WARNING :	Any malpractice or any attempt to commit any kind of malpractice in the Examination will DISQUALIFY THE CANDIDATE.

PAPER – II MATHEMATICS

Version Code	B1	Question Booklet Serial Number	
Time : 150 Minutes		Number of Questions : 120	Maximum Marks : 480
Name of Candidate			
Roll Number			
Signature of Candidate			

INSTRUCTIONS TO THE CANDIDATE

1. Please ensure that the **VERSION CODE shown at the top of this Question Booklet is the same as that shown in the OMR Answer Sheet issued to you.** If you have received a Question Booklet with a different **VERSION CODE**, please get it replaced with a Question Booklet with the same VERSION CODE as that of the OMR Answer Sheet from the invigilator. **THIS IS VERY IMPORTANT.**

2. Please fill in the items such as name, signature and roll number of the candidate in the columns given above. Please also write the Question Booklet Sl. No. given at the top of this page against item 4 in the OMR Answer Sheet.

3. Please read the instructions given in the OMR Answer Sheet for marking answers. Candidates are advised to strictly follow the instructions contained in the OMR Answer Sheet.

4. This Question Booklet contains 120 questions. For each question, five answers are suggested and given against (A), (B), (C), (D) and (E) of which, only one will be the **Most Appropriate Answer**. Mark the bubble containing the letter corresponding to the 'Most Appropriate Answer' in the OMR Answer Sheet, by using either **Blue or Black ball - point pen only.**

5. Negative Marking: In order to discourage wild guessing, the score will be subject to penalization formula based on the number of right answers actually marked and the number of wrong answers marked. Each correct answer will be awarded FOUR marks. One mark will be deducted from the total score for each incorrect answer. More than one answer marked against a question will be deemed as incorrect answer and will be negatively marked.

IMMEDIATELY AFTER OPENING THIS QUESTION BOOKLET, THE CANDIDATE SHOULD VERIFY WHETHER THE QUESTION BOOKLET ISSUED CONTAINS ALL THE 120 QUESTIONS IN SERIAL ORDER. IF NOT, REQUEST FOR REPLACEMENT.

DO NOT OPEN THE SEAL UNTIL THE INVIGILATOR ASKS YOU TO DO SO.

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PLEASE ENSURE THAT THIS BOOKLET CONTAINS 120 QUESTIONS SERIALLY NUMBERED FROM 1 TO 120 (Printed Pages : 32)

1. If $n(A) = 43$, $n(B) = 51$ and $n(A \cup B) = 75$, then $n\left((A-B) \bigcup (B-A)\right) =$

(A) 53 (B) 45 (C) 56 (D) 66 (E) 46

2. If A and B are non-empty sets such that $A \supset B$, then

(A) $B' - A' = A - B$ (B) $B' - A' = B - A$ (C) $A' - B' = A - B$

(D) $A' \cap B' = B - A$ (E) $A' \cup B' = A' - B'$

3. The domain of the function $f(x) = \dfrac{\log_2(x+3)}{x^2+3x+2}$ is

(A) $\mathbb{R} - \{-1, -2\}$ (B) $\mathbb{R} - \{-1, -2, 0\}$ (C) $(-3, -1) \cup (-1, \infty)$

(D) $(-3, \infty) - \{-1, -2\}$ (E) $(0, \infty)$

4. If $*$ is defined by $a * b = a - b^2$ and $\oplus$ is defined by $a \oplus b = a^2 + b$, where a and b are integers, then $(3 \oplus 4) * 5 =$

(A) 164 (B) 38 (C) –12 (D) –28 (E) 144

Space for rough work

5. Let $X = \{1, 2, 3, \cdots, 10\}$ and $A = \{1, 2, 3, 4, 5\}$. Then the number of subsets B of X such that $A - B = \{4\}$ is

(A) 2^5
(B) 2^4
(C) $2^5 - 1$
(D) 1
(E) $2^4 - 1$

6. The number of functions that can be defined from the set $A = \{a, b, c, d\}$ into the set $B = \{1, 2, 3\}$ is equal to

(A) 12
(B) 24
(C) 64
(D) 81
(E) 256

7. Let x_1 and y_1 be real numbers. If z_1 and z_2 are complex numbers such that $|z_1| = |z_2| = 4$, then $|x_1 z_1 - y_1 z_2|^2 + |y_1 z_1 + x_1 z_2|^2 =$

(A) $32(x_1^2 + y_1^2)$
(B) $16(x_1^2 + y_1^2)$
(C) $4(x_1^2 + y_1^2)$
(D) 32
(E) $32(x_1^2 + y_1^2)\,|z_1 + z_2|^2$

Space for rough work

8. Let z_1 and z_2 be complex numbers such that $z_1 + i(\overline{z}_2) = 0$ and $\arg(\overline{z}_1\, z_2) = \dfrac{\pi}{3}$. Then $\arg(\overline{z}_1)$ is

(A) $\dfrac{\pi}{3}$ (B) π (C) $\dfrac{\pi}{2}$ (D) $\dfrac{5\pi}{12}$ (E) $\dfrac{5\pi}{6}$

9. If z is a complex number such that $z + |z| = 8 + 12i$, then the value of $|z^2|$ is equal to

(A) 228 (B) 144 (C) 121 (D) 169 (E) 189

10. The value of $\dfrac{1}{i} + \dfrac{1}{i^2} + \dfrac{1}{i^3} + \cdots + \dfrac{1}{i^{102}}$ is equal to

(A) $-1 - i$ (B) $-1 + i$ (C) $1 - i$ (D) $1 + i$ (E) $1 - 2i$

11. Let the complex numbers z_1, z_2, z_3 and z_4 denote the vertices of a square taken in order. If $z_1 = 3 + 4i$ and $z_3 = 5 + 6i$, then the other two vertices z_2 and z_4 are respectively

(A) $5+4i,\ 5+6i$ (B) $5+4i,\ 3+6i$ (C) $5+6i,\ 3+5i$

(D) $3+6i,\ 5+3i$ (E) $5+5i,\ 5+3i$

Space for rough work

12. The quadratic equation $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ has equal roots if

(A) $a \neq b, b=c$ (B) $a=b, b \neq c$ (C) $a \neq b, b \neq c$

(D) $a=b=c$ (E) $a+b+c=0$

13. Let α and β be the roots of the equation $px^2+qx+r=0$. If p, q, r are in A.P. and $\alpha+\beta=4$, then $\alpha\beta$ is equal to

(A) -9 (B) 9 (C) -5 (D) 5 (E) -4

14. If the roots of the equation $x^2+px+c=0$ are 2, -2 and the roots of the equation $x^2+bx+q=0$ are -1, -2, then the roots of the equation $x^2+bx+c=0$ are

(A) $-3, -2$ (B) $-3, 2$ (C) $1, -4$ (D) $-5, 1$ (E) $-1, 4$

15. If a is a root of the equation $x^2-3x+1=0$, then the value of $\frac{a^3}{a^6+1}$ is equal to

(A) $\frac{1}{14}$ (B) $\frac{1}{15}$ (C) $\frac{1}{16}$ (D) $\frac{1}{17}$ (E) $\frac{1}{18}$

Space for rough work

16. If the sum of the roots of the equation $(a+1)x^2+(2a+3)x+(3a+4)=0$, where $a \neq -1$, is -1, then the product of the roots is

(A) 4 (B) 2 (C) 1 (D) –2 (E) –4

17. The product of the roots of the equation $x|x|-5x-6=0$ is equal to

(A) 36 (B) – 36 (C) – 18 (D) 6 (E) – 6

18. If $a_1 = 4$ and $a_{n+1} = a_n + 4n$ for $n \geq 1$, then the value of a_{100} is

(A) 19804 (B) 18904 (C) 18894 (D) 19904 (E) 19894

19. If the first term of a G.P. is 729 and its 7^{th} term is 64, then the sum of first seven terms is

(A) 2187 (B) 2059 (C) 1458

(D) 2123 (E) 1995

Space for rough work

20. Let a_1, a_2, a_3, a_4 be in A.P. If $a_1 + a_4 = 10$ and $a_2 . a_3 = 24$, then the least term of them is

(A) 1 (B) 2 (D) 4 (E) 5

21. The 100th term of the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, $\cdots$ is

(A) 12 (B) 13 (C) 14 (D) 15 (E) 16

22. Let S_n denote the sum of first n terms of an A.P. If $S_4 = -34$, $S_5 = -60$ and $S_6 = -93$, then the common difference and the first term of the A.P. are respectively

(A) –7, 2 (B) 7, –4 (C) 7, –2 (D) –7, –2 (E) 4, –7

23. An A.P. has the property that the sum of first ten terms is half the sum of next ten terms. If the second term is 13, then the common difference is

(A) 3 (B) 2 (C) 5 (D) 4 (E) 6

Space for rough work

24. If ${}^{n}C_{r-1} = 36$ and ${}^{n}C_{r} = 84$, then

(A) $13r - 3n - 3 = 0$ (B) $10r - 3n - 30 = 0$ (C) $10r + 3n - 3 = 0$

(D) $10r - 3n + 3 = 0$ (E) $10r - 3n - 3 = 0$

25. If $\left(1 + x + x^2\right)^n = 1 + a_1 x + a_2 x^2 + \cdots + a_{2n} x^{2n}$, then $2a_1 - 3a_2 + \cdots - (2n+1)a_{2n} =$

(A) n (B) $-n$ (C) $n+1$ (D) $-n-1$ (E) $-n+1$

26. Let t_n denote the n^{th} term in a binomial expansion. If $\frac{t_6}{t_5}$ in the expansion of $(a+b)^{n+4}$ and $\frac{t_5}{t_4}$ in the expansion of $(a+b)^n$ are equal, then n is

(A) 9 (B) 11 (C) 13 (D) 15 (E) 17

27. How many numbers with no more than three digits can be formed using only the digits 1 through 7 with no digit used more than once in a given number?

(A) 259 (B) 249 (C) 257 (D) 252 (E) 269

Space for rough work

28. If $8!\left[\frac{1}{3!}+\frac{5}{4!}\right] = {}^9P_r$, then the value of r is equal to

(A) 4 (B) 5 (C) 3 (D) 2 (E) 1

29. Choose 3, 4, 5 points other than vertices respectively on the sides *AB, BC* and *CA* of a triangle *ABC*. The number of triangles that can be formed using only these points as vertices is

(A) 220 (B) 217 (C) 215 (D) 210 (E) 205

30. The value of x satisfying the equation $\begin{vmatrix} \cos 2x & \sin 2x & \sin 2x \\ \sin 2x & \cos 2x & \sin 2x \\ \sin 2x & \sin 2x & \cos 2x \end{vmatrix} = 0$ and $x \in \left[0, \frac{\pi}{4}\right]$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{16}$ (D) $\frac{\pi}{3}$ (E) $\frac{\pi}{8}$

31. If $\begin{vmatrix} 32 & 24 & 16 \\ 8 & 3 & 5 \\ 4 & 5 & 3 \end{vmatrix} = \lambda \begin{vmatrix} 1 & 3 & 2 \\ 2 & 3 & 5 \\ 1 & 5 & 3 \end{vmatrix}$, then the value of λ is equal to

(A) 4 (B) 8 (D) 24 (E) 32

Space for rough work

32. The value of x, for which the matrix $A=\begin{bmatrix} e^{x-2} & e^{7+x} \\ e^{2+x} & e^{2x+3} \end{bmatrix}$ is singular, is equal to

(A) 9 (B) 8 (C) 7 (D) 6 (E) 5

33. If $A=\begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}$, then the value of the determinant $\left|A^{2013}-3A^{2012}\right|$ is equal to

(A) 8 (B) −8 (C) 9 (D) 7 (E) −7

34. If A is a square matrix of order 3 such that $A^2+A+4I=0$, where 0 is the zero matrix and I is the unit matrix of order 3, then

(A) A is singular and $A+I$ is non-singular

(B) A is non-singular and $A+I$ is non-singular

(C) A is non-singular and $A+I$ is singular

(D) A is singular and $A+I$ is singular

(E) A is non-singular and $A-I$ is singular

35. If t_5, t_{10} and t_{25} are 5th, 10th, and 25th terms of an A.P. respectively, then the value of

$\begin{vmatrix} t_5 & t_{10} & t_{25} \\ 5 & 10 & 25 \\ 1 & 1 & 1 \end{vmatrix}$ is equal to

(A) −40 (B) 1 (C) −1 (D) 0 (E) 40

Space for rough work

36. If x satisfies the inequalities $x+7<2x+3$ and $2x+4<5x+3$, then x lies in the interval

(A) $(-\infty, 3)$ (B) $(1, 3)$ (C) $(4,\infty)$ (D) $(-\infty,-1)$ (E) $(3, 4)$

37. The set of points (x, y) satisfying the inequalities $x+y\le 1$, $-x-y\le 1$ lie in the region bounded by the two straight lines passing through the respective pair of points

(A) $\{(1,0),(0,1)\}$ and $\{(-1,0),(0,-1)\}$

(B) $\{(1,0),(1,1)\}$ and $\{(-1,0),(0,-1)\}$

(C) $\{(-1,0),(0,-1)\}$ and $\{(1,0),(-1,1)\}$

(D) $\{(1,0),(0,-1)\}$ and $\{(-1,0),(0,1)\}$

(E) $\{(1,0),(1,1)\}$ and $\{(-1,0),(-1,-1)\}$

38. If p : 2 plus 3 is five and
q : Delhi is the capital of India
are two statements, then the statement
"Delhi is the capital of India and it is not that 2 plus 3 is five" is

(A) $\sim p \vee q$ (B) $\sim p \wedge q$ (C) $p\wedge \sim q$ (D) $p\vee \sim q$ (E) $\sim p\wedge \sim q$

39. Let f be a function from a set X to a set Y. Consider the following statements:

P: For each $x\in X$, there exists unique $y\in Y$ such that $f(x)=y$.

Q: For each $y\in Y$, there exists $x\in X$ such that $f(x)=y$.

R: There exist $x_1, x_2\in X$ such that $x_1\neq x_2$ and $f(x_1)=f(x_2)$.

The negation of the statement "f is one-to-one and onto " is

(A) P or not R (B) R or not P (C) R or not Q
(D) P and not R (E) R and not Q

Space for rough work

40. Which one of the following is a statement?

(A) Close the door.
(B) Good evening sir.
(C) Bring the book.
(D) Mumbai is the capital of India.
(E) What are you doing?

41. If $\cos\alpha + \sin\alpha = \frac{3}{4}$, then $\sin^6\alpha + \cos^6\alpha =$

(A) $\frac{877}{1024}$ (B) $\frac{777}{1024}$ (C) $\frac{878}{1024}$

(D) $\frac{789}{1024}$ (E) $\frac{878}{1064}$

42. If $\cos x + \cos^2 x = 1$, then the value of $\sin^4 x + \sin^6 x$ is equal to

(A) $-1 + \sqrt{5}$ (B) $\frac{-1-\sqrt{5}}{2}$ (C) $\frac{1-\sqrt{5}}{2}$ (D) $\frac{-1+\sqrt{5}}{2}$ (E) $1-\sqrt{5}$

43. If $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right) = \frac{\pi}{2}$, where $xy < 1$, then

(A) $x - y - xy = 1$ (B) $x - y + xy = 1$ (C) $x + y - xy = 1$
(D) $x + y + xy = 1$ (E) $y - x - xy = 1$

Space for rough work

44. If $\tan\left(\frac{\theta}{2}\right)=\frac{2}{3}$, then $\sec\theta =$

(A) $\frac{13}{5}$ (B) $\frac{13}{3}$ (C) $\frac{3}{13}$ (D) $\frac{5}{13}$ (E) $\frac{12}{13}$

45. $\tan\left(3\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right)=$

(A) $-\frac{13}{46}$ (B) $-\frac{11}{46}$ (C) $-\frac{7}{46}$ (D) $-\frac{4}{23}$ (E) $-\frac{9}{46}$

46. If $\sin^{16}\alpha=\frac{1}{5}$, then the value of $\frac{1}{\cos^2\alpha}+\frac{1}{1+\sin^2\alpha}+\frac{2}{1+\sin^4\alpha}+\frac{4}{1+\sin^8\alpha}$ is equal to

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

47. If $0<x<\frac{\pi}{2}$, then $\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$ is equal to

(A) $2\sec x$ (B) $2\cos x$ (C) $\sec x$ (D) $\cos x$ (E) $2\sin x$

Space for rough work

48. If $\tan^{-1}(-x)+\cos^{-1}\left(\frac{-1}{2}\right)=\frac{\pi}{2}$, then the value of x is equal to

(A) $\sqrt{3}$ (B) $\frac{-1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $-\sqrt{3}$ (E) 1

49. If $ab>-1$, $bc>-1$ and $ca>-1$, then the value of

$\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right)$ is equal to

(A) -1 (B) $\cot^{-1}(a+b+c)$ (C) $\cot^{-1}(abc)$

(D) 0 (E) $\tan^{-1}(a+b+c)$

50. The orthocentre of a triangle formed by the lines $x-2y=1$, $x=0$ and $2x+y-2=0$ is

(A) (0, 1) (B) $(1, 0)$ (C) $(-1,-2)$ (D) $(1, 2)$ (E) $(0, 0)$

51. Let O be the origin and A be the point (64, 0). If P, Q divide OA in the ratio 1 : 2 : 3, then the point P is

(A) $\left(\frac{32}{3},0\right)$ (B) (32, 0) (C) $\left(\frac{64}{3},0\right)$ (D) (16, 0) (E) $\left(\frac{16}{3},0\right)$

Space for rough work

52. The locus of a point which is equidistant from the points (1, 1) and (3, 3) is

(A) $y = x + 4$ (B) $x + y = 4$ (C) $x = 2$ (D) $y = 2$ (E) $y = -x$

53. The value of a for which the points (9, 5), (1, 2), (a, 8) are collinear is equal to

(A) 17 (B) 8 (C) 7 (D) 71 (E) 1

54. A straight line with slope 3 intersects a straight line with slope 6 at the point (30, 40). Then the difference between the y-intercepts of the straight lines is

(A) 60 (B) 70 (C) 80 (D) 90 (E) 100

55. If the equation $3x + 3y + 5 = 0$ is written in the form $x\cos\alpha + y\sin\alpha = p$, then the value of $\sin\alpha + \cos\alpha$ is

(A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $-\sqrt{2}$ (D) $-\frac{1}{\sqrt{2}}$ (E) $\sqrt{3}$

56. The points on the line $x + y = 4$ lying at a unit distance from the line $4x + 3y - 10 = 0$ are

(A) (–7, 11), (3, 1) (B) (7, –11), (3, –1) (C) (–7, 11), (–3, 7)

(D) (7, –3), (11, –7) (E) (2, 2), (3, 1)

Space for rough work

57. If the straight lines $y=2x$, $y=2x+1$, $y=-7x$, $y=-7x+1$ form a parallelogram, then the area of the parallelogram (in square units) is

(A) $\frac{1}{3}$ (B) $\frac{2}{9}$ (C) $\frac{1}{9}$ (D) $\frac{1}{4}$ (E) 9

58. The straight lines $x-4y+7=0$ and $3x-12y+11=0$ are tangents to a circle. The radius of the circle is

(A) $\frac{10}{3\sqrt{17}}$ (B) $\frac{5}{3\sqrt{7}}$ (C) $\frac{15}{\sqrt{17}}$ (D) $\frac{5}{3\sqrt{17}}$ (E) $\frac{5}{3\sqrt{13}}$

59. If the circles $x^2+y^2+ax-6y+4=0$ and $x^2+y^2-12x+32=0$ touch externally each other and if the distance between the centres is equal to 5, then the values of a are

(A) 2, 3 (B) –2, –3 (C) –4, –20 (D) –7, –6 (E) –6, –4

60. The equation of the circle which touches the lines $x=0$, $y=0$, and $4x+3y=12$ is

(A) $x^2+y^2-2x-2y-1=0$ (B) $x^2+y^2-2x-2y+3=0$

(C) $x^2+y^2-2x-2y+2=0$ (D) $x^2+y^2-2x-2y+1=0$

(E) $x^2+y^2-2x-2y-3=0$

Space for rough work

61. The difference between radii of smaller and larger circles, whose centres lie on the circle $x^2+y^2+4x-6y-23=0$ and pass through the point (5, 12) is

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

62. If $x=2$ is the equation of the directrix of the parabola $y^2=-4ax$, then the focus of the parabola is

(A) (0, 1) (B) (2, 0) (C) (–2, 0) (D) (0, –1) (E) (–1, 0)

63. The locus of the point which moves so that its distance from the fixed point $(-2,3)$ equals its distance from the line $x+6=0$ is

(A) $y^2-6y+8x=0$ (B) $y^2-3y+2x+5=0$ (C) $y^2-3y+2x-5=0$

(D) $y^2-6y+8x+23=0$ (E) $y^2-6y-8x-23=0$

64. The eccentricity of the hyperbola $4x^2-y^2-8x-8y-28=0$ is

(A) 3 (B) $\sqrt{5}$ (C) 2 (D) $\sqrt{7}$ (E) $\sqrt{2}$

Space for rough work

65. If the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{25}=\frac{1}{13}$ coincide, then the value of b^2 is

(A) 5 (B) 12 (C) 13 (D) 17 (E) 21

66. Let S_1, S_2 be the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{8}=1$. If $A(x, y)$ is any point on the ellipse, then the maximum area of the triangle AS_1S_2 (in square units) is

(A) $2\sqrt{2}$ (B) $2\sqrt{3}$ (C) 8 (D) 4 (E) 16

67. If $\vec{p}$ and $\vec{q}$ are non-collinear unit vectors and $|\vec{p}+\vec{q}|=\sqrt{3}$, then $(2\vec{p}-3\vec{q})\cdot(3\vec{p}+\vec{q})$ is equal to

(A) 0 (B) $\frac{1}{3}$ (C) $-\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2}$

68. The triangle formed by the three points whose position vectors are $2\hat{i}+4\hat{j}-\hat{k}$, $4\hat{i}+5\hat{j}+\hat{k}$ and $3\hat{i}+6\hat{j}-3\hat{k}$ is

(A) an equilateral triangle

(B) a right angled triangle but not isosceles

(C) an isosceles triangle but not right angled triangle

(D) a right angled isosceles triangle

(E) a scalene triangle

Space for rough work

69. If (1, 2, 4) and (2, –3λ, –3) are the initial and terminal points of the vector $\hat{i}+5\hat{j}-7\hat{k}$, then the value of λ is equal to

(A) $\frac{7}{3}$ (B) $\frac{-7}{3}$ (C) $\frac{-5}{3}$ (D) $\frac{5}{3}$ (E) $\frac{-7}{5}$

70. Let $\vec{u} = 5\vec{a} + 6\vec{b} + 7\vec{c}$, $\vec{v} = 7\vec{a} - 8\vec{b} + 9\vec{c}$ and $\vec{w} = 3\vec{a} + 20\vec{b} + 5\vec{c}$, where $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-zero vectors. If $\vec{u} = l\,\vec{v} + m\,\vec{w}$, then the values of l and m, respectively, are

(A) $\frac{1}{2}, \frac{1}{2}$ (B) $\frac{1}{2}, -\frac{1}{2}$ (C) $-\frac{1}{2}, \frac{1}{2}$ (D) $\frac{1}{3}, \frac{1}{3}$ (E) $-\frac{1}{2}, \frac{1}{5}$

71. If $\vec{\alpha} = 3\hat{i} - \hat{k}$, $|\vec{\beta}| = \sqrt{5}$ and $\vec{\alpha}\cdot\vec{\beta} = 3$, then the area of the parallelogram for which $\vec{\alpha}$ and $\vec{\beta}$ are adjacent sides is

(A) $\frac{\sqrt{17}}{2}$ (B) $\frac{\sqrt{14}}{2}$ (C) $\frac{\sqrt{7}}{2}$ (D) $\sqrt{41}$ (E) $\sqrt{50}$

72. If $3\vec{p}+2\vec{q} = \hat{i}+\hat{j}+\hat{k}$ and $3\vec{p}-2\vec{q} = \hat{i}-\hat{j}-\hat{k}$, then the angle between $\vec{p}$ and $\vec{q}$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) π

73. If the vectors $\overrightarrow{PQ} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ and $\overrightarrow{PR} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle PQR, then the length of the median through P is

(A) $\sqrt{14}$ (B) $\sqrt{15}$ (C) $\sqrt{17}$ (D) $\sqrt{18}$ (E) $\sqrt{19}$

Space for rough work

74. The point of intersection of the straight line $\frac{x-2}{2}=\frac{y-1}{-3}=\frac{z+2}{1}$ with the plane $x+3y-z+1=0$ is equal to

(A) (3, –1, 1) (B) (–5, 1, –1) (C) (2, 0, 3) (D) (5, –1, 3) (E) (4, –2, –1)

75. If the lines $\frac{2x-1}{2}=\frac{3-y}{1}=\frac{z-1}{3}$ and $\frac{x+3}{2}=\frac{z+1}{p}=\frac{y+2}{5}$ are perpendicular to each other, then p is equal to

(A) 1 (B) -1 (C) 10 (D) $-\frac{7}{5}$ (E) -19

76. The point $P(x, y, z)$ lies in the first octant and its distance from the origin is 12 units. If the position vector of P makes $45°$ and $60°$ with the x-axis and y-axis respectively, then the co-ordinates of P are

(A) $\left(3\sqrt{3},\ 6,\ 3\sqrt{2}\right)$ (B) $\left(4\sqrt{3},\ 8,\ 4\sqrt{2}\right)$ (C) $\left(6\sqrt{2},\ 6,\ 6\right)$

(D) $\left(6,\ 6,\ 6\sqrt{2}\right)$ (E) $\left(4\sqrt{2},\ 8,\ 4\sqrt{3}\right)$

77. The distance between the planes $\vec{r}\cdot\left(\hat{i}+2\hat{j}-2\hat{k}\right)+5=0$ and $\vec{r}\cdot\left(2\hat{i}+4\hat{j}-4\hat{k}\right)-16=0$ is

(A) 3 (B) $\frac{11}{3}$ (C) 13 (D) – 13 (E) $\frac{13}{3}$

Space for rough work

78. If the straight lines $\frac{x+1}{2}=\frac{-y+1}{3}=\frac{z+1}{-2}$ and $\frac{x-3}{1}=\frac{y-\lambda}{2}=\frac{z}{3}$ intersect, then the value of λ is

(A) $-\frac{5}{8}$ (B) $-\frac{17}{8}$ (C) $-\frac{13}{8}$ (D) $-\frac{15}{8}$ (E) $-\frac{21}{8}$

79. If the angle θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{p}\ z+4=0$ is such that $\sin\theta=\frac{1}{3}$, then the value of p is

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

80. The ratio in which the plane $y-1=0$ divides the straight line joining $(1,-1,3)$ and $(-2,5,4)$ is

(A) $1:2$ (B) $3:1$ (C) $5:2$ (D) $1:3$ (E) $3:2$

81. Equation of the line passing through $\hat{i}+\hat{j}-3\hat{k}$ and perpendicular to the plane $2x-4y+3z+5=0$ is

(A) $\frac{x-1}{2}=\frac{1-y}{-4}=\frac{z-3}{3}$ (B) $\frac{x-1}{2}=\frac{1-y}{4}=\frac{z+3}{3}$ (C) $\frac{x-2}{1}=\frac{y+4}{1}=\frac{z-3}{3}$

(D) $\frac{x-1}{-2}=\frac{1-y}{-4}=\frac{z-3}{3}$ (E) $\frac{x-1}{2}=\frac{y-1}{4}=\frac{z-3}{-3}$

Space for rough work

82. Five dice are tossed. What is the probability that the five numbers shown will be different?

(A) $\frac{5}{54}$ (B) $\frac{5}{18}$ (C) $\frac{5}{27}$ (D) $\frac{5}{81}$ (E) $\frac{5}{36}$

83. If the events A and B are independent and if $P(\overline{A}) = \frac{2}{3}$, $P(\overline{B}) = \frac{2}{7}$, then $P(A \cap B)$ is equal to

(A) $\frac{4}{21}$ (B) $\frac{3}{21}$ (C) $\frac{5}{21}$ (D) $\frac{2}{21}$ (E) $\frac{1}{21}$

84. If the variance of 1, 2, 3, 4, 5, $\cdots$, x is 10, then the value of x is

(A) 9 (B) 13 (C) 12 (D) 10 (E) 11

85. Mean of 10 observations is 50 and their standard deviation is 10. If each observation is subtracted by 5 and then divided by 4, then the new mean and standard deviation are

(A) 22.5, 2.5 (B) 11.25, 2.5 (C) 11.5, 2.5 (D) 11, 2.5 (E) 11.75, 2.5

86. Let $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x. If $a = \sqrt{2011^2 + 2012}$, then the value of $f(a)$ is equal to

(A) 2010 (B) 2011 (C) 2012 (D) 2013 (E) 2014

Space for rough work

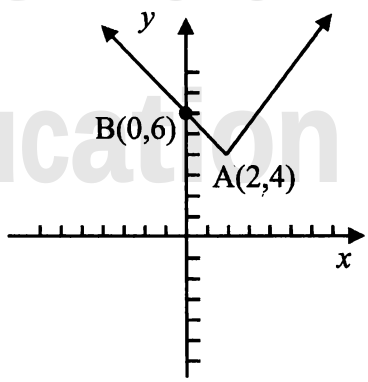
87. $\lim\limits_{x \to 3} \dfrac{\sqrt{x}-\sqrt{3}}{\sqrt{x^2-9}}$ is equal to

(A) 1 (B) 3 (C) $\sqrt{3}$ (D) $-\sqrt{3}$ (E) 0

88. $\lim\limits_{x \to 0} \dfrac{e^{x^2}-\cos x}{x^2} =$

(A) $\dfrac{3}{2}$ (B) $\dfrac{1}{2}$ (C) 1 (D) $-\dfrac{3}{2}$ (E) $-\dfrac{1}{2}$

89. In the given figure, the angle at A is $\frac{\pi}{\ }$. Then the graph represents the function



(A) $y=|2x-4|+4$ (B) $y=-|x|+6$ (C) $y=|4x-6|+2$

(D) $y=|x+2|+4$ (E) $y=|x-2|+4$

Space for rough work

90. If the function $f(x)=\begin{cases}\dfrac{x^2-(k+2)x+2k}{x-2} & \text{for } x\neq 2\\ 2 & \text{for } x=2\end{cases}$ is continuous at $x=2$, then k is

(A) $-\frac{1}{2}$ (B) -1 (C) 0 (D) $\frac{1}{2}$ (E) 1

91. If $xe^{xy}+ye^{-xy}=\sin^2 x$, then $\frac{dy}{dx}$ at $x=0$ is

(A) $2y^2-1$ (B) $2y$ (C) y^2-y (D) y^2+1 (E) y^2-1

92. If $y=\tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)$, then $\frac{dy}{dx}$ at $x=1$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) $\frac{-1}{2}$ (E) $\frac{3}{2}$

93. If $f(x)=\cos^{-1}\left(\frac{2\cos x+3\sin x}{\sqrt{13}}\right)$, then $\left[f'(x)\right]^2$ is equal to

(A) $\sqrt{1+x}$ (B) $1+2x$ (C) 2 (D) 1 (E) 0

94. If $u=\tan^{-1}\left(\frac{\sqrt{1-x^2}-1}{x}\right)$ and $v=\sin^{-1}x$, then $\frac{du}{dv}$ =

(A) $\sqrt{1-x^2}$ (B) $-\frac{1}{2}$ (C) 1 (D) $-x$ (E) -2

Space for rough work

95. If $y=\dfrac{1}{1+x+x^2}$, then $\dfrac{dy}{dx}$ is equal to

(A) $y^2(1+2x)$ (B) $\dfrac{-(1+2x)}{y^2}$

(D) $-y(1+2x)$ (E) $-y^2(1+2x)$

96. If $g(x)$ is the inverse of $f(x)$ and $f'(x)=\dfrac{1}{1+x^3}$, then $g'(x)$ is equal to

(A) $g(x)$ (B) $1+g(x)$ (C) $1+(g(x))^3$ (D) $\dfrac{1}{1+(g(x))^3}$ (E) 0

97. If f is defined and continuous on [3, 5] and if f is differentiable at $x = 4$ and $f'(4)=6$, then the value of $\lim\limits_{x\to 0}\dfrac{f(4+x)-f(4-x)}{4x}=$

(A) 0 (B) 2 (C) 3 (D) 4 (E) 6

98. The equation of the tangent to the curve $\sqrt{\dfrac{x}{a}}+\sqrt{\dfrac{y}{b}}=1$ at the point (x_1, y_1) is $\dfrac{x}{\sqrt{ax_1}}+\dfrac{y}{\sqrt{by_1}}=k$. Then the value of k is

(A) 2 (B) 1 (C) 3 (D) 7 (E) $\sqrt{2}$

Space for rough work

99. The slope of the normal to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is

(A) $\frac{6}{7}$ (B) $-\frac{6}{7}$ (C) $\frac{7}{6}$ (D) $-\frac{7}{6}$ (E) $-\frac{1}{2}$

100. If the slope of $y = 3x^2 + ax^3$ is maximum at $x = \frac{1}{2}$, then the value of a is equal to

(A) 2 (B) 1 (C) –1 (D) –2 (E) 3

101. If $y = 4x - 5$ is a tangent to the curve $y^2 = px^3 + q$ at (2, 3), then $p + q$ =

(A) –5 (B) 5 (C) –9 (D) 9 (E) 0

102. The point on the curve $y = 5 + x - x^2$ at which the normal makes equal intercepts is

(A) $(1,5)$ (B) $(0,-1)$ (C) $(-1,3)$ (D) $(0,3)$ (E) $(0,5)$

103. If the point (a, b) on the curve $y = \sqrt{x}$ is closest to the point $(1, 0)$, then the value of ab is

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{1}{4}$ (D) $\frac{\sqrt{2}}{4}$ (E) 1

Space for rough work

104. The image of the interval [–1, 3] under the mapping $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=4x^3-12x$ is

(A) [8, 72] (B) [0, 72] (C) [–8, 72] (D) [0, 8] (E) [–8, 8]

105. $\int \frac{1}{x(\log x)\log(\log x)}\,dx =$

(A) $\log(\log x)+C$ (B) $\log|\log(x\log x)|+C$ (C) $\log|\log|\log(\log x)||+C$

(D) $\log|x\log(\log x)|+C$ (E) $\log|\log(\log x)|+C$

106. $\int \frac{3^x}{\sqrt{1-9^x}}\,dx$ is equal to

(A) $(\log 3)\sin^{-1}(3^x)+C$ (B) $\frac{1}{3}\sin^{-1}(3^x)+C$ (C) $\left(\frac{1}{\log 3}\right)\sin^{-1}(3^x)+C$

(D) $\frac{1}{9}\sin^{-1}(3^x)+C$ (E) $\sin^{-1}(3^x)+C$

107. The value of the integral $\int \frac{\cos x}{\sin x+\cos x}dx$ is equal to

(A) $x+\log|\sin x+\cos x|+C$ (B) $\frac{1}{2}\left[x+\log|\sin x+\cos x|\right]+C$

(C) $\log|\sin x+\cos x|+C$ (D) $\frac{x}{2}+\log|\sin x+\cos x|+C$

(E) $x+\frac{1}{2}\log|\sin x+\cos x|+C$

Space for rough work

108. $\int\left(27e^{9x}+e^{12x}\right)^{1/3}dx$ is equal to

(A) $(1/4)(27+e^{3x})^{1/3}+C$
(B) $(1/4)(27+e^{3x})^{2/3}+C$
(C) $(1/3)(27+e^{3x})^{4/3}+C$
(D) $(1/4)(27+e^{3x})^{4/3}+C$
(E) $(3/4)(27+e^{3x})^{4/3}+C$

109. $\int\frac{4dx}{x^2\sqrt{4-9x^2}}=$

(A) $\sqrt{4-9x^2}+C$
(B) $\frac{-2}{3}\sqrt{4-9x^2}+C$
(C) $\frac{-\sqrt{4-9x^2}}{x}+C$
(D) $\frac{2}{3}\frac{\sqrt{4-9x^2}}{x}+C$
(E) $-3\sqrt{4-9x^2}+C$

110. $\int e^{-x}\operatorname{cosec}x\,(1+\cot x)dx=$

(A) $e^{-x}\operatorname{cosec}x+C$
(B) $-e^{-x}\operatorname{cosec}x+C$
(C) $-e^{-x}(\operatorname{cosec}x+\cot x)+C$
(D) $-e^{-x}(\operatorname{cosec}x-\tan x)+C$
(E) $-e^{-x}\sec x+C$

111. $\int\frac{\sin x+\cos x}{e^{-x}+\sin x}dx=$

(A) $\log|1-e^{x}\sin x|+C$
(B) $\log|1+e^{-x}\sin x|+C$
(C) $\log|1+e^{x}\sin x|+C$
(D) $\log|1-e^{-x}\sin x|+C$
(E) $\log|1+e^{2x}\sin x|+C$

Space for rough work

112. The solution of $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{12}$ is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

113. Area bounded by the curve $y = \log(x-2)$, x-axis and $x = 4$ is equal to

(A) 2 log 2 + 1 (B) log 2 – 1 (C) log 2 + 1 (D) 1 – 2 log 2 (E) 2 log 2 – 1

114. Let $f(x) = x^2 - 2$. If $\int_3^6 f(x)dx = 3f(c)$ for some $c \in (3, 6)$, then the value of c is equal to

(A) $\sqrt{12}$ (B) $\sqrt{21}$ (C) $\sqrt{19}$ (D) $\sqrt{17}$ (E) $\sqrt{13}$

115. $\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{1+2\cos^2 x} dx$ is equal to

(A) $\frac{1}{2}\log 2$ (B) $\log 2$ (C) $\frac{1}{2}\log 3$ (D) $\log 3$ (E) $\frac{1}{3}\log 3$

116. $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x} =$

(A) 1 (B) π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$ (E) 0

Space for rough work

117. The solution of the differential equation $\log x \frac{dy}{dx} + \frac{y}{x} = \sin 2x$ is

(A) $y \log|x| = C - \frac{1}{2}\cos x$

(B) $y \log|x| = C + \frac{1}{2}\cos 2x$

(C) $y \log|x| = C - \frac{1}{2}\cos 2x$

(D) $xy \log|x| = C - \frac{1}{2}\cos 2x$

(E) $y \log|x| = Cx - \frac{1}{2}\cos 2x$

118. If $xy = A\sin x + B\cos x$ is the solution of the differential equation $x\frac{d^2y}{dx^2} - 5a\frac{dy}{dx} + xy = 0$ then the value of a is equal to

(A) $\frac{2}{5}$

(B) $\frac{5}{2}$

(C) $\frac{-2}{5}$

(D) $\frac{-5}{2}$

(E) $\frac{1}{2}$

119. The solution of the differential equation $\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$ is

(A) $y = e^{3x} + C$

(B) $y = 2e^{2x} + C$

(C) $y = e^x + C$

(D) $y = e^{4x} + C$

(E) $y = 9e^{3x} + C$

120. The order and degree of the differential equation of the family of circles of fixed radius r with centres on the y-axis, are respectively

(A) 2, 2

(B) 2, 3

(C) 1, 1

(D) 3, 1

(E) 1, 2

Space for rough work

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