

MATHEMATICS

1. The number of elements in a finite field is always a
- prime number
 - even number
 - number $= p^n$ where $n \geq 1$ and p is a prime number
 - multiple of 6
2. The value of $i^{1/3}$ are
- $-i, \frac{1 \pm \sqrt{3}}{2}$
 - $i, \frac{1 \pm \sqrt{3}}{2}$
 - $-i, \frac{\sqrt{3} \pm 1}{2}$
 - $i, \frac{\sqrt{3} \pm 1}{2}$
3. If A and B are both sets having n elements then the number of surjective (onto) functions from A to B is
- n^2
 - n^n
 - $n!$
 - $n^2 - n$
4. If $A \xrightarrow{f} B \xrightarrow{g} C$ are functions such that the composed function $g \circ f : A \rightarrow C$ is injective (one-one), then
- both f and g are injective
 - g is injective but f need not be
 - f is injective but g need not be
 - neither f nor g need be injective
5. The least number of elements in a group having a proper non-abelian subgroup is
- 8
 - 12
 - 24
 - 120
6. Let F be a field containing 11 elements. Which one of the following is correct?
- If α is a non-zero element of F, then $5\alpha \neq 0$
 - $\alpha^5 = 1$ for every non-zero $\alpha \in F$ where 1 is the multiplicative identity of F
 - $\alpha^{10} = 1$ for all $\alpha \in F$
 - Let α be a non-zero element of F. It is possible to find a proper subset S of F such that $\alpha \in S$ and βS for any $\beta \in F, s \in S$
7. The number of subgroups of a cyclic group with 100 elements is
- 2
 - 5
 - 10
 - 100
8. Which one of the following is correct?
- Let $G = S_n$ be the permutation group on n symbols. Then for all $\sigma, \tau \in G$ $(\sigma \tau)^2 = \sigma^2 \tau^2$
 - If in a group G $(xy)^2 = x^2 y^2$ for all $x, y \in G$ then G must be cyclic
 - Let $G = S_3$. Let $A = \{\sigma \in G : \sigma^3 = \text{identity}\}$. Then A is a subgroup of S_3 .
 - Let $G = S_4$. Let $B = \{\sigma \in S_4 : \sigma^4 = \text{identity but } \sigma^2 \neq \text{identity}\}$. Then B contains exactly 7 elements
9. The number of roots of the equation $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$ in Z_7 is
- 1
 - 2
 - 3
 - 4
10. Which one of the following statements is correct where $R[x]$ denotes the polynomial ring in the one variable x over a ring R:
- if R is a field then $R[x]$ is a field
 - if R is an integral domain, then $R[x]$ is a field
 - if R is a field then $R[x]$ is an integral domain
 - Every integral domain is a field.

11. Which of the following rings are integral domains?

1. Z_{60}
2. Z_{71}
3. Z_{82}
4. Z_{97}

Select the correct answer using the codes given below:

- a. 1 and 2
- b. 2 and 3
- c. 2 and 4
- d. 3 and 4

12. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is

- a. zero
- b. two
- c. three
- d. four

13. If α is a complex number such that $\alpha^2 + \alpha + 1 = 0$, then α^{31} is equal to

- a. α
- b. α^2
- c. 1
- d. 0

14. If $x = \frac{1+i}{\sqrt{2}}$, then the value of $x^{1+i} + x^{1-i} - 2$ will be

- a. 0
- b. 1
- c. -1
- d. i

15. If the roots of the equation $x^3 + px^2 + qx + r = 0$ are α, β, γ , then

- a. $\alpha + \beta + \gamma = -p$
- b. $\alpha^2 + \beta^2 + \gamma^2 = -q - r^2$
- c. $\alpha^3 + \beta^3 + \gamma^3 = -q^3 - r$
- d. $\alpha^3 + \beta^3 + \gamma^3 = -p^3 - r$

16. The equation whose roots are cubes of roots of the equation $x^3 + x + 1 = 0$ is

- a. $x^3 + 3x^2 + 4x - 1 = 0$
- b. $x^3 + 3x^2 + 4x + 1 = 0$
- c. $x^3 + 3x^2 - 4x + 1 = 0$
- d. $x^3 - 3x^2 + 4x + 1 = 0$

17. Let $S = \{p + q\sqrt{2} : p, q \in \mathbb{Q}\}$. If '+' and '.' are the usual operations of addition and multiplication of real numbers then $(S, +, \cdot)$ is

- a. not a commutative ring
- b. a commutative ring but not an integral domain
- c. an integral domain but not a field
- d. a field

18. If $Z = x + iy$ and $\left| \frac{1-iZ}{Z-i} \right| = 1$, then Z lies on

- a. x-axis
- b. y-axis
- c. a circle with radius unity
- d. on line $y = x$

19. Square matrix A of order n over \mathbb{R} has rank n . Which one of the following statements is NOT correct?

- a. A^{-1} has rank n
- b. A has n linearly independent columns
- c. A is non-singular
- d. A is singular

20. If C is a non-singular matrix and

$$B = C \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} C^{-1} \text{ then}$$

- a. $B^2 = I$
- b. $B^2 = 0$
- c. $B^3 = I$
- d. $B^3 = 0$

21. If $E(\theta) = \begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{pmatrix}$, θ and ϕ differ by an odd multiple of $\pi/2$, then $E(\theta)E(\phi)$ is a

- a. null matrix
- b. unit matrix
- c. diagonal matrix
- d. none of these

22. Suppose $X = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}, b - c = 4$

$$Y = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \mid a = b + c \right\} \text{ and}$$

$$Z = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \mid b = 0, c = d \right\}$$

Which of these subsets of the vector space \mathbb{R}^4 is/are subspace(s)?

- X only
- Y and Z
- X, Y and Z
- X and Z

23. Let $\omega (\neq 1)$ be a cube root of 1 in \mathbb{C} . Then

the determinant of the matrix $\begin{bmatrix} 1 & 1 & \omega \\ 1 & 1 & \omega^2 \\ \omega^2 & \omega & 1 \end{bmatrix}$ is

- ω
- -3
- $-\omega$
- ω^2

24. Suppose P is an $n \times n$ matrix such that k^{th} diagonal element of PP^T is zero. Consider the following statements:

- the k^{th} row of P is zero
- the k^{th} row of PP^T is zero
- the k^{th} column of P is zero
- the k^{th} column of PP^T is zero

Select the correct answer using the codes given below:

- 1 and 3
- 1, 2 and 4
- 1, 3 and 4
- 1, 2 and 3

25. If $X = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ then the rank of $X^T X$,

where X^T denotes the transpose of X, is

- 0
- 2
- 3
- 4

26. Let T be a linear transformation from a 3-dimensional vector space V into a 2-dimensional vector space W. Then T

- can be both injective nor surjective
- can be neither injective nor surjective
- can be surjective but cannot be injective
- can be injective but cannot be surjective

27. Let W_1, W_2, W_3 be subspaces of a vector space V such that $W_3 \subset W_1$. Then which one of the following is correct?

- $W_1 \cap (W_2 + W_3) = W_2 + W_1 \cap W_3$
- $W_1 \cap (W_2 + W_3) = W_1 + W_2 \cap W_3$
- $W_1 \cup (W_2 + W_3) = W_2 + W_1 \cup W_3$
- None of the above

28. Let ξ_1, ξ_2 and ξ_3 be vectors of a vector space V over the field F. If r and s are arbitrary elements of F and the set $(\xi_1, \xi_2, r\xi_1 + s\xi_2 + \xi_3)$ is linearly dependent, then (ξ_1, ξ_2, ξ_3) is

- linearly dependent set
- a null set
- linearly independent set
- none of the above

29. Which of the following mapping $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ is not a linear mapping?

- $(x_1, x_2) \rightarrow (x_2, x_1)$
- $(x_1, x_2) \rightarrow (x_1 + x_2, x_2)$
- $(x_1, x_2) \rightarrow (x_1 + 1, x_2)$
- $(x_1, x_2) \rightarrow (0, 0)$

30. R is the set of real numbers. In \mathbb{R}^2 , let f_1 and f_2 the two transformation defined by

$$f_1(x, y) = (0, y)$$

$$f_2(x, y) = (y, x)$$

Then the product of the mapping $f_2 \circ f_1(x, y)$ gives the projection of x-y plane on the

- y-axis
- x-axis
- Line $y = x$
- Line $y = -x$

31. The dimension of the subspace of \mathbb{R}^3 spanned by $(-3, 0, 1)$, $(1, 2, 1)$ and $(3, 0, -1)$

- 0

- b. 1
c. 2
d. 3
32. If V is a vector space over an infinite field F such that $\dim V = 2$, then the number of distinct subspaces V has is
a. 2
b. 3
c. 4
d. infinite
33. Consider the following linear transformation from the vector space \mathbb{R}^2 into the vector space \mathbb{R}^3 :
 $T(x, y) = (-x, -y, 3x + 8y, 9x - 11y)$
Then the rank and nullity of T are respectively
a. 2 and 0
b. 1 and 3
c. 2 and 1
d. 2 and 3
34. $\vec{a}, \vec{b}, \vec{c}$ are vectors of magnitude 3, 4 and 5 respectively. If \vec{a} is perpendicular to $\vec{b} + \vec{c}$, \vec{b} to $\vec{c} + \vec{a}$ and \vec{c} to $\vec{a} + \vec{b}$, then the magnitude of $(\vec{a} + \vec{b} + \vec{c})$ is
a. $-5\sqrt{2}$
b. $5\sqrt{2}$
c. 7
d. 0
35. If the two vectors are collinear, then their vector product is
a. One
b. the product of their moduli
c. equal to zero
d. none of the above
36. $(\vec{a} \times \vec{b}) + \vec{c} \times (\vec{a} \times \vec{b}) + \vec{c} \times (\vec{a} \times \vec{c})$ is equal to
a. $3\vec{a}$
b. $5\vec{a}$
c. $-\vec{a}$
d. $2\vec{a}$
37. The plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 3$, then the point of contact is
a. (1, 4, 1)
b. (-1, 4, -2)
c. (1, 4, 2)
d. (-1, -4, -2)
38. The vertical angle of the cone $x^2 + y^2 - z^2 = 0$ is
a. $\pi/4$
b. $\pi/3$
c. $\pi/2$
d. $2\pi/3$
39. If t is real number, then the vector equation of straight line AB through two given points A and B with position vectors \vec{a} and \vec{b} , respectively is
a. $\vec{r} = (1-t)\vec{a} + t\vec{b}$
b. $\vec{r} = \vec{a} + t\vec{b}$
c. $\vec{r} = (1+t)\vec{a} + t\vec{b}$
d. $\vec{r} = \vec{a} + t\vec{b}$
40. $\vec{a}, \vec{b}, \vec{a} + \vec{b}$ are the vectors of the same magnitude all vectors being proper vectors, then the angle between \vec{a} and \vec{b} is
a. 0
b. $\pi/2$
c. $\pi/3$
d. $2\pi/3$
41. The equation of the cylinder whose generators are parallel to the line $x/1 = y/1 = z/1$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 0$ is
a. $x^2 + y^2 + z^2 = 1$
b. $x^2 + y^2 + 2z^2 + yz + xz = 0$
c. $x^2 + 2y^2 + 3z^2 - 4yz - 2xz = 1$
d. $x^2 + y^2 + z^2 + yz + zx = 0$
42. The points (1, -2), (3, 2), (1, 2) and (-1, 0) are the vertices of a
a. Rectangle
b. Square
c. Parallelogram
d. None of the above
43. If e and e' are the eccentricities of two conjugate hyperbolas, then
a. $e^2 + e'^2 = 1$
b. $e^2 + e'^2 = 1$

c. $\frac{1}{e^2} + \frac{1}{e^2} = 1$

d. $\frac{1}{e^2} + \frac{1}{e^2} = \frac{1}{2}$

44. The value of $|\vec{a} + \vec{b}|$ is

a. $\sqrt{(\vec{a} \cdot \vec{a})(\vec{b} \cdot \vec{b}) - (\vec{a} \cdot \vec{b})^2}$

b. $\sqrt{(\vec{a} \cdot \vec{a})(\vec{b} \cdot \vec{b}) - 2(\vec{a} \cdot \vec{b})^2}$

c. $\sqrt{a^2 + b^2 + 2(\vec{a} \cdot \vec{b})}$

d. $\sqrt{(\vec{a} \cdot \vec{a})(\vec{b} \cdot \vec{b}) + (\vec{a} \cdot \vec{b})^2}$

45. The line $3x + 4y - 24 = 0$ cuts the x-axis at A and y-axis at B. Then the in center of the triangle OAB, where O is the origin is

a. (1, 2)

b. (2, 2)

c. (12, 12)

d. (2, 12)

46. The equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle is

a. $x^2 + y^2 + z^2 - 2x + 4y - 6z + 10 = 0$

b. $x^2 + y^2 + z^2 - 2x - 4y + 4z + 7 = 0$

c. $x^2 + y^2 + z^2 - 6x - 4y - 4z + 9 = 0$

d. $x^2 + y^2 + z^2 - 6x + 4y + 4z - 9 = 0$

47. $|\vec{a} \cdot \vec{b}| = |\vec{a}| \cdot |\vec{b}|$, where \vec{a} and \vec{b} are any vectors. This equality

a. always holds

b. never holds

c. holds only where $\vec{a} = \vec{b} = 0$

d. holds only when $\vec{a} = k\vec{b}$, $k \geq 0$ or one of \vec{a} and \vec{b} is zero

48. If \vec{a} is the position vector of a given point A and \vec{r} is the position vector of any point P in space with respect to origin O then the equation $(\vec{r} - \vec{a}) \cdot \vec{r} = 0$ representing the locus of the point P is

a. the sphere with OA as diameter

b. the plane which passes through A and is perpendicular to OA

c. the straight line OA

d. the plane perpendicular to the axis of α

49. For the given sequence $\left\{ (-1)^n \left(1 + \frac{1}{n} \right) \right\}$ which one of the following statements is correct?

a. Limit superior = limit inferior

b. Neither limit superior nor limit inferior exists

c. Limit superior = 1 and limit inferior = -1

d. Limit superior = 1 and limit inferior = 0

50. Asymptote of the curve $x^3 + y^3 - 3axy = 0$ is

a. $x + y + a = 0$

b. $x + y - a = 0$

c. $x + a = 0$

d. $y + a = 0$

51. Consider the following statements:

1. the length of the subnormal to the parabola $y^2 = 4ax$ at any point is $2a$

2. the length of the subtangent to the parabola $y^2 = 4ax$ varies as the abscissa of the point of contact.

3. The equation of the tangent at the origin to the parabola $y^2 = 4ax$ is x-axis.

Which of the statements are correct?

a. 1, 2 and 3

b. 1 and 2

c. 1 and 3

d. 2 and 3

52. Let $f(x) = x^2 - 4x + 3$. The following statements are associated with f:

1. f is increasing in $(2, \infty)$

2. f is decreasing in $(-\infty, -2)$

3. f has a stationary point at $x = 2$

Which of these statements are correct?

a. 1 and 2

b. 1 and 3

c. 2 and 3

d. 1, 2 and 3

53. The value of c in Rolle's theorem, where $-\pi/2 < c < \pi/2$ and $f(x) = \cos x$, is equal to

a. $\pi/4$

b. $\pi/3$

c. π

d. 0

54. Let $f(x) = x^p \sin 1/x$, when $x \neq 0$ and $f(x) = 0$, when $x = 0$. Then $f(x)$ can be differentiated at the origin provided p is
- Any real number
 - Zero
 - One
 - An integer greater than 1
55. A function $f(x)$ is differentiable in the closed and bounded interval $[p, q]$. The differential $f'(p)$ and $f'(q)$ are such that $f'(p) < 0$ and $f'(q) > 0$. Then $f(x)$ will assume on $[p, q]$
- neither infimum nor supremum
 - supremum and not infimum
 - infimum and not supremum
 - none of the above
56. Consider the following statements:
- a differentiable function is continuous
 - a continuous function is differentiable
 - a continuous function on a closed interval $[a, b]$ of finite length is uniformly continuous on $[a, b]$
- Which of these statements are correct?
- 1, 2 and 3
 - 2 and 3
 - 1 and 3
 - 1 and 2
57. Let $f(x)$ be defined on \mathbb{R} by
- $$f(x) = \begin{cases} x, & \text{if } x \text{ is a rational number} \\ 5-x, & \text{if } x \text{ is an irrational number} \end{cases}$$
- Then the function is continuous in the interval
- No point
 - All points
 - 2 points
 - one point
58. $A \subset \mathbb{R}$, \mathbb{R} is the field of real numbers. $F(A)$ denote the collection of all real valued functions defined on A . The two functions $g, f \in F(A)$ are such that they satisfy the algebraic operation $(f + g)(x) = f(x) + g(x)$, $x \in A$. Then with respect to this operation the set $F(A)$ is
- Always closed
 - Never closed
 - Closed only for selected functions
 - None of the above
59. Given the following limits:
- $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
 - $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$
 - $\lim_{x \rightarrow 0} \frac{\log(x+1)}{x} = 1$
- Which of these are correct?
- 1, 2 and 3
 - 1 and 2
 - 1 and 3
 - 2 and 3
60. For a bounded function $f(x)$ the upper and lower bounds are M and m and $f(x)$ is R-integrable on the interval $[a, b]$. Given $\int_a^b f(x) dx = \mu(b-a)$ such that $m < \mu < M$, μ in $[a, b]$ can
- always be determined
 - never be determined
 - only be conditionally determined
 - be determined only for $a < \mu < b$
61. For the curve
- $$x = f(\theta) \sin \theta + f'(\theta) \cos \theta$$
- $$\text{and } y = f(\theta) \cos \theta - f'(\theta) \sin \theta,$$
- Where derivatives are with respect to θ , the length of arc is
- $(\theta) + f''(\theta) + C$
 - $f'(\theta) + f''(\theta) + C$
 - $(\theta) - f'(\theta) + C$
 - $f'(\theta) - f''(\theta) + C$
62. The length of the curve $y = \log \sec x$ from $x = 0$ to $x = \pi/3$ is equal to
- $\log(3 + \sqrt{3})$
 - $\log(2 + \sqrt{3})$
 - $\log(x + \sqrt{3})$
 - $\log(2 \cos x)$
63. For the function $f(x, y) = x^5 F(y/x)$, the value of the differential $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$ is equal to
- $5x^5 F(y/x)$

- b. $5x^4F(y/x)$
 c. $4x^5F(y/x)$
 d. $4x^4F(y/x)$
64. The only double point of the curve $(x-2)^2 - y(y-1)^2 = 0$ is :
 a. $(2, \frac{1}{3})$
 b. $(0, 0)$
 c. $(2, 1)$
 d. $(2, 2)$
65. If $u = x^2 + y^2 + z^2$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$ is equal to:
 a. $4u$
 b. u^2
 c. u
 d. $2u$
66. Let $A_n = \int_0^{\pi/4} \tan^n x \, dx$. Then the value of $A_{10} + A_8$ is
 a. $1/8$
 b. $1/9$
 c. $1/10$
 d. $-1/9$
67. The arc of parabola $y^2 = 4ax$ between the points, where the latus rectum and a line parallel to the latus rectum and a distance twice the distance of latus rectum from the vertex meet the parabola, is revolved about the x-axis. Then the area of the surface of revolution so formed is
 a. $\pi\sqrt{5}$
 b. $2\pi/5(\sqrt{3}-1)$
 c. $4\pi/5(3\sqrt{3}-2\sqrt{2})$
 d. $8\pi/5(3\sqrt{3}-2\sqrt{2})$
68. Given the sequence $(\frac{n}{n+1})_{n=1}^{\infty}$ and an arbitrarily small positive number ϵ . Then the value of a positive integer N such that $|\frac{n}{n+1} - 1| < \epsilon$ whenever $n \geq N$ must satisfy the relation
 a. $N \leq (\frac{1}{\epsilon} - 1)$
 b. $N < (\frac{1}{\epsilon} - 1)$
 c. $N > (\frac{1}{\epsilon} - 1)$
 d. $N \geq (\frac{1}{\epsilon} - 1)$
69. The general solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = \frac{e^{2x}}{x}$ is
 a. $C_1 + C_2x \log x - \frac{x^4}{2}e^{2x}$
 b. $C_1 + C_2x + x \log x - \frac{x^4}{2}e^{2x}$
 c. $C_1e^{2x} + C_2e^{-2x} + \frac{x^2}{2} \log x - \frac{x^2}{2}$
 d. $C_1e^{2x} + C_2e^{-2x} + \frac{x^2}{2} \log x - x$
70. The solution of the equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 24y = 0$ with $y(0) = 0, y'(0) = 3$ is
 a. $e^x + e^{2x}$
 b. $e^x - e^{2x}$
 c. $e^x - e^{-2x}$
 d. $e^x + e^{-2x}$
71. The singular solution of the differential equation $y = xp + a\sqrt{1+p^2}$ is a
 a. parabola
 b. hyperbola
 c. circle
 d. straight line
72. The general solution of the differential equation $(\sin y - y \sin xy) dx + (x \cos y - x \sin xy) dy = 0$ is
 a. $x \cos y + \sin xy = \lambda$
 b. $x \cos y - \sin xy = \lambda$
 c. $x \sin y + \cos xy = \lambda$
 d. $x \sin y - \cos xy = \lambda$
73. The solution of the equation $\frac{dy}{dx} = \frac{2x - y + 1}{x + 2y - 3}$ is
 a. $xy + y^2 + x^2 + 3y - x = c$
 b. $xy + y^2 - x^2 - 3y - x = c$
 c. $xy + y^2 + x^2 - 3y - x = c$
 d. $xy + y^2 - x^2 - 3y + x = c$

74. If in a culture of yeast y_0 is the amount of y east present initially, and the rate of growth dy/dt is proportional to the amount y at time t and if y doubles in one day then the amount expected after 3 days equals
- $3y_0$
 - $8y_0$
 - $9y_0$
 - $27y_0$
75. If the rate of growth is proportional to the amount x of the substance present and $dx/dt = kx$, then x is equal to (with C_1 constant)
- $C_1 e^{-kt}$
 - $C_1 e^{kt}$
 - $C_1 e^{-2kt}$
 - $C_1 e^{2kt}$
76. The general solution of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x} \cos x$ is
- $(C_1 + C_2 x + \sin x)e^{-x}$
 - $(C_1 + C_2 x - \sin x)e^{-x}$
 - $(C_1 + C_2 x + \cos x)e^{-x}$
 - $(C_1 + C_2 x - \cos x)e^{-x}$
77. The orthogonal trajectories of the family $y^2 = 4ax + 4a^2$ is the family
- $x^2 = 4ay + 4a^2$
 - $y^2 = 4ay + 4a^2 x$
 - $y^2 = 4ax + 4a^2$
 - $x^2 = 4ax + 4a^2$
78. A singular solution of the differential equation $y^2(1 + (dy/dx)^2) = R^2$ is
- $y = R/2$
 - $y = R$
 - $y = R/2$
 - $y = 2R$
79. Two scale pans each of mass 4kg are connected by string passing over a pulley. Two masses whose sum is 10 kg are to be placed in the two scale pans so as to produce an acceleration of $8/9g$ ($g =$ acceleration due to gravity). Two masses are
- 1 kg and 9 kg
 - 2 kg and 8kg
 - 3kg and 7kg
 - 4 kg and 6kg
80. Two equal rods each of length $2a$ and weight W are freely jointed at one end. The system is placed in a vertical position with the other ends of the two rods on a smooth horizontal plane and the middle points of the rods joined by a string of length l which is kept taut in equilibrium position. The tension of the string is
- $\frac{Wl}{2a}$
 - $\frac{Wl}{\sqrt{4a^2 - l^2}}$
 - $\frac{2Wl}{\sqrt{4a^2 - l^2}}$
 - $\frac{Wl}{\sqrt{4a^2 - l^2}}$
81. The velocity of a boat relative to water $2\hat{i} + \hat{j}$ and that of water relative to earth is $2\hat{i} + 2\hat{j}$, where \hat{i}, \hat{j} represent velocities of one km on hour due East and North respectively. The velocity of boat relative to earth is
- 3 km an hour due East
 - $2\sqrt{2} + \sqrt{5}$ km on hour at 45° North of East
 - $\sqrt{17}$ km an hour at $\tan^{-1}(4)$ North of East
 - $\sqrt{17}$ km an hour at $\tan^{-1}(4)$ due North
82. A ladder 5 meter long rests on a rough horizontal plane and against a smooth vertical wall. The ladder is about to slip on the rough plane when its lower end is at a distance 4 metres from the wall. The coefficient of friction is
- $2/3$
 - $1/2$
 - $3/5$
 - $4/5$
83. C.G. of any three equal particles placed at the vertices of a triangle is at the
- incentre of the triangle
 - orthocentre of the triangle
 - circumcentre of the triangle
 - centroid of the triangle

84. A particle acted upon by three forces whose magnitudes are proportional to (i) 3, 5, 8; (ii) 5, 7, 10; and (iii) 5, 7, 14; can be arranged at rest
- only in case (ii)
 - only in case (i)
 - only in cases (i) and (ii)
 - in all the three cases
85. Two parallel like forces P and Q act at a distance α apart and have a resultant which acts at a point C. A couple of moment M and arm of length β is combined with these like parallel forces. The resultant will be force
- $P+Q+\frac{M}{\alpha}$ acting at C
 - $P+Q$ acting at a distance $M/(P+Q)$ from C
 - $P+Q$ acting at a distance $\frac{\alpha M}{\beta(P+Q)}$ from C
 - $P+Q$ acting at a distance $\frac{M}{\alpha}$ from C
86. P and Q are two like parallel forces. If P is moved parallel to itself through a distance of 4 cm, then the resultant of P and Q moves through a distance of
- 2 cm
 - 4 cm
 - $\frac{4Q}{P+Q}$ cm
 - $\frac{4P}{P+Q}$ cm
87. The resultant of the forces P and Q is R. If Q is doubled then R gets doubled in magnitude. And again doubled if Q is reversed. Then P^2 , Q^2 and R^2 are in the ratio
- 2 : 1 : 3
 - 3 : 2 : 2
 - 2 : 3 : 2
 - 2 : 3 : 3
88. Two forces P and Q where $P=2Q$ acts at a point in directions at right angles to each other. Their resultant will be inclined to P at angle θ such that
- $0 < \theta < \pi/6$
 - $\pi/6 < \theta < \pi/4$
 - $\pi/4 < \theta < \pi/3$
 - $\pi/3 < \theta < \pi/2$
89. A stone falling vertically from rest travels half of its total path in the last second of its fall. The total time of its fall is
- less than 2 seconds
 - equal to 2 seconds
 - greater than 2 seconds but less than 3 seconds
 - greater than 3 seconds
90. An elastic ball hits a horizontal floor vertically with a speed of u and is allowed to strike the floor a further two times after which it rebounds from the floor with a velocity $27/64 u$. Neglecting the resistance of air, the coefficient of elasticity between the ball and floor is
- $1/5$
 - $3/4$
 - $3/4$
 - $1/16$
91. A particle of unit mass oscillates in a straight path under a force of attraction proportional to its distance from a fixed point O. If its velocity at distance a, b from O be v_1 and v_2 respectively, then the period of oscillation is
- $2\pi \sqrt{\frac{b^2 - a^2}{v_1^2 - v_2^2}}$
 - $2\pi \sqrt{\frac{v_1^2 - v_2^2}{b^2 - a^2}}$
 - $2\pi \sqrt{\frac{b^2 - a^2}{v_1^2 + v_2^2}}$
 - $2\pi \sqrt{\frac{b^2 + a^2}{v_1^2 + v_2^2}}$
92. Consider the algorithm :
- $i \leftarrow 10$
 - $i \leftarrow (i - 1)$
 - if $i > 0$, go to step 2
 - Print i
- The looping of statements 2-3 during execution, occurs
- 9 times
 - 10 times
 - 11 times

- d. 12 times
93. 51 letters of a certain language are to be uniquely coded using strings of 6 bits. In this system, how many possible non-printable control characters can be uniquely coded?
- 0
 - 8
 - 13
 - 77

94.



Which of the following logical operations is equivalent to the given connection?

- $A+(B \cdot C)$
 - $B \cdot C$
 - $A \cdot (B+C)$
 - $B+C$
95. Given the bit strings $p = 11011$ and $q = 10101$, which of the following is the value of $(\text{NOT } p) \text{ AND } q$?
- 00100
 - 11111
 - 10101
 - 10001
96. Given that the binary equivalent of decimal 12 is stored in a one-byte register. Which one of the following is the content of this register after the shift-left operation is performed 3 times?
- 0000101
 - 0010000
 - 0000000
 - 0000000

97. Assertion(A): If the position of the resultant of two parallel forces which are like, remains unaltered when the forces are interchanged, the forces are reciprocal of each other.

Reason (R): The resultant of two like parallel forces P and Q acting at A and B respectively acts at a point C on AB such that $P \times AC = Q \times BC$.

- Both A and B are true and R is the correct explanation of A
- Both A and R are true but R is not a correct explanation of A .
- A is true but R is false
- A is false but R is true

98. Let $n \geq 3$ and let the complex numbers $\alpha_1, \alpha_2, \dots, \alpha_n$ be the roots of $x^n - 1$ with $\alpha_1 = 1$.

Assertion (A): For any positive integer $t > 1$, $(t - \alpha_2)(t - \alpha_3) \dots (t - \alpha_n)$ is a positive integer.

Reason (R): For any positive integer t , $(t - \alpha_2)(t - \alpha_3) \dots (t - \alpha_n)$ equals $(t^{n-1} - 1)$.

- Both A and B are true and R is the correct explanation of A
- Both A and R are true but R is not a correct explanation of A .
- A is true but R is false
- A is false but R is true

99. Let q_1, q_2 be non-zero rational numbers. Let $q_1 = n_1/m$ and $q_2 = n_2/m$ where $n_1, n_2 \in \mathbb{Z}$, $m \in \mathbb{Z}$ and $m > 0$. Let $H = \{xq_1 + yq_2 : x, y \in \mathbb{Z}\}$ (\mathbb{Z} denotes the set of all integers)

Assertion (A): H forms a subgroup of additive group of rationals and is cyclic.

Reason (R): Any subgroup of a cyclic group is cyclic $H \cong \mathbb{Z}_q$ where $q = l/m$ and $l =$ least common multiple of n_1 and n_2 .

- Both A and B are true and R is the correct explanation of A
- Both A and R are true but R is not a correct explanation of A .
- A is true but R is false
- A is false but R is true

100. Assertion (A): $(189)_{10} = (10111101)_2$.

Reason (R): The binary number can be expressed in powers of its base 2 as $\sum_{i=1}^n b_i 2^{i-1}$ where b_i is the i^{th} bit (from the right).

- Both A and B are true and R is the correct explanation of A
- Both A and R are true but R is not a correct explanation of A .
- A is true but R is false
- A is false but R is true