5023

Register Number | | | | | | |

Part III — MATHEMATICS

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 200

SECTION - A

N. B. : i) *All* questions are compulsory.

ii) Choose the most suitable answer from the given four alternatives. $40 \times 1 = 40$

1. If $[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}] = 8$ then $[\vec{a},\ \vec{b},\ \vec{c}]$ is

1) 4 2) 16

3) 32 4) -4.

2. The equation of the plane passing through the point (2, 1, − 1) and the line of intersection of the planes $\vec{r}\cdot(\vec{i}+3\vec{j}-\vec{k}) = 0$ and $\vec{r}\cdot(\vec{j}+2\vec{k}) = 0$ is

1) $x+4y-z = 0$ 2) $x+9y+11z=0$

3) $2x+y-z+5=0$ 4) $2x-y+z=0$.

3. The two lines $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z-1}{2}$ are

1) parallel 2) intersecting

3) skew 4) perpendicular.

4. The angle between the two vectors $\vec{a}$ and $\vec{b}$ if $\left| \vec{a} \times \vec{b} \right| = \vec{a} \cdot \vec{b}$, is

1) $\frac{\pi}{4}$
2) $\frac{\pi}{3}$
3) $\frac{\pi}{6}$
4) $\frac{\pi}{2}$.

5. Chord AB is a diameter of the sphere $\left| \vec{r} - \left(2\vec{i} + \vec{j} - 6\vec{k} \right) \right| = \sqrt{18}$ with coordinate of A as (3, 2, – 2). The coordinate of B is

1) (1, 0, 10)
2) (– 1, 0, – 10)
3) (– 1, 0, 10)
4) (1, 0, – 10).

6. The equations of the latus rectum of $\frac{x^2}{16} + \frac{y^2}{9} = 1$, are

1) $y = \pm \sqrt{7}$
2) $x = \pm \sqrt{7}$
3) $x = \pm 7$
4) $y = \pm 7$.

7. The eccentricity of the hyperbola $12y^2 - 4x^2 - 24x + 48y - 127 = 0$ is

1) 4
2) 3
3) 2
4) 6.

8. The length of the latus rectum of the rectangular hyperbola $xy = 32$ is

1) $8\sqrt{2}$
2) 32
3) 8
4) 16.

9. A spherical snowball is melting in such a way that its volume is decreasing at rate of 1 cm^3/min. The rate at which the diameter is decreasing when the diameter is 10 cms, is

1) $-\frac{1}{50\pi}$ cm/min
2) $\frac{1}{50\pi}$ cm/min
3) $-\frac{11}{75\pi}$ cm/min
4) $-\frac{2}{75\pi}$ cm/min.

10. The surface area of a sphere when the volume is increasing at the same rate as its radius, is

1) 1
2) $\frac{1}{2\pi}$
3) 4π
4) $\frac{4\pi}{3}$.

11. The area of the region bounded by the graph of $y = \sin x$ and $y = \cos x$ between $x = 0$ and $x = \frac{\pi}{4}$ is

1) $\sqrt{2} + 1$
2) $\sqrt{2} - 1$
3) $2\sqrt{2} - 2$
4) $2\sqrt{2} + 2$.

12. The curved surface area of a sphere of radius 5, intercepted between two parallel planes of distance 2 and 4 on the same side from the centre is

1) 20π
2) 40π
3) 10π
4) 30π.

13. The arc length of the curve $y = f(x)$ from $x = a$ to $x = b$ is

1) $\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

2) $\int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dx$

3) $2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

4) $2\pi \int_a^b y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dx$.

14. Solution of $\frac{dx}{dy} + mx = 0$, where $m < 0$ is

1) $x = ce^{my}$

2) $x = ce^{-my}$

3) $x = my + c$

4) $x = c$.

15. The differential equation $\left(\frac{dx}{dy}\right)^2 + 5y^{1/3} = x$ is

1) of order 2 and degree 1

2) of order 1 and degree 2

3) of order 1 and degree 6

4) of order 1 and degree 3.

16. In the set of integers under the operation $*$ defined by $a * b = a + b - 1$, t identity element is

1) 0

2) 1

3) a

4) b.

17. $\mu_2 = 20$, $\mu_2' = 276$ for a discrete random variable X. Then, the mean of th random variable X is

1) 16

2) 5

3) 2

4) 1.

18. The random variable X follows normal distribution, $f(x) = ce^{-\frac{1}{2}\frac{(x-100)^2}{25}}$. Then the value of c is

1) $\sqrt{2\pi}$ 2) $\frac{1}{\sqrt{2\pi}}$

3) $5\sqrt{2\pi}$ 4) $\frac{1}{5\sqrt{2\pi}}$

19. A discrete random variable X has probability mass function $p(x)$, then

1) $0 \le p(x) \le 1$ 2) $p(x) \ge 0$

3) $p(x) \le 1$ 4) $0 < p(x) < 1$.

20. In 16 throws of a die, getting an even number is considered a success. Then the variance of the successes is

1) 4 2) 6

3) 2 4) 256.

21. The rank of the diagonal matrix

$$\begin{bmatrix} 1 & & & & \\ & 2 & & & \\ & & 0 & & \\ & & & -4 & \\ & & & & 0 \end{bmatrix}$$ is

1) 0 2) 2

3) 3 4) 5.

22. If A is a scalar matrix with scalar $k \neq 0$, of order 3, then A^{-1} is

1) $\frac{1}{k^2} I$ 2) $\frac{1}{k^3} I$

3) $\frac{1}{k} I$ 4) kI.

23. In a system of three linear non-homogeneous equations with three unknowns, if $\Delta = 0$ and $\Delta_x = 0$, $\Delta_y \neq 0$ and $\Delta_z = 0$ then the system has

1) unique solution

2) two solutions

3) infinitely many solutions

4) no solution.

24. In the system of three linear equations with three unknowns, in the non-homogeneous system, $\rho(A) = \rho(A, B) = 2$; then the system

1) has unique solution

2) reduces to two equations and has infinitely many solutions

3) reduces to a single equation and has infinitely many solutions

4) is inconsistent.

25. The area of the parallelogram having a diagonal $3\vec{i} + \vec{j} - \vec{k}$ and a side $\vec{i} - 3\vec{j} + 4\vec{k}$ is

1) $10\sqrt{3}$ 2) $6\sqrt{30}$

3) $\frac{3}{2}\sqrt{30}$ 4) $3\sqrt{30}$.

26. The modulus and amplitude of the complex number $\left[e^{3 - i\pi/4}\right]^3$ are respectively

1) $e^9, \frac{\pi}{2}$ 2) $e^9, -\frac{\pi}{2}$

3) $e^6, -\frac{3\pi}{4}$ 4) $e^9, -\frac{3\pi}{4}$.

27. If P represents the variable complex number z, and if $|2z-1| = 2|z|$, then the locus of P is

1) the straight line $x = \frac{1}{4}$

2) the straight line $y = \frac{1}{4}$

3) the straight line $z = \frac{1}{2}$

4) the circle $x^2 + y^2 - 4x - 1 = 0$.

28. If $-i+2$ is one root of the equation $ax^2 - bx + c = 0$, then the other root is

1) $-i-2$ 2) $i-2$

3) $2+i$ 4) $2i+i$.

29. Polynomial equation $P(x) = 0$ admits conjugate pairs of imaginary roots only if the coefficients are

1) imaginary 2) complex

3) real 4) either real or complex.

30. The line $4x + 2y = c$ is a tangent to the parabola $y^2 = 16x$ then c is

1) -1 2) -2

3) 4 4) -4.

31. The curve $y = -e^{-x}$ is

1) concave upward for $x > 0$

2) concave downward for $x > 0$

3) everywhere concave upward

4) everywhere concave downward.

32. If f has a local extremum at a and if $f'(a)$ exists then

1) $f'(a) < 0$
2) $f'(a) > 0$
3) $f'(a) = 0$
4) $f''(a) = 0$.

33. If $u = \log\left(\frac{x^2 + y^2}{xy}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is

1) 0
2) u
3) $2u$
4) u^{-1}.

34. The curve $a^2y^2 = x^2(a^2 - x^2)$ has

1) only one loop between $x = 0$ and $x = a$
2) two loops between $x = 0$ and $x = a$
3) two loops between $x = -a$ and $x = a$
4) no loop.

35. The value of $\int_0^{\pi/4} \cos^3 2x \, dx$ is

1) $\frac{2}{3}$
2) $\frac{1}{3}$
3) 0
4) $\frac{2\pi}{3}$.

36. If $f'(x) = \sqrt{x}$ and $f(1) = 2$, then $f(x)$ is

1) $-\frac{2}{3}(x\sqrt{x} + 2)$
2) $\frac{3}{2}(x\sqrt{x} + 2)$
3) $\frac{2}{3}(x\sqrt{x} + 2)$
4) $\frac{2}{3}x(\sqrt{x} + 2)$.

37. The differential equation corresponding to $xy = c^2$ where c is an arbitrary constant, is

1) $xy'' + x = 0$
2) $y'' = 0$
3) $xy' + y = 0$
4) $xy'' - x = 0$.

38. If p is true and q is false then which of the following is not true ?

1) $p \to q$ is false
2) $p \vee q$ is true
3) $p \wedge q$ is false
4) $p \leftrightarrow q$ is true.

39. If truth values of p is T and q is F, then which of the following are having the truth value T ?

i) $p \vee q$

ii) $\sim p \vee q$

iii) $p \vee \sim q$

iv) $p \wedge \sim q$.

1) (i), (ii), (iii) only
2) (i), (ii), (iv) only
3) (i), (iii), (iv) only
4) (ii), (iii), (iv) only.

40. Which of the following is not a binary operation on R ?

1) $a * b = ab$

2) $a * b = a - b$

3) $a * b = \sqrt{ab}$

4) $(a * b) = \sqrt{a^2 + b^2}$

SECTION – B

N. B. : i) Answer any *ten* questions.

ii) Question No. **55** is compulsory and choose any *nine* questions from the remaining. $10 \times 6 = 60$

41. Examine the consistency of the system

$$x + y + z = 7$$

$$x + 2y + 3z = 18$$

$$y + 2z = 6.$$

If it is consistent then solve by using rank method.

42. Verify that $\left(A^{-1}\right)^T = \left(A^T\right)^{-1}$ for the matrix $A = \begin{bmatrix} -2 & -3 \\ 5 & -6 \end{bmatrix}$.

43. Find the vector and Cartesian equations of the sphere whose centre is (1, 2, 3) and which passes through the point (5, 5, 3).

44. Show that the points representing the complex number ($7 + 9i$), ($-3 + 7i$) and ($3 + 3i$) form a right-angled triangle on the Argand diagram.

45. Prove that $(1 + i)^n + (1 - i)^n = 2^{\frac{n+2}{2}} \cos \frac{n\pi}{4}$ ($n \in N$).

46. The headlight of a motor vehicle is a parabolic reflector of diameter 12 cm and depth 4 cm. Find the position of bulb on the axis of the reflector for effective functioning of the headlight.

47. Verify Lagrange's law of mean for the function $f(x) = 2x^3 + x^2 - x - 1$, on $[0, 2]$.

48. i) Obtain the Maclaurin's series for e^x.

ii) Find the critical numbers of $x^{3/5}(4 - x)$.

49. Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ if $w = x^2 + y^2$ where $x = u^2 - v^2$ and $y = 2uv$ by using chain rule for partial derivatives.

50. Solve $\left(D^2 + 14D + 49\right)y = e^{-7x} + 4$.

51. Show that $p \leftrightarrow q \equiv ((\sim p) \vee q) \wedge ((\sim q) \vee p)$.

52. State and prove reversal law on inverses of a group.

53. i) If $F(x) = \frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1} x\right)$, $-\infty < x < \infty$ is a distribution function of a continuous variable X, find $P(0 \leq x \leq 1)$.

ii) The difference between the mean and the variance of a binomial distribution is 1 and the difference between their squares is 11. Find n.

54. Alpha particles are emitted by a radioactive source at an average rate of 5 in a 20 minutes interval. Using Poisson distribution, find the probability that there will be

i) 2 emissions

ii) at least 2 emissions

in a particular 20 minute interval. $\left(e^{-5} = 0\cdot0067 \right)$.

55. a) i) If $\vec{a} = \vec{i} + \vec{j} + 2\vec{k}$ and $\vec{b} = 3\vec{i} + 2\vec{j} - \vec{k}$, find $\left(\vec{a} + 3\vec{b} \right) . \left(2\vec{a} - \vec{b} \right)$.

ii) The volume of the parallelopiped whose edges are represented by $-12\vec{i} + \lambda\vec{k}$, $3\vec{j} - \vec{k}$, $2\vec{i} + \vec{j} - 15\vec{k}$ is 546.

Find the value of λ.

OR

b) Evaluate : $\int_{-\pi/2}^{\pi/2} x \sin x \, dx$, using properties of definite integrals.

SECTION – C

N. B. : i) Answer any *ten* questions.

ii) Question No. **70** is compulsory and choose any *nine* questions from the remaining. 10 × 10 = 100

56. A bag contains 3 types of coins namely Re. 1, Rs. 2 and Rs. 5. There are 30 coins amounting to Rs. 100 in total. Find the number of coins in each category.

57. Find the vector and Cartesian equations of the plane passing through the point $(2, -1, -3)$ and parallel to the lines $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z-3}{-4}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-2}{2}$.

58. Derive the equation of the plane in the intercept form (both Cartesian and vector forms).

59. P represents the variable complex number z. Find the locus of P if

$$Im\left(\frac{2z+1}{iz+1}\right) = -2.$$

60. Find the axis, vertex, focus, equation of directrix, equation of latus rectum and length of the latus rectum for the parabola $x^2 - 6x - 12y - 3 = 0$ and hence draw the graph.

61. The ceiling in a hallway 20 ft wide is in the shape of a semi-ellipse and 18 ft high at the centre. Find the height of the ceiling 4 feet from either wall if the height of the side walls is 12 ft.

62. Let P be a point on the curve $y = x^3$ and suppose that the tangent at P intersects the curve again at Q. Prove that the slope at Q is four times the slope at P.

63. The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of the printed material on the poster is fixed at 384 cm^2, find the dimensions of the poster with the smallest area.

64. If $u = \tan^{-1}\left(\frac{x}{y}\right)$ then verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

65. Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$.

66. Find the volume of the solid obtained by revolving the area of the triangle whose sides are having the equations $y = 0$, $x = 4$ and $3x - 4y = 0$, about x-axis.

67. Radium disintegrates at a rate proportional to the amount present. If 5% of the original amount disintegrates in 50 years, how much will remain at the end of 100 years ? [Take A_0 as the initial amount]

68. Show that $\left(Z_7 - \{ [0] \}, \cdot_7 \right)$ forms a group.

69. Find c, μ and σ^2 of the normal distribution whose probability function is given by $f(x) = ce^{-x^2 + 3x}$, $-\infty < x < \infty$.

70. a) Find the eccentricity, centre, foci and vertices of the hyperbola

$x^2 - 4y^2 + 6x + 16y - 11 = 0$ and hence draw the diagram.

OR

b) Show that the equation of the curve whose slope at any point is equal to $y + 2x$ and which passes through the origin is $y = 2\left(e^x - x - 1\right)$.