MATHEMATICS

(Two hours and a half)

Answers to this Paper must be written on the paper provided separately. You will **not** be allowed to write during the first **15** minutes. This time is to be spent in reading the question paper. The time given at the head of this Paper is the time allowed for writing the answers.

Attempt all questions from Section A and any four questions from Section B. All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks. The intended marks for questions or parts of questions are given in brackets []. Please refer to the mathematical tables in your school textbook.

SECTION A (40 MARKS)

Attempt all questions from this Section.

Question 1: 3 + 3 + 4 = 10 marks

(a) Find the rate per cent at which a sum of money becomes $\frac{125}{64}$ of itself in 3 years.

(b) Solve the following inequation:

 $\frac{1}{2} \left(\frac{3x}{2} + 4 \right) \geq \frac{1}{3} \left(x - 6 \right)$, x $\in \mathbb{R}$

(c) In trapezium ABCD, AB | |DC and DC=2AB. FE drawn parallel to AB cuts AD in F and BC in E such that 4BE=3EC. Diagonal DB intersects EF at G. Prove that 7FE=10AB.

Question 2: 3 + 3 + 4 = 10 marks

(a) A man invested **Rs.**45,000 in 15% **Rs.**100 shares quoted at **Rs.**125. When the market value of these shares rose to **Rs.**140, he sold some shares, just enough to raise **Rs.**8,400. Calculate:

(i) the number of shares he still holds;

(ii) the dividend due to him on those remaining shares.

(b) Using ruler and compass only, construct a regular hexagon of side 2.5 cm. Draw its all lines of symmetry.

(c) (i) Find the third proportion of $\frac{a}{b} + \frac{b}{a}$ and $\sqrt{a^2 + b^2}$.

(ii) Using the properties of proportion, find the value of $\frac{x+a}{x-a} + \frac{x+b}{x-b}$ if $x = \frac{2ab}{a+b}$

Question 3: 3 + 3 + 4 = 10 marks

(a) The triangle ABC where A(1,2), B(4,8) and C(6,8) is reflected in the x-axis to triangle A'B'C'. The triangle A'B'C' is reflected in the origin to triangle A''B''C''. Write down the coordinates of A''B''C''. Write down a single transformation that maps ABC onto A''B''C''.

(b) The largest area of an equilateral triangle, which can be formed by a given length of wire, is $484\sqrt{3}$ cm². If the same wire is bent twice, first to form a circle and then a square; find the ratio of the areas of circle and square.

(c) (i) If x=a sin A cos C, y=a sin A sin C, z=a cos A, show that $a^2=x^2+y^2+z^2$.

(ii) If $(\cot\theta + \tan\theta) = m$ and $(\sec\theta - \cos\theta) = n$, prove that $(m^2n)^{2/3} - (mn^2)^{2/3} = 1$

Question 4: 3 + 3 + 4 = 10 marks

(a) A box contains 24 balls out of which some are green and others are red. If a ball is drawn from the box, the probability of getting a green ball is $\frac{2}{3}$. Find the number of red balls in the box.

(b) A man invests a sum of money in the ratio 16:9 in two schemes which give compound interests p.a. He invests for 2 years in the first scheme and for 4 years in the second scheme. If the rate of interest p.a. in the second scheme is 20% and the amount receivable in both the schemes are equal then find the rate of interest p.a. in the first scheme.

(c) (i) Find the coordinates of points which trisect the line segment joining (1,-2) and (-3,4).

(ii) Determine the ratio in which the line 3x + y - 9 = 0 divides the segment joining the points (1,3) and (2,7).

SECTION B (40 MARKS)

Attempt any four questions from this Section.

Question 5: 3 + 3 + 4 = 10 marks

(a) 'A' produces cycles at a cost of **Rs**.855 each. The chain of seller is like (A sells to B, B sells to C, C sells to D) and all live in the same state. The tax rate is 10%, and the profit is **Rs**.100 at each stage of the selling chain. Find the total amount of VAT.

(b) Draw a triangle PQR in which QT=4 cm, PR-QR=2 cm and \perp PQR=75°. Locate a point X such that X is equidistant from the sides QP and QR. Measure the length of PX to the nearest cm.

(c) (i) ABC is a right-angled triangle in which $\ \ ABC=90^{\circ}$. BD is the perpendicular from B to AC and BD=5 cm. If AD:DC=2:1 then find the ratio of AB and BC.

(ii) The product of the presnt ages of the father and his only son is 520 year². After 14 years the age of the father will become double of the age of the son. Find their present ages.

Question 6: 3 + 3 + 4 = 10 marks

(a) Given that $x \in I$, solve the inequation and graph the solution on the number line: $3 \ge \frac{x-4}{2} + \frac{x}{3} \ge 2$

(b) Manu has a five years recurring deposit account and deposits Rs.240 per month. If he receives Rs.17,694 at the time of maturity, find the rate of interest.

(c) (i) Two congruent circles with centers O and P intersect at A and B. CBD is a line segment and EBF is a tangent to the circle with centre P. Prove that CE=BD.

(ii) Prove that any four vertices of a regular pentagon are concyclic.

Question 7: 3 + 3 + 4 = 10 marks

(a) A fire at a building B is reported to two fire stations F_1 and F_2 10 km apart from each other. F_1 observes that the fire is at an angle of 50° from it and F_2 observes that it is at an angle of 45° from it. Which station should send its team and how much distance has it to travel?

(b) Prove that a rhombus inscribed in a circle, is a square.

(c) (i) In a \triangle ABC, AD is the altitude through A where D is on BC. If D has coordinates (1,1), and the equations of AB and AC are x = y = 0 and 2x – y = 3 respectively then find the equation of BC.

Question 8: 3 + 3 + 4 = 10 marks

(a) A man has a choice to invest in hundred rupee shares of two firms at **Rs**.120 or at **Rs**.132. The first firm pays a dividend of 5% per annum and the second firm pays a dividend of 6% per annum. Find:

(i) Which company is giving better return?

(ii) If a man invests **Rs**.26,400 with each firm, how much will be the difference between the annual returns from the two firms?

(b) Using ruler and compass only,

(i) Construct a triangle ABC with base AB=7 cm, BC=6.5 cm and $\ \ CAB=60^{\circ}$.

(ii) In the same figure, draw a circle which passes through the points A, B and C and mark its centre O.

(iii) Draw a perpendicular from O to AB which meets AB in D.

(c) Let $A = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 3 \\ 1 & -3 \end{bmatrix}$, find $A^2 - A + BC$

Question 9: 3 + 3 + 4 = 10 marks

(a) Two vertices of an isosceles triangle are (2,0) and (2,5). Find the third vertex if the length of equal side is 3.

(b) Three coins are tossed simultaneously. Find the probability of getting:

- (i) no tail
- (ii) 3 heads
- (iii) no head

(c) In a bullet the gunpowder is to be filled up inside the metallic enclosure. The metallic enclosure is made up of a cylindrical base and conical top with a base of radius 5 cm. The ratio of height of cylinder and cone is 3:2. A cylindrical hole is drilled into the metal solid with height two-third the height of metal solid. What should be the radius of the hole, so that the volume of the hole (in which gunpowder is to be filled up) is on-third the volume of the metal solid after drilling?

Question 10: 3 + 3 + 4 = 10 marks

(a) If sin A + cos A = a, tan A + cot A = b, show that $\frac{1}{2}(a^2 - 1) = \frac{1}{b}$.

(b) (i) Find $a^2 - 3a + 7$, if (2a + 6) : (3a - 7) = 4 : 5.

(ii) The ratio of income of tgwo persons is 9 : 7 and the ratio of their expenditures is 4 : 3. Each of them saves **Rs.**200 yearly. Find their yearly income.

(c) (i) If 2x + 1 is a factor of $2x^3 - x^2 - 5x - a$ then solve the equation $2x^3 - x^2 - 5x = 2$. (ii) If $(a^2 + b^2 + c^2) (x^2 + y^2 - z^2) = (a^2 + b^2 - c^2) (x^2 + y^2 + z^2)$, prove that $\sqrt{p + q} : z = \sqrt{r + s} : c$ where $p = x^2$, $q = y^2$, $r = a^2$ and $s = b^2$.

Question 11: 4 + 6 = 10 marks

(a) Points (3,0) and (-1,0) are invariant points under reflection in the line L_1 . Points (0,-3) and (0,1) are invariant points under reflection in the line L_2 .

(i) Name the lines L_1 and L_2 . Write their equations also.

(ii) Write down the images of points P (3,4) and Q(-5,-2) on reflection in L_1 . Name the images as P' and Q' respectively.

(iii) Write down the images of points P and Q on reflection in L_2 . Name the images as P'' and Q'' respectively.

(iv) State or describe a single transformation that maps P' onto P''.

(b) The marks obtained by 100 students in Maths are given in the form of a frequency distribution:

Marks	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Number of students	2	10	25	31	11	12	2	1

Draw the ogive ("More than") for the above frequency distribution.