MATHEMATICS: Max.Marks : 70

SECTION I

Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices A), B), C) and D) out of which **ONLY ONE is correct.**

41. Let $f(x)$ be a real valued function satisfying $af(x)+bf(-x)=px^2+qx+r$, Where a and b are distinct real numbers and p, q and r are non-zero real numbers. Then $f(x)=0$ will have real solution when

A) $\left(\frac{a+b}{a-b}\right)^2 \le \frac{a^2}{4pr}$

B) $\left(\frac{a+b}{a-b}\right)^2 \le \frac{4pr}{q^2}$

C) $\left(\frac{a+b}{a-b}\right)^2 \ge \frac{q^2}{4pr}$

D) $\left(\frac{a+b}{a-b}\right)^2 \ge \frac{4pr}{q^2}$

42. A line $L=\sqrt{3}x-y+2-\sqrt{3}=0$ is rotated through an angle 60^0 about A(1, 2) in counter clockwise direction. Let L_1 be new position of L and $B(\alpha,\beta)$ be a point on L_1 which is at a distance of 5 units from A. If C(h, k) is a point such that area of ΔABC is $\frac{5}{4}$ sq. units then maximum value of $\sqrt{3}h+k$ is equal to

A) $\sqrt{3}+1$

B) $\sqrt{3}\left(\sqrt{3}+1\right)$

C) $\sqrt{3}\left(\sqrt{3}-1\right)$

D) $\sqrt{3}\left(\sqrt{3}+2\right)$

43. The sum $\frac{3}{1!+2!+3!}+\frac{4}{2!+3!+4!}+\text{------}+\frac{2012}{2010!+2011!+2012!}=$

A) $\frac{1}{2012!}$

B) $1-\frac{1}{2012!}$

C) $\frac{1}{2}-\frac{1}{2012!}$

D) $\frac{1}{2011!-2012!}$

44. A truck is to be driven 300 km on a highway at a constant speed of x kmph. Speed rules of the highway require that $30 \le x \le 60$. The fuel costs Rs. 10 per liter and is consumed at the rate of $2+\frac{x^2}{600}$ liters per hour. The wages of the driver are Rs.200 per hour. The most economical speed to drive the truck, in kmph, is

A)30

B)60

C) $30\sqrt{33}$

D) $20\sqrt{33}$

45.

$If\ a=1+\frac{x^3}{\lfloor 3}+\frac{x^6}{\lfloor 6}+\frac{x^9}{\lfloor 9}+......,b=x+\frac{x^4}{\lfloor 4}+\frac{x^7}{\lfloor 7}+......$

$and\ c=\frac{x^2}{\lfloor 2}+\frac{x^5}{\lfloor 5}+\frac{x^8}{\lfloor 8}+.....then\ a^3+b^3+c^3-3abc=$

A)0

B)1

C) e^x

D)-1

46. Let $f:R\to R$ be a positive non-decreasing function with $\lim_{x\to\infty}\frac{f\left(\frac{x-x^3}{6}\right)}{f(x)}=1$, then $\lim_{x\to\infty}\frac{f(\sin x)}{f(x)}=$

A) 1

B) $\pi/2$

C) 1/6

D) $\pi/36$

47. Which of the following statements is/are true?

A) The no. of zeroes of $2^x - x^2 - 1 = 0$ is 3

B) The no. of zeroes of $2^x \ln 2 - 2x = 0$ is 3

C) The no. of zeroes of $2^x - x^2 - 1 = 0$ is 2

D) The no. of zeroes of $2^x \ln 2 - 2x = 0$ is 1

48. If the equation $z^2+z+\alpha=0$ has a purely imaginary root and α lies on the circle |z| = 1 then the imaginary part of that root, is (are)

A) $\pm\sqrt{2}$ B) 0

C) $\pm\sqrt{2-\sqrt{2}}$ D) $\pm\sqrt{\frac{\sqrt{5}-1}{2}}$

49. The reciprocal of the distance between two points, one on each of the lines $\frac{x-2}{3}=\frac{y-4}{2}=\frac{z-5}{5}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$

A) cannot be less than 9

B) having minimum value $5\sqrt{3}$

C) cannot be greater than $\sqrt{78}$

D) cannot be $2\sqrt{19}$

50. Let X be a set containing n elements. The no. of all the ordered triplets (A, B, C) such that C is a subset of B and B is a proper subset of A where $A\subseteq X$, is

A) 4^n B) 3^n

C) 4^n-3^n D) 3^n-2^n

SECTION II

Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions.** Each question has four choices A), B), C) and D) out of which **ONE or MORE are correct.**

51. Let $\vec{a}=a_1\vec{i}+a_2\vec{j}+a_3\vec{k}, \vec{b}=b_1\vec{i}+b_2\vec{j}+b_3\vec{k}$ and $\vec{c}=c_1\vec{i}+c_2\vec{j}+c_3\vec{k}$ be three non zero vectors such that $\vec{c}$ is perpendicular to both the vectors $\vec{a}$ *and* $\vec{b}$. If the angle between $\vec{a}$ *and* $\vec{b}$ is $\frac{\pi}{6}$ and $|\vec{a}|,|\vec{b}|,|\vec{c}|$, are the roots of $x^3-11x^2+38x-40=0$ then value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ is

A) 10 B) -10

C) -20 D) 20

52. Let $f(x)=\begin{cases} \int_0^x (x+|1-t|)\,dt, & if\ x>2 \\ 5x+1, & if\ x\le 2 \end{cases}$ then

A) $f(x)$ is not continuous at x=2

B) $f(x)$ is continuous but not differentiable at x=2

C) $f(x)$ is differentiable everywhere

D) the right derivative of $f(x)$ at x=2 does not exist

53. Consider the matrix equation $X^2=I$, I *being* 2×2 matrix, 'X' is a real matrix (all the elements being real). Which of the following is / are correct?

A) The equation has exactly two solutions

B) The equation has infinitely many Solution

C) $x=\begin{bmatrix} \sqrt{2}-\sqrt{3} & 4\sqrt{2} \\ \frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}} & \sqrt{3}-\sqrt{2} \end{bmatrix}$ is a possible Solution

D) $x=\begin{bmatrix} \sqrt{3}-\sqrt{2} & \sqrt{\frac{3}{2}}-1 \\ 4 & \sqrt{2}-\sqrt{3} \end{bmatrix}$ is a possible Solution

54. Two real numbers x and y are selected at random. Given that $0\le x\le 1, 0\le y\le 1$. Let A be the event that $y^2\le x$ and B be the event that $x^2\le y$. Then

A) $P(A\cap B)=\frac{1}{3}$

B) A and B are exhaustive events

C) A and B are mutually exclusive

D) A and B are independent events

55. If $f(x) = kx^2 - m$ such that $-4 \le f(1) \le -1, -1 \le f(2) \le 5$ then which of the following statements is/are true

A) $f(3)_{max} = 19$

B) $f(3)_{min} = -2$

C) $f(3)_{max} = 20$

D) $f(3)_{min} = -1$

SECTION III
Integer Answer Type

This section contains **5 questions**. The answer to each question is single digit integer, ranging from 0 to 9 (*both inclusive*).

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56. The solution set of $3x^{\log_3^4} + 4^{\log_3^x} = 64$ *is* ______

57. Let A be a 2×3 matrix whereas B be a 3×2 matrix. If $\det(AB) = 4$, then the value of $\det(BA)$ is

58. If $\int_{1/2}^{2}\left(1 + x - \frac{1}{x}\right)e^{x + \frac{1}{x}}\,dx = I$, then $\frac{4}{3}e^{-5/2}I =$

59. Let f(x) be a polynomial with leading coefficient unity and satisfying the equation

$(x - 16)\,f(2x) = 16\,(x - 1)\,f(x)$, then $\frac{f(0)}{128} =$

60. Non-zero vectors $\vec{a}, \vec{b}, \vec{c}$ satisfy $\vec{a}.\vec{b} = 0$; $(\vec{b} - \vec{a}).(\vec{b} + \vec{c}) = 0$ and $2|\vec{b} + \vec{c}| = |\vec{b} - \vec{a}|$. If $\vec{a} = \mu\vec{b} + 4\vec{c}$, then the value of μ is