

61. A ladder of length 15 m moves with its ends always touching the vertical wall and the horizontal floor. Determine the equation of the locus of a point P on the ladder, which is 6 mts from the end of the ladder in contact with the floor.

62. Find the eccentricity, centre, foci and vertices of the following hyperbola and draw the diagram :

$$12x^2 - 4y^2 - 24x + 32y - 127 = 0$$

63. Let P be a point on the curve $y = x^3$ and suppose that the tangent line at P intersects the curve again at Q. Prove that the slope at Q is four times the slope at P.

64. Show that the volume of the largest right circular cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ of volume of the sphere.

65. Trace the curve : $y = x^3$.

66. Find the area of the region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ between the two latus rectums.

67. Find the surface area of the solid generated by revolving the cycloid $x = a(t + \sin t)$, $y = a(1 + \cos t)$ about its base (x-axis).

68. Solve : $\left(x^2 + y^2\right) dx + 3xy\, dy = 0$.

69. Obtain k, μ and σ^2 of the normal distribution whose probability distribution function is given by

$$f(x) = ke^{-2x^2+4x}, \quad -\infty < X < \infty.$$

70. a) A bank pays interest by continuous compounding, that is by treating the interest rate as the instantaneous rate of change of principal. Suppose in an account interest accrues at 8% per year compounded continuously. Calculate the percentage increase in such an account over one year.

[Take $e^{0\cdot 08} \approx 1\cdot 0833$]

OR

b) Show that the set G of all positive rationals forms a group under the composition $*$ defined by $a * b = \frac{ab}{3}$ for all $a, b \in G$.