SOLUTIONS & ANSWERS FOR KERALA ENGINEERING ENTRANCE EXAMINATION-2013 – PAPER II
VERSION – B1

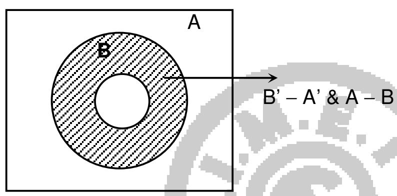
[MATHEMATICS]

1. Ans: 56

Sol: $n(A \cap B) = n(A) + n(B) - n(A \cup B)$
$= 19$
$n(A - B) \cup n(B - A)$
$= n(A \cup B) - n(A \cap B)$
$= 75 - 19$
$= 56.$

2. Ans: B' – A' = A – B

Sol:



3. Ans: $(-3, \infty) - \{-1, -2\}$

Sol: $\log_2(x + 3) > 0 \Rightarrow x \in (-3, \infty)$
$x^2 + 3x + 2 \neq 0 \Rightarrow (x + 1)(x + 3) \neq 0$
$\Rightarrow x \neq -1, x \neq -2$
$\therefore x \in (-3, \infty) - \{-1, -2\}$

4. Ans: –12

Sol: $(3 \oplus 4) * 5 = (3^2 + 4) * 5$
$= 13 * 5$
$= 13 - 5^2 = 13 - 25 = -12.$

5. Ans: 2^5

Sol: $X - \{4\} - \{1, 2, 3, 5\}$
$= \{6, 7, 8, 9, 10\}$
Number of subsets of $\{6, 7, 8, 9, 10\}$
$= 2^5$

6. Ans: 81

Sol: $A = \{1, b, c, d\}$, $B = \{1, 2, 3\}$
Element a can have an image in 3 ways (1, 2, or 3). Similar is the case for b, c
$\therefore$ total number of ways $= 3 \times 3 \times 3 \times 3$
$= 81$

7. Ans: $32\left(x_1^2 + y_1^2\right)$

Sol: $x_1^2|z_1|^2 + y_1^2|z_2|^2 + y_1^2|z_1|^2 + x_1^2|z_2|^2$
$= 2\left(x_1^2 + y_1^2\right)\left(4^2\right)$
$= 32\left(x_1^2 + y_1^2\right)$

8. Ans: $\frac{5\pi}{12}$

Sol: $z_1 = -i\,\bar{z}_2$
$\arg(z_1) = \arg(-i) + \arg\left(\bar{z}_2\right)$
$\Rightarrow \arg(z_1) + \arg(z_2) = -90°$ -----(1)
$\arg\left(\bar{z}_1\right) + \arg\left(\bar{z}_2\right) = \frac{\pi}{3}$
$\Rightarrow \arg z_2 - \arg z_1 = 60°$ -----(2)
(1) – (2) $\Rightarrow 2\arg(z_1) = -150°$
$\arg(z_1) = -75$
$\Rightarrow \arg\left(\bar{z}_1\right) = 75° = \frac{5\pi}{12}.$

9. Ans: 169

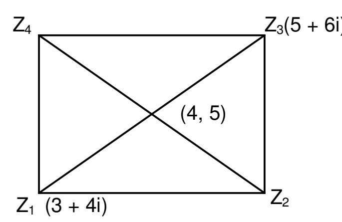
Sol: $x + iy + \sqrt{x^2 + y^2} = 8 + 12i$
$\Rightarrow y = 12$
$x + \sqrt{x^2 + 144} = 8$
$\Rightarrow x = -5$
$z = -5 + 12i$
$|z|^2 = 25 + 144 = 169.$

10. Ans: $-1 - i$

Sol: $\dfrac{\frac{1}{i}\left(1 - \left(\frac{1}{i}\right)^{102}\right)}{1 - \frac{1}{i}} = \dfrac{\frac{1}{i}(1 + 1)}{1 + i} = -1 - i$

11. Ans: 5 + 4i, 3 + 6i

Sol:



By inspection

12. Ans: a = b = c

Sol: Given expression
$= 3x^2 - 2(a + b + c)x + ab + ac + bc = 0$
Since roots are equal
$4(a + b + c)^2 - 4 \times 3(ab + bc + ac) = 0$
$4[(a^2 + b^2 + c^2) + 2ab + 2ac + 2bc] - 12(ab + bc + ac) = 0$

$4(a^2 + b^2 + c^2) - 4(ab + ac + bc) = 0$

$\frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right] = 0$

$a = b = c$.

13. Ans: -9

Sol: $\alpha + \beta = \frac{-q}{p}$ $\quad 4 = \frac{-q}{p}$ $\quad q = -4p$

p, q, r in A.P

$2q = p + r \quad -8p = p + r$

$r = -9p$

$\alpha\beta = \frac{r}{p} = \frac{-9p}{p} = -9$.

14. Ans: $-4, 1$

Sol: $p = 0, c = -4, b = 3, q = 2$

$x^2 + bx + c = 0 \Rightarrow x^2 + 3x - 4 = 0$

$(x + 4)(x - 1) = 0 \Rightarrow x = -4$ or 1

15. Ans: $\frac{1}{18}$

Sol: $a^2 - 3a + 1 = 0 \Rightarrow a^2 + 1 = 3a$

$a + \frac{1}{a} = 3$

$a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)$

$= 27 - 3 \times 3 = 18$

$\therefore \frac{a^3}{a^6 + 1} = \frac{1}{a^3 + \frac{1}{a^3}} = \frac{1}{18}$.

16. Ans: 2

Sol: sum of the roots $= -\left(\frac{2a+3}{a+1}\right) = -1$

i.e. $-2a - 3 = -a - 1 \Rightarrow a = -2$

product of roots $= \frac{3a+4}{a+1}$

$= \frac{-6+4}{-2+1} = \frac{-2}{-1} = 2$

17. Ans: 36

Sol: If $x > 0$ $(x^2 - 5x - 6) = 0$

$\Rightarrow (x - 6)(x - 1) = 0 \Rightarrow x = 6$

If $x < 0$. $-x^2 - 5x - 6 = 0$

$\Rightarrow x^2 + 5x + 6 = 0 \Rightarrow (x + 2)(x + 3) = 0$

$\Rightarrow x = -2$ or -3.

18. Ans: 19804

Sol: $a_{n+1} - a_n = 4n$ put n = 1, 2, …..99

$a_2 - a_1 + a_3 - a_2 + a_4 - a_3 + \ldots + a_{100} - a_{99}$

$= 4(1 + 2 + \ldots\ldots + 99)$

$a_{100} - a_1 = 4 \times \frac{99 \times 100}{2}$

$a_{100} = 4 + 2(99 \times 100)$

$= 19804$.

19. Ans: 2059

Sol: $a = 729$, $ar^6 = 64$ $\quad r = \frac{2}{3}$

$S = \frac{a(1-r^7)}{1-r} = 2059$

20. Ans: 2

Sol: $a_2 + a_3 = 10$

$a_2 a_3 = 24$

$a_2 = 4, \; a_3 = 6$

21. Ans: 14

Sol: $1^{st} \rightarrow 1$

$2^{nd} \rightarrow 2$

$4^{th} \Rightarrow 3$

$7^{th} \Rightarrow 4$

$11^{th} \Rightarrow 5$

………

$100^{th} \Rightarrow ?$

1, 2, 4, 7, 11, …….

$a_n = 1 + \frac{(n-1)n}{2} = \frac{n^2 - n + 2}{2}$

If n = 14 then $a_n = 92$

i.e, 92^{nd} term 14

If n = 15 then $a_n = 106$

i.e, 106^{th} term 15

$\therefore$ 100^{th} term is 14

22. Ans: $-7, 2$

Sol: $\frac{4}{2}(2a + 3d) = -34 ----(1)$

$\frac{5}{2}(2a + 4d) = -60 ----(2)$

Solving, $d = -7$ and $a = 2$

23. Ans: 2

Sol: $\frac{10}{2}(2a + 9d) = \frac{1}{2}.\frac{10}{2}(2a + 29d)$

$4a + 18d = 2a + 29d \Rightarrow 2a = 11d$

$a + d = 13$

$a = 13 - d$

$2(13 - d) = 11d \Rightarrow 26 - 2d = 11d$

$13d = 26 \Rightarrow d = 2$.

24. Ans: $10r - 3n - 3 = 0$

Sol: $\frac{^nC_{r-1}}{^nC_r} = \frac{r}{n-r+1} = \frac{36}{84} = \frac{3}{7}$

$7r = 3n - 3r + 3$

$10r = 3n + 3 \Rightarrow 10r - 3n - 3 = 0$.

25. Ans: $-n$

Sol: $(1 + x + x^2)^n x$
$= x + a_1x^2 + a_2x^3 + \ldots + a_{2n}x^{2n+1}$
Differentiating both sides
$(1 + x + x^2)^n + x \cdot n(1 + x + x^2)^{n-1}(1 + 2x)$
$= 1 + 2a_1x + 3a_2x^2 + \ldots\ldots + (2n + 1) \cdot a_{2n} \cdot x^{2n}$
Put $x = -1$
$2a_1 - 3a_2 + \ldots\ldots -(2n + 1)a_{2n} = -n$

26. Ans: 15

Sol: $(a + b)^{n+4} \rightarrow t_6 = t_{5+1} = {}^{n+4}C_5 a^{n-1} b^5$
$t_5 = t_{4+1} = {}^{n+4}C_4\, a^{n-2}\, b^6$
$(a + b)^n \rightarrow t_5 = t_{4+1} = {}^nC_4\, a^{n-4}\, b^4$
$t_4 = t_{3+1} = {}^nC_3 a^{n-3}\, b^3$
$\frac{t_6}{t_5} = \frac{t_5}{t_4}$ i.e $\frac{n}{5} = \frac{n-3}{4} \Rightarrow n = 15$

27. Ans: 259

Sol: □ □ □

$7 \times 6 \times 5 = 210$
Numbers can be with 1 digit, 2 digit and 3 digits are 1, 2, 3, 4, 5, 6, 7
Required numbers $= 7P_1 + 7P_2 + 7P_3$
$= 259$

28. Ans: 5

Sol: $8!\left[\frac{1}{3!}+\frac{5}{4!}\right] = \frac{8!}{3!}\left[1+\frac{5}{4}\right] = \frac{8!}{3!}\times\frac{9}{4} = \frac{9!}{24}$

$\frac{9!}{24} = {}^9P_r = (9-r)! = 4!$
$\Rightarrow 9 - r = 4 \Rightarrow r = 5.$

29. Ans: 205

Sol: ${}^{12}C_3 - \left[{}^3C_3 + {}^4C_3 + {}^5C_3\right] = 205$

30. Ans: $\frac{\pi}{8}$

Sol: $.R_1 + R_2 + R_3$ $(2\sin2x + \cos2x)$

$$\begin{vmatrix} 1 & 1 & 1 \\ \sin 2x & \cos 2x & \sin 2x \\ \sin 2x & \sin 2x & \cos 2x \end{vmatrix} = 0$$

$C_2 - C_1$, $C_3 - C_1$

$$\begin{vmatrix} 1 & 0 & 0 \\ \sin 2x & \cos 2x - \sin 2x & 0 \\ \sin 2x & 0 & \cos 2x - \sin 2x \end{vmatrix}$$

$= 0$
$(\cos 2x - \sin 2x)^2 = 0$
$\cos2x = \sin2x$
$x = \frac{\pi}{8}.$

31. Ans: 32

Sol: $D_1 = 8\begin{vmatrix} 4 & 3 & 2 \\ 8 & 3 & 5 \\ 4 & 5 & 3 \end{vmatrix}$

$= 8 \times 4\begin{vmatrix} 1 & 3 & 2 \\ 2 & 3 & 5 \\ 1 & 5 & 3 \end{vmatrix}$

$\Rightarrow \lambda = 32.$

32. Ans: 8

Sol: $e^{3x+1} = e^{9+2x}$
$3x + 1 = 9 + 2x$
$x = 8.$

33. Ans: -7

Sol: $\left|A^{2013} - 3A^{2012}\right| = \left|A^{2012}\right| \left|A - 3I\right|$

$= \left|A\right|^{2012} \left|A - 3I\right| = (1)\times\begin{vmatrix} 0 & 7 \\ 1 & -1 \end{vmatrix}$

$= 1 \times (0 - 7) = -7.$

34. Ans: A is non-singular and A + I is non-singular

Sol: $A(A + I) = -4I$
$|A| \cdot |A + I| = 4$
Both A and A + I are non singular.

35. Ans: 0

Sol: Shortcut method

$\begin{vmatrix} 1 & 1 & 1 \\ 5 & 10 & 5 \\ 1 & 1 & 1 \end{vmatrix} = 0.$

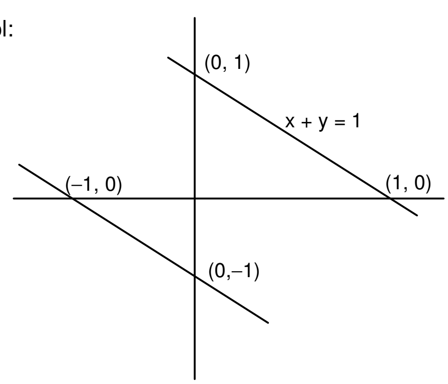
36. Ans: $(4, \infty)$

Sol: $x + 7 < 2x + 3$
$4 < x \Rightarrow x > 4$
$2x + 4 < 5x + 3$
$\Rightarrow 1 < 3x \Rightarrow 3x > 1 \Rightarrow x > \frac{1}{3}$

$\Rightarrow x > \frac{1}{3} \Rightarrow x > 4$
$(4, \infty).$

37. Ans: $\{(1, 0), (0, 1)\}$ and $\{(-1, 0), (0, -1)\}$

Sol:



{(1, 0), (0, 1)} and {(–1, 0), (0, –1)}

38. Ans: $\sim p \wedge q$

Sol: p : 2 plus 3 is five
~p : it is not that 2 plus 3 is five
∴ Delhi is the capital of India and it is not that 2 plus 3 is five
is $\sim p \wedge q$.

39. Ans: R or not Q

Sol: R or not Q

40. Ans: Mumbai is the capital of India.

Sol: Mumbai is the capital of India.

41. Ans: $\frac{877}{1024}$

Sol: $\cos\alpha + \sin\alpha = \frac{3}{4}$

$\cos^6\alpha + \sin^6\alpha$

$= (\cos^2\alpha + \sin^2\alpha)^3 - 3\sin^2\alpha\cos^2\alpha$
$(\sin^2\alpha + \cos^2\alpha)$

$= 1 - 3\sin^2\alpha\cos^2\alpha$
$\left|(\cos\alpha + \sin\alpha)^2 - 2\sin\alpha\cos\alpha\right|$

$= 1 - 3\sin^2\alpha\cos^2\alpha\left[\frac{9}{16} - 2\sin\alpha\cos\alpha\right]$

$(\cos\alpha + \sin\alpha)^2 = \cos^2\alpha + \sin^2\alpha + 2\sin\alpha\cos\alpha$

$= \frac{9}{16}$

$\sin\alpha\cos\alpha = \frac{-7}{16} \times \frac{1}{2} = \frac{-7}{32}$

$= 1 - 3\left[\frac{49}{1024}\right]\left[\frac{9}{10} + \frac{7}{16}\right]$

$= 1 - \frac{147}{1024}\left[\frac{9}{16} + \frac{7}{16}\right]$

$= 1 - \frac{147}{1024} = \frac{877}{1024}$.

42. Ans: $\frac{-1+\sqrt{5}}{2}$

Sol: $\cos^2\alpha + \cos x = 1$

$\sin^4 x\,(1 + \sin^2 x) = 2$

$\cos x = \frac{-1 \pm \sqrt{1+4}}{2}$

$= \frac{-1 \pm \sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}$

$\sin^2 x(1 + \sin^2 x)$

43. Ans: $x + y + xy = 1$

Sol. Put $x = \tan\theta$ and $y = \tan\phi$

$\therefore \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\phi}{1+\tan^2\phi}\right)$

$= \frac{\pi}{2}$

$\cos^{-1}(\cos 2\theta) + \cos^{-1}(\cos 2\phi) = \frac{\pi}{2}$

$2(\theta + \phi) = \frac{\pi}{2}$

$\left(\tan^{-1}x + \tan^{-1}y\right) = \frac{\pi}{4}$

$\frac{x+y}{1-xy} = 1$; $x + y + xy = 1$

44. Ans: $\frac{13}{5}$

Sol: $\tan\frac{\theta}{2} = \frac{2}{3}$

$\cos\theta = \frac{1-\frac{4}{9}}{1+\frac{4}{9}} = \frac{5}{13}$

$\Rightarrow \sec\theta = \frac{13}{5}$.

45. Ans: $-\frac{9}{46}$

Sol: Put $\tan^{-1}\left(\frac{1}{5}\right) = x$

$\tan\left[3x - \frac{\pi}{4}\right] = \frac{\tan 3x - \tan\frac{\pi}{4}}{1 + \tan 3x}$

$\left[\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}\right]$

$\therefore \tan\left[3x - \frac{\pi}{4}\right] = \frac{\frac{37}{55} - 1}{1 + \frac{37}{55}} = \frac{-9}{46}$.

46. Ans: 10

Sol: $\frac{1}{\cos^2\alpha} + \frac{1}{1+\sin^2\alpha} + \frac{2}{1+\sin^4\alpha} + \frac{4}{1+\sin^8\alpha}$

$\frac{1}{1-\sin^2\alpha} + \frac{1}{1+\sin^2\alpha} + \frac{2}{1+\sin^4\alpha} + \frac{4}{1+\sin^8\alpha}$

$\frac{2}{1-\sin^4\alpha} + \frac{2}{1+\sin^4\alpha} + \frac{4}{1+\sin^8\alpha}$

$= \frac{4}{1-\sin^8\alpha} + \frac{4}{1+\sin^8\alpha} = \frac{4\times 2}{1-\sin^{16}\alpha}$

$= \frac{8}{1-\frac{1}{5}} = 10.$

47. Ans: 2 secx

Sol: $\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$= \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}+\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$

$= \frac{1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}+1+\tan^2\frac{x}{2}-2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$

$= \frac{2\sec^2\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}.\cos^2\frac{x}{2} = 2\sec x.$

48. Ans: $\frac{1}{\sqrt{3}}$

Sol: $-\tan^{-1}x+\left(\pi-\frac{\pi}{3}\right)=\frac{\pi}{2}$

$-\tan^{-1}x+\frac{2\pi}{3}=\frac{\pi}{2}$

$-\tan^{-1}x=\frac{\pi}{2}-\frac{2\pi}{3}$

$=\frac{3\pi-4\pi}{6}=\frac{-\pi}{6}$

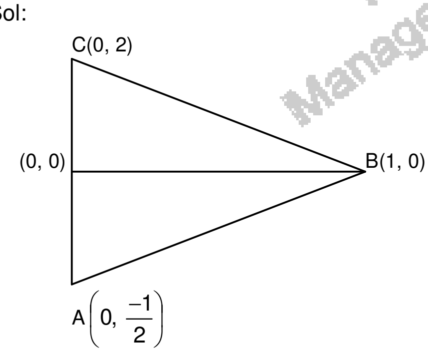
$\therefore x = \frac{1}{\sqrt{3}}$

49. Ans: 0

Sol: $\cot^{-1}b - \cot^{-1}a + \cot^{-1}c - \cot^{-1}b + \cot^{-1}a - \cot^{-1}c = 0$

50. Ans: (1, 0)

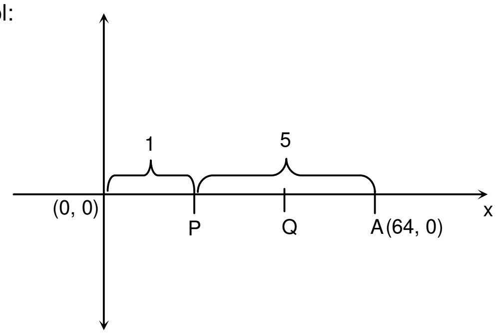
Sol:



AB perpendicular to AC
Orthocentre is the meeting point of AB and OB ie. (1, 0)

51. Ans: $\left(\frac{32}{3}, 0\right)$

Sol:



P is $\left(\frac{5\times0+1\times64}{5+1}, 0\right)$

$\left(\frac{64}{6}, 0\right)=\left(\frac{32}{3}, 0\right)$

52. Ans: x + y = 4

Sol: $(x-1)^2+(y-1)^2=(x-3)^2+(y-3)^2$
$x^2-2x+1+y^2-2y+1$
$= x^2-6x+9+y^2-6y+9$
$2-2x-2y=-6x-6y+18$
$4x+4y=16$
$x+y=4.$

53. Ans: a = 17

Sol: $\begin{vmatrix}9 & 5 & 1\\1 & 2 & 1\\a & 8 & 1\end{vmatrix}=0$

$9[2-8]-5[1-a]+1[8-2a]=0$
$-51+3a=0$
$3a=51$
$a=17.$

54. Ans: 90

Sol: $m_1 = 3$
$m_2 = 6$
$y = 3x + c_1$
$y = 6x + c_2$
since the point is (30, 40)
we have $c_1 = -50$
$40 = 180 + c_2$
$c_2 = +140 - 50$
$= +90.$

55. Ans: $-\sqrt{2}$

Sol: given $3x + 3y + 5 = 0$
$x\cos\alpha + y\sin\alpha = p$
$\Rightarrow \left(\frac{-1}{\sqrt{2}}\right)x+\left(\frac{-1}{\sqrt{2}}\right)y=\frac{5}{3\sqrt{2}}$

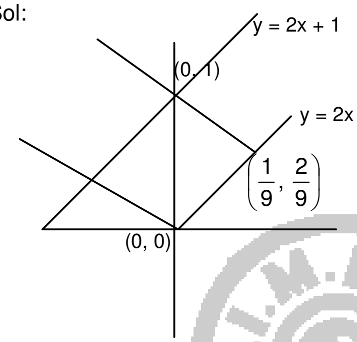
$\sin\alpha+\cos\alpha=\frac{-1}{\sqrt{2}}+\frac{-1}{\sqrt{2}} = -\sqrt{2}.$

56. Ans: (–7, 11), (3, 1)

Sol: Let (α, β) be the points on x + y = 4
The perpendicular distance to
4x + 3y − 10 = 0 is ± 1
i.e, $\frac{4\alpha + 4\beta - 10}{\sqrt{25}} = \pm 1$
Also, α + β = 4
Solving for (α, β) the points are
(–7, 11), (3, 1).

57. Ans: $\frac{1}{9}$

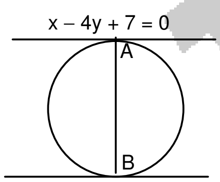
Sol:



Area of a parallelogram
= 2 ×
$\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$
$= \left[\frac{1}{9}(1) + 0\right] = \frac{1}{9}$

58. Ans: $\frac{5}{3\sqrt{17}}$

Sol:



3x –12y + 11 = 0, $x - 3y - \frac{11}{3} = 0$

Diameter = $\frac{\left|7 - \frac{11}{3}\right|}{\sqrt{1^2 + 4^2}} = \frac{10}{3\sqrt{17}}$

Radius = $\frac{5}{3\sqrt{17}}$

59. Ans: (–4, –20)

Sol: The centres are $\left(\frac{-a}{2}, 3\right), (6, 0)$

$\left(6 + \frac{a}{2}\right)^2 + 9 = 25$

$36 + \frac{a^2}{4} + 6a + 9 = 25$

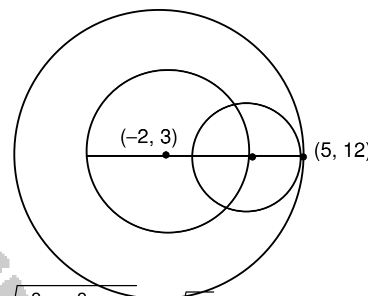
$45 + \frac{a^2}{9} + 6a = 25$
$a^2 + 24a + 80 = 0$
(a + 4) (a + 20) = 0; a = –4, –20

60. Ans: $x^2 + y^2 - 2x - 2y + 1 = 0$

Sol: Let radius be a
The perpendicular distance to
4x + 3y = 12 is $\frac{4a + 3a - 12}{\sqrt{25}} = -a$
7a – 12 = –5a
a = 1
$(x - 1)^2 + (y - 1)^2 = 1$
$x^2 + y^2 - 2x - 2y + 1 = 0.$

61. Ans: 12

Sol:



$\sqrt{2^2 + 3^2 + 23} = \sqrt{36} = 6$
Dia = 12cm
Difference between the radii is the diameter of the given circle.

62. Ans: (–2, 0)

Sol: For parabola $y^2 = -4ax$
Focus is (–a,0)
Here x = 2
∴ Focus is (–2, 0)

63. Ans: $y^2 - 6y - 8x - 23 = 0$

Sol: For a parabola
$\frac{SP}{PM} = 1 [\because e = 1]$
$SP^2 = PM^2$
$SP^2 = (x + 2)^2 + (y - 3)^2$
$PM^2 = (x + 6)^2$
$(x + 6)^2 = (x + 2)^2 + (y - 3)^2$
i.e $y^2 - 6y - 8x - 23 = 0$

64. Ans: $\sqrt{5}$

Sol: $4x^2 - 8x - y^2 - 8y - 28 = 0$
$4(x^2 - 2x) - (y^2 - 8y) - 28 = 0$
$4(x^2 + 2x + 1 - 1) - (y^2 - 8y + 16 - 16) - 28 = 0$

$4(x-1)^2 - 4 - (y - 4)^2 + 16 - 28 = 0$

$4(x-1)^2 - (y-4)^2 = 16$

$$\frac{(x-1)^2}{4}-\frac{(y-4)^2}{16}=1$$

$$\frac{x^2}{4}-\frac{y^2}{16}=1$$

$b^2 = 16$, $a^2 = 4$

$16 = 4(e^2 - 1)$

$4 = e^2 - 1$

$e^2 = 5$

$e = \sqrt{5}$

65. Ans: 12

Sol: given $\frac{x^2}{\frac{144}{13}}-\frac{y^2}{\frac{25}{13}}=1$

$a^2 = \frac{144}{13}$ $b^2 = \frac{25}{13}$

$c^2 = a^2 + b^2 = \frac{144}{13}+\frac{25}{13}=13$

For an ellipse, $c^2 = a^2 - b^2$

$13 = 25 - b^2$

$b^2 = 25 - 13 = 12$

66. Ans: 8

Sol: $a^2 = 16$; $a = 4$

$b^2 = 8$; $b = \sqrt{8}$

$e = \frac{1}{\sqrt{2}}$

Area is maximum when vertex is at (0, b)

$\therefore$ maximum area $= \frac{1}{2} 2ae \times b = 8$

67. Ans: $\frac{-1}{2}$

Sol: $|p+q| = \sqrt{3}$

$p^2 + q^2 + 2pq = 3$

$2pq = 1$; $pq = \frac{1}{2}$

$(2p - 3q)(3p + q)$

$= 6p^2 + 2pq - 9pq - 3q^2$

$= 6 - 3 - 7pq = 3 - 7pq = \frac{3-7}{2}=\frac{-1}{2}$

68. Ans: Right angled isosceles triangle

Sol: $AB = 2\bar{i}+\bar{j}+2\bar{k} = \sqrt{4+1+4}=3$

$BC = -\bar{i}+\bar{j}-4\bar{k} = \sqrt{1+1+16}=\sqrt{18}$

$CA = - -\bar{i}-2\bar{j}+2\bar{k} = \sqrt{1+4+4}=3$

$\therefore$ Triangle is isosceles right angled triangle

69. Ans: $\frac{-7}{3}$

Sol: $(2-1)\vec{i}+(-3\lambda-2)\vec{j}+(-3-4)\vec{k}$

$= \vec{i}+5\vec{j}-7\vec{k}$

$\therefore 5 = -3\lambda - 2$

$-3\lambda = 7$

$\lambda = \frac{-7}{3}$

70. Ans: $\left(\frac{1}{2},\frac{1}{2}\right)$

Sol: $\bar{u} = 5\bar{a}+6\bar{b}+7\bar{c}$

$\bar{v} = 7\bar{a}-8\bar{b}+9\bar{c}$

$\bar{w} = 3\bar{a}+20\bar{b}+5\bar{c}$

$\ell\left[7\bar{a}-8\bar{b}+9\bar{c}\right]+m\left[3\bar{a}+20\bar{b}+5\bar{c}\right]$

$= 5\bar{a}+6\bar{b}+7\bar{c}$

$7\ell + 3m = 5$; $-8\ell + 20m = 6$;

$9\ell + 5m = 7$

$\ell = \frac{1}{2}$ m; $= \frac{1}{2}$

71. Ans: $\sqrt{41}$

Sol: $\bar{\alpha}.\bar{\beta} = 3$, $|\beta| = \sqrt{5}$

$|\alpha| = \sqrt{3^2+1^2} = \sqrt{10}$

$\alpha\beta = \sqrt{50}$

$\bar{\alpha}.\bar{\beta} = \alpha\,\beta\cos\theta$

$3 = \sqrt{10}\times\sqrt{5}\cos\theta \Rightarrow \cos\theta = \frac{3}{\sqrt{50}}$

$\left|\bar{\alpha}\times\bar{\beta}\right| = \alpha\beta\sin\theta = \alpha\beta\sqrt{1-\cos^2\theta}$

$= \sqrt{50}\times\sqrt{1-\frac{9}{50}}$

$= \sqrt{50}\times\frac{\sqrt{41}}{\sqrt{50}} = \sqrt{41}$

72. Ans: $\frac{\pi}{2}$

Sol: $3\vec{p}+2\vec{q} = \vec{i}+\vec{j}+\vec{k}$

$3\vec{p}+2\vec{q} = \vec{i}-\vec{j}-\vec{k}$

$6p = 2\vec{i}$; $4q = 2\vec{j}+2\vec{k}$

$\vec{p} = \frac{1}{3}i$; $\vec{q} = \frac{\vec{j}}{2}+\frac{+\vec{k}}{2}$

$p \,.\, q = |p||q|\cos\theta = 0$

$\therefore \theta = \frac{\pi}{2}$

73. Ans: $\sqrt{18}$

Sol: $\frac{1}{2}\left|\overline{PR}+\overline{PQ}\right|$

$= \frac{1}{2}|2i + 2j + 8k|$

$= \frac{1}{2}\sqrt{4+4+64} = \sqrt{18}$

74. Ans: (4, –2, –1)

Sol: Any point on the line
$(2\lambda + 2, -3\lambda + 1, \lambda - 2)$
i,e on the plane $x + 3y - z + 1 = 0$
i.e $2\lambda + 2 + 3(-3\lambda + 1) - (\lambda - 2) + 1 = 0$
$\Rightarrow \lambda = 1$
Points (4, –2, –1)

75. Ans: 1

Sol: $\frac{x - \frac{1}{2}}{1} = \frac{y-3}{-1} = \frac{z-1}{3}$ & $\frac{x+3}{2} = \frac{y+2}{5} = \frac{z+1}{P}$

Since lines are perpendicular
$1 \times 2 + -1 \times 5 + 3 \times P = 0$
P = 1

76. Ans: $(6\sqrt{2}, 6, 6)$

Sol: $\sqrt{x^2 + y^2 + z^2} = 12$ direction cosinθ
cos45°, cos60°, cos60°

$x = r\cos\alpha = 12 \times \frac{1}{\sqrt{2}} = 6\sqrt{2}$,

$y = r\cos\beta = 12 \times \frac{1}{2} = 6$, $z = r\cos 60 = 6$

$P(x, y, z) \rightarrow (6\sqrt{2}, 6, 6)$

77. Ans: $\frac{13}{3}$

Sol: $x + 2y - 2z + 5 = 0$ & $2x + 4y - 4z - 16 = 0$

distance $= \frac{5-(-8)}{\sqrt{1^2+2^2+2^2}} = \frac{13}{3}$

78. Ans: $\lambda = \frac{-5}{8}$

Sol: $\frac{x-(-1)}{2} = \frac{y-1}{-3} = \frac{z-(-1)}{-2}$

$\frac{x-3}{1} = \frac{y-\lambda}{2} = \frac{z-0}{3}$

$\begin{vmatrix} 4 & \lambda-1 & 1 \\ 2 & -3 & -2 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow \lambda = \frac{-5}{8}$

79. Ans: $p = \frac{5}{3}$

Sol: $\sin\theta = \frac{2 + -2 + 2\sqrt{P}}{\sqrt{4+4+1}\sqrt{4+1+P}} = \frac{1}{3}$

$P = \frac{5}{3}$

80. Ans: 1: 2

Sol: Ratio $= -\left[\frac{-1-1}{5-1}\right] = \frac{2}{4}$

= 1 : 2

81. Ans: $\frac{x-1}{2} = \frac{1-y}{4} = \frac{z+3}{3}$

Sol: $\frac{x-1}{2} = \frac{y-1}{-4} = \frac{z+3}{3}$

ie $\frac{x-1}{2} = \frac{1-y}{4} = \frac{z+3}{3}$

82. Ans: $\frac{5}{54}$

Sol: Total no of cases $= 6^5$
no: of favourable case = 6!
= 720

$\therefore$ required probability $= \frac{720}{6^5}$

$= \frac{5}{54}$

83. Ans: $\frac{5}{21}$

Sol: $P(A) = \frac{1}{3}, P(B) = \frac{5}{7}$

$P(A \cap B) = \frac{5}{21}$

84. Ans: 11

Sol: $\sqrt{\frac{n^2-1}{12}} = 10$

$n^2 - 1 = 120$

$n^2 = 121$

n = 11

85. Ans: 11.25, 2.5

Sol: New mean $= \frac{500-50}{4 \times 10}$

= 11.25

New S. D $= \frac{10}{4} = 2.5$

86. Ans: 2011

Sol: a = 2011.5
f(a) = 2011

87. Ans: 0

Sol: $\lim\limits_{x\to 3} \dfrac{x-3}{\sqrt{(x+3)(x-3)\left(\sqrt{x}+\sqrt{3}\right)}}$

$\lim\limits_{x\to 3} \sqrt{\dfrac{x-3}{x+3}} \quad \dfrac{1}{\left(\sqrt{x}+\sqrt{3}\right)}$

$= 0$

88. Ans: $\dfrac{3}{2}$

Sol: $\lim\limits_{x\to 0} \dfrac{e^{x^2}+2x+\sin x}{2x}$

$\lim\limits_{x\to 0} 2\dfrac{\left[x\,e^{x^2}\ \ 2x+e^{x^2}\right]+\cos x}{2}$

$= \dfrac{3}{2}$

89. Ans: y = |x –2| +4

Sol: y = |x –2| +4

90. Ans: 0

Sol: $\lim\limits_{x\to 2} (x-k) = 2-k$

$2-k = 2 \Rightarrow k = 0$

91. Ans: $y^2 - 1$

Sol: $x\ e^{xy}\left(y + x\dfrac{dy}{dx}\right) + e^{xy} +$

$y\ e^{-xy}\left(-x\dfrac{dy}{dx} - y\right) + e^{-xy}\dfrac{dy}{dx}$

$= 2\sin x\cos x$

$1-y^2 + \dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = y^2 - 1$

92. Ans: $\dfrac{3}{2}$

Sol: $y = \tan^{-1}\left(\dfrac{x+x-1}{1+x(x-1)}\right) = \tan^{-1}x +$

$\tan^{-1}(x-1)$

$\dfrac{dy}{dx} = \dfrac{1}{1+x^2} + \dfrac{1}{1+(x-1)^2}$

$\left.\dfrac{dy}{dx}\right/_{x=1} = \dfrac{1}{2} + 1 = \dfrac{3}{2}$

93. Ans: 1

Sol: $f(x) = \alpha - x$, where $\cos\alpha = \dfrac{2}{\sqrt{13}}$

$\sin\alpha = \dfrac{2}{\sqrt{13}}$

$f'(x) = -1$

$[f'(x)]^2 = 1$

94. Ans: $\dfrac{-1}{2}$

Sol: $x = \sin\theta \Rightarrow u = -\dfrac{\theta}{2}$

$= -\dfrac{\sin^{-1}x}{2}$

$\dfrac{du}{dv} = \dfrac{-1}{2}$

95. Ans: $-y^2\ (1 + 2x)$

Sol: $y\ (x^2+x+1) = 1$

$\dfrac{dy}{dx} = -y^2\ (2x+1)$

96. Ans: $1 + [g(x)]^3$

Sol: $g[f(x)] = x$

$g'[f(x)]\ .\ f'(x) = 1$

$g'[f(x)] = \dfrac{1}{f'(x)} = 1 + x^3$

Replace x by $f^{-1}(x)$

$g'\left[f\left\{f^{-1}(x)\right\}\right] = 1 + \left[f^{-1}(x)\right]^3$

$g'(x) = 1 + [g(x)]^3$.

97. Ans: 3

Sol: $\underset{x0}{Lt}\left[\dfrac{f'(4+x)+f'(4-x)}{4}\right] = \dfrac{2f'(4)}{4} = \dfrac{6}{2} = 3$

98. Ans: 1

Sol: $\dfrac{1}{\sqrt{a}} \times \dfrac{1}{2\sqrt{x}} + \dfrac{1}{\sqrt{b}} \times \dfrac{1}{2\sqrt{y}}\dfrac{dy}{dx} = 0$

$\left.\dfrac{dy}{dx}\right/_{(x_1, y_1)} = \dfrac{-\sqrt{by_1}}{\sqrt{ax_1}}$

$y - y_1 = \dfrac{-\sqrt{by_1}}{\sqrt{ax_1}}\ (x - x_1)$

$\dfrac{x}{\sqrt{ax_1}} + \dfrac{y}{\sqrt{by_1}} = \dfrac{y_1}{\sqrt{by_1}} + \dfrac{x_1}{\sqrt{ax_1}} = 1$

99. Ans: $\dfrac{-7}{6}$

Sol: $\dfrac{dy}{dx} = \dfrac{4t-2}{2t+3}$

$$= \left.\frac{1}{\frac{dy}{dx}}\right|_{t=2} \text{ is } \frac{-7}{6}$$

100. Ans: –2

Sol: $\frac{dy}{dx} = 6x + 3ax^2$

6+6ax at $x = \frac{1}{2}$

= 6+3a = 0 ⇒ a = –2

101. Ans: –5

Sol: $2y\frac{dy}{dx} = 3px^2$

6 = 3p × 4

$2p\frac{dy}{dx} = 2p$

2p = 4, p = 2

9 = 8p + q = 16 + q

q = –7, p + q = –5

102. Ans: (0, 5)

Sol: x + y = a ⇒ x' = –1

$-\frac{dx}{dy} = \frac{1}{2x-1} = -1$

2x – 1 = 1 ⇒ x = 0

y = 5

103. Ans: $\frac{\sqrt{2}}{4}$

Sol: $D^2 = (a-1)^2 + \left(\sqrt{a}\right)^2$

$\frac{d\left(D^2\right)}{da} = 2(a-1) + 1 = 0 \Rightarrow$

$a = \frac{1}{2}, b = \frac{1}{\sqrt{2}}$

$ab = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

104. Ans: [–8, 72]

Sol: $f(x) = 4x^3 - 12x$

f'(x) = 0 ⇒ x = 1 or –1

f(–1) = 8

f(1) = –8

f(3) = 72.

105. Ans: log |log (logx)| + C

Sol: log (logx) = t

$\int \frac{dx}{x(\log x)\log(\log x)}$

= log |log (logx)| + C

106. Ans: $\left(\frac{1}{\log 3}\right)\sin^{-1}\left(3^x\right) + C$

Sol: $3^x = \sin t \Rightarrow 3^x \log 3\, dx = \cos t\, dt$

$\frac{1}{\log 3}\int \frac{\cos t\, dt}{\cos t} = \frac{1}{\log 3}\sin^{-1}(3^x) + C$

107. Ans: $= \frac{1}{2}\left[x + \log\left|\sin x + \cos x\right|\right] + C$

Sol: $\frac{1}{2}\int\left(\frac{\cos x + \sin x + \cos x - \sin x}{\sin x + \cos x}\right)dx$

$= \frac{1}{2}\left[x + \log\left|\sin x + \cos x\right|\right] + C$

108. Ans: $\frac{1}{4}\left(27 + e^{3x}\right)^{\frac{4}{3}} + C$

Sol: $27 + e^{3x} = t$

$\frac{1}{3}\int t^{\frac{1}{3}}dt = \frac{1}{4}\left(27 + e^{3x}\right)^{\frac{4}{3}} + C$

109. Ans: $\frac{-\sqrt{4-9x^2}}{x} + C$

Sol: 3x = 2sinθ

$I = \frac{2}{3}\int \frac{\cos\theta d\theta}{\frac{4}{9}\sin^2\theta \times 2\cos\theta}$

= 3 (–cotθ)

$= \frac{-3\sqrt{4-9x^2}}{3x} + C$

$= \frac{-\sqrt{4-9x^2}}{x} + C$

110. Ans: $-e^{-x}$ cosec x + C

Sol: $\int e^{-x}\left(-\cos ecx + \frac{d}{dx}(-\cos ecx)\right)dx$

$= -e^{-x}$ cosec x + C

111. Ans: log|1 + e^x sin x| + C

Sol: $\int \frac{e^x(\sin x + \cos x)}{1 + e^x \sin x}dx$

= log|1 + e^x sin x| + C

112. Ans: 2

Sol: $\left[\sec^{-1} t\right]_{\sqrt{2}}^{x} = \frac{\pi}{12}$

$\sec^{-1}x = \frac{\pi}{12} + \frac{\pi}{4} = \frac{\pi}{3}$

x = 2

113. Ans: $2 \log 2 - 1$

Sol: $\int_3^4 \log(x-2)dx$

$= [\log(x-2)x]_3^4 + [x + 2\log|x-2|]_3^4$

$= 4\log 2 - 1 - 2\log 2$

$= 2\log 2 - 1$

114. Ans: $\sqrt{21}$

Sol: $\left(\frac{x^3}{3} - 2x\right)_3^6 = 3f(c)$

$f(c) = 19$

$c^2 - 2 = 19,\ c^2 = 21,\ c = \sqrt{21}$

115. Ans: $\frac{1}{2}\log 3$

Sol: $-\frac{1}{2}\int_3^1 \frac{dt}{t}$

$= \frac{1}{2}(\log t)_1^3$

$= \frac{1}{2}\log 3$

116. Ans: $\frac{\pi}{4}$

Sol: $I = \frac{1}{2}\int_0^{\frac{\pi}{2}} dx = \frac{\pi}{4}$

117. Ans: $y \log|x| = C - \frac{1}{2}\cos 2x$

Sol: $\frac{dy}{dx} + \frac{1}{\log x}\ \frac{y}{x} = \frac{\sin 2x}{\log x}$

$e^{\int pdx} = \log x$

$y \log|x| = \int \sin 2x\, dx$

$= -\frac{\cos 2x}{2} + C$

118. Ans: $\frac{-2}{5}$

Sol: $x\frac{dy}{dx} + y = A\cos x - B\sin x$

$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$

$-5a = 2$ or $a = \frac{-2}{5}$

119. Ans: $y = e^{3x} + C$

Sol: $.\ dy = 3e^{3x}\ \frac{[1+e^{2x}]dx}{1+e^{2x}}$

$= 3e^{3x}$

$y = e^{3x} + C$

120. Ans: order 1, degree 2

Sol: $x^2 + (y-a)^2 = r^2$

$2x + 2(y-a)\ \frac{dy}{dx} = 0$

$y - a = -\frac{x}{\frac{dy}{dx}}$

$x^2 + \frac{x^2}{\left(\frac{dy}{dx}\right)^2} = r^2$

$x^2\left[\left(\frac{dy}{dx}\right)^2 + 1\right] = r^2\left(\frac{dy}{dx}\right)^2$