MATHEMATICS

1. Consider the following statements:

1. For any natural number n (i.e., $n \in N$), let $I_n = \{r \in R: -n \le r \le n\}$. Then $\bigcup_{n \in N} I_n = R$.
2. For any positive real number r, let $I_r = \{x \in R: -r < x < r\}$. Then $\bigcap_{\substack{r \in R \\ r > 0}} I_r$ contains exactly one element.

Which of the statements given above is/are correct?

a. 1 only
b. 2 only
c. Both 1 and 2
d. Neither 1 nor 2

Ans. a

2. If $x^2 + x + 1 = 0$, then what is the value of $\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \ldots + \left(x^{27} + \frac{1}{x^{27}}\right)^2$?

a. 27
b. 36
c. 45
d. 54

Ans. d

3. If $x + iy = \frac{3}{2 + \cos\theta + i\sin\theta}$, then what is $x^2 + y^2$ equal to?

a. $\frac{3}{5 + 4\cos\theta}$
b. $\frac{9}{5 + 4\cos\theta}$
c. $\frac{3}{5 + 4\sin\theta}$
d. Independent of θ

Ans. b

4. Consider the following four numbers where ω is a cube root of unity:

1. Principal branch of i^i
2. $\omega + \omega^2$
3. $\omega + i\omega$
4. $\omega^2 + i\omega^2$

How many of the above are real numbers?

a. One
b. Two
c. Three
d. Four

Ans. b

5. Consider the polynomial $x^3 + 2x + 1$ over integers. If α, β, γ are roots of $x^3 + 2x + 1 = 0$, then what is the value of $\alpha^2 + \beta^3 + \gamma^3$?

a. 9
b. –4
c. –6
d. –9

Ans. d

6. If the set of integers with the operation defined by $m * n = m + n - 1$ forms a group, what is the inverse of m?

a. –m
b. 2m
c. m
d. 1 – m

Ans. c

7. In (Z, +), nZ denotes the subgroup of all integral multiples of n. If $pZ = jZ \cap kZ$, then what is p equal to?

a. jk
b. j + k
c. LCM of j and k
d. GCD of j and k

Ans. c

8. In a group (G, *), if $(a * b)^{-1} = a^{-1} * b^{-1}$ for all a, $b \in G$, then what is G?

a. An Abelian
b. Finite
c. Cyclic
d. None of the above

Ans. a

9. Consider the following statements in respect of a finite group G:

1. $O(a) = O(a^{-1})$ for all $a \in G$
2. $O(a) = O(bab^{-1})$ for all a, $b \in G$

Which of the statements given above is/are correct?

a. 1 only
b. 2 only
c. Both 1 and 2
d. Neither 1 nor 2

Ans. c

10. Let H be a group of order 23. What is the number of subgroups of H?

a. 2^{23}
b. 23!
c. 2
d. 1

Ans. c

11. Which one of the following is correct?

if $\alpha, \beta \in S_n$, then $\alpha^{-1}\beta^{-1}\alpha\beta$ is an even permutation

a. for all $\alpha, \beta \in S_n$
b. only if α, β are odd
c. only if α, β are even
d. only if one of α, β is odd and other is even

Ans. a

12. Let M denote the set of all 2×2 matrices over the reals. Addition and multiplication on M are as follows:

$A=(a_{ij})$ and $B=(b_{ij})$, then $A + B = (c_{ij})$, where $c_{ij} = a_{ij} + b_{ij}$, and $A \cdot B = (d_{ij})$, where $d_{ij} = a_{ij}b_{ij}$

Then which one of the following is valid for $(M, +, \cdot)$?

a. M is a field
b. M is an integral domain which is not a field
c. M is a commutative ring which is not an integral domain
d. M is a non-commutative ring

Ans. c

13. Consider Z[x], the set of all polynomials with integer coefficients and $Q[\sqrt{2}]$, the set of all real numbers of the form $a + b\sqrt{2}$ with a, b rational numbers. Which of the following is correct about Z[x] and $Q[\sqrt{2}]$?

a. Both are rings, but only one has unity
b. Both are commutative rings, but only one is an integral domain
c. Both are integral domains, but only one is a field
d. Both are fields

Ans. c

14. Consider the field R of real numbers and the following spaces:

1. Set of real-valued functions on [0, 1] having discontinuity at $x = 1/2$
2. $\{(x, y) \in R^2 : x > 0\}$

Which of the above is/are vector space(s)?

a. 1 only
b. 2 only
c. Both 1 and 2
d. Neither 1 nor 2

Ans. d

15. Consider the real vector space $V = R^3$ and following of its subsets:

1. $S = \{(x, y, z) \in V : x = y = 0\}$
2. $T = \{(x, y, z) \in V : x = 0\}$
3. $W = \{(x, y, z) \in V : z \neq 0\}$

Which one of the following statements is correct?

a. S, T and W are subspaces
b. Only S, W are subspaces
c. Only T, W are subspaces
d. Only S, T are subspaces

Ans. d

16. Let V be a vector space over the field F of dimension n. Consider the following statements:

1. Every subset of V containing n elements is a basis of V
2. No linearly independent subset of V contains more than n elements

Which of the above statements is/are correct?

a. 1 only
b. 2 only
c. Both 1 and 2
d. Neither 1 nor 2

Ans. b

17. If $T: R^3 \to R^3$ is a linear transformation such that $T(1, 0, 0) = (2, 3, 1)$ and $T(1, 1) = (3, 0, 2)$, then which one of the following statements is correct?

a. $T(x, y) = (x + y, 2x + y, 3x - 3y) \forall (x, y) \in R^2$
b. $T(x, y) = (2x + y, 3x - 3y, x + y) \forall (x, y) \in R^2$
c. $T(x, y) = (2x - y, 3x + 3y, x - y) \forall (x, y) \in R^2$
d. $T(x, y) = (x - y, 2x - y, 3x + 3y) \forall (x, y) \in R^2$

Ans. b

18. Let $T: R^3 \to R^2$ be the linear transformation given by

$T(x, y, z) = (x, y) \forall (x, y, z) \in R^3$

With respect to standard basis of R^3 and the basic $\{(1, 0), (1, 1)\}$ of R^2, what is the matrix representation of T?

a. $\begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 0 \end{bmatrix}$
b. $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
c. $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
d. $\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$

Ans. c

19. Let $T: R^3 \to R^3$ be defined by $T(x, y, z) = (x, y, 0)$ and $S: R^2 \to R^2$ be defined by $S(x, y) = (2x, 3y)$, be linear transformations on the real vector spaces R^3 and R^2, respectively. Then which one of the following is correct?

a. T and S are both singular
b. T and S are both non-singular
c. T is singular and S is non-singular
d. S is singular and T is non-singular

Ans. c

20. Consider the linear transformation $T: R^4 \to R^4$ given by

$T(x, y, z, u) = (x, y, 0, 0) \forall (x, y, z, u) \in R^4$

Then which one of the following is correct?

a. Rank of T > Nullity of T
b. Nullity of T > Rank of T
c. Rank of T = Nullity of T = 3
d. Rank of T = Nullity of T = 2

Ans. d

21. If the value of the determinant $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$ is positive, then which one of the following is correct?

a. $abc \geq -8$
b. $abc > -8$
c. $abc < -8$
d. $abc \leq -8$

Ans. b

22. If A and B are square matrices of same order, then what is $(A+B)^3$ equal to?

a. $A^3 + B^3 + 3AB(A + B)$
b. $A^3 + B^3 - 3AB(A + B)$
c. $A^3 + B^3 + 3A^2B + 3B$
d. $A^3 + B^3 + A^2B + B^2A + AB^2 + BA^2 + ABA + BAB$

Ans. d

23. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} 1 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then matrix A is equal to which one of the following?

a. $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$
b. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
c. $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$
d. $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

Ans. a

24. If A and B are two odd order skew-symmetric matrices such that AB = BA, then what is the matrix AB?

a. An orthogonal matrix
b. A skew-symmetric matrix
c. A symmetric matrix
d. An identity matrix

Ans. c

25. If for a matrix A, $A^2 + I = 0$, where I is the identity matrix, then A is equal to which one of the following?

a. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
b. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
c. $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$
d. $\begin{bmatrix} -1 & \\ & -1 \end{bmatrix}$

Ans. b

26. If $A = \begin{bmatrix} 0 & \alpha \\ \beta & 0 \end{bmatrix}$ then when is $A^3 + A = 0$?

a. If $\alpha\beta = 0$
b. If $\alpha\beta = \frac{1}{2}$
c. If $\alpha\beta = 1$
d. If $\alpha\beta = -1$

Ans. d

27. A straight line drawn through the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$ meets the coordinate axes in A and B. What is the locus of the midpoint of AB?

a. $2(a + b)xy = ab(x + y)$
b. $a(x + y) = b(x - y)$
c. $ab(x - y) = x + y$
d. $(a + b)xy = ab(x + y)$

Ans. a

28. What is the area of the rhombus enclosed by the lines $ax \pm by \pm c = 0$?

a. $\frac{|ab|}{c^2}$
b. $\frac{|ab|}{2c^2}$
c. $\frac{2c^2}{|ab|}$

d. $\frac{c^2}{|ab|}$

Ans. c

29. If (4, 4) is one end of a focal chord of the parabola $x^2 = 4y$, then what is the other end?

a. (–2, 1)
b. (–1, 1/4)
c. (–1, –1/4)
d. (–4, 4)

Ans. b

30. What is the ratio between the y-coordinates of a point on ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and its corresponding point on the auxiliary circle?

a. $\frac{a}{b}$
b. $\frac{b}{a}$
c. $\frac{a^2}{b^2}$
d. $\frac{b^2}{a^2}$

Ans. b

31. Match List-I with List-II and select the correct answer using the code given below the Lists:

List-I (Straight Line)

A. $x + 1 = 0$
B. $y = x + 1$
C. $x - y - 3 = 0$
D. $4x - 3y - 4 = 0$

List-II (Its Nature Pertains to the parabola $y^2 = 4x$)

1. Tangent
2. Focal chord
3. Normal
4. Directrix

Code:

a. A-1, B-3, C-1, D-2
b. A-2, B-4, C-3, D-4
c. A-4, B-1, C-3, D-2
d. A-2, B-3, C-1, D-4

Ans. c

32. What is the area of the circle $x^2 + y^2 + z^2 - 4x + 2y - 2z - 58 = 0 = 2x - 3y + 6z - 62$?

a. 15 π sq. unit
b. 16 π sq. unit
c. 49 π sq. unit
d. 64 π sq. unit

Ans. a

33. What is the shortest distance from the point (1, 2, –1) to the surface of the sphere $x^2 + y^2 + z^2 = 24$?

a. $3\sqrt{6}$
b. $2\sqrt{6}$
c. $\sqrt{6}$
d. 2

Ans. c

34. What is the equation of the axis of the cylinder when the equation of the guiding circle of the right circular cylinder is $x^2 + y^2 + z^2 - 1 = 0 = x - y + z - 3$?

a. $x = -y = z$
b. $x = y = -z$
c. $x = y = z$
d. $-x = y = z$

Ans. a

35. Planes through Z and Y axes include a constant angle. What is the locus of their line of intersection?

a. Plane
b. Sphere
c. Cone
d. Cylinder

Ans. c

36. Given $|\vec{a}+\vec{b}| = |\vec{a}| + |\vec{b}|$, where $\vec{a}$ and $\vec{b}$ are vectors. Then which one of the following is correct?

The above relation

a. always holds
b. never holds
c. holds only when $\vec{a} = \vec{0}$ and $\vec{b} = \vec{0}$
d. holds in general if $\vec{a} = k\vec{b}$, k = constant

Ans. d

37. What are the values of x for which the angle between the vectors $\vec{a} = x\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = 2x\hat{i} + x\hat{j} - \hat{k}$ is acute, and the angle between the vector $\vec{b}$ and the axis of x is obtuse?

a. 2, 3 only
b. All $x > 0$
c. All $x < 0$
d. None of the above

Ans. d

38. Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} - c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then which one of the following is correct?

a. c is the arithmetic mean of a and b
b. c is the geometric mean of a and b
c. c is the harmonic mean of a and b

d. c is the reciprocal of harmonic mean of a and b

Ans. b

39. Let α, β, γ be distinct real numbers. Which one of the following is correct?

The points with position vectors

$\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}, \beta\hat{i}+\gamma\hat{j}+\alpha\hat{k}; \gamma\hat{i}+\alpha\hat{j}+\beta\hat{k}$

a. are collinear

b. form an equilateral triangle

c. form a scalene triangle

d. form a right-angled triangle

Ans. b

40. A vector $\vec{r}$ is equally inclined with the coordinate axes. If the tip of $\vec{r}$ is in the positive octant and $|\vec{r}| = 6$, then what $\vec{r}$?

a. $6(\hat{i}+\hat{j}+\hat{k})$

b. $2\sqrt{3}(-\hat{i}+\hat{j}+\hat{k})$

c. $2\sqrt{3}(\hat{i}+\hat{j}-\hat{k})$

d. $2\sqrt{3}(\hat{i}+\hat{j}+\hat{k})$

Ans. d

41. What is the angle between the line $\vec{r}=(2\hat{i}-\hat{j}+\hat{k})+\lambda(-\hat{i}+\hat{j}+\hat{k})$ and the plane $\vec{r}\cdot(3\hat{i}+2\hat{j}-\hat{k})=4$?

a. $\cos^{-1}\left(\frac{2}{\sqrt{42}}\right)$

b. $\cos^{-1}\left(\frac{-2}{\sqrt{42}}\right)$

c. $\sin^{-1}\left(\frac{2}{\sqrt{42}}\right)$

d. $\sin^{-1}\left(\frac{-2}{\sqrt{42}}\right)$

Ans. d

42. If D is the set of all real x such that $1-e^{\left(\frac{1}{x}\right)-1}$ is positive, then what is D equal to?

a. $(-\infty, 0) \cup (1, \infty)$

b. $(-\infty, 0) \cup [1, \infty)$

c. $(1, \infty)$

d. $(-\infty, 1)$

Ans. a

43. If $f(x)$ is an odd periodic function with period 2, then what is the value of $f(4)$?

a. 0

b. 2

c. –4

d. 4

Ans. a

44. Given the function

$$f(x)=\lim_{n\to\infty} n\left(x^{\frac{1}{n}}-1\right), x>0$$

If f satisfies $f\left(\frac{1}{x}\right)=kf(x)$, then what is/are the value(s) of k?

a. 1

b. –1

c. 0

d. ±1

Ans. b

45. The function $f(x)=\frac{1}{1-x}$. What is the number of points of discontinuity of the composite function $y = f^{[n]}(x)$, where $f^{[n]}(x) = f \circ f \circ f \ldots$ (n times)?

a. 1

b. 2

c. n

d. 3n

Ans. a

46. The function $y=xe^{-x^2/2}$ satisfies the equation $y'=\left(\frac{?}{x}-x\right)y$. What is the value of λ?

a. 1

b. 0

c. –1

d. None of the above

Ans. a

47. If $f(x)=\begin{cases} x^{\left(\frac{1}{k}\right)-1}\cos\left(\frac{1}{x}\right), & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$

then under what condition is $f(x)$ differentiable at $x = 0$?

a. $k>\frac{1}{2}$

b. $k\geq\frac{1}{2}$

c. $k\leq\frac{1}{2}$

d. $k<\frac{1}{2}$

Ans. d

48. If a man of 2 m height is moving away from a lamppost of 10 m height, at the rate of 2 m/s, then what is the rate of increase of the length of his shadow?

a. $\frac{1}{2}$ m/s

b. $\frac{2}{5}$ m/s

c. 1 m/s

d. 2 m/s

Ans. a

49. What is the value of c if $y^2 = x$ and $xy = c^3$ cut orthogonally?

a. $\sqrt{2}$

b. $\frac{1}{\sqrt{2}}$

c. 2

d. $\frac{1}{2}$

Ans. b

50. What is the equation of the normal to the curve $y = (1+x)^y + \sin^{-1}(\sin^2 x)$ at $x = 0$?

a. $x + y = 1$

b. $x - y = 1$

c. $-x + y = 1$

d. $x + y = -1$

Ans. a

51. What is the value of a for which the function

$f(x) = a \sin(x) + \frac{1}{3}\sin(3x)$

has an extremum at $x = \frac{\pi}{3}$?

a. 2

b. 1

c. -1

d. 0

Ans. a

52. Which one of the following is correct?

The equation

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n = 0$

has at least one root between 0 and 1, if

a. $\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + \ldots + a_{n-1} = 0$

b. $\frac{a_0}{n-1} + \frac{a_1}{n-2} + \frac{a_2}{n-3} + \ldots + a_{n-2} = 0$

c. $na_0 + (n-1)a_1 + (n-2)a_2 + \ldots a_{n-1} = 0$

d. $\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + \ldots + a_n = 0$

Ans. d

53. If f(x) and g(x) are differentiable function for $0 \le x \le 1$ such that

$f(1) - f(0) = k[g(1)-g(0)]$, $k \neq 0$

then there exists c satisfying $0 < c < 1$

What is $\frac{f'(c)}{g'(c)}$ equal to?

a. 2k

b. k

c. -k

d. $\frac{1}{k}$

Ans. b

54. In how many points is the curve [illegible] nth degree cut by its asymptotes?

a. n

b. 1

c. n-2

d. n-3

Ans. c

55. What is $\int_{-a}^{a} f($[illegible] equal to?

a. $\int_0^a [$[illegible]$] dx$

b. $\int_0^a [f(x) - f(-x)] dx$

c. $2\int_0^a f(x)dx$

d. 0

Ans. a

56. If $\int_0^x t^2 f(t)dt = x + \int_x^1 f(t)dt$ then that is the value of $f\left(\frac{1}{2}\right)$?

a. $\frac{1}{5}$

b. $\frac{4}{5}$

c. $\frac{3}{5}$

d. $\frac{2}{5}$

Ans. b

57. If $I_1 = \int_{1-k}^{k} xf[x(1-x)]dx$ and $I_2 = \int_{1-k}^{k} f[x(1-x)]dx$ where $2k-1 > 0$, then what is the value of $\frac{I_1}{I_2}$?

a. $\frac{1}{2}$

b. 1

c. 2

d. k

Ans. a

58. Let

$$\int e^{\sec x}\left[\sec x \tan x f(x) + + \left(\sec x \tan x + \sec^2 x\right)\right]dx$$

$$= e^{\sec x} f(x) + c$$

Then what is f(x) equal to?

a. $\sec x + \tan x$

b. $\sec x - \tan x$

c. $-x \sec x + \tan x$

d. $\sec x - x \tan x$

Ans. a

59. If the coordinate x = a divides that area bounded by the curve $y = 1 + \frac{8}{x^2}$ and the coordinates x = 2, x = 4 into two equal parts, then what is the value of a?

a. $\frac{2}{\sqrt{3}}$

b. $2\sqrt{3}$

c. $\sqrt{2}$

d. $2\sqrt{2}$

Ans. d

60. What is the length of curve $y = \frac{\left(x^2+2\right)^{\frac{3}{2}}}{3}$ fro

$x = 0$ to $x = 3$?

a. 10

b. 12

c. 3π

d. 6π

Ans. b

61. What is the volume of b sol generated by revolving the region bet y-axis and the curve $x = \frac{2}{y}$, y = 4, about the y-axis?

a. 3π

b.

c. 3

d. 6

Ans. a

62. Which one of the following statements is correct?

The differential equation

$$\left(\frac{dy}{xy}\right)^2 + 5y^{\frac{1}{3}} = x$$

is

a. linear equation of order 2 and degree 1

b. non-linear equation for order 1 and degree 2

c. non-linear equation of order 1 and degree 6

d. linear equation of order 1 and degree 6

Ans. b

63. What is the solution of differential equation

$$\frac{dy}{dx} = \frac{y}{x} + \frac{\varphi\left(\frac{y}{x}\right)}{\varphi'\left(\frac{y}{x}\right)}?$$

a. $\varphi\left(\frac{y}{x}\right) = kx$

b. $\varphi\left(\frac{y}{x}\right) = ky$

c. $x\varphi\left(\frac{y}{x}\right) = k$

d. $y\varphi\left(\frac{y}{x}\right) = k$

for some con t k

Ans. a

64. In the ferenial equation

$(x - y)dx - 2xydy = 0$

Which one of the following substitutions makes variables separable?

a. $y = \sqrt{vx}$

b. $y = vx$

c. $y^2 = v$

d. $y^2 = vx$

Where v is a function of x

Ans. c

65. What is the general solution of $2x\frac{dy}{dx} = 10x^3y^3 + y$?

a. $y^4 = cx^2 - 4x^3$

b. $y^{-4} = \frac{c}{x^2} + 4x^3$

c. $y^4 = \frac{c}{x^2} + 4x^3$

d. $y^{-4} = \frac{c}{x^2} - 4x^3$

Ans. d

66. What is the equation of orthogonal trajectories of the family of concentric circle $x^2 + y^2 = a^2$, where a is a parameter?

a. $y^2 = 4ax$

b. $x^2 - y^2 = c^2$

c. $y = cy$

d. $x + y = a$

Ans. c

67. What are the general and singular solutions, respectively of the differential equation $(xp - y)^2 = p^2 - 1$?

a. $(cx + y)^2 = c^2 + 1, x = \pm 1$
b. $(cx - y)^2 = c^2 - 1, x^2 + y^2 = 1$
c. $(cx - y)^2 = c^2 - 1, x^2 - y^2 = 1$
d. $(x - cy)^2 = 1 - c^2, x^2 \pm y^2 = 1$

Ans. c

68. $m = 2$ is a double root and $m = -1$ is another root of the auxiliary equation of a homogenous differential equation with constant coefficient. What is the differential equation?

a. $(D^3 + 3D^2 + 4)y = 0$
b. $(D^3 + 3D^2 - 4)y = 0$
c. $(D^3 - 3D^2 + 4)y = 0$
d. $(D^3 - 3D^2 - 4)y = 0$

Ans. c

69. $y_1 = x^m$ and $y_2 = x^n$, where m and n are constants, are the two solutions of a second order differential equation with constant coefficients. What is the condition under which $y = c_1y_1 + c_2y_2$ is the general solution of the same equation?

a. $m \neq n$
b. $m = n$
c. $m = -n$ only
d. $m + n = 1$ only

Ans. d

70. What is the solution of $\frac{d^2y}{dx^2} - y = \cosh x$?

a. $y = A\cos x + B\sin x + \frac{xe^x}{4} + \frac{e^{-x}}{4}$
b. $y = Ae^x + Be^{-x} + \frac{x\cosh x}{2}$
c. $y = Ae^x + Be^{-x} + \frac{\sinh x}{2}$
d. $y = Ae^x + Be^{-x} + \frac{x\sinh x}{2}$

Ans.

71. The resultant of two forces $\vec{P}$ and $\vec{Q}$ acting at a point is $\vec{F}_1$. If the force $\vec{Q}$ is replaced by $2\vec{Q}$, the resultant is $\vec{F}_2$. What is $|\vec{F}_1 - \vec{F}_2|$?

a. $|3\vec{Q}|$
b. $|\vec{P} + 2\vec{Q}|$
c. $|\vec{Q} + \vec{P}|$
d. $|\vec{Q}|$

Ans. d

72. If a force $\vec{F}$ acting at a point of a rigid body and a coplanar couple of moment $\vec{M}$ on the body are combined into a single force, then what is the perpendicular distance from the point to the line of action of the resultant force?

a. $\frac{M}{2F}$
b. $\frac{M}{F}$
c. $\frac{2M}{F}$
d. $\frac{F}{M}$

Ans. b

73. $\vec{P}$ and $\vec{Q}$ are like parallel forces acting on a rigid body. If $\vec{Q}$ is moved parallel to itself through a distance x, then the resultant of $\vec{P}$ and $\vec{Q}$ moves through what distances?

a. $\frac{Q}{P+Q}$
b. $\frac{P}{P+Q}$
c. $\frac{(P+Q)x}{Q}$
d. $\frac{(P+Q)x}{P}$

Ans. a

74. Three forces acting on a rigid body are represented in magnitude, direction and line of action by the sides of a triangle of length 4, 5 and x unit and are equivalent to a couple whose moment is equal to 12 unit. What is the value of x?

a. 2
b. 3
c. 4
d. 6

Ans. b

75. For concurrent forces of magnitudes P, $\left(2\sqrt{2}+1\right)$P, 2P and 2P act in the directions of East, North, South-West and South-East, respectively. What is the resultant of the forces?

a. A force of magnitude $2\sqrt{2}$P in the North-West direction
b. A force of magnitude $\sqrt{2}$P in the North-East direction
c. A force of magnitude 2P in the South direction

d. A force of magnitude P in the East direction

Ans. b

76. What are the coordinates of the mass centre of a uniform semicircular lamina of radius a placed with its base on the x-axis and centre at the origin?

a. $(0, 0)$

b. $\left(0, \frac{a}{2}\right)$

c. $\left(0, \frac{3a}{8}\right)$

d. None of the above

Ans. d

77. A particle executes simple harmonic motion of amplitude A along the x-axis. At $t = 0$, the position of the particle is $x = \frac{A}{2}$ and it moves along the positive x-direction. If the equation is written as $x = A\sin(\omega t + \delta)$, then what is the value of the phase constant δ?

a. $\frac{\pi}{2}$

b. $\frac{\pi}{6}$

c. $\frac{\pi}{4}$

d. $\frac{\pi}{3}$

Ans. b

78. A car starts from rest and moves along a straight road with a constant acceleration f m/(minute)^2. What is the ratio of the distance covered by the car in the 3rd minute to that covered in the 1st minute?

a. 3 for all f

b. 5 for all f

c. 7 for all f

d. Depends on the value of f

Ans. b

79. A stone is thrown horizontally with a velocity $\sqrt{gh}$ from the top of a tower of height h. At what distance will it strike the horizontal ground through the foot of the tower?

a. 2h

b. 3h

c. 6h

d. 8h

Ans. a

80. If a particle with initial velocity u is projected upward in a direction included at an angle 60^0 to the horizontal, then what is its velocity when it is at the greatest height?

a. u

b. 2u

c. $\frac{u}{2}$

d. $\frac{\sqrt{3}u}{2}$

Ans. c

81. The mass of the sun is derived by considering the earth to travel in a circular orbit of radius r and with period T about the sun. Assuming universal gravitational constant to be G, the calculated mass of the sun is $\frac{kr^3}{GT^2}$. What is the value of k?

a. $\frac{4\pi^2}{3}$

b. $\frac{4}{3}$

c. $4\pi^2$

d. 4π

Ans. c

82. A body falls from rest freely under gravity. If the speed is v when it has lost an amount P of gravitational potential energy, then what is the mass of the body?

a. $\frac{P}{2v^2}$

b. $\frac{P}{v^2}$

c. $\frac{P}{g}$

d. $\frac{2P}{v^2}$

Ans. d

83. Three particles A, B, C move in space. Which one of the following shows that they move as rigid body?

a. The particles move such that AB = BC = CA always

b. The centre of mass of the three particles is always at rest

c. Distance between A and B, and that between B and C remain unchanged and the angle ABC is unchanged

d. A, B and C move uniformly towards B, C and A, respectively

Ans. c

84. Forces of 2, 4 and 3 units act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$, respectively of an equilateral triangle ABC. What is the resultant of these forces?

a. $\sqrt{3}$ units
b. 3 units
c. $\sqrt{29}$ units
d. 9 units

Ans. a

85. Which one of the following is the memory that is programmed at the time it is manufactured?

a. RAM
b. EROM
c. EPROM
d. ROM

Ans. d

86. Match List-I with List-II and select the correct answer using the code given below the lists:

List I
(Octal Number)
A. 743_8
B. 657_8
C. 735_8
D. 641_8

List-II
(Binary Equivalent)
1. 111011101_2
2. 110100001_2
3. 111100011_2
4. 110101111_2

Codes:
a. A2, B4, C1, D3
b. A3, B1, C4, D2
c. A2, B1, C4, D3
d. A3, B4, C1, D2

Ans. d

87. Which one of the following operators gives the value as 1 when both the operands are true?

a. NOR
b. OR
c. AND
d. None of the above

Ans. a

88. What is the number of 16K x 1RAMs required to obtain a memory with a word capacity of 16K and a word length of eight bits?

a. Eight
b. Sixteen
c. Twenty-four
d. Thirty-two

Ans. a

89. An assembler in a computer system prepares which one of the following?

a. Machine language programme from a symbolic language programme
b. Object programme
c. It assembles computer instructions and data in the machine
d. None of the above

Ans. a

90. What is the number of bits required to encode all letter (26), 10 symbols and all digits (10)?

a. 5
b. 6
c. 4
d. 3

Ans. b

91. The volume of a spherical balloon is increasing at the rate of 2 cm^3/s. What is the rate of change of its surface area when its radius is 6 cm?

a. 6 cm^2/s
b. 8 cm^2/s
c. 10 cm^2/s
d. 12 cm^2/s

Ans. b

92. Which one of the following functions is continuous at origin ?

a. $f(x)=\cos\left(\frac{1}{x}\right)$, when $x \neq 0$, $f(0)=0$

b. $f(x)=\sin x \sin\left(\frac{1}{x}\right)$, when $x \neq 0$, $f(0)=0$

c. $f(x)=x+\sin\left(\frac{1}{x}\right)$, when $x \neq 0$, $f(0)=1$

d. $f(x)=x\sin\left(\frac{1}{x}\right)$, when $x \neq 0$, $f(0)=1$

Ans. b

93. Let Z be the ring of integers and $a \in Z$. Which one of the following is correct?

a. $a^2 = 0 \pmod 4$ only
b. $a^2 = 1 \pmod 4$ only
c. $a^2 = 2 \pmod 4$ only
d. $a^2 = 0 \pmod 4$ or $a^2 = 1 \pmod 4$

Ans. d

94. Consider the vector space V over the filed of real numbers spanned by the set
S = {(0, 1, 0, 0), (1, 1, 0, 0), (1, 0, 1, 0), (0, 0, 1, 0), (1, 1, 1, 0), (1, 0, 0,0)}
What is the dimension of V?

a. 1
b. 2

c. 3
d. 4
Ans. c

95. If $\vec{a}=\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}+\hat{k}$ are given vectors, then a vector $\vec{b}$ satisfies $(\vec{a}\times\vec{b})+\vec{c}=0$ and $\vec{a}\cdot\vec{b}=3$. What is $\vec{b}$ equal to

a. $\hat{i}+\hat{j}-2\hat{k}$
b. $\hat{i}-\hat{j}-2\hat{k}$
c. $-\hat{i}+\hat{j}+2\hat{k}$
d. $\hat{i}+\hat{j}+2\hat{k}$

Ans. a

Directions :

Each of the following Five (5) items consists of two statements, one labeled as 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers to these items using the code given below:

Codes:

(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

96. Assertion (A): Any polynomial equation $f(x)=0$ of odd degree, with real coefficients, has at least one real root.

Reason (R): Complex roots of a polynomial equation $f(x)=0$, with real coefficients, occur in conjugate pairs.

Ans. a

97. Let $G=S_5$ be the permutation group on the set $\{1, 2, 3, 4, 5\}$. Let $\sigma\in G$ be given by

$$\sigma=\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 2 & 3 & 4 & 5 & 1\end{pmatrix}$$

Assertion (A): For any integer n, $\sigma^n\in\{e,\sigma,\sigma^2,\sigma^3,\sigma^4\}$, where e is the identity element of S_5.

Reason (R): $\sigma^{-31}=\sigma^3$

Ans. c

98. Assertion (A): Let $G=N\cup\left\{\frac{1}{n}:n\in N\right\}$, where N is the set of all positive integers. The G is a group with respect to the usual multiplication.

Reason (R): Multiplication in both N and $\left\{\frac{1}{n}:n\in N\right\}$ is well-defined and associative

$1\in G$ and $a\in G\Rightarrow\frac{1}{a}\in G$

Ans. d

99. Let G be an Abelian group. Let $H_1=\{e, g_1\}$, $H_2=\{e, g_2\}$, where $g_1\neq g_2$, be two subgroups of G, each of order 2.

Assertion (A): $H=\{e, g_1, g_2, g_1g_2\}$ is a subgroup of G, and H is not cyclic.

Reason (R): If T is finite group of order n and if T is cyclic, then T will contain at least one element t with O(t) = n i.e. n is the least positive integer with the property that $t^n=e$)

Ans. a

100. Assertion (A): $y=x^3$ has no local maxima or minima.

Reason (R): $\frac{dy}{dx}\geq 0$ for all x.

Ans. a