GCHAPTER 1 **Introduction /Basic concept**

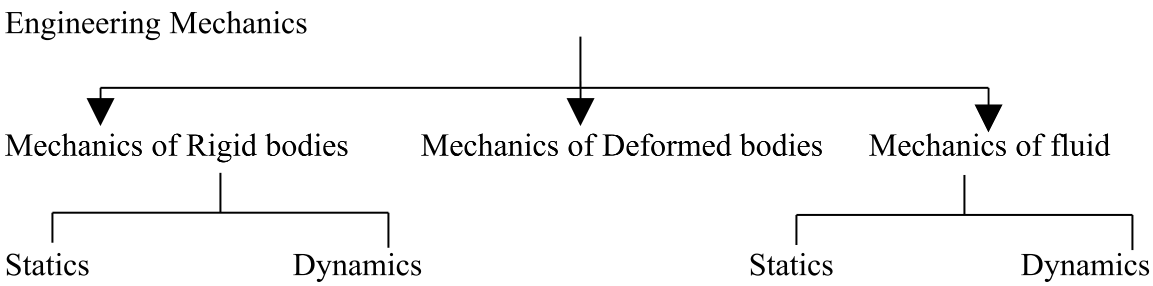
MECHANICS:

Mechanics can be defined as the branch of physics concerned with the state of rest or motion of bodies that subjected to the action of forces. **OR**

It may be defined as the study of forces acting on body when it is at rest or in motion is called mechanics.

Classification of Mechanics

The engineering mechanics are classified as shown



BRANCHES OF MECHANICS:

Mechanics can be divided into two branches.

1. Static. 2. Dynamics.

a) Statics

It is the branch of mechanics that deals with the study of forces acting on a body in equilibrium. Either the body at rest or in uniform motion is called statics

b) **Dynamics**:

It is the branch of mechanics that deals with the study of forces on body in motion is called dynamics. It is further divided into two branches.

i) Kinetics ii) kinematics.

i) **Kinetics**

It is the branch of the dynamics which deals the study of body in motion under the influence of force i.e. is the relationship between force and motion are considered or the effect of the force are studied

ii) **Kinematics**:

It is the branch of the dynamics that deals with the study of body in motion with out considering the force.

Fundamental concept

The following are the fundamental concept used in the engineering mechanics

1. Force

In general force is a Push or Pull, which creates motion or tends to create motion, destroy or tends to destroys motion. In engineering mechanics force is the action of one body on another. A force tends to move a body in the direction of its action,

A force is characterized by its point of application, magnitude, and direction, i.e. a force is a vector quantity.

Units of force

The following force units are frequently used.

A. Newton

The S.I unit of force is Newton and denoted by N. which may be defined as $1N = 1\ kg.\ 1\ m/s^2$

B. Dynes

Dyne is the C.G.S unit of force. $1\ Dyne = 1\ g.\ 1\ cm/s^2$

One Newton force = 10^2 dyne

C. Pounds

The FPS unit of force is pound. $1\ lb_f = 1\ lb_m.\ 1 ft/s^2$

One pound force = 4.448 N

One dyne force = 2.248×10^{-6} lbs

2. Space

Space is the geometrical region occupied by bodies whose positions are described by linear and angular measurement relative to coordinate systems. For three dimensional problems there are three independent coordinates are needed. For two dimensional problems only two coordinates are required.

3. Particle

A particle may be defined as a body (object) has mass but no size (neglected), such body cannot exists theoretically, but when dealing with problems involving distance considerably larger when compared to the size of the body. For example a bomber aeroplane is a particle for a gunner operating from ground.

In the mathematical sense, a particle is a body whose dimensions are considered to be near zero so that it analyze as a mass concentrated at a point. A body may tread as a particle when its dimensions are irrelevant to describe its position or the action of forces applied to it. For example the size of earth is insignificant compared to the size of its orbits and therefore the earth can be modeled as a particle when studying its orbital motion. When a body is idealized as a particle, the principles of mechanics reduce to rather simplified form since the geometry of the body will not be involved in the analysis of the problem.

4. Rigid Body

A rigid body may be defined a body in which the relative positions of any two particles do not change under the action of forces means the distance between two points/particles remain same before and after applying external forces.

As a result the material properties of any body that is assumed to be rigid will not have to be considered while analyzing the forces acting on the body. In most cases the actual deformations occurring in the structures, machines, mechanisms etc are relatively small and therefore the rigid body assumption is suitable for analysis

Basic quantities

In engineering mechanics length, mass, time and force are basic quantities

1. Length

In engineering mechanics length is needed to locate the position of a particle and to describe the size of physical system. Some important length conversions factors

1cm = 10 mm 1 m = 100 cm 1 m = 1000 mm

1 m = 3.2808' (feet) 1 m = 39.37 Inch 1 Mile = 1.609 km

2. Mass

Mass is the property of matter by which we can compare the action of one body with that of another. This property manifests itself as gravitational attraction between two bodies and provides a quantitative measure of the resistance of matter to a change in velocity. Some important mass conversion factors are given below

$$1 \text{ Kg} = 2.204 \text{ lb}_m$$

3. Time

Time is the measure of the succession of events and is a basis quantity in dynamic. Time is not directly involved in the analysis of statics problems but it has importance in dynamics.

Systems of units

In engineering mechanics length, mass, time and force are the basic units used therefore; the following are the units systems are adopted in the engineering mechanics

1. International System of Units (SI):

In SI system of units the basic units are length, time, and mass which are arbitrarily defined as the meter (m), second (s), and kilogram (kg). Force is the derived unit.

$1\text{N} = 1 \text{ kg. } 1 \text{ m/s}^2$

2. CGS systems of units

In CGS system of units, the basic units are length, time, and mass which are arbitrarily defined as the centimeter (cm), second (s), and gram (g). Force is the derived units $1 \text{ Dyne} = 1 \text{ g. } 1 \text{ cm/s}^2$

3. British systems of units

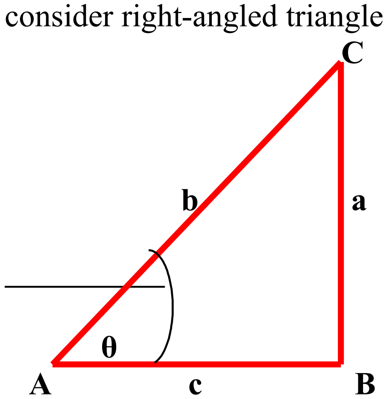
In CGS system of units, the basic units are length, time, and mass which are arbitrarily defined as the centimeter (cm), second (s), and gram (g). Force is the derived units $1 \text{ lb} = 1\text{lbg. } 1\text{ft/s}^2$

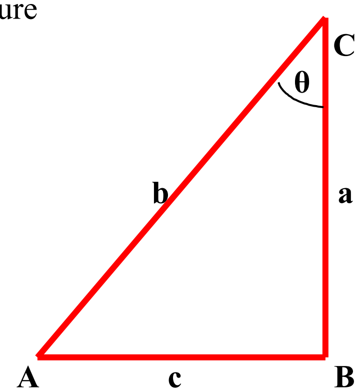
4. U.S. Customary Units

The basic units are length, time, and force which are arbitrarily defined as the foot (ft), second (s), and pound (lb). Mass is the derived unit,

Trigonometry

The measurement of the triangle sides and angles is called trigonometry. Let us consider right-angled triangle ABC as shown in figure





Than the following ratio can be considered for both the triangles

Sin θ = per/hyp = a/b
Cos θ = base/hyp = c/b

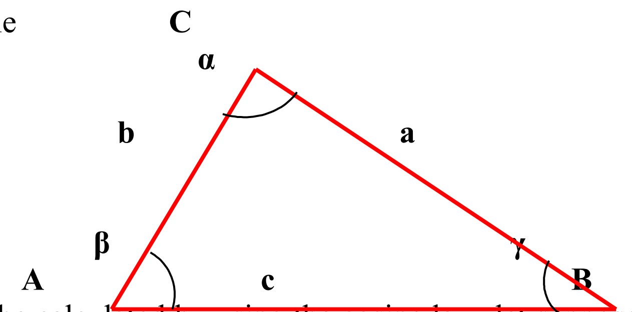
Sin θ = per/hyp = c/b
Cos θ = base/hyp =a/b

$\text{Tan } \theta = \text{per/base} = a/c$ $\text{Tan } \theta = \text{per/base} = c/a$

The any side of the right angled triangle may be calculated by

$b^2 = a^2 + b^2$

Similarly consider the following Triangle



The any side of the triangle can be calculated by using the cosine law, let suppose we have to calculate the side "AC" that is "b" then

$$b = \sqrt{a^2 + c^2 - (2bc)\cos \gamma}$$

Similarly, to calculate sides "AB" that is "c" and "AC" that is "a" then by using the cosine lay as below

$$c = \sqrt{a^2 + b^2 - 2ab\cos \alpha}$$

And $$a = \sqrt{c^2 + b^2 - 2cb\cos \beta}$$

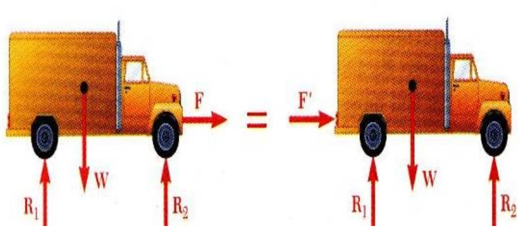
The sides of the triangle ABC can be calculated by using the sin law

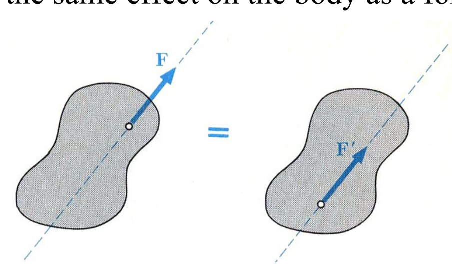
$$\frac{a}{\text{Sin } \beta} \qquad \frac{b}{\text{Sin } \gamma} \qquad \frac{c}{\text{Sin } \alpha}$$

<u>Principle of transmissibility of forces</u>

The state of rest of motion of a rigid body is unaltered if a force acting in the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force.

For example the force F acting on a rigid body at point A. According to the principle of transmissibility of forces, this force has the same effect on the body as a force F applied at point B.





The following two points should be considered while using this principle.

1. In engineering mechanics we deal with only rigid bodies. If deformation of the body is to be considered in a problem. The law of transmissibility of forces will not hold good.
2. By transmission of the force only the state of the body is unaltered, but not the internal stresses which may develop in the body

Therefore this law can be applied only to problems in which rigid bodies are involved

SCALAR AND VECTOR QUANTITY

Scalar quantity

Scalar quantity is that quantity which has only magnitude (numerical value with suitable unit) **or**

Scalars quantities are those quantities, which are completely specified by their magnitude using suitable units are called scalars quantities. For example mass, time, volume density, temperature, length, age and area etc

The scalars quantities can be added or subtracted by algebraic rule e.g.

7kg + 8kg = 15 kg sugar Or 4 sec + 5 sec = 9 sec

Vector quantity

Vector quantity is that quantity, which has magnitude unit of magnitude as well as direction, is called vector quantity. **Or**

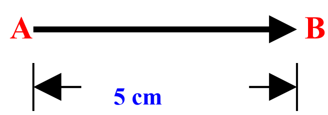
Vector quantities are those quantities, which are completely specified by their magnitude using suitable units as well directions are called vector quantities. For example velocity, acceleration, force, weight, displacement, momentum and torque etc are all vector quantities. Vector quantity can be added, subtracted, multiplied and divided by particular geometrical or graphical methods.

VECTOR REPRESENTATION

A vector quantity is represented graphically by a straight line the length of line gives the magnitude of the vector and arrowhead indicates the direction.

For example we consider a displacement (d) of magnitude 10 km in the direction of east. Hence we cannot represent 10 km on the paper therefore we select a suitable scale shown in fig. Scale 1 cm = 2 km

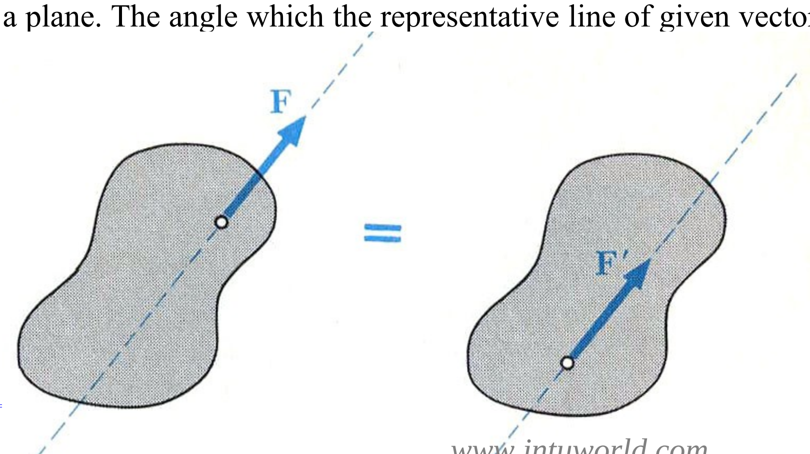
So we draw a line of length 5 cm which show the magnitude of vector quantity that is 10 km while the arrow indicates the direction form origin to east ward as shown in fig.

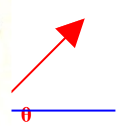


Point A is called tail that shows the origin.
Point B is called head, which shows the direction of vector quantity.
The length of line is the magnitude of the vector quantity.

RECTANGULAR CO-ORDINATE SYSTEM

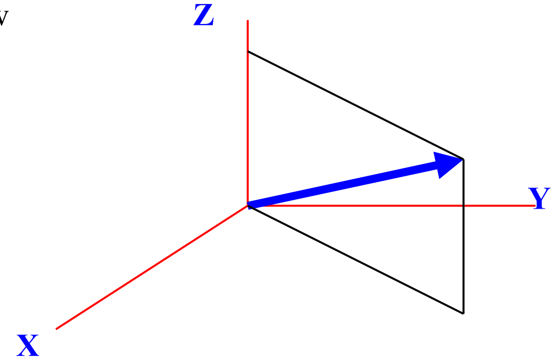
Two lines at right angle to each other are known as co-ordinate axes and their point of intersection is called origin. The horizontal line is called x-axis while vertical line is called y-axis. Two co ordinate systems are used to show the direction of a vector is a plane. The angle which the representative line of given vector makes with + ve x axis in





Y'

In space the direction of vector requires the 3rd axis that is Z-axis. The direction of the vector in space is specified by three angles named α, β, and γ with X, Y Z axes respectively as show



EXERCIS 1

Show the following vectors graphically from 1 to 6

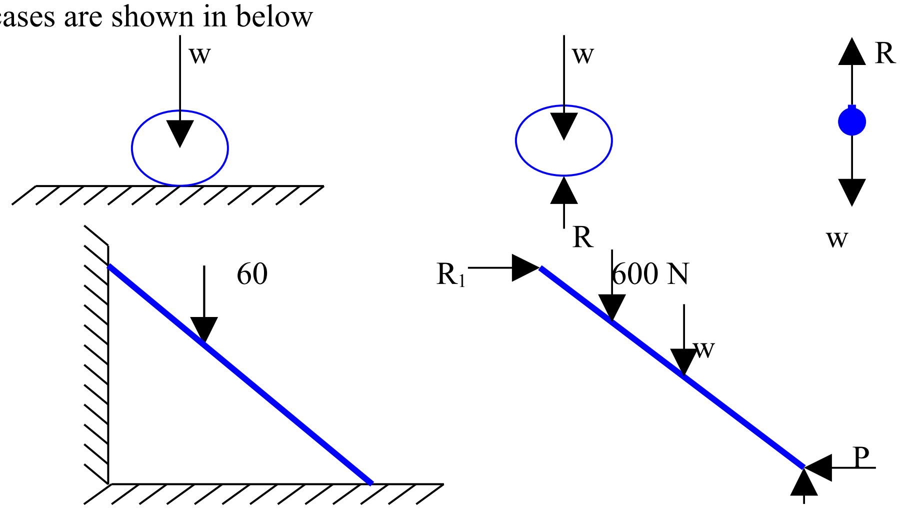
1.	Force	15 kN	45^0	with x-axes.
2.	Displacement	75 km	30^o	north of east
3.	Velocity	60 km\h	90^o	with x-axes.
4.	Velocity	5 km\h	45^o	with horizontal axes
5.	Force	20 kN	135^o	with x-axes.
6.	Displacement	40 k m		north-east.

7. A crow flies northward from pole A to pole B and covers distance of 8 km. It then flies eastward to pole C and covers 6 km. find the net displacement and direction of its flight. **Ans: 10 km 53º north of east**

8. A traveler travels 10 km east 20 km north 15 km west and 8 km south. Find the displacement of the traveler from the starting point. **Ans: 13 km 23º north west**

<u>Free body diagram</u>

A diagram or sketch of the body in which the body under consideration is freed from the contact surface (surrounding) and all the forces acting on it (including reactions at contact surface) are drawn is called free body diagram. Free body diagram for few cases are shown in below



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Procedure of drawing Free Body Diagram

To construct a free-body diagram, the following steps are necessary:

Draw Outline Shape

Imagine that the particle is cut free from its surroundings or isolated by drawing the outline shape of the particle only

Show All Forces

Show on this sketch all the forces acting on the particle. There are two classes of forces that act on the particle. They can be active forces, which tend to set the particle in motion, or they can be reactive forces which are the results of the constraints or supports that tend to prevent motion.

Identify Each Force

The forces that are known should be labeled complete with their magnitudes and directions. Letters are used to represent the magnitudes and directions of forces that are not known.

Method of Problem Solution

Problem Statement

Includes given data, specification of what is to be determined, and a figure showing all quantities involved.

Free-Body Diagrams

Create separate diagrams for each of the bodies involved with a clear indication of all forces acting on each body.

Fundamental Principles

The six fundamental principles are applied to express the conditions of rest or motion of each body. The rules of algebra are applied to solve the equations for the unknown quantities.

Solution Check:

1. Test for errors in reasoning by verifying that the units of the computed results are correct
2. Test for errors in computation by substituting given data and computed results into previously unused equations based on the six principles.
3. Always apply experience and physical intuition to assess whether results seem "reasonable"

Numerical Accuracy

The accuracy of a solution depends on

1. Accuracy of the given data.
2. Accuracy of the computations performed. The solution cannot be more accurate than the less accurate of these two.
3. The use of hand calculators and computers generally makes the accuracy of the computations much greater than the accuracy of the data. Hence, the solution accuracy is usually limited by the data accuracy.

CHAPTER 2. **SYSTEM OF FORCES:**

Force

In general force is a Push or Pull, which creates motion or tends to create motion, destroy or tends to destroys motion. In engineering mechanics force is the action of one body on another. A force tends to move a body in the direction of its action,

A force is characterized by its point of application, magnitude, and direction, i.e. a force is a vector quantity.

Force exerted on body has following two effects

1. The **external effect**, which is tendency to change the motion of the body or to develop resisting forces in the body
2. The **internal effect**, which is the tendency to deform the body.

If the force system acting on a body produces no external effect, the forces are said to be in **balance** and the body experience no change in motion is said to be in **equilibrium**.

Units of force

The following force units are frequently used.

A. Newton

The S.I unit of force is Newton and denoted by N. which may be defined as

$1N = 1\ kg.\ 1\ m/s^2$

B. Dynes

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One pound force = 4.448 N
One dyne force = 2.248×10^{-6} lbs

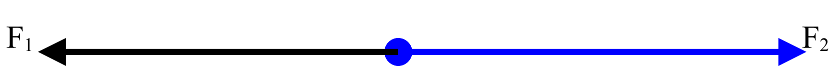
Systems of forces

When numbers of forces acting on the body then it is said to be system of forces

Types of system of forces

1. Collinear forces:

In this system, line of action of forces act along the same line is called collinear forces. For example consider a rope is being pulled by two players as shown in figure

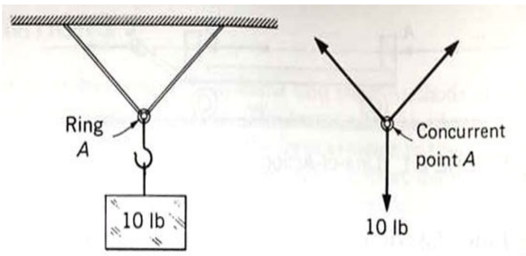


2. Coplanar forces

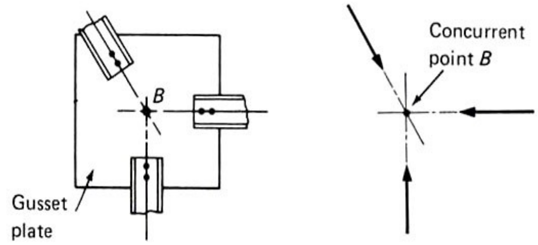
When all forces acting on the body are in the same plane the forces are coplanar

3. Coplanar Concurrent force system

A concurrent force system contains forces whose lines-of action meet at same one point. Forces may be **tensile (pulling)** or Forces may be ***compressive (pushing)***



Two wires supporting a load Forces acting on ring



Truss joint Forces acting on joint

4. Non Concurrent Co-Planar Forces

A system of forces acting on the same plane but whose line of action does not pass through the same point is known as non concurrent coplanar forces or system for example a ladder resting against a wall and a man is standing on the rung but not on the center of gravity.

5. Coplanar parallel forces

When the forces acting on the body are in the same plane but their line of actions are parallel to each other known as coplanar parallel forces for example forces acting on the beams and two boys are sitting on the sea saw.

6. Non coplanar parallel forces

In this case all the forces are parallel to each other but not in the same plane, for example the force acting on the table when a book is kept on it.

ADDITION OF FORCES

ADDITION OF (FORCES) BY HEAD TO TAIL RULE

To add two or more than two vectors (forces), join the head of the first vector with the tail of second vector, and join the head of the second vector with the tail of the third vector and so on. Then the resultant vector is obtained by joining the tail of the first vector with the head of the last vector. The magnitude and the direction of the resultant vector (Force) are found graphically and analytically.

RESULTANT FORCE

A resultant force is a single force, which produce same affect so that of number of forces can produce is called resultant force

COMPOSITION OF FORCES

The process of finding out the resultant Force of given forces (components vector) is called composition of forces. A resultant force may be determined by following methods

1. Parallelogram laws of forces or method
2. Triangle law of forces or triangular method
3. polygon law of forces or polygon method

A) PARALLELOGRAM METHOD

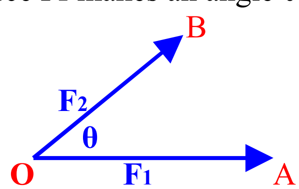
According to parallelogram method 'If two forces (vectors) are acting simultaneously on a particle be represented (in magnitude and direction) by two adjacent sides of a parallelogram, their resultant may represent (in magnitude and direction) by the diagonal of the parallelogram passing through the point. OR

When two forces are acting at a point such that they can by represented by the adjacent sides of a parallelogram then their resultant will be equal to that diagonal of the parallelogram which passed through the same point.

The magnitude and the direction of the resultant can be determined either graphically or analytically as explained below.

Graphical method

Let us suppose that two forces F_1 and F_2 acting simultaneously on a particle as shown in the figure (a) the force F_2 makes an angle θ with force F_1



First of all we will draw a side OA of the parallelogram in magnitude and direction equal to force F_1 with some suitable scale. Similarly draw the side OB of parallelogram of same scale equal to force F_2, which makes an angle θ with force F_1. Now draw sides BC and AC parallel to the sides OA and BC. Connect the point O to Point C which is the diagonal of the parallelogram passes through the same point O and hence it is the resultant of the given two forces. By measurement the length of diagonal gives the magnitude of resultant and angle α gives the direction of the resultant as shown in fig (A).

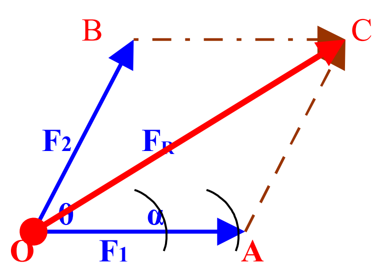


fig (A)

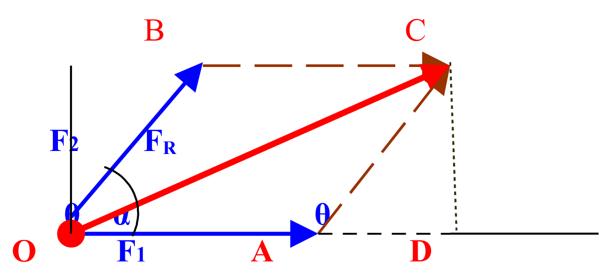


Fig (B)

Analytical method

In the paralleogram OABC, from point C drop a perpendicular CD to meet OA at D as shown in fig (B)

In parallelogram OABC,

$OA = F_1$ $OB = F_2$ Angle $AOB = \theta$

Now consider the ΔCAD in which

Angle $CAD = \theta$ $AC = F_2$

By resolving the vector F_2 we have,

$CD = F_2 \sin\theta$ and $AD = F_2 \text{ Cosine } \theta$

Now consider ΔOCD

Angle $DOC = \alpha$. Angle $ODC = 90^o$

According to Pythagoras theorem

$(Hyp)^2 = (per)^2 + (base)^2$

$OC^2 = DC^2 + OD^2$.

$OC^2 = DC^2 + (OA + AD)^2$

$F_R{}^2 = F^2 \sin^2\theta + (F_1 + F_2 \text{ Cosine } \theta)^2$

$F_R{}^2 = F^2{}_2 \sin^2\theta + F^2{}_1 + F^2{}_2 \cos^2\theta + 2 F_1 F_2 \text{ Cosine } \theta$.

$F_R{}^2 = F^2{}_2 \sin^2\theta + F^2{}_2 \cos^2\theta + F^2{}_1 + 2 F_1 F_2 \text{ Cosine } \theta$.

$F_R{}^2 = F^2{}_2 (\sin^2\theta + \cos^2\theta) + F^2{}_1 + 2 F_1 F_2 \text{ Cosine } \theta$.

$F_R{}^2 = F^2{}_2 (1) + F^2{}_1 + 2 F_1 F_2 \text{ Cosine } \theta$.

$F_R{}^2 = F^2{}_2 + F^2{}_1 + 2 F_1 F_2 \text{ Cosine } \theta$.

$F_R{}^2 = F^2{}_1 + F^2{}_2 + 2 F_1 F_2 \text{ Cosine } \theta$.

$$\mathbf{F_R = \sqrt{F^2{}_1 + F^2{}_2 + 2 F_1 F_2 \text{ Cosine } \theta}.}$$

The above equation gives the magnitude of the resultant vector.

Now the direction of the resultant can be calculated by

$$\sin\alpha = \frac{CD}{OC} = \frac{F_2 \sin\theta}{F_R} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad 1 \qquad \text{OR}$$

$$\tan\alpha = \frac{CD}{OD} = \frac{F_2 \sin}{F_1 + F2 \text{ Cosine } \theta} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad 2$$

The above two equation gives the direction of the resultant vector that is α.

B) TRIANGLE METHOD OR TRIANGLE LAW OF FORCES

According to triangle law or method" If two forces acting simultaneously on a particle by represented (in magnitude and direction) by the two sides of a triangle taken in order their resultant is represented (in magnitude and direction) by the third side of triangle taken in opposite order. OR

If two forces are acting on a body such that they can be represented by the two adjacent sides of a triangle taken in the same order, then their resultant will be equal to the third side (enclosing side) of that triangle taken in the opposite order.

The resultant force (vector) can be obtained graphically and analytically or trigonometry.

Graphically

Let us consider two forces F_1 and F_2 acting on the particle the force F_1 is horizontal while the force F_2 makes an angle θ with force F_1 as shown in fig (A). Now draw lines OA and AB to some convenient scale in magnitude equal to F_1 and F_2. Join point O to point B the line OB will be the third side of triangle, passes through the same point O and hence it is the resultant of the given two forces. By measurement the length of OB gives the magnitude of resultant and angle α gives the direction of the resultant as shown in fig (B).

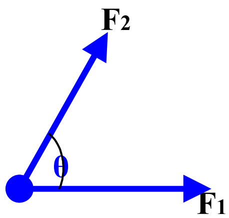


Fig (A)

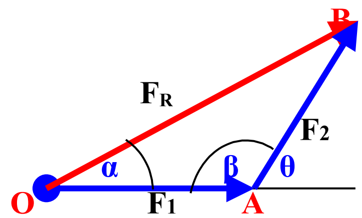


Fig (B)

ANALYTICAL OR TRIGONOMETRIC METHOD

Now consider Δ AOB in which

Angle AOB = α which is the direction of resultant vector OB makes with horizon anal axis.

Angle OAB = 180º - θ. As we know

Angle AOB + Angle OAB + Angle ABO = 180º.

By putting the values we get

α + 180º -θ + angle ABO = 180º

Angle ABO = α-θ

By applying the sine law to the triangle ABO

$$\frac{OA}{\text{Sin B}} = \frac{AB.}{\text{Sin O}} = \frac{OB}{\text{Sin A}}$$

$$\frac{F_1}{\sin(\theta - \alpha)} = \frac{F_2}{\sin \alpha} = \frac{F_R}{\sin(180 - \theta)}$$

Note

It is better to calculate the resultant of F_1 and F_2 by using cosine law we get

$$F_R = \sqrt{F^2_1 + F^2_2 + 2\, F_1\, F_2 \,\text{Cosine}\, \beta.}$$

Where $\beta = 180 - \theta$

And the direction of resultant may be determined by using sine law

$$\frac{F_1}{\sin \gamma} = \frac{F_2}{\sin \alpha} = \frac{F_R}{\sin \beta}$$

C) POLYGON METHOD

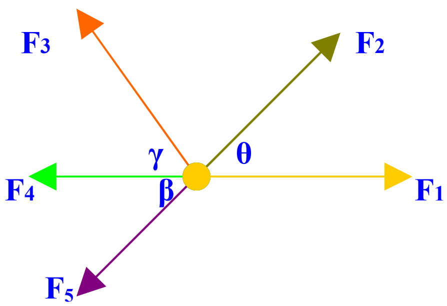
According to this method" if more then two forces acting on a particle by reprehend by the sided of polygon taken in order their resultant will be represented by the closing side of the polygon in opposite direction" OR

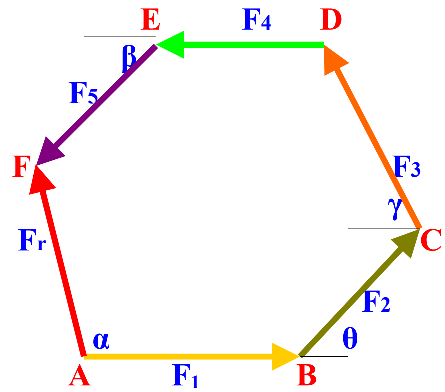
If more than two forces are acting on a body such that they can by represented by the sides of a polygon Taken in same order, then their resultant will be equal to that side of the polygon, which completes the polygon (closing side taken in opposite order.

The resultant of such forces can be determined by graphically and analytically.

Graphically:

Consider the following diagram in which number of forces acting on a particle.





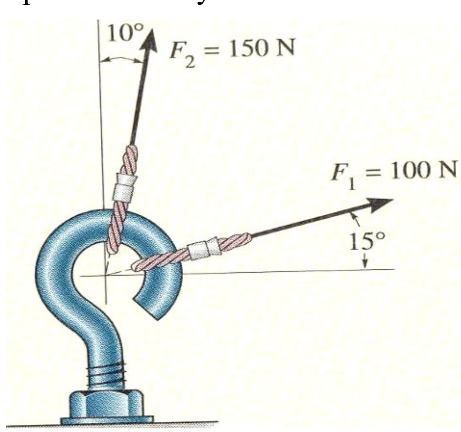
Starting from A the five vectors are plotted in turns as shown in fig by placing the tail end of each vector at the tip end of the preceding one. The arrow from A to the tip of the last vector represents the resultant of the vectors with suitable scale.In this polygon the side AF represents the resultant of the given components and α shows the direction. By measurement of AF will give the resultant and α give direction of given scale

Analytically

The resultant and direction can be determined by solving it step-by-step analytically using formulas of parallelogram, triangle law or trigonometry

<u>EXAMPLE</u>

The screw eye is subjected to two forces F_1 and F_2 as shown in fig. Determine the magnitude and direction of the resultant force by parallelogram by using the graphical or analytical method.



Draw the free body diagram of the given fig.

Given $F_1 = 100$ N $F_2 = 150$ N $\theta_1 = 15^\circ$ $\theta_2 = 10^\circ$

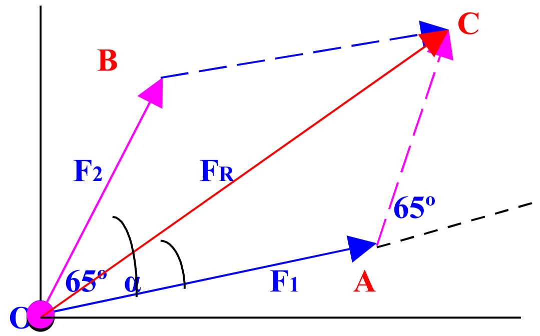
Required Resultant = F_R =?

Solution Angle AOB = 90 -15 – 10 = 65º

<u>A) Graphically</u>

Scale 20 N = 1 cm.

Now draw parallelogram OABC with rule and protractor according to scale as shown in diagram.



By measuring

$OC = F_R = 10.6$ cm $= 10.6 \times 20 = 212$ N

$\alpha = 54^\circ$ with x axis

Result **Resultant = 212 N** **Direction = 54 with x axis**

B Analytical method

We know that

$$Fr = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \text{ Cosine } \theta}.$$

Putt the value and θ= 65º

$$Fr = \sqrt{(100)^2 + (150)^2 + 2\,(100)\,(150) \text{ Cosine}65^\circ}$$

$Fr = 212.55$ N.

We also know that $\sin \alpha = \dfrac{F_2 \sin \theta.}{R}$

$$\sin \alpha = \frac{150 \sin 65^\circ}{212.55}$$

$$\alpha = \sin^{-1} \frac{150 \sin 65^\circ}{212.5}$$

$\alpha = 39.665^\circ$ with force F_1

$39.665^\circ + 15^\circ$

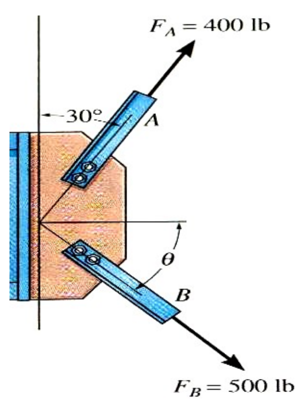
$= 54.665^\circ$ with x axis.

Result **Resultant = 212.55 N** **Direction = 54.665º with x axis**

.

EXAMPLE 3

The plate is subjected to the forces acting on member A and B as shown. If θ = 60º determine the magnitude of the resultant of these forces and its direction measured from clockwise from positive x-axis. Adopt triangle method graphically and analytically.



Given

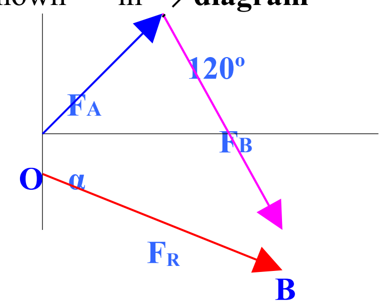
$F_A = 400N$ $F_B = 500N$ $\theta_1 = 30^o$ with Y axis $\theta_2 = 60^o$ with positive x axis

Required Resultant F_R =? Direction = α =?

Solution the angle between two forces 60 + (90 – 30) = 120º

A: Graphically Scale 100 lb = 1 cm

Now draw triangle OAB with suitable scale with the help of scale and protractor as shown in **diagram A**



By measurement we get,

OB = F_R = 4.6 cm x 100 = 460 lb Angle BOA = 70º α = 10º

Result Resultant = 460 lb Direction = 10º

B Analytically:

According to cosine law for given triangle AOB

$$F_R = \sqrt{F^2_A + F^2_B - 2(F_A)(F_B)(\text{cosine } \theta)}$$

$$F_R = \sqrt{(400)^2 + (500)^2 - 2(400)(500)(\text{cosine }(180-120)}$$

F_R = 458.257 lb

According to sine law for given triangle AOB

$$\frac{F_B}{\text{Sin } \alpha} = \frac{F_R}{\text{Sin }(180-\theta)}$$

$$\frac{500}{\text{Sin } \alpha} = \frac{458.257}{\text{Sin }(180-\theta)}$$

$$\text{Sin } \alpha = \frac{500 \text{ Sin }(180-\theta)}{458.257}$$

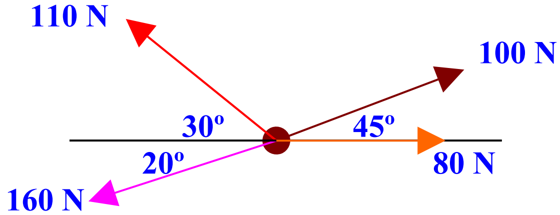
α = 70.89º with force F_A

And α = 70.89º -60º = 10 with x axis

Result Resultant = 458.257 lb & Direction = 10.89º

Example 4

Four forces act on a body at point O as shown in fig. Find their resultant.



Given

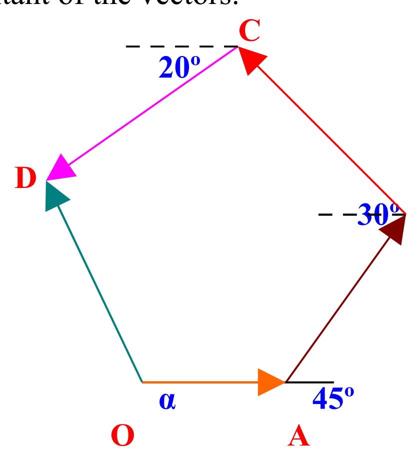
$F_1 = 80$ N $\theta_0 = 0$ at x axis
$F_2 = 100$ N $\theta_1 = 45^o$ with x axis
$F_3 = 110$ N $\theta_2 = 30^o$ with –ve x axis
$F_4 = 160$ N $\theta_3 = 20^o$ with –x axis

Required

Resultant = F_R =? Direction = α =?

Sol: Graphically Scale 20 N = 1 cm.

Starting from O the four vectors are plotted in turn as shown in fig by placing the tail end of each vector at the tip end of the preceding one. The arrow from O to the tip of the last vector represents the resultant of the vectors.



By measurement

The resultant OB = F_R = x 20 = 124 N

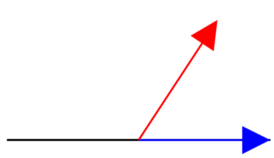
The direction of the resultant = = 143º with + ve x axis.

Result: Resultant = 119 N Direction = 143º

EXERCISE 2.1

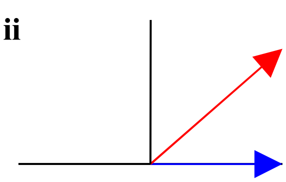
1. Find the resultant and the direction of the following diagram.

i
6lb



8 N

ii



60º

4 lb

Ans: 8.718 lb & 36.585º

42º

5 N

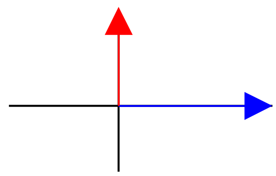
Ans: 12.18 N & 26.07º

iii

24 N

N

30 N



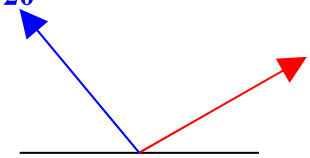
140º

30º

Ans 26 N & 67.38º

iv

20

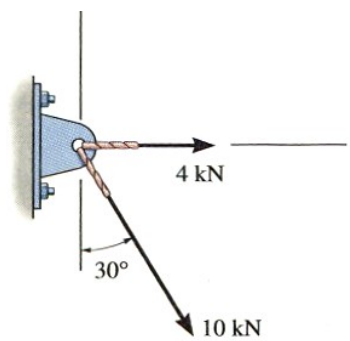


10 N

Ans: 29.826 N & 69.059º with x-axis.

2 Determine the magnitude and direction of the resultant force as shown in fig

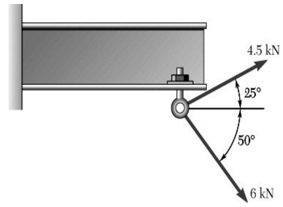
Ans: 12.489 N & 43.902º



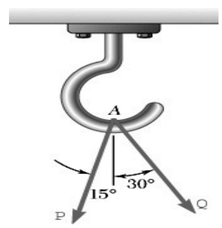
3 Determine the magnitude and the direction of the resultant of two forces 7 N and 8 N acting at a point with an included angle of 60º with between them. The force of 7 N being horizontal

4. Determine the magnitude and direction of the resultant of two forces 20 N and 30 N acting at a point with an included angle of 40º between them. The force 30 N being horizontal

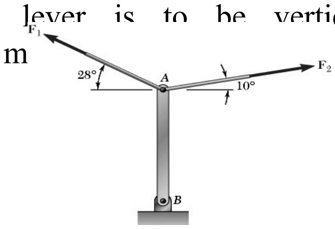
5. Two forces are applied to an eye bolt fastened to a beam. Determine the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.



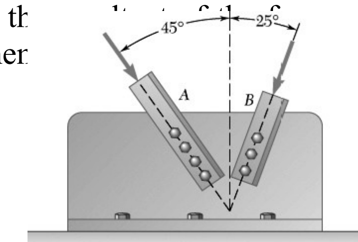
6. Two forces **P** and **Q** are applied as shown at point *A* of a hook support. Knowing that *P* =15 lb and *Q* = 25 lb, determine the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.



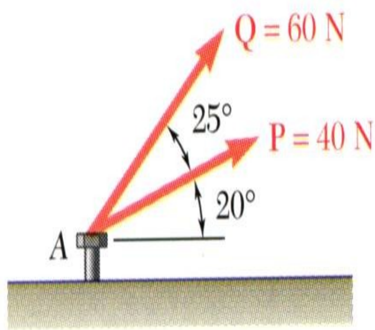
7. Two control rods are attached at A to lever AB. knowing that the force in the left-hand rod is F_1 = 120 N, determine (a) the required force F_2 in the right-hand rod if the resultant of the forces exerted by the rods on the lever is to be vertical, (b) the corresponding m



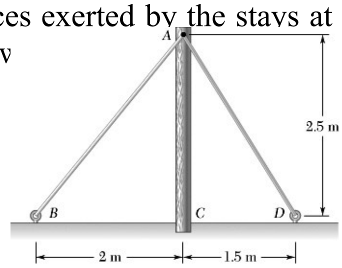
8. Two structural members *A* and *B* are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 30 kN in member *A* and 20 kN in member *B*, determine, the magnitude and direction of th … applied to the bracket by mer



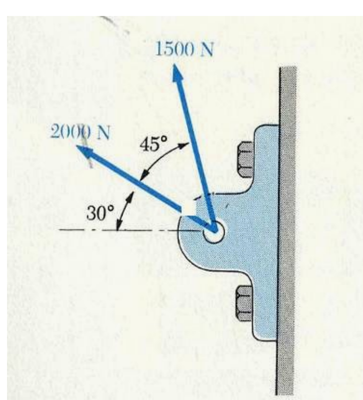
9. The two forces P and Q act on bolt A as shown in diagram. Find their resultant and direction



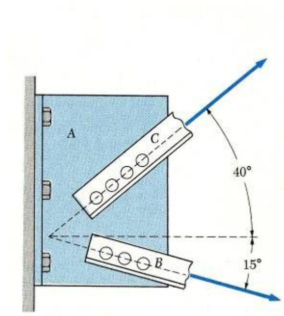
10. The cable stays AB and AD help support pole AC. Knowing that the tension is 500 N in AB and 160 N in AD, determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at A using (a) the parallelogram lav



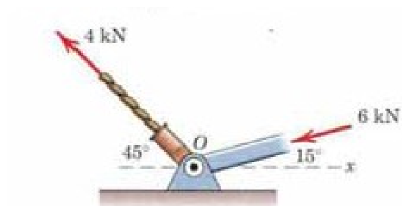
11. Determine the magnitude and direction of the resultant of the two forces**.**



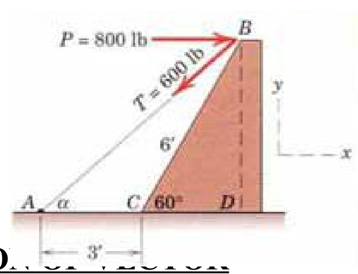
12. Two structural members B and C are riveted to the bracket A. Knowing that the tension in member B is 6 kN and the tension in C is 10 kN, determine the magnitude and direction of the resultant force acting on the bracket.



13. The two structural member one in tension and other in compression, exerts on point O, determine the resultant and angle θ



14. The force P and T act on body at point B replace them with a single force

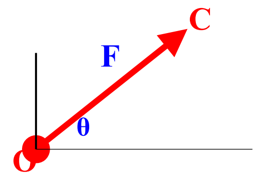


RESOLUTION OF VECTOR

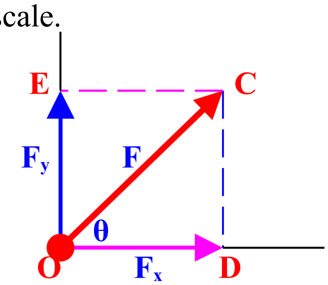
The processes of finding the components of given vector (resultant) is called resolution of vector. Or The processes of splitting up of single vector into two or more vector is called resolution of the vector A vector can be resolved into two or more vectors which have the same combined affect as that the effect of original vector

RESOLUTION OF VECTOR INTO RECTANGULAR COMPONENTS

If vector is resolved into such components which are at right angles (perpendicular) to each other then they are called the rectangular components of that vector, now let us consider a resultant vector F to be resolved into two components which makes an angle θ with horizontal axes as shown in fig.



Now draw a line OC to represent the vector in magnitude, which makes an angle θ with x-axis with some convenient scale. Drop a perpendicular CD at point C which meet x axis at point D, now join point O to point D, the line OD is called horizontal component of resultant vector and represents by F_x in magnitude in same scale. Similarly draw perpendicular CE at point C, which will meet y-axis at point E now join O to E. The line OE is called vertical component of resultant vector and represents by F_y in magnitude of same scale.



Analytically or trigonometry

In ΔCOD Angle COD = θ Angle ODC = 90° OC = F

OD = F_x OE = CD = F_y

We know that

$$\text{Cosine } \theta = \frac{OD}{OC} \qquad \text{Cosine } \theta = \frac{F_x}{F}$$

And $\mathbf{F_x = F \text{ Cosine } \theta}$

Similarly we have

$$\text{Sin } \theta = \frac{DC}{OC} \qquad \text{Sin } \theta = \frac{F_y}{F}$$

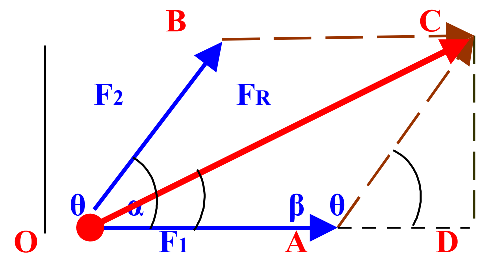
And $\mathbf{F_y = F \text{ Sine } \theta}$

RESOLVING OF A FORCE INTO TWO COMPONENTS WHICH ARE NOT MUTUALLY AT RIGHT ANGLE TO EACH OTHER

If a force or vector is to be required to resolved into such components which are not at right angle to each other then it can be determined in reverse manner as we find the resultant vector of given components by Parallelogram method, Triangle method or Trigonometry

A) Parallelogram method

Now consider a force F_R, which is resolved into components F_1 and F_2. The force F makes an angle α with force F_1 and force F_2 makes an angle θ with component F_1, so we can make a parallelogram with suitable scale as shown in fig.



We can also determine the components of force F by analytically as we know that direction of the resultant vector can be determined by

$$\sin\alpha = \frac{F_2 \sin\theta}{F_R} \quad \text{OR} \qquad (1)$$

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \text{Cosine}\,\theta} \qquad (2)$$

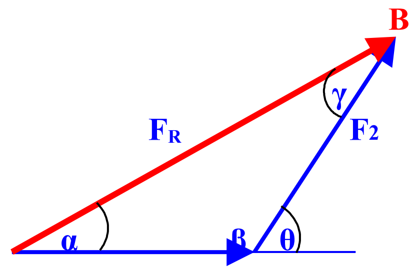
So we can find F_2 from equation 1

$$F_2 = \frac{F_R \sin\alpha}{\sin\theta}$$

Similarly from equation 2

$$F_1 = \frac{F_2 \sin\theta}{\tan\alpha} - F_2 \text{Cosine}\,\theta$$

B) Triangle method: Now consider a force F, which is resolved into components F_1 and F_2. The force F makes an angle α with force F_1 and force F_2 makes an angle θ with component F_1, so we can make a triangle with some suitable scale as shown in fig.



O **F_1** **A**

By measurements we get the components F_1 and F_2.
Similarly we can find the components F_1 and F_2 by using the following formula

$$\frac{F_1}{\sin \gamma} = \frac{F_2}{\sin \alpha} = \frac{F_R}{\sin \beta}$$

For component F_1

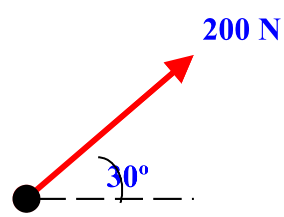
$$F_1 = \frac{F_R \sin \gamma}{\sin \beta}$$

For component F_2

$$F_2 = \frac{F_R \sin \alpha}{\sin \beta}$$

EXAMPLE 5

Resolve the force 200 N into components along x and y direction and determine the magnitude of components.



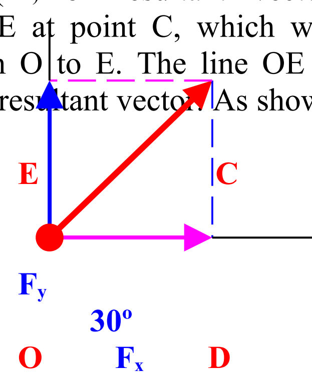
Given: Force = F = 200 N Direction = θ = 30°

Required Horizontal components = F_x =?
Vertical components = F_y =?

Solution

A) **Graphically** Scale 1 cm = 20 N

Now draw a line OC to represent t vector in magnitude with given scale, which makes an angle 30° with x-axis. Drop a perpendicular CD at point C which meet x axis at point D, now join point O to point D, the line OD is called horizontal component (F_x) of resultant vector. Similarly draw perpendicular CE at point C, which will meet y-axis at point E now join O to E. The line OE is called vertical component (F_y) of resultant vector. As shown in fig



By measuring we get

$OD = F_x = 8.6 \text{ cm} \times 20 = 172 \text{ N}$

OE = Fy = 5 cm x 20 = 100 N

Result: $F_x = 173.20$ N $F_y = 100$ N

B) Analytically

We know that $F_x = F \cos\theta$ = 200 cosine30

$F_x = 173.20$ N

We also know that

$F_y = F \sin\theta$ = 200 Sin 30 $F_y = 100$ N

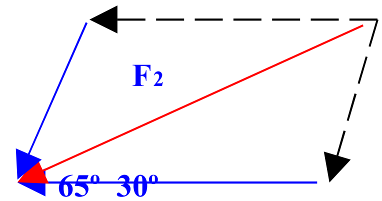
Result: $F_x = 173.20$ N $F_y = 100$ N

EXAMPLE 6

A push of 40 N acting on a point and its line of action are inclined at an angle of 30º with the horizontal. Resolve it along horizontal axis and another axis which is inclined at an angle of 65º with the horizontal.

B

F



D

Given Force = F = 40 N Direction = θ = 30º

Direction = α = 65º

Required Force component = F_1 =? Force component = F_2 =?

Solution **Graphical Method**

Let Scale 10 N = 1 cm

Now draw the parallelogram ABCD with given scale as shown in fig

By measurement AD = F_1 = 2.5 x 10 = 25 N

AC = F_2 = 2.3 x 10 = 23 N

Result $F_1 = 25$ N $F_2 = 23$ N

Analytically

We have $F_2 = \frac{F \sin\alpha}{\sin\theta} = \frac{40 \sin 30}{\sin 65^\circ}$

F_2 = 22.06 N

Similarly from equation

$F_1 = \frac{F_2 \sin\theta}{\tan\alpha} - F_2 \cos\theta$

$F_1 = \frac{22.06 \sin 65}{\tan 30} - 22.06 \cos 65$

$F_1 = 25.32$ N

Result **$F_1 = 25.32$ N** **$F_2 = 22.06$ N**

EXERCISE 2.2

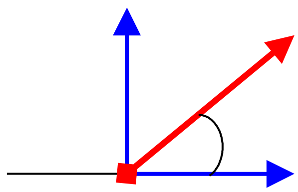
1. Resolve the given forces as shown in following diagrams into components F_1 and F_2

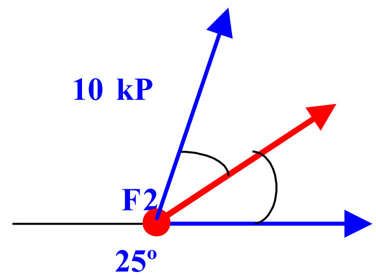
i **ii**

–

200 N

30º



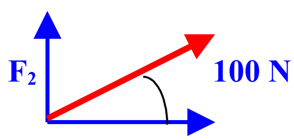


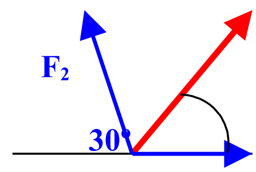
30º **F_1**

F1

iii **iv**

156 lb





26º

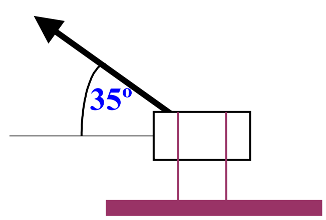
F_1

34º **F_1**

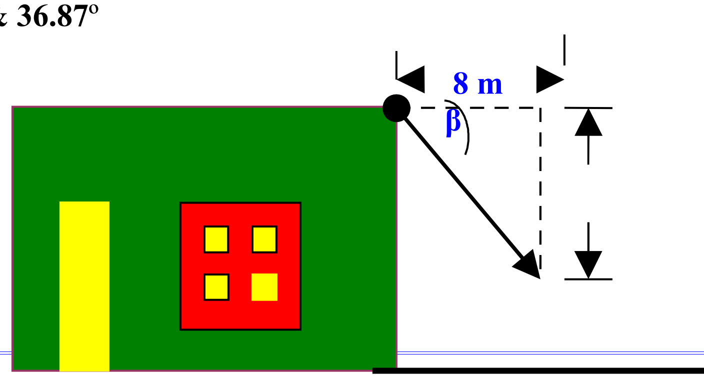
2. A force of 800 N is exerted on a bolt A as shown in fig. Determine the horizontal and vertical components of force.

800 N

Ans: 655.32 N & 458.816N



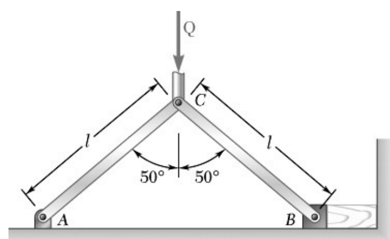
4. A man pull with force of 300 N on a rope attached to a building as shown in fig, what are the horizontal and vertical components of the force exerted by the rope at point A
Ans: 180 N & 36.87º



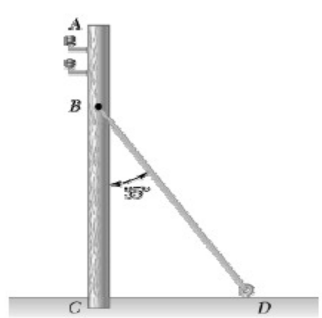
6 m

5 While emptying a wheel barrow, a gardener exerts on each handle AB a force **P** directed along line CD. Knowing that **P** must have a 135-N horizontal component, determine (a) the magnitude of the force **P**, (b) its vertical component

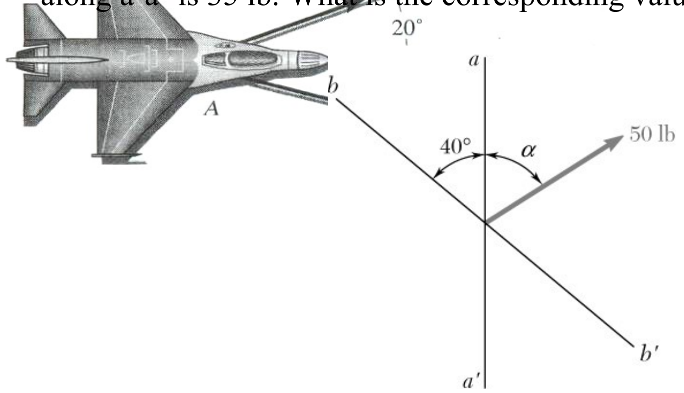
6 Member *CB* of the vise shown exerts on block *B* a force P directed along line *CB*. Knowing that P must have a 260-lb horizontal component, determine (*a*) the magnitude of the force P, (*b*) its vertical component.



7. The guy wire BD exerts on the telephone pole AC a force P directed along BD. Knowing that P has a 450-N component along line AC, determine (a) the magnitude of the force P, (b) its component in a direction perpendicular to AC.

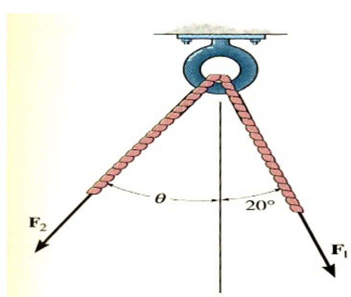


8. The 50-lb force is to be resolved into components along lines a-a' and b-b' knowing that the component along a-a' is 35 lb. What is the corresponding value



9. The ring shown in fig is subjected to two forces F_1 and F_2. if it is required that the resultant forces have a magnitude of I kN and are directed vertically downward. Determine the magnitude of F_1 and F_2 provided that θ = 30º

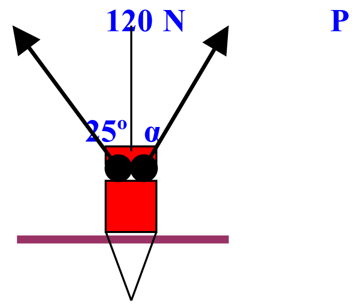
Ans: 652.704 N & 446.47 N



10. A jet aircraft is being towed by two trucks B and C. Determine the magnitude of two forces F_B and F_C. If the force has a magnitude of F_R = 10 KN and it is directed along positive x-axis. Set θ = 15º
Ans: 5.693 K N & 4.512 KN

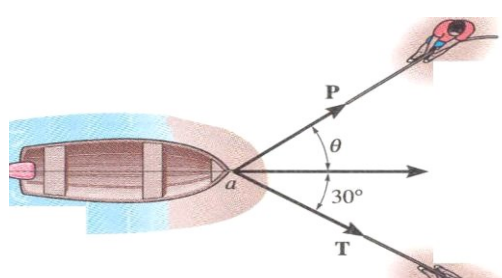
11. A stake is pulled out of the ground by means of two ropes as shown. Knowing that the tension in one rope is 120 N, determine by trigonometry the magnitude and direction of the force P so that the resultant is a vertical force of 160 N.

Ans: 72.096 N & 44.703º



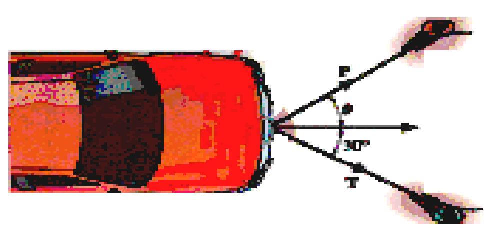
12. The boat is to be pulled onto the shore using two ropes, determine the magnitude of two forces T and P acting in each rope in order to develop a resultant force of 80 lb in direction along the keel as shown in fig. take $\theta = 40^{\circ}$

Ans: 42.567 lb & 54.723 lb



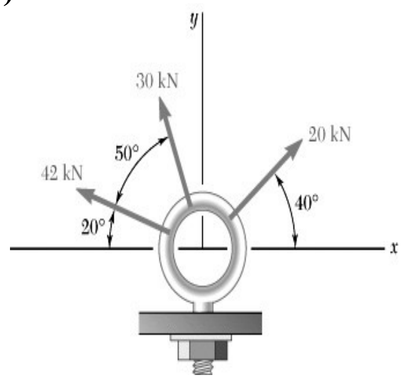
13. A disabled automobile is pulled by means of two ropes as shown. Knowing that the tension in the rope P is 500 lb, determine the tension in rope T and the value of so that the resultant force exerted is as 800 lb force directed along the axis of the automobile

Ans: 442.020 N



15. Find the x and y components of each force and determine the resultant and direction

i)



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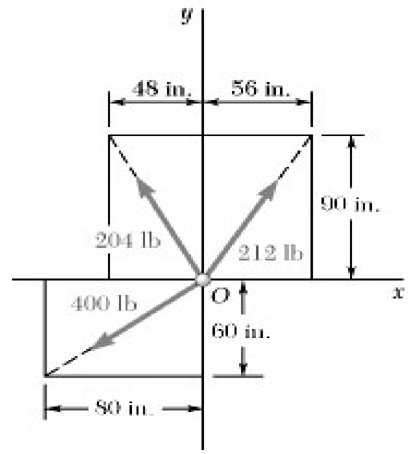

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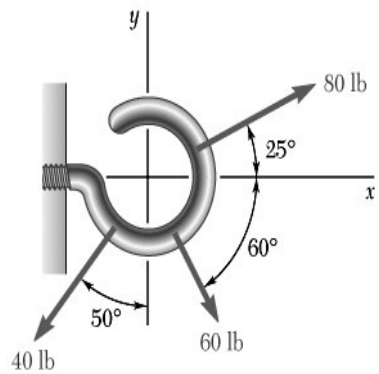

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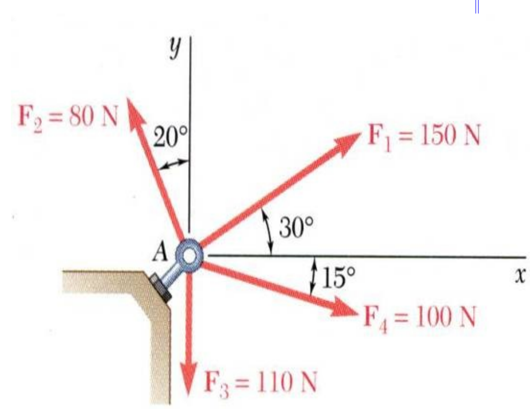

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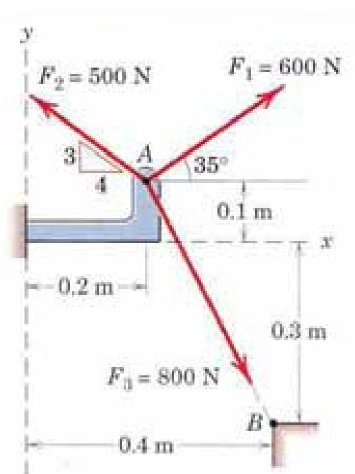
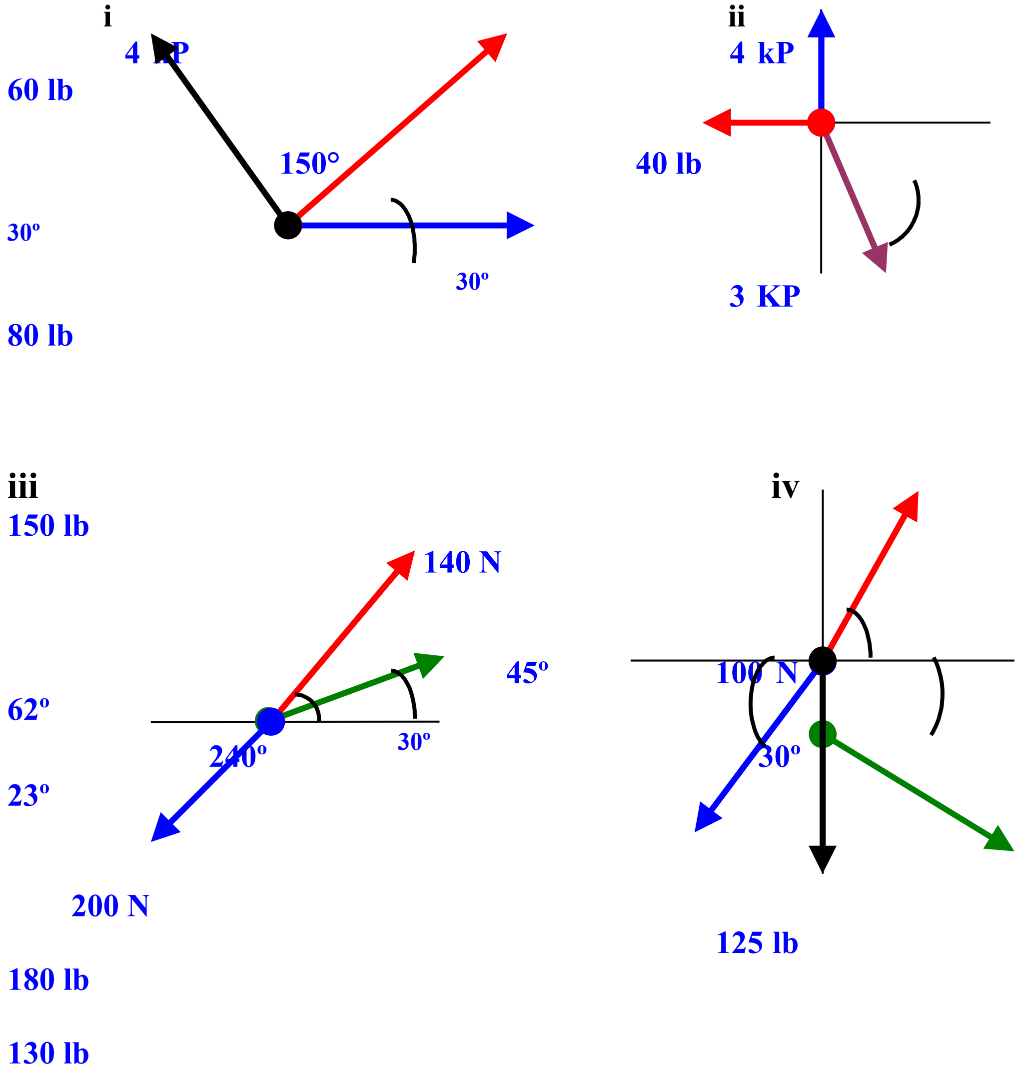
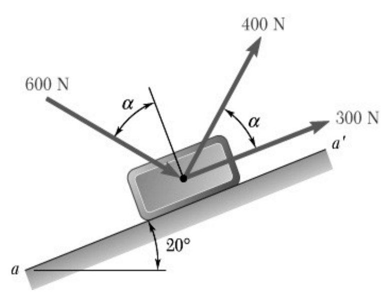


16. Find the resultant and direction of following forces as shown in diagram by resolving method.



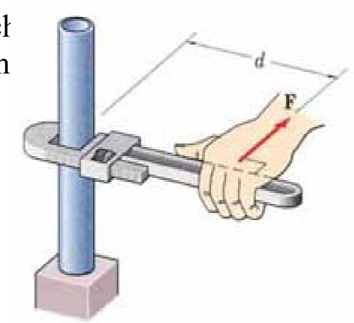
17. Knowing that α = 35°, determine the resultant of the three forces shown.



CHAPTER 3 Moment of a force

The tendency of a force to move the body in the direction of its application a force can tend to rotate a body about an axis. This axis may be any line which is neither intersects nor parallel to the line of the action of the force. This rational tendency of force is know as the moment of force.

As a familiar example of the concept of moment, consider the pipe wrench as shown in figure (a). One effect of the force applied perpendicular to the handle of the wrench is the tendency to rotate the pipe about its vertical axis. The magnitude of this tendency depends on both the magnitude of the force and the effective length d of the wrench handle. Common experience shown that a pull which is not perpendicular to the wrencl an the right angle pull. Mathem force (moment) is calculated by oment arm (d)



(a)

Moment about a point

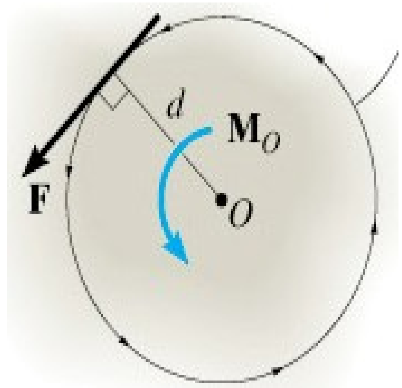
Consider following body (two dimensional) acted by a force F in its plane. The magnitude of moment or tendency of the force to rotate the body about the axis O_O perpendicular to the plane of the body is proportional both to the magnitude of the force and to the moment arm d, therefore magnitude of the moment is defined as the product of force and moment arm.

Moment = Force x moment arm

M = Fd

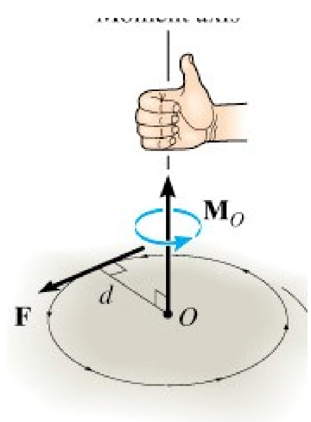
Where d = moment arm and F = magnitude of force

Moment arm is defined as the perpendicular distance between axis of rotation and the line of action of force.



<u>**Direction of moment of a force**</u>

The direction Mo is specified using the “right-hand rule”. To do this the fingers of the right hand are curled such that they follow the sense of rotation, which would occur if the force could rotate about point O. The thumb then point along the moment axis so that it gives the direction and sense of the moment vector, which is upward and perpendicular to the shaded plane containing **F** and **d**.



<u>**CLOCK WISE AND ANTI CLOCK WISE MOMENTS**</u>

The moment are classified as clockwise and anticlockwise moment according to the direction in which the force tends to rotate the body about a fixed point

Clockwise Moment

When the force tends to rotate the body in the same direction in which the hands of clock move is called clockwise moment the clockwise moment is taken as positive or other wise mentioned.

Anticlockwise Moment

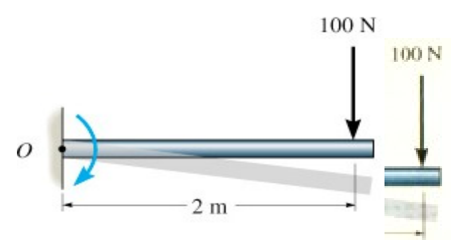
When the force tends to rotate the body in the opposite direction in which the hands of clock move is called anti clockwise moment which is taken as negative or other wise mentioned

Unit of moment

S.I unit	is	N.m.	(Newton. meter)
F.P.S unit	is	lb. ft	(Pound. foot)
G.G.S unit	is	dyne.cm	(dyne. Centimeter) etc

Example 1

Determine the moment of the force about point "O" for following diagram.



(a)

1 **Given** Force=100 N
Moment arm=2m

Required Mo=?

Working formula: - Mo=Force x Moment arm.

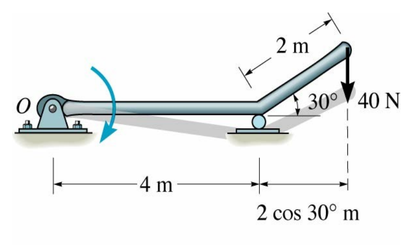
Sol putt the values in first w, f

Mo= F x r = 100 x 2

Mo= 200N.m.

Result: - Moment = 200N.m Direction = clock wise

2



(c)

Given

Force = 40lb

Required; Mo =?

W.F, Mo = F x d.

Sol

By geometry of fig

Moment arm = 4ft + 2cos 30º = 5.73ft

Put the value in W.F.

Mo = F x r

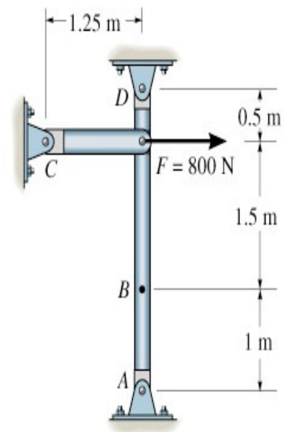
Mo = 40 x 5.73

Mo = 229.282lb.ft

Resultant Moment = 229.282 lb.ft Direction = clock wise

Example2

Determine the moment of the force 800 N acting on the frame about points A, B, C and D.



Given

Force = F = 800 N

Required M_A=? M_B? M_C=? M_D=?

Working formula

Moment =force x moment arm.

Sol Solve this question step by step

Now first consider the Point A.

M_A = F x r

M_A = 800 x (1.5+1)

M_A = 2000 N.m clock wise_______ I

Now Moment about B

M_B = F x r = 800x 5

M_B = 1200 N m clock wise_________ (2)

From (1) and (2) it is evidence that when force remain constant then moment varies with moment arm that is moment depends upon moment arm. Similarly it can be proved that moment about any point varies with force when moment arm remain same.

Now consider point C

Moment = Force x distance

Mc = 800 x 0

Mc = 0. ________ (3)

As the line of action of force passes through point C that is point of application it shows that the line of action should be perpendicular to the point i.e. "C"

Now consider the point D.

M_D = F x r.

M_D = 800 x 0.5

M_D= 400 N.m

Result

M_A =2000 N.m clock wise Or
M_A = + 200 N.m

M_B = 1200 N.m clock wise Or
M_B = + 1200 N.m

M_C = O.
Mc = O

M_D =. 400 N.m anti clock wise
M_D = - 400N.m

Note: - The positive sign shows that the moment is clock wise direction and it is also proved that moment defends upon following two factors.

1. The magnitude of the force
2. The perpendicular distance from the line of action of the force to the fixed point or line of the body about which it rotates.

PRINCIPLE OF MOMENT/ VARIGNON'S THEOREM

It is stated that the moment of a force about a point is equal to the sum of the moments of the force components about the point. Or the moment produce by the resultant force is equal to the moment produce by the force components.

Mathematically $M_{F_o} = \sum M_o$

Moment produce by the force F about any point O = Moment produce due to force components. Let us consider a force F acting at a point A and this force create the moment about point O which is r distance away from point A as shown in fig (a)

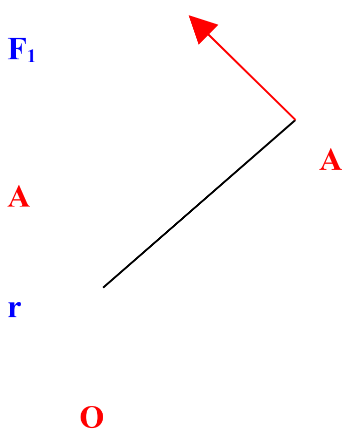


fig (a)

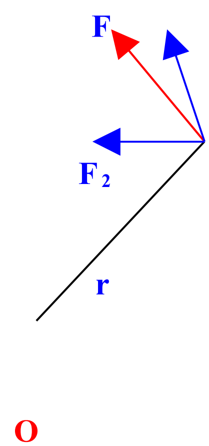


fig (b)

The moment produce due to Force F is given by

$$M_{F_o} = F \times r \qquad 1$$

Now resolve the force into its components F_1 and F_2 in such a way that

$$F = F_1 + F_2 \quad \text{as shown in fig (b)}$$

The moment produce by these components about O is given by

$$\sum M_o = 0$$

$\sum M_o$ = moment produce due to force F_1 + moment produce due to force component F_2

$$\sum M_o = F_1 \times r + F_2 \times r = (F_1 + F_2)\, r$$

Put $F = F_1 + F_2$ in the above formula

$\sum M_o = F \times r$ ________________ 2

By comparing the equation 1 and by equation 2

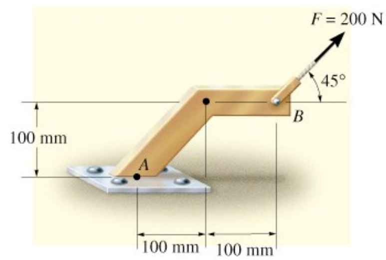
$M_{F_o} = \sum M_o$

The above equation shows that moment produce by the Force (resultant) is equal to the moment produce by components F_1 and F_2.

Note the above equation is important application to solution of problems and proofs of theorems. Such it is often easier to determine the moments of a force's components rather than the moment of the force.

EXAMPLE 3

A 200 N force acts on the bracket as shown determine the moment of force about "A"



Given $F = 200N$ $\theta = 45^o$

Required $M_A = ?$

Solution Resolve the force into components F_1 am F_2

$F_1 = F \cos\theta$ $F_1 = 200$ cosine 45^o

$F_1 = 141.42N$.

$F_2 = F \sin\theta$ $F_2 = 200 \sin 45^o$

$F_2 = 2.468N$.

We know that $M_A = 0$

M_A = moment produce due to component F_1+ moment produce due to component F_2.

$M_A = F_1 \times r_1 + F_2 \times r_2$.

Let us consider that clock wise moment is + ve.

$M_A = F_1 \times r_1 + F_2 \times r_2$

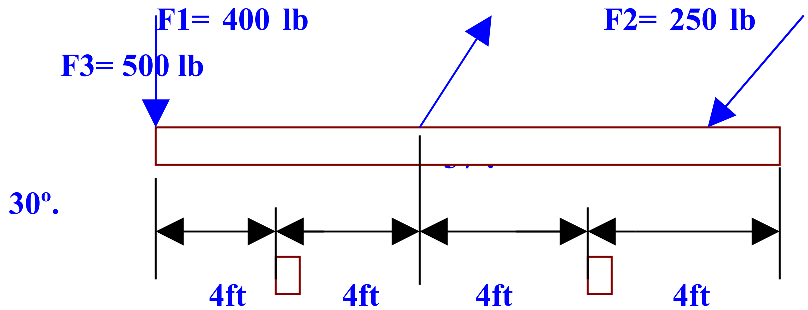
$M_A = - 141.42 \times 0.1 + 2.468 \times (0.1 + 0.1)$

$M_A = - 13.648$ N

$M_A =$ 13 .648 N anti clock wise.

EXAMPLE 2.4

Determine the moment of each of three forces about B on the beam.



Given

$F_1 = 400lb$ $F_2 = 250$ lb $F_3 = 500lb$

$r_1 = 4$ Ft $r_2 = 4$ Ft $r_3 = 4$ Ft $r_4 = 4$ Ft

Required Moment about B = M_B =?

Solution

Moment due to force F1 about B:

Consider clockwise moment is positive

$M_B = 400 \times (4+4+4)$

M_B = 48, 00 lb .ft

Moment due to vertical component of F2

$M_B = F2 \sin \theta \times r$

$M_B = 250 \text{ Sin } 37 \times 4$

M_B = 601.815lb ft clock wise

Moment due to vertical component of F3

$M_B = F3 \sin \theta \times R$

$M_B = 500 \times \text{Sin } 30 \times 4$

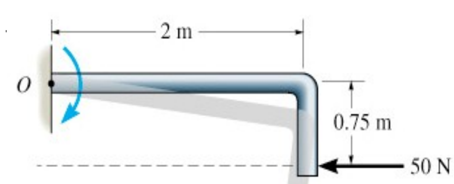
M_B = 601.815lb clock wise

Result **M_B = 48, 00 lb .ft 601.815lb, 601.815lb**

EXERCISE

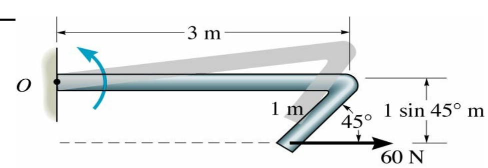
1. Find the moment of the force about "O" as shown in diagram

i



(b)

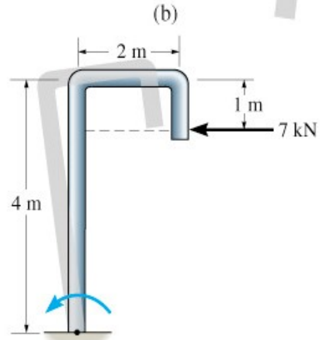
ii



(d)

cloc

iii



(e)

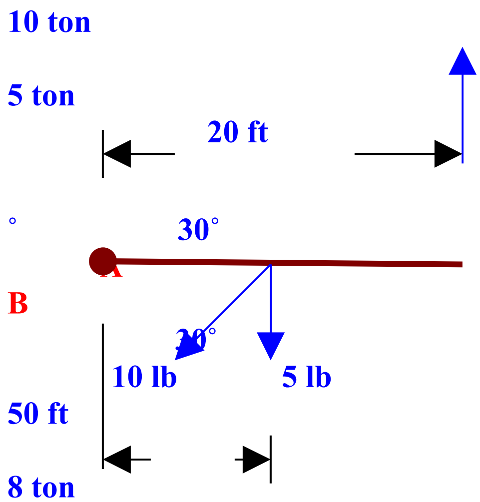
ise

Ans : 42.426

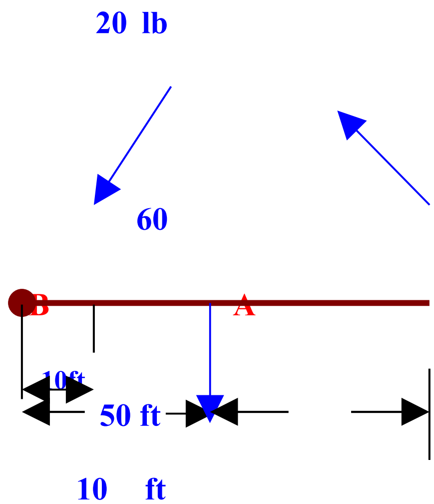
Ans: 21 kN m

2. Find the moment of each force about A as shown in the following force system.

i

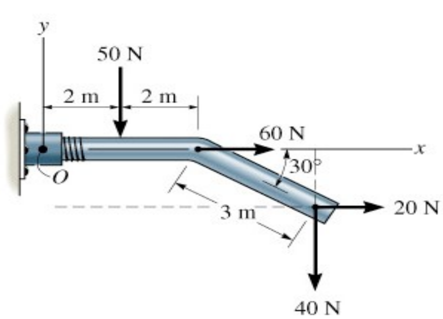


ii

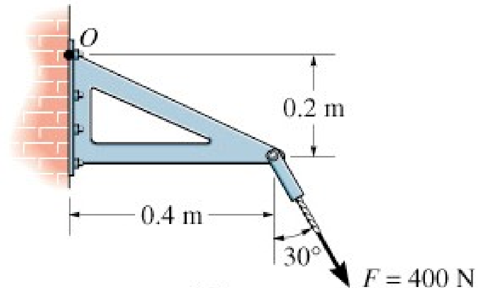


Ans: 300 lb ft anti clockwise **Ans: 236.603 ton ft anti clock wise**

3. Determine the resultant moment of four forces acting on the rod about "O" as shown is diagram. **Ans: 333.92 N m clock wise**

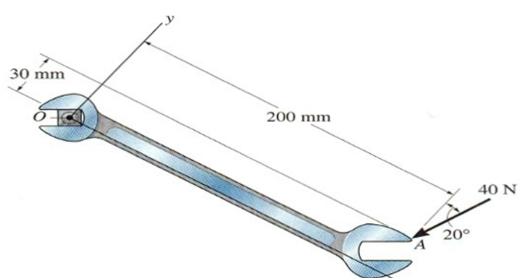


4. The Force F acts at the end of angle bracket shown determine th … “O”
Ans : 98.56 cl

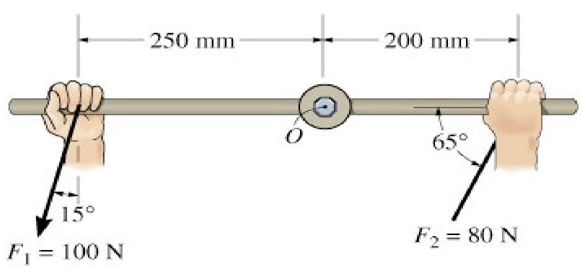


(a)

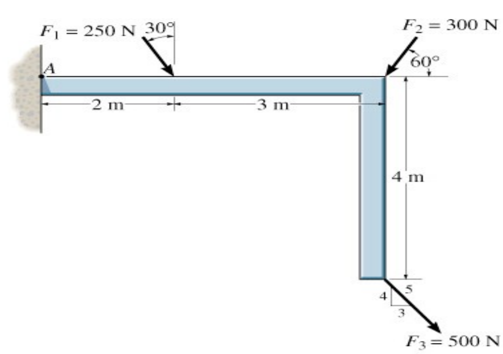
5. A force of 40N is applied to the wrench. Determent the moment …)”
Ans: 7.107 c



6. The wrench is used to loosen the bolt. Determine the moment of each force about the bolt’s axis passing through point O. **(Ans: 24.1 N-m, 14.5 N-m)**

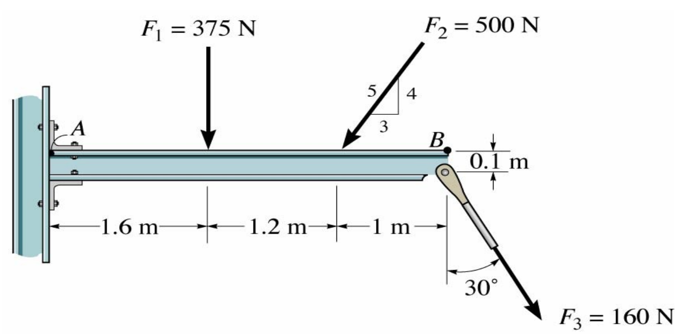


7. Determine the moment of each of the three forces about point A. … orce as a whole, ar … ts. … : 433 N- … 800 N-

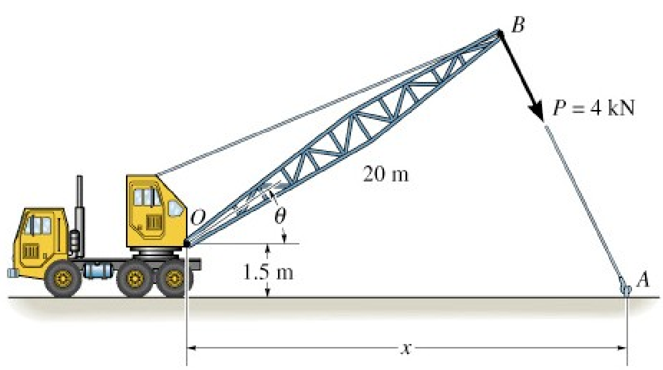


8. Determine the moment about point A of each of the three forces

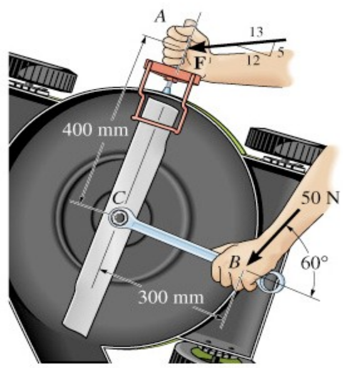
Ans: 600 N-m, 1.12 KN-m, 518 N-m



9. The towline exerts a force of P = 4 kN at the end of the 20 m long crane boom. If θ = 30º, determine the displacement x of the hook at A so that the force creates a maximum moment about point O. What is this moment? **(Ans: 24.0 m, 80 kN-m)**

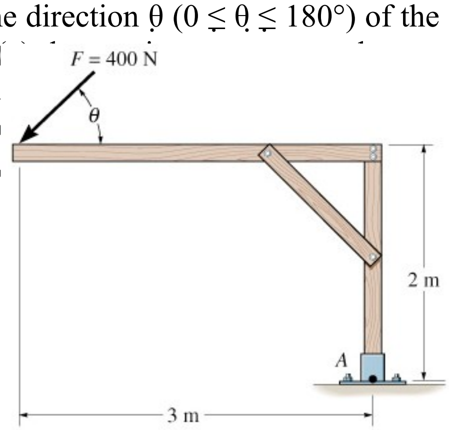


10. The tool at A is used to hold a power lawnmower blade stationary while the nut is being loosened with the wrench. If a force of 50 N is applied to the wrench at B in the direction shown, determine the moment it creates about the nut at C. What is the magnitude of force F at A so that it creates the opposite moment about C?



(Ans: 13.0, N-m, 35.2, N-m)

11. Determine the direction θ ($0 \leq \theta \leq 180°$) of the force F so that it produces (... ɔint A and (b) the minimu ... npute the moment in each ... **ns: 56.3°, 146°, 1442 N-m,**



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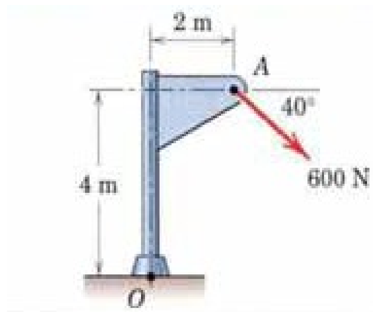
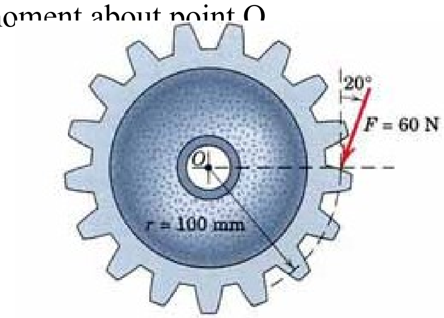
12. Calculate the magnitude of moment about base point O by five different ways

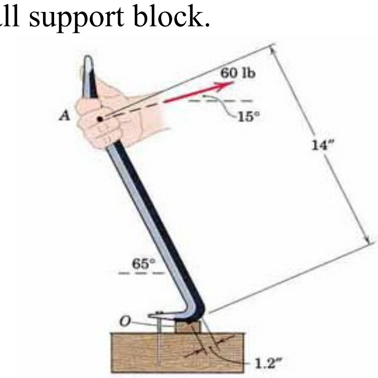


13. A force F of magnitude 400 N is applied. Determine the magnitude of moment about point O



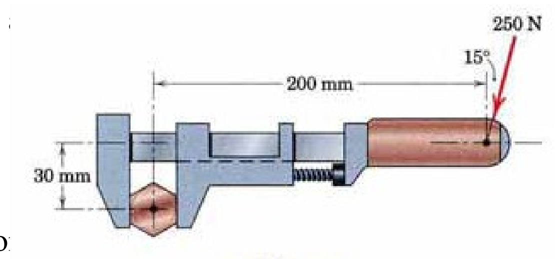
N- m

14. A pry bar is used to remove nail as shown. Determine the moment of the force 60 lb about point O of contact between the pry and the small support block.

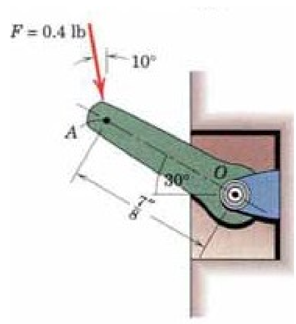


Ans: 70 lb-ft CW

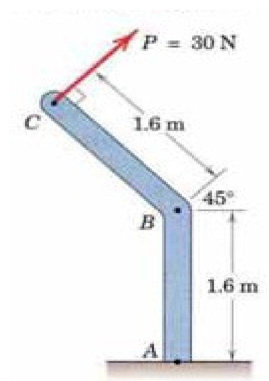
15. Calculate the moment of force 250 N on the handle of monkey wrench



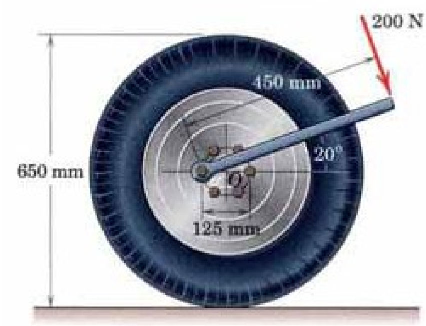
16. Compute the mo
of the wall switch toggle



17. The 30 N force P is applied perpendicular to the portion BC of the bent bar. Determine the moment of P about point A and B.



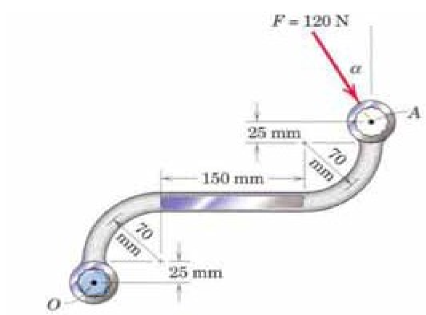
18. A force of 200 N is applied to the end of the wrench to tighten a flange bold which holds the wheel to the axle. Determine the moment M produced by this force about the center O of the wheel for the poison of the wrench shown **Ans: 78.3 N-m CW**



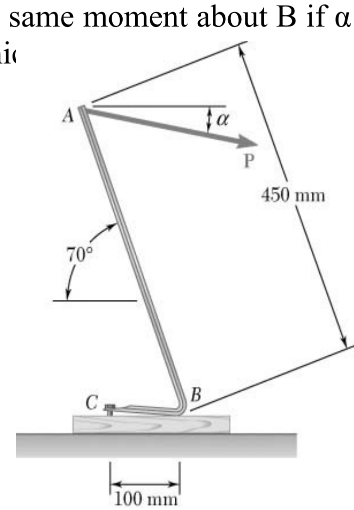
19. The 120 N forces is applied as shown to one end of the curved wrench. If α = 30°, calculate the moment of F about the center O of the bolt. Determine the value of α which would maximize the moment about O state the value of this maximum moment

Ans: 41.5 N=

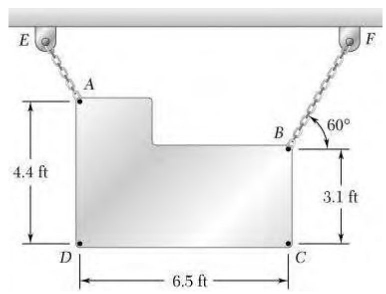
m CW 32.2° 41.6 N-m CW



20. It is known that a vertical force of 800 N is required to remove the Nail at C form the board. As the nail first starts moving determine (a) the moment about B of the force exerted on the nail (b) the magnitude of the force P which creates the same moment about B if α = 10° (c) the smallest force whi… …nt about B



20. A sign is suspended from two chains AE and AF. Knowing that the tension in BF is 45 lb, determine (a) the moment about A of the force exert by the chain at B (b) the smallest d force applied AT C which creates the same moment about A

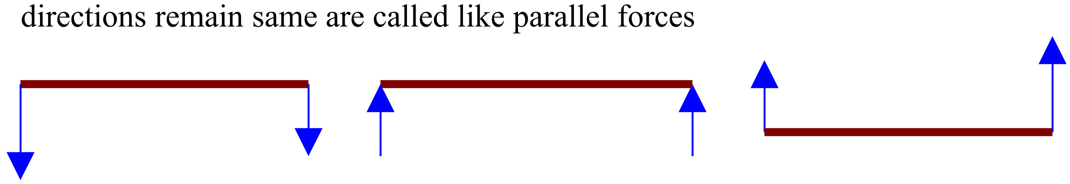


PARALLEL FORCES

When the lines of action of Forces are parallel to each other are called parallel forces the parallel forces never meet to each other. There are two types of parallel forces as discussed as under

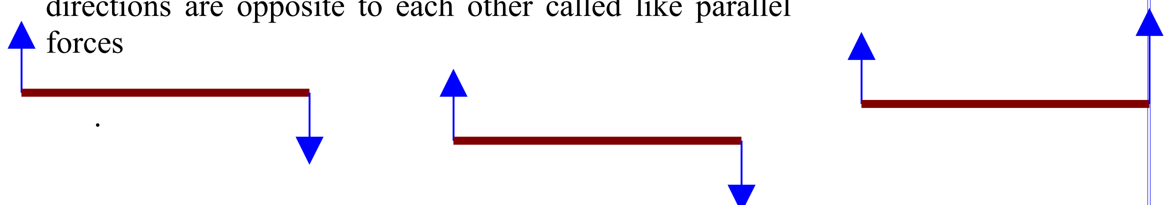
1. **Like parallel forces**

   When two parallel forces acing in such away that their directions remain same are called like parallel forces



2. **Un like parallel forces**

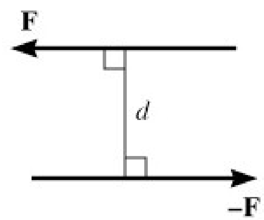
   When two parallel forces acing in such away that their directions are opposite to each other called like parallel forces



COUPLE

When two parallel forces that have the same magnitude but opposite direction is known as couple. The couple is separated by perpendicular distance. As matter of fact a couple is unable to produce any straight-line motion but it produces rotation in the body on which it acts. So couple can be defined as unlike parallel forces of same magnitude but opposite direction which produce rotation about a specific direction and whose resultant is zero

.



APPLICATION OF COUPLE

1. To open or close the valves or bottle head, tap etc
2. To wind up a clock.
3. To Move the paddles of a bicycle
4. Turning a key in lock for open and closing.

Couple Arm

The perpendicular distance between the lines of action of the two and opposite parallel forces is known as arm of the couple.

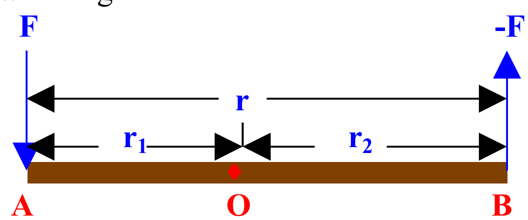
Moment of couple or couple moment

The moment of the couple is the product of the force (one of the force of the two equal and opposite parallel forces) and the arm of the couple. Mathematically

Moment of couple = force x arm of couple

Moment of couple = F x r

Let us find the resultant moment of couple about a point O on the couple arm AB as shown in fig



Moment about O

$\sum M$ = Moment about O due to F + moment about O due to –F

$$\sum M = -F \times r_1 + (-F \times r_2)$$
$$\sum M = -F \times r_1 - F \times r_2$$
$$\sum M = -F (r_1 + r_2)$$
$$\sum M = F (r_1 + r_2) \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_} 1$$

From diagram $r = r_1 + r_2$ put in equation 1

$$\sum M = F \times r$$

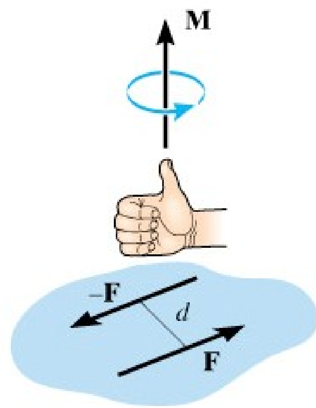
So the moment produce by the two unlike parallel forces is equal to moment produce by one of the force of the two equal and opposite parallel forces.

Therefore

The moment of couple = force x couple arm.

Direction of couple

The direction and sense of a couple moment is determined using the right hand rule, where the thumb indicates the direction when the fingers are curled with the sense of rotation caused by the two forces.



CLASSIFICATION OF COUPLE

The couplet are classified as clockwise couple and anticlockwise couple

1. Clockwise couple

A couple whose tendency is to rotate the body in a clockwise direction is known as clockwise couple

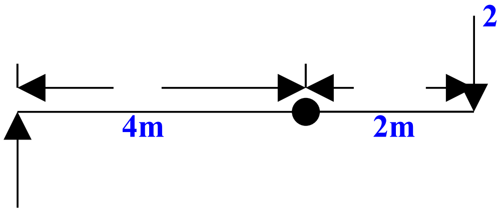
2. Anticlockwise couple

A couple whose tendency is to rotate the body in anticlockwise direction is known as anticlockwise couple

EXAMPLE 8

Determine the moment of couple acting on the moment shown

00 N



200 N

Given

F_1=200 N L_1=4m F_2=200 N L_2 = 2m.

Required Moment of couple = M =?

Working Formula M = F x r.

Solution

Put the values in working formula

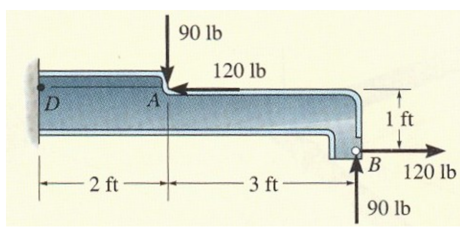
M= 200(4+2)

M=1200 N. m

Result **M= 1200 N. m**

<u>EXAMPLE 9</u>

Determine the moment of couple acting on the moment shown.



Given F_1=F_2 =90lb F_3 = F_4 = 120lb.

Required Moment of couple = M=?

Solution The moment of couple can be determined at any point for example at A, B or D.

Let us take the moment about point B

$M_B = \sum F R.$

$M_B = -F_1 \times r_1 - F_2 \times r_2 .$

M_B = - 90(3) – 120 (1)

M_B = - 390 lb ft

Result **M_B = M_A=M_D =390 lb .ft counter clock wise.**

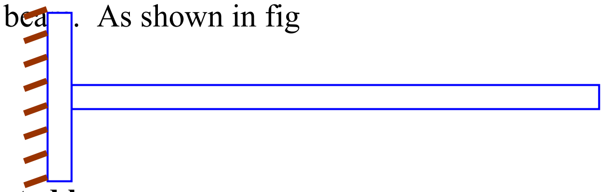
Moment of couple = 390 lb.ft count cloche wise

<u>BEAM</u> A beam is a long straight bar having a constant cross-sectional area. Beams are classified as

1 Cantilever beam 2 Simply supported beam
3 Over hanging beam 4 Rightly fixed or built in beam
5 Continuous beam.

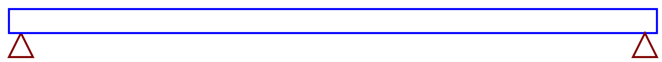
1. Cantilever beam

A beam, which is fixed at one and free at the other end, is called cantilever beam. As shown in fig



2. Simply supported beam

A beam which is pinned (pivoted) at one end and roller support at other end is called simply supported beam. As shown in fig

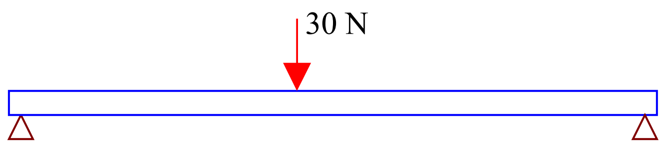


LOAD

The external applied force is called load. Load is in the form of the force or the weight of articles on the body is called load.

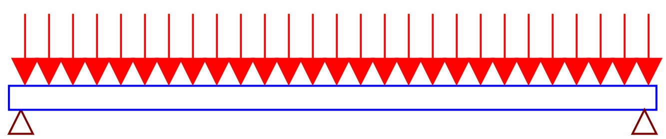
1. Concentrated or Point load

A load, which is applied through a knife-edge, is called point or concentrated load.



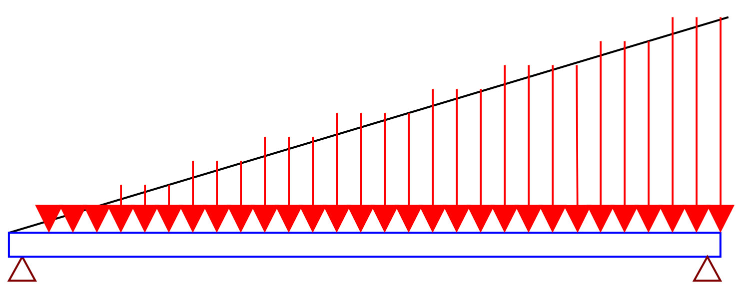
2. Uniformly distributed load

A load which is evenly distributed over a part or the entire length of beam is called uniformly distributed load or U D.L



3. Uniformly varying load

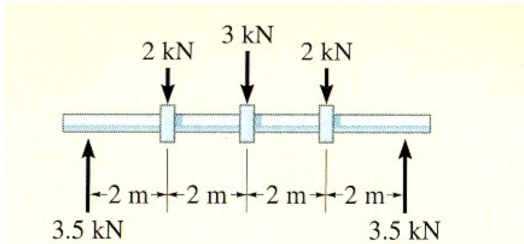
The load whose intensity varies lineally along the length of beam over which it is applied is called uniformly varying load.



Note

Any beam may be point, uniformly distributed and uniformly varying load

EXAMPLE 10 Find the reaction of the shaft at point shown**.**



(b)

Given Span = L = 8m x = 2m, y = 2m, Z = 2m

F_1 = 2 KN F_2 = 3 KN F_3= 2 kN.

Required Shear force and moment diagram

Solution Take moment about "A" also consider the upward force and clock wise moment is positive

$\sum M_A = 0$

R_E (L) – F3 (x + y + z) – F_2 (x + y) – F_1 (x) + R_A (0) = 0.

R_E (8) - 2 (6) – 3 (4) – 2 (2) + 0 = 0

R_E = 3.5 KN

Now for R_A we can calculate by

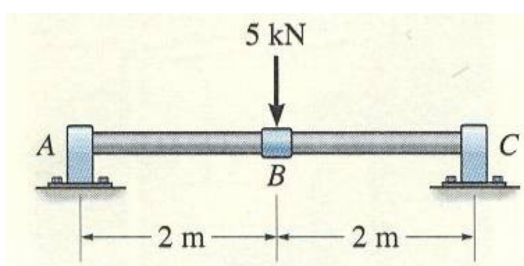
$\sum F = 0$

$R_A - F_1 - F_2 - F_3 + R_E = 0$

R_A - 2 - 3 - 2 + 3.5 = 0

EXAMPLE 2.11

Find reaction at A and C for shaft shown. The support at A is a thrust bearing and support C is a Journal bearing. Also draw shear force bending moment diagram.



Given Span = L = 4m. Load = P = 5 kN.

Required R_A =? R_C =?

Solution Take moment about "A" also considers upward force and clockwise moment is positive.

$\sum M_A = 0$

$R_c (L) – P (x) + R_A (0) = 0.$

$R_c (4) – 5 (2) = 0$

$\mathbf{R_c = 2.5\ k\ N}$

To calculate the reaction at point A

$\sum F = 0$

$R_A - P + R_c = 0$

$R_A – 5 + 2.5 = 0$ $\mathbf{R_A = 2.5\ k\ N}$

EXAMPLE 2

Find the reaction of a simply supported beam 6m long is carrying a uniformly distributed load of 5kN/m over a length of 3m from the right hand.

Given_

$P = 5\ k\ N/m$ $L = 6\ m$ $Y = 3m, Z = 3m.$

Required Reaction at A & B = R_A & R_B =?

Solution first of all we will change the uniformly distributed load into the point load

= 5 x 3= 15 kN

Take moment about A also consider that the upward force or load and clockwise moment is positive.

$\sum M_A = 0$

$R_c (L) – P (y + z/2) + R_A (0) = 0$

$R_B (6) – (15) (3 + 1.5) + R_A (0) = 0$

$\mathbf{R_B = 11.25\ kN}$

To calculate the reaction at point A

$\sum F = 0$

$R_A - P + R_B = 0$

$R_A -15 - 11.25$

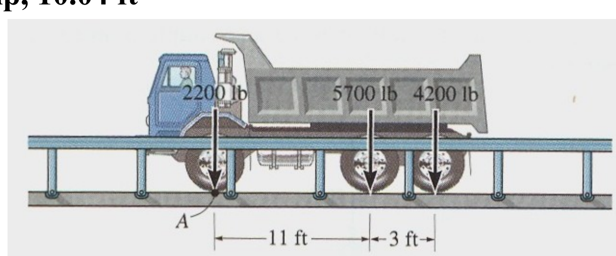
$\mathbf{R_A = 3.75.kN}$

Exercise 2

6. Find the moment of couple shown what must the force of a couple balancing this couple having arm of length of 6ft.
Ans: 36 lb ft, 6 lb

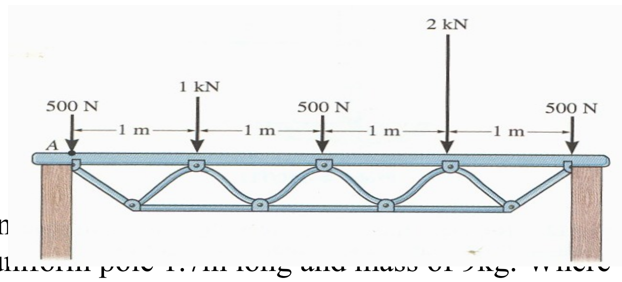
7. The tires of a truck exert the forces shown on the deck of the bridge replace this system of forces by an equivalent resultant force and specify its measured form point A.

Ans: 12.1 kip, 10.04 ft



8. The system of parallel forces acts on the top of the Warne truss. Determine the equivalent resultant force of the system and location measured from point A

Ans: 4.5

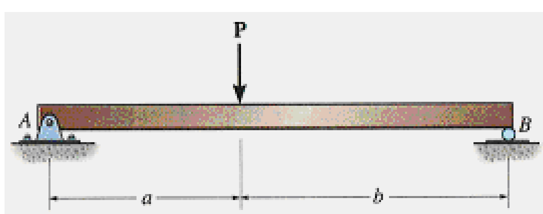


9. A man an
means of a uniform pole 1.7m long and mass of 9kg. Where the weight must placed so that the man may carry twice as mush of weight as that boy.
Ans: 111.18 N, .04646 m

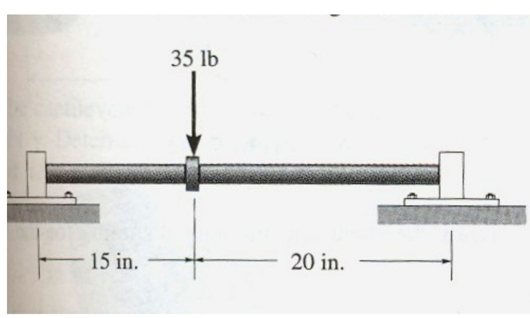
10. Two unlike parallel forces of magnitude 400 N and 100 N acting in such a way that their lines of action are 150 mm apart. Determine the magnitude of the resultant force and the point at which it acts.

Ans: 300 N & 50 mm

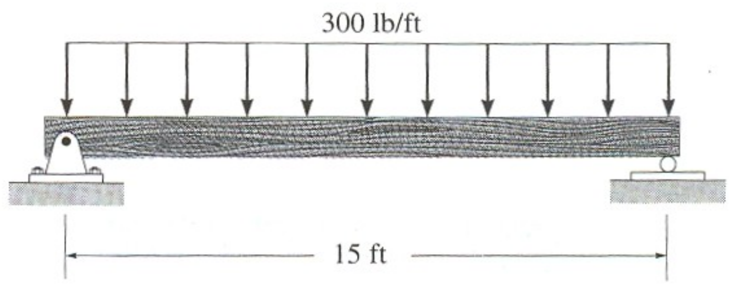
11. Find reaction at point A and B for the beam shown set P= 600lb a = 5ft b = 7ft.



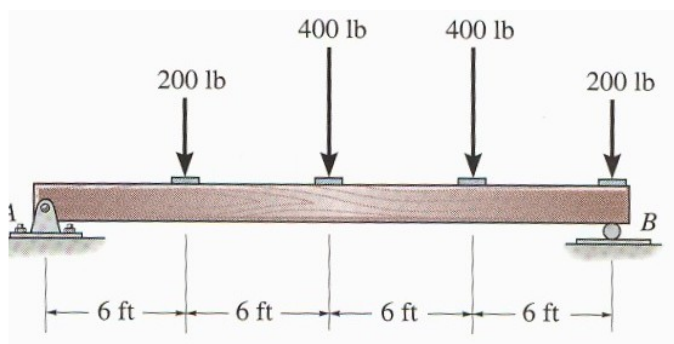
12. Find the reaction at the points for the beam as shown



13 Find the reaction at the points as shown in diagram



14 Find the reaction at the points as shown in diagram



CHAPTER 3 EQUILIBRIUM OF PARTICLE AND BODY

Equilibrium of a Particle

When the resultant of all forces acting on a particle is zero, the particle is said to be in equilibrium.

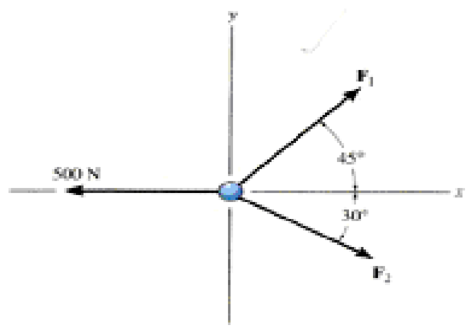
A particle which is acted aupon two forces

Newton's First Law:

If the resultant force on a particle is zero, the particle will remain at rest or will continue at constant speed in a straight line.

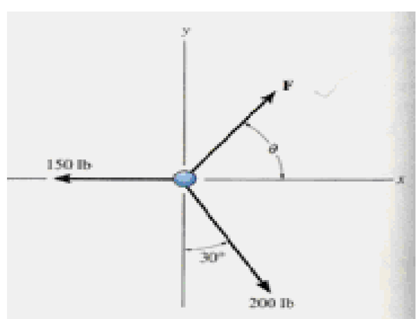
Exercise

1. Determine the magnitude of F_1 and F_2 so that the partial is in equilibrium



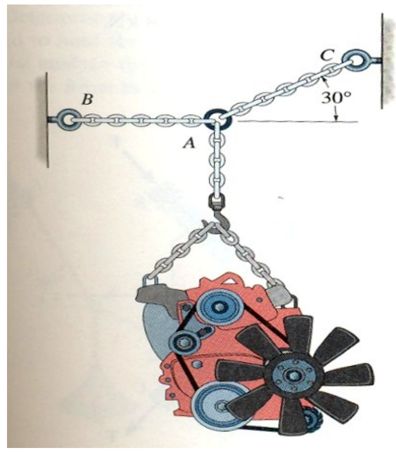
12. Determine the magnitude and direction of F_1 and F_2 so that the partial is in equilibrium

Ans: **42.567 lb & 54.723 lb**



13. Determine the maximum weight of the engine that can be supported without exceeding a tension of 450 lb in chain AC and 480 lb in chain AC.

Ans: 240 lbs


B
C
30°
A

EQUILIBRIUM

A particle is in equilibrium if it is at rest if originally at rest or has a constant velocity if originally in motion. The term equilibrium or static equilibrium is used to describe an object at rest. To maintain equilibrium it is necessary to satisfy Newton's first law of motion, which requires the resultant force acting on particle to be equal to zero. That is

$$\sum F = 0 \longrightarrow A$$

Where $\sum F$ = Sum of all the forces acting on the particle which is necessary condition for equilibrium. This follows from Newton's second law of motion, which can be written as

$$\sum F = ma.$$

Put in equation A $\quad ma = 0$

Therefore the particle acceleration a = 0. Consequently the particle indeed moves with constant velocity or at rest.

METHODS FOR THE EQUILIBRIUM OF FORCES

There are many methods of finding the equilibrium but the following are important

1. Analytical Method 2. Graphical Method

1. Analytical method for the equilibrium of forces

The equilibrium of forces may be studied analytically by Lami's theorem as discussed under

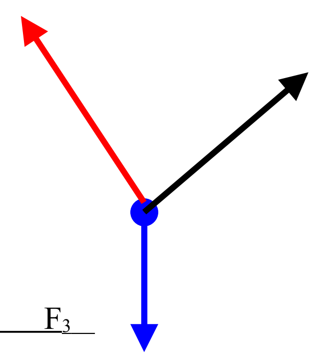
LAMI'S THEOREM

It states, "If there are three forces acting at a point be in equilibrium then each force is proportional to the sine of the angle between the other two forces".

Let three force F_1, F_2 and F_3 acting at a point and the opposite angles to three forces are γ , β, and α as shown in figure

F_2

F_1



Mathematically

α

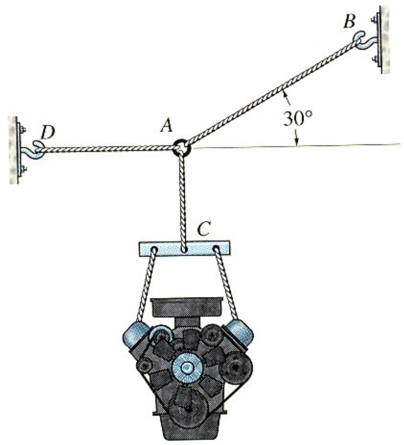
β **γ**

$$\frac{F_1}{\sin\beta} = \frac{F_2}{\sin\gamma} = \frac{F_3}{\sin\alpha}$$

F_3

EXAMPLE 7

Determine the tension in cables AB and AD for equilibrium of the 25



T_{AB}

30º

T_{AD}

W

Given Mass of Engine = 250 Kg. Angle = θ = 30º

Required: Tension in the cable = T_{AB} =? T_{AD} =?

Working Formulas $\frac{T_{AD}}{\text{Sin } \alpha} = \frac{T_{AB}}{\text{Sin } \beta} = \frac{W}{\text{Sin } \gamma}$

Solution

We know that W = mg.

W = 250 x 9.81 = 2452.5 N = 2452.5/1000

W = 2.453 KN

From the geometry of diagram we have

α = 90 + 30 = 120º

β = 90º

γ = 180 – 30 = 150º

Put the values in the working formula

$\frac{T_{AD}}{\text{Sin } 120} = \frac{T_{AB}}{\text{Sin } 90} = \frac{W}{\text{Sin } 150}$

Therefore T_{AD} = 4.249 KN

Similarly T_{AB} = 4.906 KN

Result: **T_{AD} = 4.249 KN**

T_{AB} = 4.91 KN

Alternate method The same question may be solved by resolving method

Working Formulas $\sum F = 0$

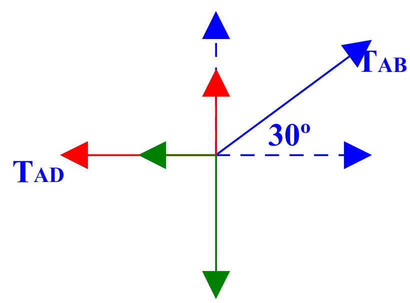
Solution

We know that W = mg.

W = 250 x 9.81 = 2452.5 N = 2452.5/1000

W = 2.453 KN

Now resolve force T_{AB}, TAD and W as shown in following diagram



#	Force	Magnitude N	Angle θ°	Horizontal Components Fx = F Cosine θ	Vertical Components Fy = F Sine θ
1	T_{AB}	-	30	T_{AB} Cosine 30 = .866 T_{AB}	T_{AB} Sine 30 = 0.5 T_{AB}
2	T_{AD}	-	0	T_{AD} Cosine 0 = T_{AD}	T_{AD} Sine 0 = 0
3	W	2452.5	90	2452.5 Cosine 90 0	2452.5 Sine 90 = 2452.5

We know that

+ → $\sum Fx = 0$

$.866\, T_{AB} - T_{AD} - 0 = 0$ ———— A

+ ↑ $\sum Fy = 0$

$0.5\, T_{AB} + 0 - 2.4525 = 0$ ———— B

From Equation A

$T_{AB} = \frac{T_{AD}}{0.866}$ ———— C

Put in equation B

$.5 \left(\frac{T_{AD}}{0.866}\right) + 0 - 2.4525 = 0$

$\mathbf{T_{AD} = 4.248\ KN}$

Put in equation C

$T_{AB} = \frac{4.248}{0.866}$

$\mathbf{T_{AB} = 4.91\ KN}$

Result: T_{AD} = 4.248 KN T_{AB} = 4.91 KN

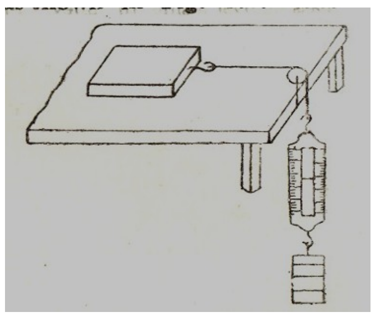
CHAPTER 3 FRICTION

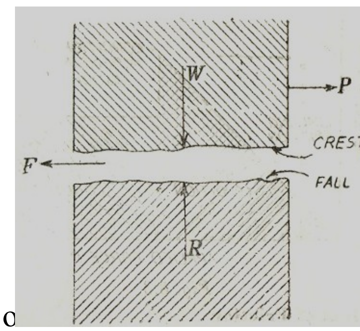
A force which prevents the motion or movement of the body is called friction or force of friction and its direction is opposite to the applied external force or motion of the body. Friction is a force of resistance acting on a body which prevents or retards motion of the body. Or

When a body slides upon another body, the property due to which the motion of one relative to the other is retarded is called friction. This force always acts tangent to the surface at points of contact with other body and is directed opposite to the motion of the body.

Explanation

Consider a block resting on, a horizontal plane surface. Attach a string to one side of the block as shown in Fig.





The other end o string is connected to the spring balance. Apply an external force on the balance. Gradually increase the magnitude of the external force. Initially the body will not move and the effect of the applied force is nullified. This is because there acts a force on the block which opposes the motion or movement of the block. The nature of this opposing force is called friction. It depends upon many factors. The major cause of friction is the microscopic roughness of the contact surfaces. No surface is perfectly smooth. Every surface is composed of crests and falls as shown in fig b. It is the interlocking of the crests of one surface into the falls of the other surface which produces the resistance against the movement of one body over the other body. When the force exerted is sufficient to overcome the friction, the movement ensures and the crests are being sheared off. This gives rise to heat and raises the local temperature. This is also the reason of the wear of the contact surfaces. This phenomenon of friction necessitates the presence o fluid film between the two surfaces to avoid wear of surfaces. The process of creating the fluid film is called lubrication.

TYPES OF FRICTION

Friction is of the following two types.

1. Static Friction

It is the friction acting on the body when the body is at the state of rest or the friction called into play before the body tends to move on the surface is called static friction. The magnitude of the static friction is equal to the applied force. It varies from zero to maximum until the movement ensures.

2. Dynamic Friction

It is the friction acting on the body when body is in motion is called dynamic friction. Dynamic friction is also known as kinetic friction. The magnitude of the dynamic friction is constant.

The dynamic friction has two types

i. Sliding Friction ii. Rolling Friction

i. Sliding friction

The sliding friction acts on those bodies, which slide over each other for example the friction between piston, and cylinder will slide friction because the motion of the motion of the piston in cylinder is sliding and there is surface contact between piston and cylinder.

ii. Rolling Friction

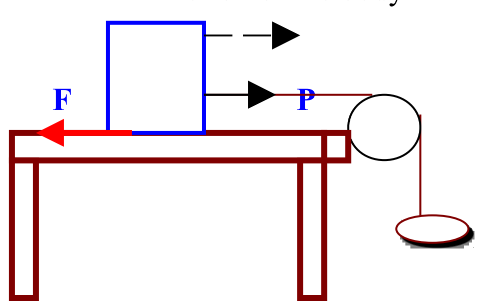
The rolling friction acts on those bodies which have point contact with each other for example the motion of the wheel on the railway track is the example of rolling motion and the friction between the wheel and railway track is rolling friction. It is experimentally found that the magnitude of the sliding friction is more than the rolling friction because in the rolling friction there is a point contact rather than surface contact.

LIMITING FRICTION

The maximum friction (before the movement of body) which can be produced by the surfaces in contact is known as limiting friction

It is experimentally found that friction directly varies as the applied force until the movement produces in the body. Let us try to slide a body of weight w over another body by a force P as shown in fig

Motion of the body



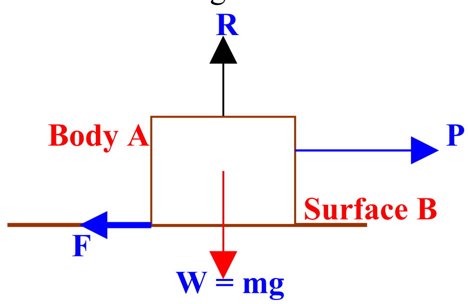
Pan

A little consideration will show that the body will not move because the friction F which prevents the motion. It shows that the applied force P is exactly balanced by the force of friction acting in the opposite direction of applied force P. if we increase the force P by increasing the weight in the pan, the friction F will adjust itself according to applied force P and the body will not move. Thus the force of friction has a property of adjusting its magnitude to become exactly equal and opposite to the applied force which tends to produce the motion.

There is however a limit beyond which the friction cannot increase. If the applied force increases this limit the force of friction cannot balance applied force and body begins to move in the direction of applied force. This maximum value of friction, which acts on body just begin to move, is known as limiting friction. It may be noted that when the applied force is less than the limiting friction the body remains at rest, and the friction is called static friction, which may have any values zero to limiting friction.

NORMAL REACTION

Let us consider a body A of weight "W" rest over another surface B and a force P acting on the body to slide the body on the surface B as shown in fig



A little concentration will show that the body A presses the surface B downward equal to weight of the body and in reaction surface B lift the body in upward direction of the same magnitude but in opposite direction therefore the body in equilibrium this upward reaction is termed as normal reaction and it is denoted by R or N.

Note

It is noted the weight W is not always perpendicular to the surface of contact and hence normal reaction R is not equal to the weight W of body in such a case the normal reaction is equal to the component of weight perpendicular to surface.

CO EFFICIENT OF FRICTION

The ratio of limiting friction and normal reaction is called coefficient of friction and is denoted by μ.

Let R = normal reaction

And F = force of friction (limiting friction)

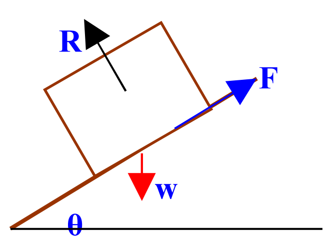
μ = Co efficient of friction

$$\frac{F}{R} = \mu$$

$$\mathbf{F = \mu R}$$

ANGLE OF FRICTION

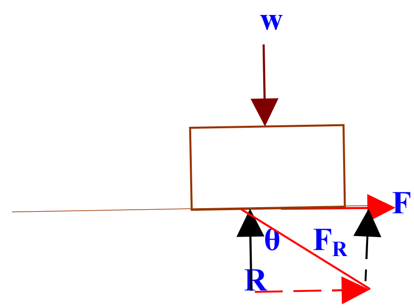
The angle of a plane at which body just begins to slide down the plane is called angle of frication. Consider a body resting on an inclined plane as shown in diagram.



The body is in equilibrium under the Acton of the following forces

1. Weight of the body acting vertically downwards = w
2. Friction force acting along upwards = F
3. Normal reaction acting at right angle to the plane =R

Let the angle of inclination be gradually increased till the body just starts sliding down the plane. This angle of inclined plane at which a body just begins to slide down the plane is called the angle of friction. And it is equal to the angle between normal reaction R and the resultant between frictional force F and normal reaction R



From diagram

$$\text{Tan } \theta = F / R$$

But $F / R = \mu$

Where μ is the co-efficient of friction,

$$\text{Tan } \alpha = \mu$$

LAWS OF FRICTION

These laws are listed below:

1. Laws of Static Friction

1 The force of friction always acts in a direction opposite to that in which the body tends to move.

2 The magnitude of force of static friction is just sufficient to prevent a body from moving and it is equal to the applied force.

3. The force of static friction does not depend upon, shape, area, volume, size etc. as long as normal reaction remains the same.

4. The limiting force of friction bears a constant ratio to normal reaction and this constant ratio is called coefficient of static friction.

2. Laws of Dynamic Friction

1 When a body is moving with certain velocity, it is opposed by a force called force of dynamic friction.

2 The force of dynamic friction comes into play during the motion of the body and as soon as the body stops, the force of friction disappears.

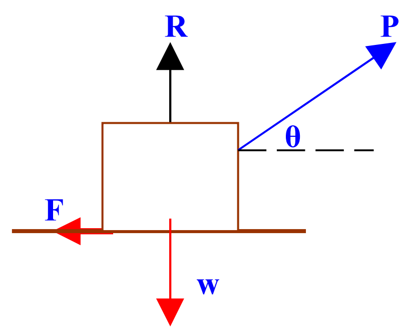
3 The force of dynamic friction is independent of area, volume, shape, size etc. of the body so long the normal reaction remains the same.

However, to some extent it varies with the magnitude of velocity of the body. Force of dynamic friction is high for low speeds and low for very high speeds.

4 The ratio of force of dynamic friction and normal reaction on the body is called coefficient of dynamic friction.

EQUILIBRIUM OF A BODY ON A ROUGH HORIZONTAL PLANE

We know that a body lying on a rough horizontal plane will remain in equilibrium but when ever a force is applied on the body it will tend to move in the direction of force. Consider a body moving on a horizontal Plane under the influence of force P which is inclined at an angle θ to the surface. As shown in fig



Where

w = weight of the body
P = applied force
α = Angle of Repose
F = friction
θ = angle of inclination of the plane the horizontal

Resolve the applied force P into its component that is

Horizontal component = P Cos θ Vertical component = P Sin θ

Now consider the horizontal & vertical equilibrium condition of the body then

$F = P \cos\theta$ ______________________ 1

And $w = R + P \sin\theta$ ___________________ 2

The value of P can be determined by following formula

$$P = \frac{w \sin\alpha}{\cos(\theta - \alpha)}$$

For minimum force P

P = W Sin α

MOTION OF BODY ON INCLINED PLANE IN UPWARD DIRECTION

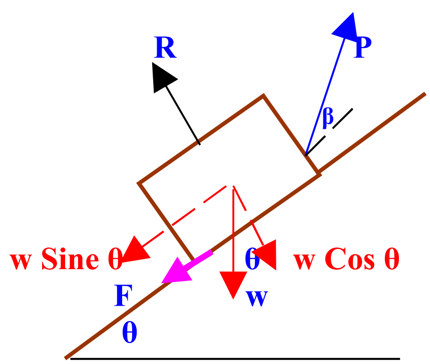
Let

W = weight of the body P = applied force

α = Angle of Repose θ = angle of inclination of the plane the horizontal

Now consider the following two cases

Case 1) When angle of inclination of the force to plane is β



Consider the forces acting on body which are parallel to the plane also consider the equilibrium of body

P cosine β = w sin θ + F

P cosine β = w sin θ + μR ____________________ 1

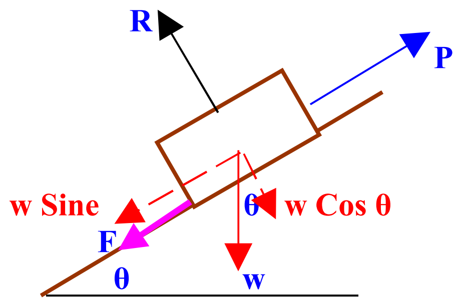
Similarly the forces acting on body normal to the plane and consider the equilibrium condition

R + P sin β = w cosine θ ____________________ 2

The magnitude of the force P can be calculated by the following formula

$$P = \frac{W \sin(\theta + \alpha)}{\text{Cosine}(\beta - \alpha)}$$

Case 2) When the force is parallel to the plane



By considering the equilibrium of the forces parallel and normal to the plane we have

$P = w \text{ Sine } \theta + F$

$P = w \text{ Sine } \theta + \mu R$ ________________ 1

And $R = w \text{ Cosine } \theta$ ________________ 2

The force P can be calculated by the following formula

$$P = \frac{W \sin(\theta + \alpha)}{\cos \alpha}$$

Motion of body on Inclined plane in downward direction

Let

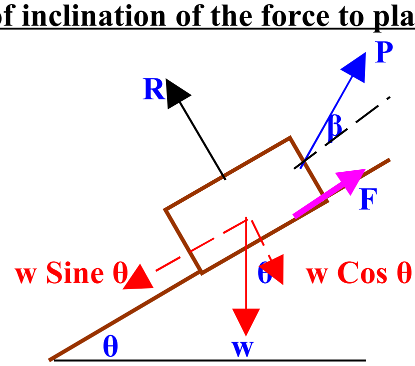
W = weight of the body P = applied force

θ = angle of inclination of the plane the horizontal

α = Angle of Repose β = angle of force P

Now consider the following two cases

Case 1 When angle of inclination of the force to plane is β



Now consider the forces acting parallel to the plane also the equilibrium of forces

$P \text{ cosine } \beta + F = w \sin \theta$

$P \text{ cosine } \beta + \mu R = w \sin \theta$ ________________ 1

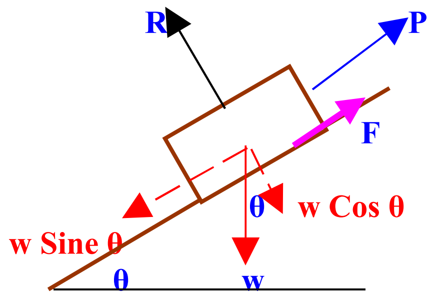
Similarly consider the force normal to the plane

$R + P \sin \beta = w \cos \theta$ ________________ 2

The magnitude of the force P can be calculated by the following formula

$$P = \frac{W \sin(\theta - \alpha)}{\cos(\beta - \alpha)}$$

Case 2 when the force is parallel to the plane



From diagram we have

P + F = w Sine θ

P + μR = w Sine θ ________________ 1

Similarly R = w Cos θ ________________ 2

The force P can be calculated by following formula

$$P = \frac{W \sin(\theta - \alpha)}{\cos \alpha}$$

EQUILIBRIUM OF LADDER

A ladder is a device which is used to climb up or down to the roof or walls. It consists of two long uprights and number of rungs which makes the steps of the ladder.

Consider a ladder which is resting on ground and leaning against walls as shown in the fig. Let

L = Length of ladder

w_1 = Weight of ladder acts at middle of the ladder

w_2= Weight of man climbing up acts at the distance x from the lower end

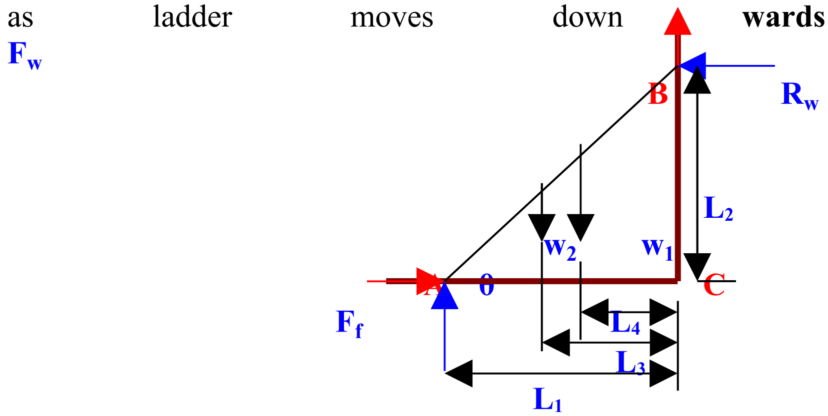
μ_f = co efficient of friction between floor and ladder

μ_w = co efficient of friction between ladder and wall

Let us suppose ladder slips down wards

F_f = friction produce between floor and ladder towards wall as ladder moves away from the wall.

F_w = friction produce between wall and ladder upwards as ladder moves down **wards**



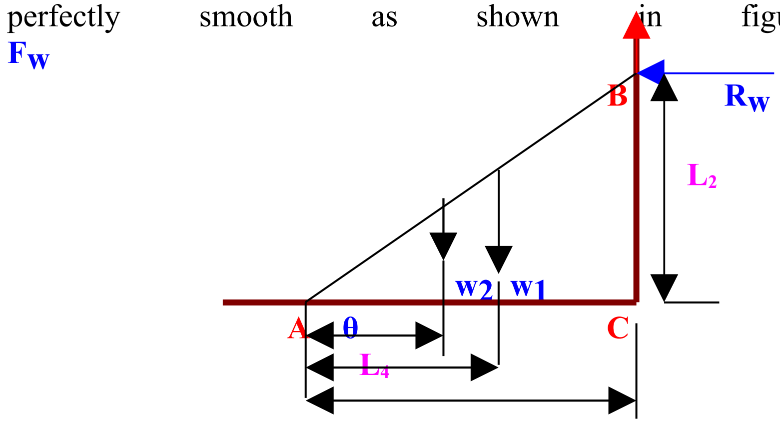
For the sake of convince we consider that the friction at B is zero i.e. the wall is perfectly smooth. Now take the moment about B.

$$R_f \times L_1 = F_f \times L_2 + w_2 \times L_3 + w_1 \times L_4$$

Where $F_f = \mu_f \times R_f$

$$\mathbf{R_f \times L_1 = (\mu_f \times R_f \times L_2) + w_2 \times L_3 + w_1 \times L_4} \qquad \text{A}$$

Similarly consider the friction at A is zero i.e. the floor is perfectly smooth as shown in figure.



L_3

L_1

Therefore $R_w \times L_2 = F_w \times L_1 + w_1 \times L_3 + w_2 \times L_4$

Where $F_w = \mu_w \times R_w$

$\mathbf{R_w \times L_2 = (\mu_w \times R_w \times L_1) + w_1 \times L_3 + w_2 \times L_4}$

_______________ A

EXAMPLE 1

A horse exerts a pull of 3 KN just to move a carriage having a mass of 800 kg. Determine the co efficient of friction between the wheel and the ground

Take g = 10 m/sec²

Given P = 3 KN Mass = m = 800 Kg g = 10 m/sec²

Required co efficient of friction = μ =?

Working formula **F = μ R**

Solution we know that W = mg

W = 800 x 10 = 8000 N

A little consideration will show that the weight of the carriage is equal to the normal reaction because that the body is horizontal to the plane as shown in fig

Therefore W = R and P = F

R

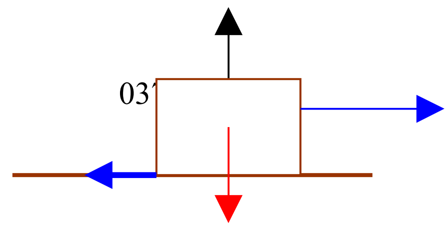
Put the values in working formula we get

300 = μ 8000

μ = 03

P

Result co efficient of friction = 0.375



F

w = mg

EXAMPLE 2

A pull of 490 N inclined at 30º to the horizontal is necessary to move a block of wood on a horizontal table. If the coefficient of friction between to bodies in contact is 0.2 what is the mass of the block

Given P = 490 N θ = 30º μ = 0.2

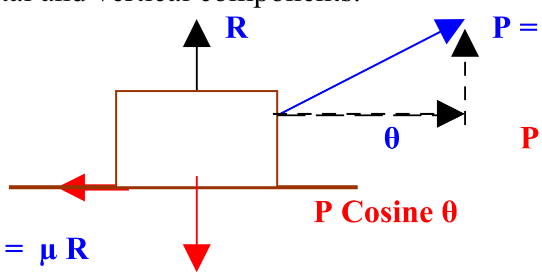
Required mass of block =?

Solution

Now consider the following diagram and also resolve the force P into horizontal and vertical components.

490 N

Sine θ



w = mg

Now apply the condition of equilibrium the forces acting in x axis is positive

+→ ∑ Fx = 0

P Cosine θ – F = 0

$P \text{ Cosine } \theta - \mu R = 0$

$490 \text{ Cosine } 30 - 0.2 \times R = 0$ Therefore

R = 2121.762

Now consider the forces acting in y axis is positive

+▲ $\sum Fy = 0$

$R + P \text{ Sine } \theta - W = 0$

$R + P \text{ Sine } \theta - mg = 0$

$2121.762 + 490 \text{ Sine } 30 - m \times 9.81 = 0$

m = 241.260 Kg

Result **mass of the wooden block = 241.260 Kg**

EXAMPLE 3

A body of mass 100 Kg rests on horizontal plane the co efficient of friction between body and the plane 0.40. Find the work done in moving the body through a distance of 20 m along the plane.

Given m = 100 Kg μ = 0.40 d = 20 m
Required work done =?
Working formula 1 W = F x d
2 Fs = μ R
Solution we know that R = W = mg
R = W = 10 x 9.81 = 98.1 N
Put the values in 2nd working formula we get
Fs = 0.40 x 98.1
Fs = 39.24 N
Now put the values in 1st working formula
W = 39.24 x 20
W = 748.8 N
Resultant **weight = 748.8 N**

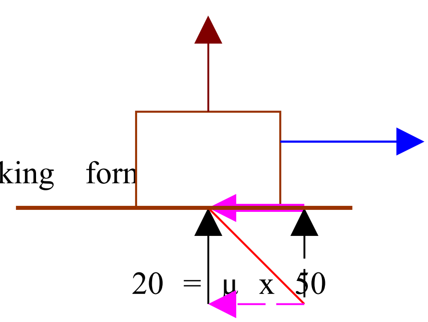
EXAMPLE 4

A weight of 50 N is resting on the horizontal table and can be moved by a horizontal force of 20 N. Find the co efficient of friction, the direction and magnitude of the resultant between normal reaction and frictional force

Given W = 50 N P = 20 N
Required co efficient of friction = μ =?
Direction = θ =?
Resultant = S =?
Working formula 1 F = μ R
w = 50 N

$$S = \sqrt{R^2 + Fs^2} \quad (2)$$

3 Tan θ = μ



Solution put the value in 1st working form
P = 20 N
Fs = μ R
20 = μ x 50
F
μ = 0.4
R
put the value in the 2nd working formula
S

$$S = \sqrt{50^2 + 20^2}$$

S = 53.85 N

Put the value in the 3rd working formula
Tan θ = μ

$\tan\theta = 0.4$

$\theta = 21.801^{o}$

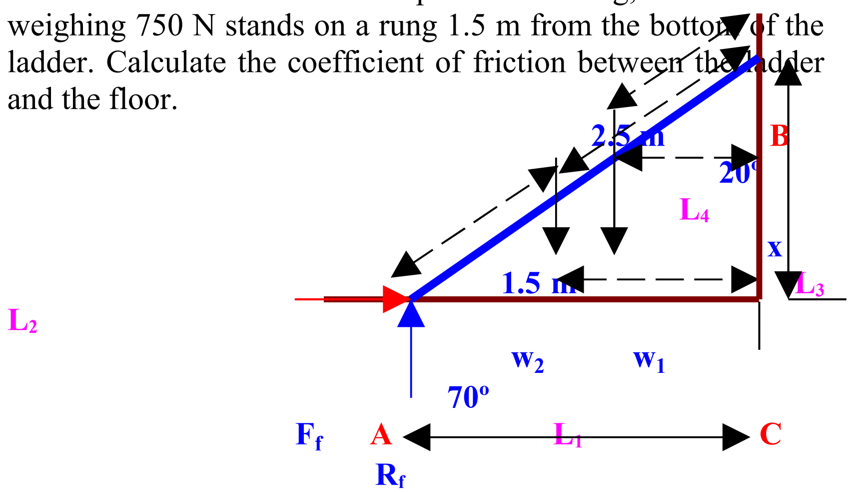
Result Co efficient of friction = $\mu = 0.4$

Direction = $\theta = 21.801^{o}$

Resultant = S = 53.85 N

EXAMPLE 5

A ladder 5 m long rests on a horizontal ground and leans against a smooth vertical wall at an angle 70º with the horizontal. The weight of the ladder is 900 N and acts at its middle. The ladder is at the point of sliding, when a man weighing 750 N stands on a rung 1.5 m from the bottom of the ladder. Calculate the coefficient of friction between the ladder and the floor.



Given Length of leader = L = 5 m weight of leader = w_1 = 900 N

Weight of man = w_2 = 750 N inclination of leader = θ = 70º

Distance covered by man from bottom = 1.5 m

Required coefficient of frication between ladder and floor = μ_f =?

Working formula $R_f \times L_1 = (\mu_f \times R_f \times L_2) + w_2 \times L_3 + w_1 \times L_4$

Solution we know that

$R_f = w_1 + w_2$

$R_f = 900 + 750$

$R_f = 1650$ N

We can calculate L_1, L_2, by considering the geometry of the figure. Now consider the triangle ABC

Cos 70 = $L_1/L = L_1/5$ L_1 = 1.7101 m

And Sin 70 = $L_2/L = L_2/5$ L_2 = 4.698 m

Similarly we can calculate the L_3 & L_4 by considering the geometry of the figure

Sin 20 = $L_4/2.5$ L_4 = 0.85 m

And Sin 20 = $L_3/5-1.5$ L_3 = 1.197 m

Put the values in the working formula to calculate the coefficient of friction between the floor and ladder

$R_f \times L_1 = (\mu_f \times R_f \times L_2) + w_2 \times L_3 + w_1 \times L_4$

$1650 \times 1.7101 = \mu_f \times 1650 \times 4.698 + 750 \times 1.197 + 900 \times 0.85$

$\mu_f = 0.149$

Resultant Coefficient of friction = μ_f = 0.15

EXERCISE 3

1. A block having a mass of 220 Kg is resting on a wooden table what is the minimum force necessary to impart motion to the block when the coefficient of friction between the block and the table is 0.25.

Ans: 539 N

2. A 12 N force is just able to slide a block of weight of 100 N on a horizontal plane board. What is the co efficient of friction? What is the least value of the inclination of the plane so as to allow the block to slide downward by self?

Ans: 0.21 and 6.0º

3. A block of wood weighing 3 lb. rests on a horizontal table. A horizontal force 1.25 lb. is just sufficient to cause it to slide. Find the coefficient of friction for the two surfaces and the angle of friction

Ans: 0.42 & 22.6º

4. A block of wood weighing 7.5 kg rests on a horizontal table and can just be moved along by a force equal to 2 kg weight. Another 3 kg is placed on the block what is the least horizontal force which will just move the block

Ans: 2.8 kg & 0.2667

5. A body of weight 6 lb rests on a horizontal table and the coefficient of friction between the two surfaces is 0.32. What horizontal force will be required to start the body moving?

Ans: 1.92 lbs

6. A block of wood of weight 2.5 kg rests on a rough horizontal board and the coefficient of friction between the surfaces is 0.4 by means of string inclined at 30º to the board. A pull is exerted on the block witch is just sufficient to make it move. Calculate the amount of the pull.

Ans: 0.938 kg

7. A body rests on a rough horizontal board. This is gradually tilted until, when it is inclined at 22º to the horizontal, the body begins to move down the plane. What is the coefficient of friction between the body and the plane? If the body weighs 2.5 what is the magnitude of the force of friction when the body begins to slip.

Ans: 0.936 N

8. A block of wood rests on an inclined plane and the coefficient of friction between it and the plane is 0.31. At what angle must the plane be inclined to the horizontal so that the block begins to move down the plane?

Ans: 17.22º

9. A block rests on a horizontal board. The board is gradually tilted upward and the block begins to slide down the board when the angle of inclination is θ_1 is 21º. After the block starts moving, it is found that it keep sliding at constant speed when the angle of tilt is 15º. Find the coefficient of static friction and the coefficient of dynamic friction between the block and the board.

Ans: 0384 & 0.268

10. A body of weight is 20 lb is placed on a rough inclined plane whose slope 37º. if the coefficient of friction between the plane and the body is 0.2 find the least force acting parallel to the plane required

To prevent the body by sliding down

Ans: 8.842 lbs

To pull the body up the plane

Ans: 15.23 lbs

11. A uniform ladder of length 3.25 m and weighing 250 N is placed against a smooth vertical wall with its lower end 1.25m from the wall. The coefficient of friction between the ladder and floor is 0.3. What is the frictional force acting on the ladder at the point of contact between the ladder and the floor?

Ans: 52.083 N

CHAPTER 4 CENTRE OF GRAVITY

The center of gravity is a point where whole the weight of the body act is called center of gravity. As we know that every particle of a body is attracted by the earth towards its center with a magnitude of the weight of the body. As the distance between the different particles of a body and the center of the earth is the same, therefore these forces may be taken to act along parallel lines. A point may be found out in a body, through which the resultant of all such parallel forces acts. This point, through which the whole resultant (weight of the body acts, irrespective of its position, is known as center of gravity (briefly written as C.G). It may be noted that every body has one and only one center of gravity.

CENTROID

The plane figures (like triangle, quadrilateral, circle etc.) have only areas, but no mass. The center of area of such figures is known as Centroid. The method of finding out the Centroid of a figure is the same as that of finding out the center of gravity of a body.

AXIS OF REFERENCE

The center of gravity of a body is always calculated with referrer to some assumed axis known as axis of reference. The axis of reference, of plane figures, is generally taken as the lowest line of the figure for calculating y and the left line of the figure for calculating x.

METHODS FOR CENTRE OF GRAVITY OF SIMPLE FIGURES

The center of gravity (or Centroid) may be found out by any one of the following methods

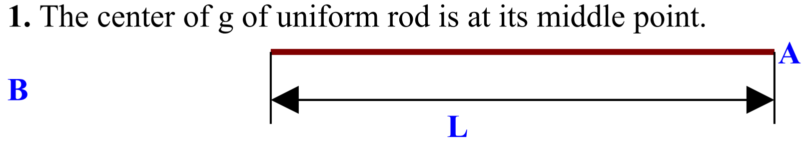
I. By geometrical considerations
2. By moments method
3. By graphical method

1 Center of Gravity by Geometrical Considerations

The center of gravity of simple figures may be found out from the geometry of the figure

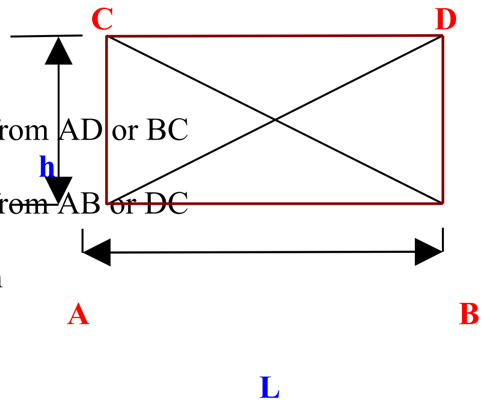
A) The center of gravity of plane figure

1. The center of g of uniform rod is at its middle point.



Center of gravity = L / 2 from point A or B

2. The center of gravity of a rectangle is at a point, where its diagonals meet each other. It is also a mid point of the length as well as the breadth of the rectangle as shown in fig

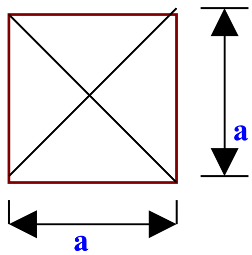


G = L/2 from AD or BC

G = h/2 from AB or DC

Area = L x h

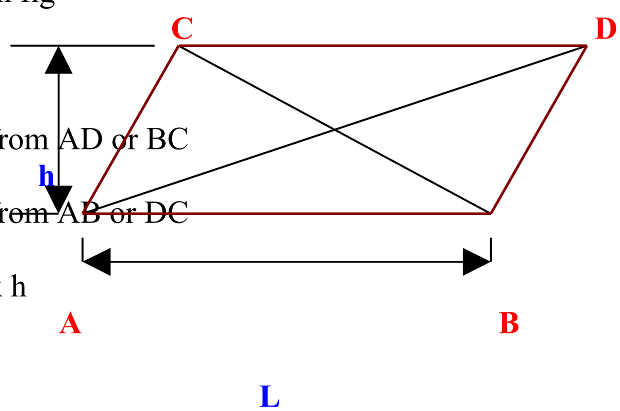
3. The center of gravity of a square is a point, where its diagonals meet each other. It is a mid point of its side as shown in fig



G = a/2 from any side

Area = 2x a

4. The center of gravity of a parallelogram is at a point, where its diagonals meet each other. It is also a mid point of the length as well as the height of the parallelogram as shown in fig

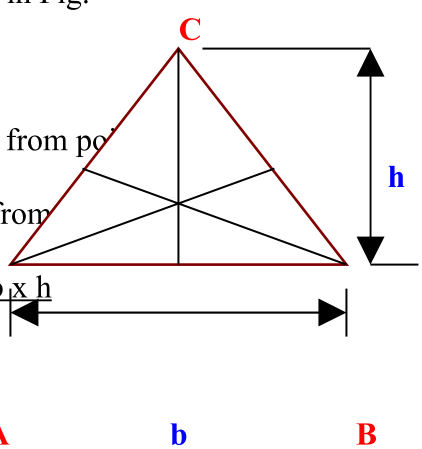


G = L/2 from AD or BC

G = h/2 from AB or DC

Area = L x h

5. The center of gravity of a triangle is at the point, where the three medians (a median is a line connecting the vertex and middle point of the opposite side) of the triangle meet as shown in Fig.

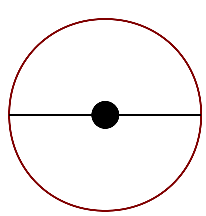


G = 2h/3 from po

G = h/3 from

Area = $\frac{b \times h}{2}$

6. The center of gravity of the circle is the center of the circle



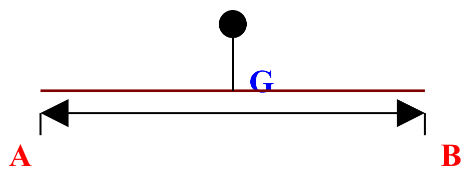
G = r or d/2 from any point from the circumference

Area = $\pi \times r^2$

7. The center of gravity of the semi circle is at a distance 4 r/3 π from diameter AB

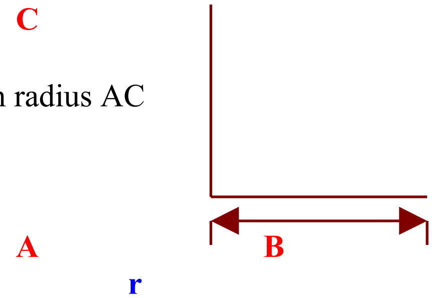
G = 4 r/3 π from diameter AB

Area = $\frac{\pi \times r^2}{2}$



8. The center of gravity of quarter circular at a distance 4 r/3 π from diameter AC

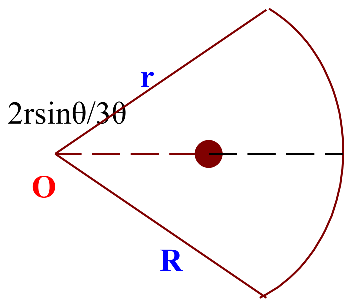
from radius AC



G = 4 r/3 π

Area = $\frac{\pi \times r^2}{4}$

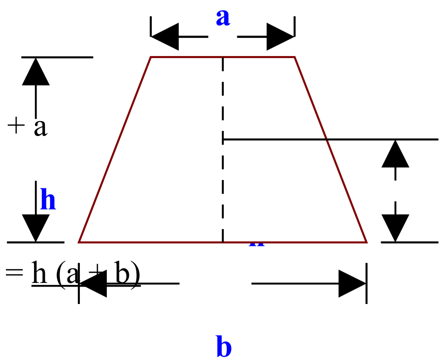
9. The center of gravity of sector is at a distance 2rsinθ/3θ from center c.



G =

Area = θ x r^2

10. The center of gravity of a trapezium is at a distance of h/3x [b+2a/b+a] form the side AB as shown in Fig.



3 b + a

Area = h (a + b)

G = h (b + 2a)

b

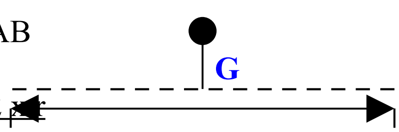
G

2

11. The center of gravity semi circular arc is at distance 2 r/ π from AB

G = 2 r/ π from AB

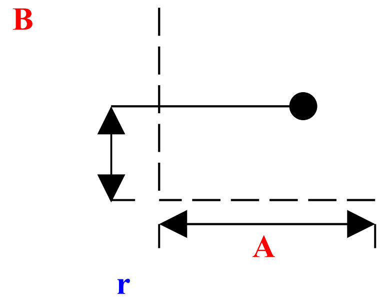
Length of Arc = 2 πr



π

A **B**

8. The center of gravity of quarter arc is at a distance 2 r/ π

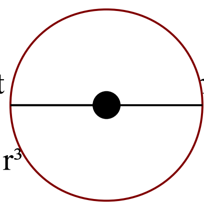


G = 2 r/ π
Length of arc AB =
rπ /2

G

B) THE CENTRE OF GRAVITY OF SOLID BODY

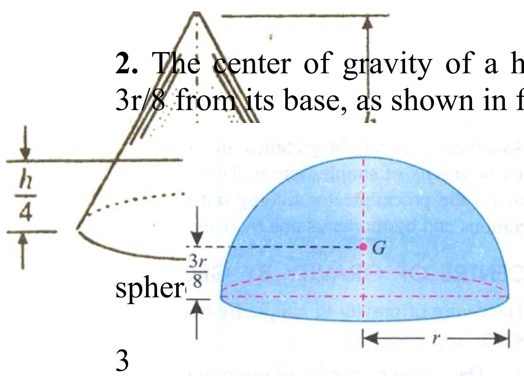
1. The center of gravity of a sphere is at a distance r from any point



G = r or d/2 from any

point mference

Volume of sphere = $\frac{4}{3}$ x π x r^3



2. The center of gravity of a hemisphere is at a distance of 3r/8 from its base, as shown in fig.

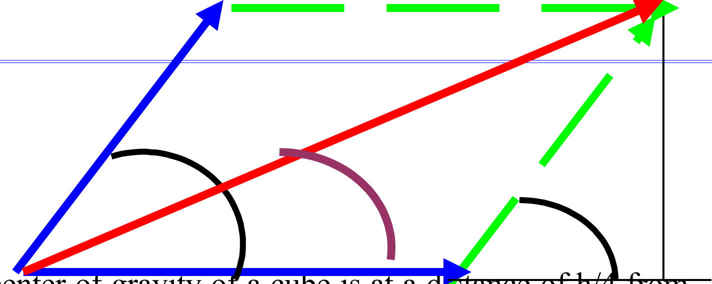
G = $\frac{3}{8}$ x r

Volume of spher

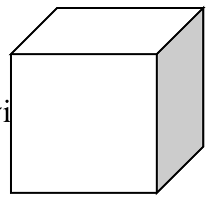
3

3. The gravity of right circular solid cone is at a distance h/4 from its base, measured along the vertical axis

G = h/4
Volume of cone = $\frac{1}{3}$ x π x r^2 x h



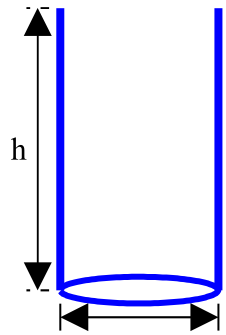
4. The center of gravity of a cube is at a distance of h/4 from every face (where h is the length of each side).



G = h/4
Volume of cube =

length x wi

5. The center of gravity of a cylinder is h/2 from diameter AB

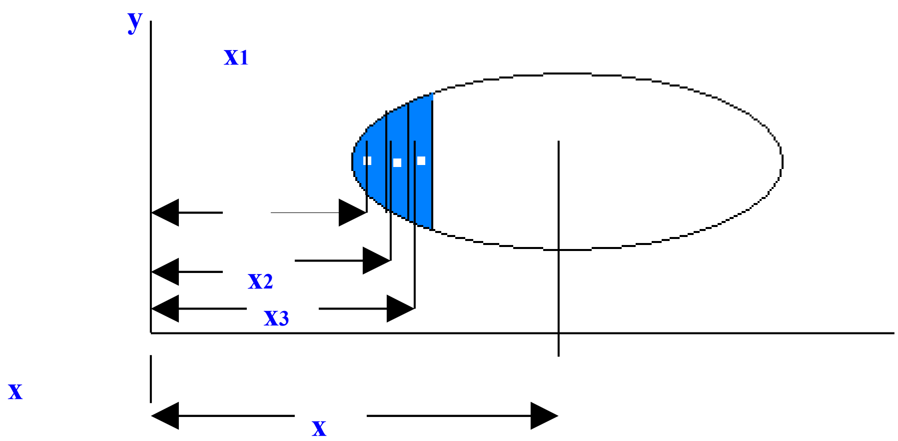


G = h/2
Volume of cylinder = π

x r^2 x h

CENTRE OF GRAVITY BY MOMENTS

The center of gravity of a body may also be found out by moments as discussed below. Consider a body of mass M whose center of gravity is required to be found out. Now divide the body into small strips of masses whose centers of gravity are known as shown in fig



Let

m_1, m_2, m_3 …… = mass of strips 1, 2, 3,

$x_1, x_2,$ and x_3… = the corresponding perpendicular distance or the center of gravity of strips from Y axis

According to principal of moment

$M\,x = m_1 x_1 + m_2 x_2 + m_3 x_3$

$M\,x = \sum m\,x$

$$\mathbf{x = \frac{\sum m\,x}{M}} \quad \text{———————— } 1$$

Where $\sum m = m_1 + m_2 + m_3 + \ldots\ldots$

And $\sum x = x_1 + x_2 + x_3 + \ldots\ldots$

Similarly

$$\mathbf{y = \frac{\sum m\,y}{M}} \quad \text{———————— } 2$$

The plane geometrical figures (such as T-section, 1-section, L-section etc.) have only areas but no mass the center of gravity of such figures is found out in the same way as that of solid bodies. Therefore the above two equations will become

$$\mathbf{x = \frac{\sum a\,x}{A}}$$

Or $$\mathbf{x = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots\ldots}{a_1 + a_2 + a_3 + \ldots\ldots}}$$

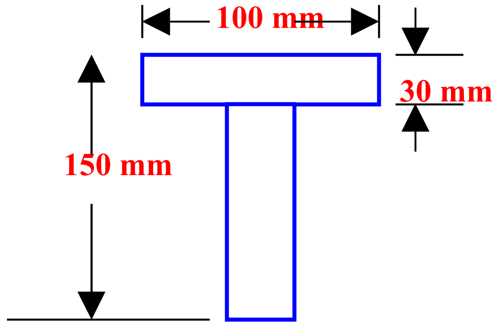
$$\mathbf{y = \frac{\sum a\,y}{A}}$$

Or $$\mathbf{y = a_1 y_1 + a_2 y_2 + a_3 y_3 + \ldots\ldots}$$

$a_1 + a_2 + a_3 + \ldots\ldots\ldots$

EXAMPLE 4

Find the center of gravity of a 100 mm x 150 mm x 30 mm T-section. As shown in the fig



Given Height = 150 mm width = 100 mm
thick ness = 30 mm

Required center of gravity = y =?

Working formulae $y = \frac{\sum a\, y}{A}$ or $y = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \ldots\ldots\ldots}{a_1 + a_2 + a_3 + \ldots\ldots\ldots}$

Solution

#	Body	Area mm²	Distance (y) mm	Area x y
1	Rectangular ABCD	a_1 = 100 x 30 = 3000	30/2 = 15	3000 x 15 = 450
2	Rectangular EFGH	a_2 = (150 – 30) x 30 = 3600	150-30/2 = 135	3600 x135= 486
		∑ = 9600		∑ = 531000

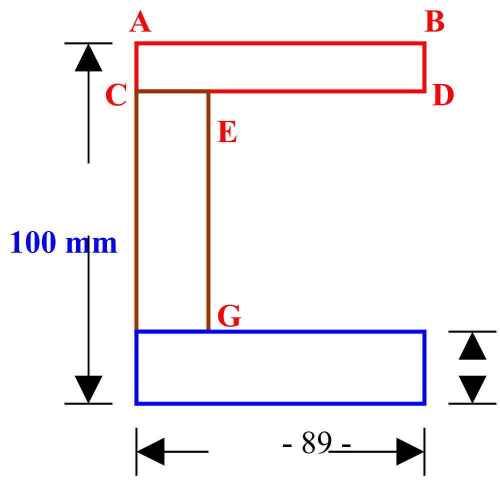
Put in the working formula

$y = \frac{\sum a\, y}{A} = \frac{531000}{9600}$ Y = 94.09 mm

Result center of gravity = 94.09 mm

EXAMPLE 2

Find the center of gravity of a channel section 100 mm x 50mm x 15 mm.



F H

15 mm

I J

50 mm

Required center of gravity =?

Working formula $x = \dfrac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots\ldots\ldots}{a_1 + a_2 + a_3 + \ldots\ldots}$

Solution Consider the rectangle ABC

Area = a_1 = 50 x 15 = 750 mm² x_1 = 50 / 2 = 25 mm

Consider the rectangle CEFG

Area = a_2 = (100 -15-15) x 15 = 1050 mm²

x_{21} = 15 / 2 = 7.5 mm

Consider the rectangle FHIJ

Area = a_3 = 50 x 15 = 750 mm² x_3 = 50 / 2 = 25 mm

Put the values in the working formula

$$x = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \frac{750 \text{ x } 25 + 1050 \text{ x } 7.5 \text{ x } 750 \text{ x } 25}{25 + 7.5 + 25}$$

x = 17 .8 mm

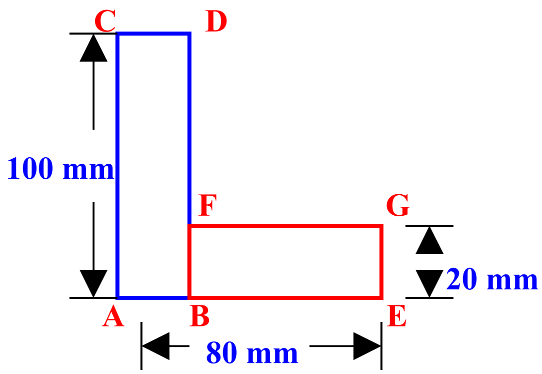
Result **Center of gravity = 17.8 mm**

<u>CENTRE OF GRAVITY OF UNSYMMETRICAL SECTIONS</u>

Sometimes, the given section, whose center of gravity is required to be found out, is not symmetrical either about x-axis or y-axis. In such cases, we have to find out both the values of center of gravity of x and y which means with reference to x axis and y axis

<u>EXAMPLE 3</u>

Find the centroid of an unequal angle section 100 mm x 80 mm x 20mm.



Required center of gravity =?

Working formula $x = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$

$$y = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

#	Body	Area mm²	Distance (x) mm	Distance (y)
1	Rectangular ABCD	a_1 = 100 x 20 = 2000	x_1 = 20/10 =10	y_1 =100/2 = 50

2	Rectangular BEFG	$a_2 = (80 – 20) \times 20 = 1200$	$x_2 = 20 – 60/2 = 50$	$y_2 = 20/2 = 10$

Put the value in the first working formula

$$x = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{10 + 60}$$

$x = 25$ mm

$$y = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{10 + 60}$$

$y = 35$ mm

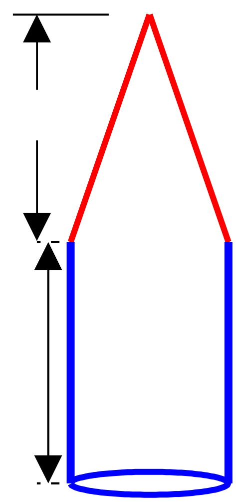
Result **x = 25 mm** **y = 35 m**

CENTRE OF GRAVITY OF SOLID BODIES

The center of gravity of solid bodies (such as hemisphere, cylinder, right circular solid cone etc) is found out in the same way as that of the plane figures. The only difference between the plane and solid bodies is that in the case of solid bodies we calculate volumes instead of areas

EXAMPLE 4

A solid body formed by joining the base of a right circular cone of height H to the equal base of right circular cylinder of height h. calculate the distance of the center of gravity of the solid from its plane face when H = 120 mm and h = 30 mm



Given cylinder height = h = 30 mm
Right circular cone = H = 120 mm

Required center of gravity =?

120 mm

Working formula

$$y = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2}$$

Solution

Consider the cylinder

30 mm

Volume of cylinder = $\pi \times r^2 \times 30 = 94.286\ r^2$

C.G of cylinder = $y_1 = 30/2 = 15$mm

Now consider the right circular cone

Volume of cone = $\pi/3 \times r^2 \times 120 = 377.143\ r^2$

C.G of cone = $y_2 = 30 + 120/4 = 60$ mm

Put the values in the formula

$$y = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2} = \frac{94.286\ r^2 \times 15 + 377.143\ r^2 \times 60}{94.286\ r^2 + 377.143\ r^2}$$

$y =$ 40.7 mm

Result center of gravity = 40.7 mm

CENTRE OF GRAVITY OF SECTIONS WITH CUT OUT HOLES

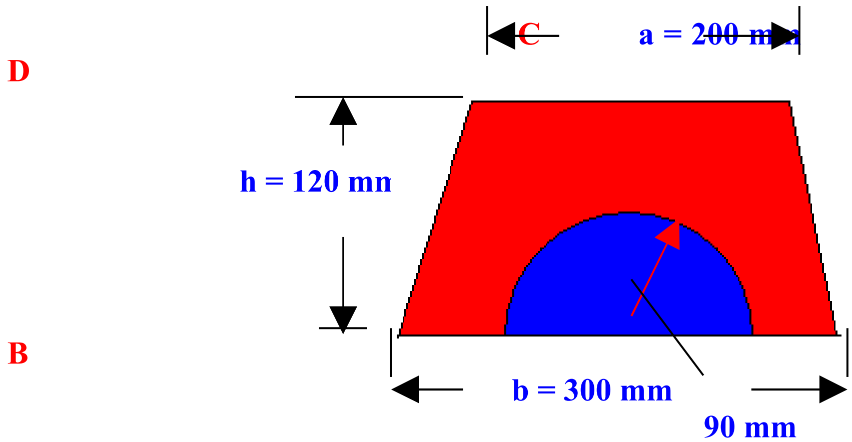
The center of gravity of such a section is found out by considering the main section; first as a complete one and then deducting the area of the cut out hole that is taking the area of the cut out hole as negative. Now substituting the area of the cut out hole as negative, in the general equation for the center of gravity, so the equation will become

$$x = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

Or $$y = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

EXAMPLE 5

A semicircles of 90 mm radius is cut out from a trapezium as shown in fig find the position of the center of gravity



Given

Trapezium ABCD

b = 300 mm a = 200 mm h = 120 mm

Semicircle radius = r = 90 mm

Working Formula $y = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$

Solution

Area of trapezium = $\frac{a + b}{2} \times h = \frac{200 + 300}{2} \times 120 = 30000$ mm²

centre of gravity of trazezium = $y_1 = \frac{h}{3} = \left[\frac{b + 2a}{b + a}\right]$

4

$y_1 = \frac{120}{3}\left(\left[\frac{300 + 2 \times 200}{300 + 200}\right]\right) = 56$ mm

4

Area of semicircle = $\frac{\text{area of the circle}}{2} = \frac{\pi r^2}{2} = \frac{\pi 90^2}{2} = 89100$ mm²

Center of gravity of the semicircle = $\frac{4r}{3\pi} = \frac{4\ 90}{3\pi} = 38.183$

Put the values in working formula

$$y = \frac{30000 \times 56 - 89100 \times 38.183}{30000 - 89100}$$

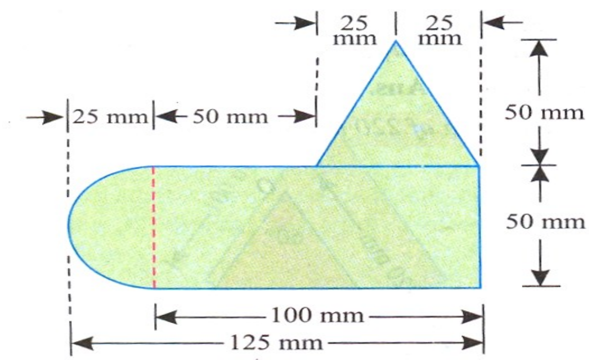
Result **Center of the gravity = 69.1 mm**

EXERCISE 4

1. An I section has the following dimensions in mm units.
Top flange = 150 x 50 Bottom flange = 300 x100 Web = 300 x 50
Find the center of gravity (centroid)

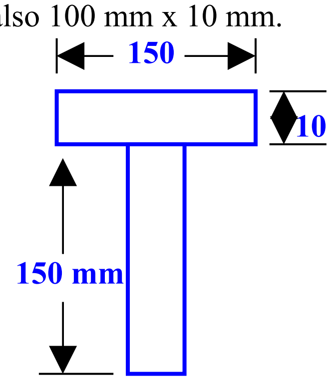
Ans: 160.7 mm

2. A uniform lamina shown in fig consists of rectangle, a semi circle and a triangle. Find the centre of gravity



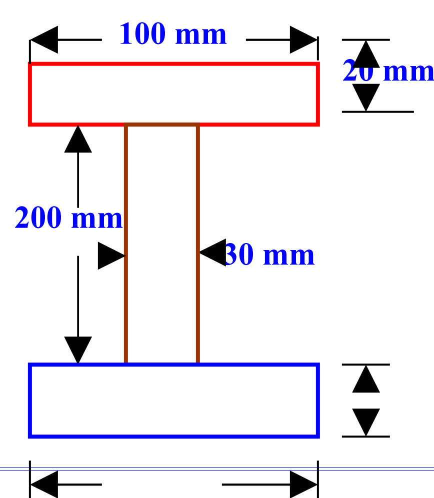
Ans: 71.1 mm

3. Find the centre of gravity of T section with flange 150 mm x 10 mm and web also 100 mm x 10 mm.



Ans: 115 mm

4. Find the center of gravity a T section with top flange 100 mm x 20 mm web 200 mm x 30 mm and the bottom flange 300 mm x 40 mm

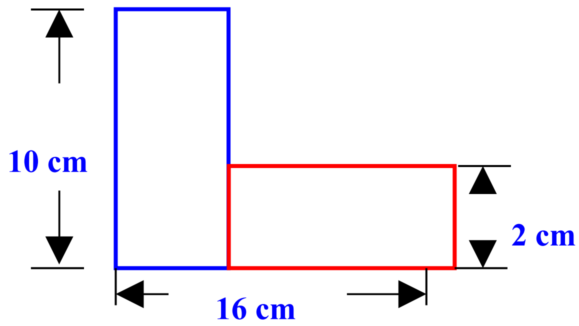


40 mm

300 mm

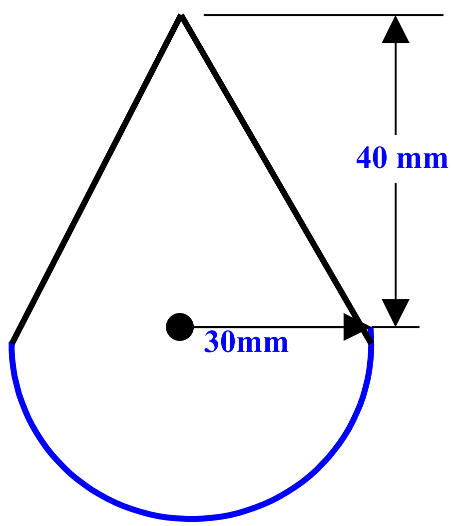
Ans: 79mm

5. Find the center of gravity of an unequal angle section 10 cm x 16 cm x 2 cm



Ans: 5.67 mm and 2.67 mm

6. A body consists of a right circular solid cone of height 40 mm and radius 30 mm placed on a solid hemisphere of radius 30 mm of the same material find the position of the center of gravity of the body

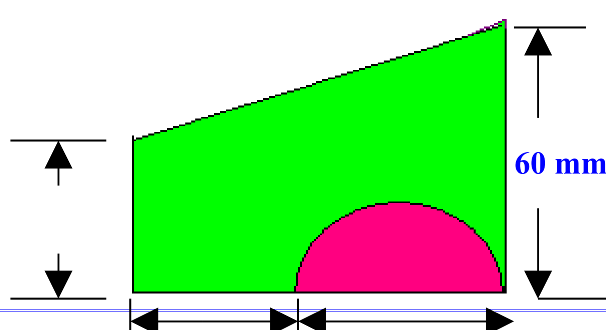


Ans: 28.4 mm

7. A hemisphere of 60 mm diameter is placed on the top of the cylinder having 60 mm diameter. Find the center of gravity of the body from the base of the cylinder if its height is 100 mm.

Ans: 60.2 mm

8. A semicircular area is removed from a trapezium as shown in fig determine the position of the center of gravity

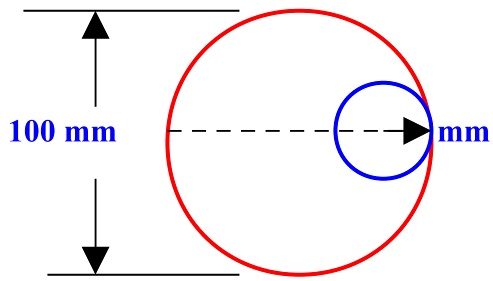


30 mm

40 mm **40 mm**

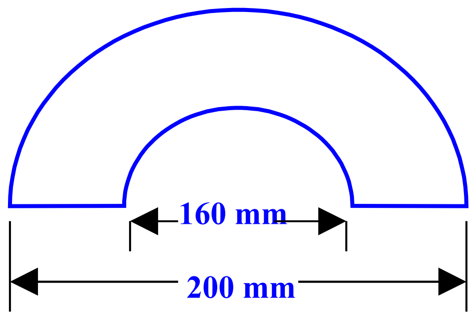
Ans

9. A circular hole of 50 mm diameter is cut out from a circular disc of 100 mm diameter as shown in fig find the center of gravity of the section



Ans: 41.7 mm

10. Find the center of gravity of a semicircular section having outer and inner diameters of 200 mm and 160 mm respectively as shown in fig.



Ans: 57.5 mm

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