Question Paper Sl. No. 3352

4321

Register Number | | | | | |

Part III — MATHEMATICS

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 200

SECTION – A

N. B. : i) *All* questions are compulsory.

ii) Choose the most suitable answer from the given four alternatives. $40 \times 1 = 40$

1. The differential equation formed by eliminating A and B from

   $y = e^{x}(A\cos x + B\sin x)$ is

   a) $y_2 + y_1 = 0$ b) $y_2 - y_1 = 0$

   c) $y_2 - 2y_1 + 2y = 0$ d) $y_2 - 2y_1 - 2y = 0$.

2. In finding the differential equation corresponding to $y = e^{mx}$ where m is the arbitrary constant, then m is

   a) $\frac{y}{y'}$ b) $\frac{y'}{y}$

   c) y' d) y.

B

[Turn over

3. If a compound statement is made up of three simple statements then the number of rows in the truth table is

a) 8 b) 6

c) 4 d) 2.

4. In the multiplicative group of n^{th} roots of unity, the inverse of w^k is ($k < n$)

a) $w^{1/k}$ b) w^{-1}

c) w^{n-k} d) $w^{n/k}$.

5. Which of the following is a contradiction ?

a) $p \vee q$ b) $p \wedge q$

c) $p \vee \sim p$ d) $p \wedge \sim p$.

6. $\lim_{x \to 0} \dfrac{a^x - b^x}{c^x - d^x} =$

a) ∞ b) 0

c) $\log \dfrac{ab}{cd}$ d) $\dfrac{\log\left(\frac{a}{b}\right)}{\log\left(\frac{c}{d}\right)}$.

7. Which of the following statements are correct (m_1 and m_2 are slopes of two lines) ?

I. If the two lines are perpendicular then $m_1 m_2 = -1$

II. If $m_1 m_2 = -1$ then the two lines are perpendicular

III. If $m_1 = m_2$ then the two lines are perpendicular

IV. If $m_1 = -\dfrac{1}{m_2}$ then the two lines are perpendicular.

a) II, III and IV b) I, II and IV

c) II and III d) I and II.

8. The curve $a^2 y^2 = x^2 \left(a^2 - x^2 \right)$ has

a) only one loop between $x = 0$ and $x = a$

b) two loops between $x = 0$ and $x = a$

c) two loops between $x = -a$ and $x = a$

d) no loop.

9. The curve $y^2 (x - 2) = x^2 (1 + x)$ has

a) an asymptote parallel to x-axis

b) an asymptote parallel to y-axis

c) asymptotes parallel to both axes

d) no asymptotes.

10. The value of $\int_0^{\pi} \sin^4 x \, dx$ is

a) $\frac{3\pi}{16}$ b) $\frac{3}{16}$

c) 0 d) $\frac{3\pi}{8}$.

11. The modulus and amplitude of the complex number $\left[e^{3 - i\pi/4} \right]^3$ are respectively

a) $e^9, \frac{\pi}{2}$ b) $e^9, -\frac{\pi}{2}$

c) $e^6, -\frac{3\pi}{4}$ d) $e^9, -\frac{3\pi}{4}$.

12. The quadratic equation whose roots are $\pm i\sqrt{7}$, is

a) $x^2 + 7 = 0$ b) $x^2 - 7 = 0$

c) $x^2 + x + 7 = 0$ d) $x^2 - x - 7 = 0$.

13. If $-i+2$ is one root of the equation $ax^2 - bx + c = 0$, the other root is

a) $-i-2$ b) $i-2$

c) $2+i$ d) $2i+i$.

14. Which of the following statements is correct ?

a) Negative complex numbers exist

b) Order relation does not exist in real numbers

c) Order relation exists in complex numbers

d) $(1+i) > (3-2i)$ is meaningless.

15. The point of intersection of the tangents at $t_1 = t$ and $t_2 = 3t$ to the parabola $y^2 = 8x$ is

a) $(6t^2, 8t)$ b) $(8t, 6t^2)$

c) $(t^2, 4t)$ d) $(4t, t^2)$.

16. Inverse of $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ is

a) $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$ b) $\begin{bmatrix} -2 & 5 \\ 1 & -3 \end{bmatrix}$

c) $\begin{bmatrix} 3 & -1 \\ -5 & -3 \end{bmatrix}$ d) $\begin{bmatrix} -3 & 5 \\ 1 & -2 \end{bmatrix}$.

17. If the rank of the matrix $\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2, then λ is

a) 1 b) 2

c) 3 d) any real number.

18. If the matrix $\begin{bmatrix} -1 & 3 & 2 \\ 1 & k & -3 \\ 1 & 4 & 5 \end{bmatrix}$ has an inverse then

a) k is any real number
b) $k = -4$
c) $k \neq -4$
d) $k \neq 4$.

19. In the system of 3 linear equations with three unknowns, if $\Delta = 0$ and one of Δ_x, Δ_y or Δ_z is non-zero, then the system is

a) consistent
b) inconsistent
c) consistent and the system reduces to two equations
d) consistent and the system reduces to a single equation.

20. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, then $(\vec{a} + \vec{b})$ is a unit vector if

a) $\theta = \frac{\pi}{3}$
b) $\theta = \frac{\pi}{4}$
c) $\theta = \frac{\pi}{2}$
d) $\theta = \frac{2\pi}{3}$.

21. Which of the following are statements ?

I. $7 + 2 < 10$

II. The set of rational numbers is finite

III. How beautiful you are !

IV. Wish you all success.

a) III and IV
b) I and II
c) I and III
d) II and IV.

22. If $f(x) = \frac{A}{\pi} \frac{1}{16 + x^2}$, $-\infty < x < \infty$ is a p.d.f. of a continuous random variable X, then the value of A is

a) 16 b) 8

c) 4 d) 1.

23. $\mu_2 = 20$, $\mu_2' = 276$ for a discrete random variable X. Then the mean of the random variable X is

a) 16 b) 5

c) 2 d) 1.

24. For a Poisson distribution with parameter $\lambda = 0{\cdot}25$, the value of the second moment about the origin is

a) 0·25 b) 0·3125

c) 0·0625 d) 0·025.

25. Which of the following is not true regarding the normal distribution ?

a) Skewness is zero

b) Mean = Median = Mode

c) The points of inflexion are at $X = \mu \pm \sigma$

d) Maximum height of the curve is $\frac{1}{\sqrt{2\pi}}$.

26. The volume of solid, when the curve $y = \sqrt{3 + x^2}$ from $x = 0$ to $x = 4$ is rotated about x-axis is

a) 100π b) $\frac{100\pi}{9}$

c) $\frac{100\pi}{3}$ d) $\frac{100}{3}$.

27. The curved surface area of a sphere of radius 5, intercepted between two parallel planes of distances 2 and 4 from the centre (on the same side) is

a) 20π b) 40π

c) 10π d) 30π.

28. $\int_0^\infty x^6 e^{-x/2}\,dx$ is

a) $\frac{\lfloor 6}{2^7}$ b) $\frac{\lfloor 6}{2^6}$

c) $2^6 \lfloor 6$ d) $2^7 \lfloor 6$.

29. Integrating factor of $\frac{dy}{dx} + \frac{1}{x \log x} \cdot y = \frac{2}{x^2}$ is

a) e^x b) $\log x$

c) $\frac{1}{x}$ d) e^{-x}.

30. Particular integral of $\left(D^2 - 4D + 4 \right) y = e^{2x}$ is

a) $\frac{x^2}{2} e^{2x}$ b) xe^{2x}

c) xe^{-2x} d) $\frac{x}{2} e^{-2x}$.

31. The sum of the distances of any point on the ellipse $4x^2 + 9y^2 = 36$ from $\left(\sqrt{5}, 0 \right)$ and $\left(-\sqrt{5}, 0 \right)$ is

a) 4 b) 8

c) 6 d) 18.

32. The eccentricity of the hyperbola $12y^2 - 4x^2 - 24x + 48y - 127 = 0$ is

a) 4 b) 3

c) 2 d) 6.

33. The equation of chord of contact of tangents from (5, 3) to the hyperbola $4x^2 - 6y^2 = 24$ is

a) $9x + 10y + 12 = 0$

b) $10x + 9y - 12 = 0$

c) $9x - 10y + 12 = 0$

d) $10x - 9y - 12 = 0$.

34. The slope of the normal to the curve $y = 3x^2$ at the point whose x-coordinate is 2, is

a) $\frac{1}{13}$ b) $\frac{1}{14}$

c) $-\frac{1}{12}$ d) $\frac{1}{12}$.

35. If $s = t^3 - 4t^2 + 7$, the velocity when the acceleration is zero, is

a) $\frac{32}{3}$ m/sec b) $-\frac{16}{3}$ m/sec

c) $\frac{16}{3}$ m/sec d) $-\frac{32}{3}$ m/sec.

36. If the projection of $\vec{a}$ on $\vec{b}$ and projection of $\vec{b}$ on $\vec{a}$ are equal, then the angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$

c) $\frac{\pi}{4}$ d) $\frac{2\pi}{3}$.

37. The area of the parallelogram having a diagonal $3\vec{i} + \vec{j} - \vec{k}$ and a side $\vec{i} - 3\vec{j} + 4\vec{k}$ is

a) $10\sqrt{3}$ b) $6\sqrt{30}$

c) $\frac{3}{2}\sqrt{30}$ d) $3\sqrt{30}$.

38. The shortest distance of the point (2, 10, 1) from the plane $\vec{r} \cdot \left(3\vec{i} - \vec{j} + 4\vec{k} \right) = 2\sqrt{26}$ is

a) $2\sqrt{26}$ b) $\sqrt{26}$

c) 2 d) $\frac{1}{\sqrt{26}}$.

39. The angle between the line $\vec{r} = \vec{a} + t\vec{b}$ and the plane $\vec{r} \cdot \vec{n} = q$ is connected by the relation

a) $\cos\theta = \frac{\vec{a} \cdot \vec{n}}{q}$

b) $\cos\theta = \frac{\vec{b} \cdot \vec{n}}{|\vec{b}|\,|\vec{n}|}$

c) $\sin\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{n}|}$

d) $\sin\theta = \frac{\vec{b} \cdot \vec{n}}{|\vec{b}|\,|\vec{n}|}$.

40. The non-parametric vector equation of a plane passing through a point whose position vector is $\vec{a}$ and parallel to $\vec{u}$ and $\vec{v}$, is

a) $[\vec{r} - \vec{a} \;\; \vec{u} \;\; \vec{v}] = 0$ b) $[\vec{r} \;\; \vec{u} \;\; \vec{v}] = 0$

c) $[\vec{r} \;\; \vec{a} \;\; \vec{u} \times \vec{v}] = 0$ d) $[\vec{a} \;\; \vec{u} \;\; \vec{v}] = 0$.

SECTION – B

N. B. : i) Answer any *ten* questions.

ii) Question No. **55** is compulsory and choose any *nine* questions from the remaining. 10 × 6 = 60

41. Find the inverse of the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$.

42. a) Find the angle between the lines

$$\vec{r} = \left(5\vec{i} - 7\vec{j}\right) + \mu\left(-\vec{i} + 4\vec{j} + 2\vec{k}\right) \text{ and}$$

$$\vec{r} = \left(-2\vec{i} + \vec{k}\right) + \lambda\left(3\vec{i} + 4\vec{k}\right).$$

b) Find the centre and radius of the sphere

$$\vec{r}^{\,2} - \vec{r}\cdot\left(4\vec{i} + 2\vec{j} - 6\vec{k}\right) - 11 = 0.$$

43. Show that the points $A(1, 2, 3)$, $B(3, -1, 2)$, $C(-2, 3, 1)$ and $D(6, -4, 2)$ are lying on the same plane.

44. Solve : $x^4 + 4 = 0$.

45. State and prove the triangle inequality of complex numbers.

46. Find the equation of the ellipse whose foci are $(2, 1)$, $(-2, 1)$ and length of the latus rectum is 6.

47. Find two numbers whose sum is 100 and product is maximum.

48. Verify Lagrange's law of mean for $f(x) = x^3$ on $[-2, 2]$.

49. Find the approximate value of $\sqrt{36 \cdot 1}$ using differentials.

50. Evaluate :

a) $\int_{-1}^{1} \log\left(\frac{3-x}{3+x}\right) dx$

b) $\int_{0}^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx.$

51. Solve : $(1+x^2)\frac{dy}{dx} + 2xy = \cos x.$

52. Show that $((\sim p) \vee (\sim q)) \vee p$ is a tautology.

53. Verify whether $(p \wedge (\sim q)) \vee ((\sim p) \vee q)$ is a tautology or contradiction.

54. Find the mean and variance for the probability density function :

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & ; \text{ if } x > 0 \\ 0 & ; \text{ otherwise.} \end{cases}$$

55. a) A pair of dice is thrown 10 times. If getting a doublet is considered a success, find the probability of

i) 4 successes

ii) no success.

OR

b) Solve the following system of equations by determinant method :

$4x + 5y = 9, \quad 8x + 10y = 18.$

SECTION – C

N. B. : i) Answer any *ten* questions.

ii) Question No. **70** is compulsory and choose any *nine* questions from the remaining. 10 × 10 = 100

56. Using Rank method, for what values of k, has the system of equations

$kx + y + z = 1, \quad x + ky + z = 1, \quad x + y + kz = 1$

i) unique solution ?

ii) more than one solution ?

iii) no solution ?

57. Find the vector and Cartesian equations of the plane passing through the points

(2, 2, – 1), (3, 4, 2) and (7, 0, 6).

58. If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = -2\vec{i} + 5\vec{k}$, $\vec{c} = \vec{j} - 3\vec{k}$, verify that

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$.

59. Assume that water issuing from the end of a horizontal pipe, 7·5 m above the ground describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2·5 m below the line of the pipe, the flow of water has curved outward 3 m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground ?

60. The orbit of the planet mercury around the sun is in elliptical shape with sun at a focus. The semi-major axis is of length 36 million miles and the eccentricity of the orbit is 0·206. Find (i) how close the mercury gets to sun, (ii) the greatest possible distance between mercury and sun.

61. Find the eccentricity, centre, foci and vertices of the hyperbola

$x^2 - 3y^2 + 6x + 6y + 18 = 0$ and draw the diagram.

62. A ladder 10 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 m/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 m from the wall ?

63. Evaluate : $\lim_{x \to 0} (\cot x)^{\sin x}$.

64. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ by using Euler's theorem prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$$

65. Find the surface area of the solid generated by revolving the arc of the parabola $y^2 = 4ax$ bounded by its latus rectum, about x-axis.

66. Find the area bounded by the curve $y = x^3$ and the line $y = x$.

67. Solve : $\left(D^2 - 5D + 6\right)y = \sin x + 2e^{3x}$.

68. Prove that the set of four functions f_1, f_2, f_3, f_4 on the set of non-zero complex numbers $C - \{0\}$ defined by

$f_1(z) = z,\ \ f_2(z) = -z,\ \ f_3(z) = \frac{1}{z},\ \ f_4(z) = -\frac{1}{z}\ \ \forall\, z \in C - \{0\}$ form an Abelian group with respect to the composition of functions.

69. The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers, find approximately the number of drivers with

i) no accidents in a year

ii) more than 3 accidents in a year $\left(e^{-3} = 0{\cdot}0498\right)$.

70. a) If α and β are the roots of the equation $x^2 - 2px + \left(p^2 + q^2\right) = 0$ and $\tan\theta = \frac{q}{y+p}$, show that $\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = q^{n-1}\frac{\sin n\theta}{\sin^n\theta}$.

OR

b) A cup of coffee at temperature 100°C is placed in a room where temperature is 15°C and it cools to 60°C is 5 minutes. Find its temperature after a further interval of 5 minutes (use Newton's law of cooling).