

# MATHEMATICS

1. Which one of the following statements is always correct for arbitrary sets A, B and C?
- If  $A \in B$  and  $B \subseteq C$ , then  $A \in C$
  - If  $A \in B$  and  $B \subseteq C$ , then  $A \in C$
  - If  $A \subseteq B$  and  $B \in C$ , then  $A \in C$
  - If  $A \subseteq B$  and  $B \in C$ , then  $A \subseteq C$
2. Let  $A = \mathbb{N} - \{1\} = \{k \in \mathbb{N}; k > 1\}$ ,  
 $X_2 = \{2k : k \in A\}$   
 $X_3 = \{3k : k \in A\}, \dots$   
 $X_j = \{jk : k \in A\}, \dots$  then what is the complement of  $\bigcup_{j=2}^{\infty} X_j$  in A equal to?
- A finite non-empty set
  - $\emptyset$
  - The set of primes
  - The set of composite numbers
3. What are the four values of  $\left(\frac{1}{2} + \frac{1}{2}\sqrt{-3}\right)^{3/4}$ ?
- $\left(\pm \frac{1}{\sqrt{2}}\right)(1 \pm i)$
  - $\left(\pm \frac{1}{\sqrt{2}}\right) \pm i$
  - $\pm 1 \pm \frac{i}{\sqrt{2}}$
  - $\pm 1 \pm \sqrt{3}i$
4. If a, b and c are three non-zero real numbers and  $a \geq b \geq c$ , then which one of the following is always correct?
- $a \geq a^2 \geq a^3$
  - $|a|b \geq |a|c$
  - $|ab| \geq |ac|$
  - $a|b| \geq a|c|$
5. Which one of the following is the set of all the real numbers x satisfying  $\|3-x\| - \|x+2\| = 5$ ?
- $[3, \infty]$
  - $]-\infty, -2]$
  - $]-\infty, -2] \cup [3, \infty]$
  - $]-\infty, -3] \cup [2, \infty]$
6. Which one of the following is an example of a polynomial equation not having any real root?
- $x^3 + 3 = 0$
  - $x^2 - x + 1 = 0$
  - $x^4 - 5 = 0$
  - $x^3 + 7 = 0$
7. What is the nature of the roots of the equation  $(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$  where a, b, c are real numbers?
- Always positive
  - Always negative
  - Always real
  - Always imaginary
8. If  $\alpha_1, \alpha_2, \dots, \alpha_n$  are the roots of the equation  $x^n + 1 = 0$ , then what is  $\{(1-\alpha_1)(1-\alpha_2)\dots(1-\alpha_n)\}$  equal to?
- 0
  - 1
  - 2
  - n
9. In the set  $G = (\mathbb{R} - \{0\}) \times \mathbb{R}$ , consider the binary operation 'o' given by  $(a, b) o (c, d) = (ac, bc + d)$ . What is the identity in g for this binary operation?
- (1, 1)
  - (1, -1)
  - (1, 0)
  - (0, 1)
10. Let G be the additive group  $Z_8$  of the residue classes modulo 8. Then what is the order of the residue class  $\bar{3}$  in G?
- 8
  - 6
  - 4
  - 2
11. On the set  $\mathbb{N}$  of natural numbers consider the binary relations
- $$a \otimes b = a^b$$
- $$a \oplus b = a + b - ab$$
- Which one of the following is correct?
- $\otimes$  is both commutative and associative
  - $\oplus$  is both commutative and associative
  - $\otimes$  is commutative and  $\oplus$  is associative

- d.  $\oplus$  is commutative and  $\otimes$  is associative
12. What is the least number of elements in a non abelian group all of whose proper subgroups are cyclic?
- 5
  - 6
  - 7
  - 8
13. What is the number of proper subgroups of the group  $(\mathbb{Z}, +)$  of integers?
- 0
  - 1
  - 2
  - infinite
14. Consider the following statements:
- Every cyclic group is abelian.
  - Every abelian group is cyclic.
  - There is at least one abelian group of order  $n$ , for every  $n \in \mathbb{N}$
  - Every group of  $<4$  is cyclic.
- Which of the statement given above are correct?
- 1, 2 and 3
  - 2, 3 and 4
  - 1, 3 and 4
  - 1 and 4 only
15. Let  $G = \{a, a^2, \dots, a^n = e\}$  be a group. What is the number of generators in  $G$ ?
- 2
  - 3
  - 4
  - 7
16. Consider the permutation:  $\alpha = (123)(145)$  on the set  $\{1, 2, 3, 4, 5\}$ . What is the permutation  $\alpha^{27}$ ?
- $(54)(132)$
  - $(14)(123)$
  - $(12)(154)$
  - $(132)(154)$
17. The matrices  $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} i & 0 \\ 0 & 0 \end{bmatrix}$  and  $\begin{bmatrix} -i & 0 \\ 0 & 0 \end{bmatrix}$  form a group with respect to matrix multiplication. Which one of the following statements about this group is correct?
- The group has no elements of order 4
  - The group has an element of order 3
  - The group is non-abelian
  - $\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$  is its own inverse.
18. Suppose that  $H$  and  $K$  are subgroups of a group  $G$  with  $O(H) = 10$  and  $O(K) = 12$ . If  $H \cap K$  is non-trivial, then what is the number of elements in the complex  $HK$ ?
- 120
  - 60
  - 30
  - 22
19. Consider the ring  $\mathbb{Z}_n = \{\overline{0}, \overline{1}, \dots, \overline{n-1}\}$  of congruent modulo  $n$  classes. Under addition and multiplication modulo  $n$ , consider the following statements:
- In  $\mathbb{Z}_6$ ,  $\overline{4}$  divides  $\overline{2}$ .
  - In  $\mathbb{Z}_8$ ,  $\overline{3}$  divides  $\overline{7}$ .
  - In  $\mathbb{Z}_{15}$ ,  $\overline{9}$  divides  $\overline{12}$ .
- Which of the statements given above are correct?
- 1 and 2, only
  - 2 and 3, only
  - 1 and 3, only
  - 1 and 2 and 3.
20. Let  $V$  be a vector space over a field  $F$ .  $\alpha \in R$  and  $u \in V$ . Which of the following statements is not correct?
- $\alpha u = \theta \Rightarrow$  either  $\alpha = 0$  or  $u = \theta$
  - $|-1u| = |-1|u$  for all  $u \in V$
  - $\alpha\theta = \theta$
  - $0u = \theta$
21. What is the dimension of the vector space over the field  $\mathbb{R}$ ?
- 1
  - Infinite
  - 2
  - 4
22. Consider the real vector space  $\mathbb{R}^3$ . The subspace  $\{(x, y, z) \in \mathbb{R}^3 \mid y = x\}$  of  $\mathbb{R}^3$  is generated by which one of the following?
- $\{(1, 1, 0), (0, 0, 1)\}$
  - $\{(1, 1, 0), (1, 0, 0)\}$
  - $\{(1, 0, 0), (0, 1, 0)\}$
  - $\{(1, 0, 1), (0, 0, 1)\}$
23. Let  $S = \{(1, 1, 0)(2i, 1, 1), (0, 1 + i, 1 - i)\}$  be a subset of the complex vector space  $\mathbb{C}^3$  and  $T = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ , be a subset of the real vector space  $\mathbb{R}^3$ . Which one of the following statements is correct?

- a. S and T are both basis  
 b. S is a basis but T is not a basis  
 c. S is not a basis but T is a basis  
 d. Neither S nor T is a basis

24. Consider the mappings:

1.  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2 \ni T(x, y, z) = (x+1, y+z)$   
 2.  $T: \mathbb{R}^2 \rightarrow \mathbb{R} \ni T(x, y) = xy$   
 3.  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2 \ni T(x, y, z) = (|x|, 0)$

Which of the above are linear transformations?

- a. 1, 2, 3  
 b. 1 and 2, only  
 c. 2 and 3, only  
 d. None of these.
25. Let  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be a linear transformation given by  $T(x, y, z) = (x/2, y/2, 0)$ . What is the rank of T?

- a. 4  
 b. 3  
 c. 2  
 d. 1

26. Let  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be a linear transformation given by  $T(x, y, z) = (x, y, 0)$ . Then the null space is generated by which one of the following?

- a.  $(0, 0, 1)$   
 b.  $(0, 1, 0)$   
 c.  $(1, 0, 0)$   
 d. None of these

27. If  $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ , then what is  $A^n$  equal to?

- a.  $\begin{bmatrix} 1+2n & -4n \\ n & -2n \end{bmatrix}$   
 b.  $\begin{bmatrix} 3^n & -4^n \\ 1 & -1^n \end{bmatrix}$   
 c.  $\begin{bmatrix} 1+3n & 1-4n \\ 1+n & 1-n \end{bmatrix}$   
 d.  $\begin{bmatrix} 1+2n & -4n \\ 1+n & 1-2n \end{bmatrix}$

28. If  $A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$  and  $MA = A^{2m}$ ,  $m \in \mathbb{N}$  for some matrix M, then which one of the following is correct?

- a.  $M = \begin{bmatrix} a^{2m} & b^{2m} \\ b^{2m} & -a^{2m} \end{bmatrix}$

b.  $M = (a^2 + b^2)^m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c.  $M = (a^m + b^m) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d.  $M = (a^2 + b^2)^{m+1} \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$

29. If  $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 3 & 2 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$  then what is  $\det(A-1)$

equal to?

- a. 2  
 b. -2  
 c.  $\frac{1}{2}$   
 d. -1/2

30. If A is a  $2 \times 2$  non-singular matrix, then what is  $\text{adj}(\text{adj} A)$  equal to?

- a.  $A^2$   
 b. A  
 c.  $A^{-1}$   
 d.  $-A$

31. If A is a  $3 \times 3$  matrix with  $\det A = 5$  and if  $B = 4A^2$ , then what is  $\det B$  equal to?

- a. 20  
 b. 100  
 c. 320  
 d. 1600

32. The lines  $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$  and  $\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$  are coplanar.

- a.  $x - y + z = 0$   
 b.  $x + y - z = 0$   
 c.  $x - 2y + z = 0$   
 d.  $x + 2y + z = 0$

33. The intersection of two parabolic cylinders  $y + 4z^2 = 1$  and  $y = 4x^2$  is a space curve. What is its projection on the  $z-x$  plane?

- a. Parabola  
 b. Circle  
 c. Ellipse  
 d. Hyperbola

34. Which one of the following lines are generators to the cone  $yz + 4zx + 3xy = 0$ ?

- a.  $x = y = z$   
 b.  $x = -y = z$   
 c.  $x = 2y = -z$

- d.  $x = y = -z$
35. What is the position vector of the centre of the sphere  $|\vec{r}|^2 + \vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) - 9 = 0$ .
- $\hat{i} + \hat{j} - \hat{k}$
  - $1/2(\hat{i} + \hat{j} - \hat{k})$
  - $1/2(-\hat{i} - \hat{j} + \hat{k})$
  - $-\hat{i} - \hat{j} + \hat{k}$
36. The point (4, 1) undergoes the following two successive transformations:
- Reflection about the line  $y = x$ .
  - Translation through a distance 2 unit along the positive x-axis.
- What are the final coordinates of the point?
- (4, 3)
  - (3, 4)
  - (1, 4)
  - (6, 1)
37. When does the equation  $ax^2 + ay^2 + az^2 + 2ux + 2vy + 2wz + d = 0$ , ( $a \neq 0$ ) represent a sphere?
- $u^2 + v^2 + w^2 - d \geq 0$
  - $u^2 + v^2 + w^2 - ad \geq 0$
  - $u^2 + v^2 + w^2 - d \leq 0$
  - $u^2 + v^2 + w^2 - ad < 0$
38. If  $\langle l_1, m_1, n_1 \rangle, \langle l_2, m_2, n_2 \rangle, \langle l_3, m_3, n_3 \rangle$  are the direction cosines of three mutually perpendicular lines then what is/are the value(s) of  $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$ ?
- 0
  - $\pm 1$
  - $\pm 2$
  - 3
39. The line  $(p + 2q)x + (p - 3q)y = p - q$  for all values of p and q passes through which one of the following points?
- (3/2, 5/2)
  - (1/5, 2/5)
  - (3/5, 3/5)
  - (2/5, 3/5)
40. What is the equation of a circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length 3a?
- $x^2 + y^2 = 9a^2$
  - $x^2 + y^2 = 6a^2$
  - $x^2 + y^2 = 4a^2$
  - $x^2 + y^2 = a^2$
41. Under which one of the following conditions does the equation  $ax^2 + by^2 + cx + cy = 0, c \neq 0$  represent a pair of straight lines?
- $a + b = 0$
  - $a + c = 0$
  - $b + c = 0$
  - $a + b + c = 0$
42. What does the curve  $x = 3\cos t - \sin t, y = 4(\cos t - \sin t)$  represent?
- Ellipse
  - Parabola
  - Hyperbola
  - Circle
43. A, B, C are fixed points and P is a variable point. Sum of vectors  $\vec{PA}$  and  $\vec{PB}$  always passes through C. What is the ratio in which C divides AB?
- 1 : 2
  - 2 : 1
  - 1 : 1
  - 1 : 3
44. If  $\vec{A} = 2\hat{i} + \hat{k}, \vec{B} = \hat{i} + \hat{j} + \hat{k}, \vec{C} = 4\hat{i} - 3\hat{j} + 7\hat{k}$  and  $\vec{r} \times \vec{B} = \vec{C} \times \vec{B}, \vec{r} \cdot \vec{A} = 0$ , then what is  $\vec{r}$ ?
- $\hat{i} - 8\hat{j} + 2\hat{k}$
  - $2\hat{i} + 8\hat{j} + 4\hat{k}$
  - $-\hat{i} + 8\hat{j} + 2\hat{k}$
  - $-2\hat{i} - 6\hat{j} + 2\hat{k}$
45. If the vectors  $\hat{i} - 2p\hat{j} + 3q\hat{k}$  and  $\hat{i} + 2p\hat{j} - 3q\hat{k}$  are orthogonal, then what is the locus of the point (p, q)?
- Circle
  - Ellipse
  - Hyperbola
  - Pair of straight lines
46. For any vector  $\vec{a}$ , what is  $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ ?
- $|\vec{a}|^2$
  - $2|\vec{a}|^2$
  - $3|\vec{a}|^2$
  - $4|\vec{a}|^2$

47. If  $\vec{a}, \vec{b}, \vec{c}$  be three unit vectors such that  $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}(\vec{b})$ , what is the angle which

$\vec{a}$  makes with  $\vec{c}$ ?

- a.  $30^\circ$   
 b.  $60^\circ$   
 c.  $90^\circ$   
 d. cannot be determined
48. What is the value of  $\lim_{x \rightarrow \infty} \{\sin(1/x) + \cos(1/x)\}^x$ ?

- a. 0  
 b.  $1/e$   
 c.  $e$   
 d. 1

49. Let  $f(x+y) = f(x) \cdot f(y)$  for all  $x$  and  $y$ , and  $f(5) = -2$ ,  $f'(0) = 3$ . What is the value of  $f'(5)$ ?

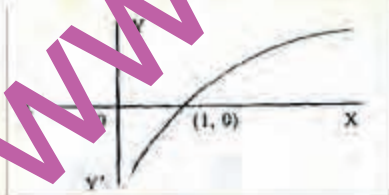
- a. 3  
 b. 1  
 c. -6  
 d. 6

50. Which one of the following functions is well-defined?

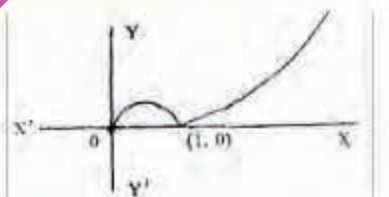
- a.  $\sqrt{1 + \sin x}$   
 b.  $\sqrt{5 \sec^2 x - 4}$   
 c.  $\sqrt{3 - \cos^2 x}$   
 d.  $\sqrt{x^2 + x^2 + \frac{1}{100}}$

51. Which one of the following graphs is the correct graph of the function  $y = x / \ln x$ ?

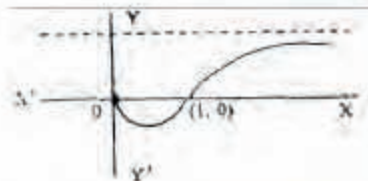
a.



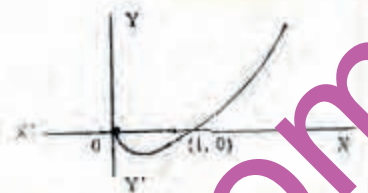
b.



c.



d.



52. The function  $f(x) = \sin^2 x - \sin x$  is defined on the open interval  $(-\pi/2, \pi/2)$  and it assumes only 1 maximum value and only 1 minimum value on this interval. Then, which one of the following must be correct?

- a.  $0 < m < 1$   
 b.  $-1 < m < 0$   
 c.  $m = 0$   
 d.  $m = 3$

53. What is the value of the integral

$$\int_0^{\pi/2} \frac{1 - \sin 3x}{1 + \sin x} dx ?$$

- a. 2  
 b.  $2\pi/3$   
 c.  $4 + \frac{3}{2}\pi$   
 d.  $\frac{3\pi}{2} - 4$

54. What is the length of an arc of the curve  $y = 1 - \ln \cos x$  intercepted between  $x = 0$  and  $x = \pi/4$ ?

- a.  $\ln(\sqrt{2} + 1)$   
 b.  $(\ln \sqrt{2} + 2)$   
 c.  $1 - (\ln \sqrt{2})$   
 d.  $(\ln \sqrt{2}) - 1$

55. Which one of the following is correct?

The function defined by  $f(x) = \ln(|x-1| + 2|x+1|)$  is a monotonically decreasing function on the largest interval defined by

- a.  $-1 < x < 1$   
 b.  $x > 1$   
 c.  $x < -1$   
 d.  $x \leq -1$

56. Let  $I_1 = \int_0^{\pi} \frac{\log\left(\frac{x+1}{x}\right)}{1+x^2} dx$  and  $I_2 = \int_0^{\pi/2} \log \sin 2t dt$

What is the value of  $(I_1 + I_2)$ ?

a.  $\frac{\pi}{2} \log 2$

b.  $\frac{-\pi \log 2}{2}$

c.  $\pi \log 2$

d.  $-\pi \log 2$

57. Let  $\frac{e^x}{e^x+1} = \frac{1}{2} + \frac{x}{4} + \alpha x^2 + \frac{x^3}{48} + \dots$

What is the value of  $\alpha$ ?

[Use  $f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \dots$ ]

a.  $\alpha = 1/16$

b.  $\alpha = 1/8$

c.  $\alpha = 0$

d.  $\alpha = -1/16$

58. Let  $f(x) = \int \sec^{2/3} x \operatorname{cosec}^{4/3} x dx$ . What is the value of  $f(x)$ ?

a.  $-3(\cot x)^{1/3} + c$

b.  $-3(\tan x)^{-2/3} + c$

c.  $3(\operatorname{cosec} x)^{1/3} + c$

d.  $3(\cos 2x)^{1/3} + c$

59. Let  $f(x)$  be continuous and differentiable over  $(a-h, a+h)$ ,  $a \in \mathbb{R}$ .

$$F(x) = \begin{cases} \left( \frac{f'(x)}{f(x) - f(a)} - \frac{1}{x-a} \right), & \text{if } x \neq a \\ a, & \text{if } x = a \end{cases}$$

what is  $F(x)$  continuous at  $x = a$ ?

a.  $\alpha = 1$

b.  $\alpha = f'(a)/f(a)$

c.  $\alpha = a$

d.  $\alpha = \frac{f'(a)}{2f(a)}$

60. Given  $f(x) = \begin{cases} x, & \text{when } 0 \leq x < 1/2 \\ 1/2, & \text{when } x = 1/2 \\ 1-x, & \text{when } 1/2 < x \leq 1 \end{cases}$

and  $G(x) = \left(x - \frac{1}{2}\right)^3, x \in \mathbb{R}$ . What is the

area in the interval  $\left[\frac{1}{2}, \frac{\sqrt{3}}{2}\right]$ ?

a.  $\frac{\sqrt{3}-1}{4-3}$

b.  $\frac{\sqrt{3}}{4} + \frac{1}{3}$

c. 0

d.  $\frac{\sqrt{3}}{12}$

61. Which one of the asymptotes of the curve

$$y = \frac{x+3}{x+2}$$

a.  $y = 1$  and  $x = -2$

b.  $y = -1$  and  $x = -2$

c.  $y = 0$  and  $x = -2$

d.  $y = 1$  and  $x = 2$

62. Let A and B be fixed points with coordinates  $(0, a)$  and  $(0, b)$  respectively and P is a variable point  $(x, 0)$  referred to rectangular axes. When is the angle APB extremum?

a.  $x = \sqrt{ab}$

b.  $x^2 = ab$

c.  $x = a + b$

d.  $x^2 = a - b$

63. What are the values of  $a$  and  $b$

respectively, if  $\lim_{x \rightarrow 0} \frac{\sin ax - \ln(e^x \cos x)}{x \sin bx} = \frac{1}{2}$ ?

a. 1, 1

b.  $1, \frac{1}{2}$

c. -1, 1

d.  $-1, \frac{1}{2}$

64.  $f(x) = \frac{\sqrt{2} \cos x - 1}{\cot x - 1}, x = \frac{\pi}{4}$

$= \alpha, x = \frac{\pi}{4}$

What should be the value of  $\alpha$  for which  $f(x)$  is continuous at  $x = \pi/4$ ?

a.  $\frac{1}{2}$

b.  $\sqrt{2}$

c.  $\frac{1}{2}$

d. 2

65. What are the order and degree respectively of the differential equation

$$\frac{d^2}{dx^2} \left\{ \left( \frac{dy}{dx} \right)^{-3/2} \right\} = 0?$$

a. 1, 4

- b. 4, 1  
c. 4, 4  
d. 1, 1
66. What is the solution of the differential equation  $xy \frac{d^2y}{dx^2} + x \left( \frac{dy}{dx} \right)^2 + y \frac{dy}{dx} = 0$ ?
- a.  $y^2 = A \ln x + B$   
b.  $y^2 = A \ln^2 x + B$   
c.  $y = A \ln x + B$   
d.  $y = A \ln^2 x + B$
67. Which equation represents the set of all curves in the  $xy$ -plane which have slope at each point  $P$  equal to the reciprocal of the slope at each point  $P$  equal to the reciprocal of the slope of the straight line through  $P$  and the origin?
- a.  $y^2 = x + c$   
b.  $x^2 + y^2 = c^2$   
c.  $x^2 - y^2 = c^2$   
d.  $xy = c^2$
68. What is the value of the particular integral of the differential equation  $(D^2 + a^2 D)y = \sin ax$ ?
- a.  $\frac{x}{2a^2} \cos ax$   
b.  $\frac{x}{2a^2} \sin ax$   
c.  $-\frac{x}{2a^2} \sin ax$   
d.  $\frac{x}{2a^2} \cos ax$
69. For non-exact differential equation integrating factor is
- a.  $1/(2xy)$   
b.  $1/(2x)$   
c.  $1/(2xy^2)$   
d.  $1/(x^2y)$
70. The differential equation  $\sin 2y \frac{dy}{dx} + 2 \tan \cos^2 y = 2 \sec x \cos^2 y$  is reduced to the linear form  $\frac{dv}{dx} + Pv = Q$ , where  $P$  and  $Q$  are function of  $x$  alone. What is the substitution?
- a.  $\tan y = v$   
b.  $\sec y = v$   
c.  $\cot y = v$   
d.  $\operatorname{cosec} y = v$
71. What is the solution of the differential equation  $\frac{dy}{dx} = (4x + y + 1)^2$ ?
- a.  $4x + y + 1 = 2 \tan(2x + y + c)$   
b.  $4x + y + 1 = 2 \tan(x + 2y + c)$   
c.  $4x + y + 1 = 2 \tan(2y + c)$   
d.  $4x + y + 1 = 2 \tan(2x + c)$
72. What is the solution of the differential equation  $\frac{dy}{dx} = e^{(x-y)} \{(e^x - e^y)\}$ ?
- a.  $e^{xy} (e^x - e^y + 1) = c$   
b.  $e^{xy} (e^y - e^x + 1) = c$   
c.  $e^{xy} (e^x - e^y + 1) = c$   
d.  $e^{xy} (e^y - e^x + 1) = c$
73. What is the degree of the differential equation for the given curve in which (subtangent)<sup>m</sup> = (Subnormal)<sup>n</sup> in Cartesian form, where  $0 < n < m$ ,  $m, n, m/n$  are integers?
- a.  $m + n$   
b.  $m - n$   
c.  $mn$   
d.  $m/n$
74. Which one of the following curves in the orthogonal trajectory of straight lines passing through a fixed point  $(a, b)$ ?
- a.  $x - a = k(y - b)$   
b.  $(x - a)(y - b) = k$   
c.  $(x - a)^2 = k(y - b)$   
d.  $(x - a)^2 + (y - b)^2 = k$
75. What is the singular solution of  $y^2(1 + y^2) = r^2$  where  $r$  is a constant?
- a.  $y^2 = 4rx$   
b.  $y^2 = 4r$   
c.  $y^2 = r^2$   
d.  $y^2 = r^3$
76. A function  $f'$  is defined as follows:  
 $f(x) = x^{1/2}$ , if  $x < 1$   
 $= (2 - x)^{1/2}$ , if  $1 \leq x < 2$   
 $= (x - 2)^{1/2}$ , if  $x \geq 2$   
What is the total number of points where derivatives do not exist?
- a. 2  
b. 3  
c. 3

d. none of these

77.  $f(x) = x^6 - x - 1, x \in [1, 2]$

Consider the following statements:

1.  $f$  is increasing on  $[1, 2]$ .
2.  $f$  has a root in  $[1, 2]$ .
3.  $f$  is decreasing on  $[1, 2]$ .
4.  $f$  has no root in  $[1, 2]$ .

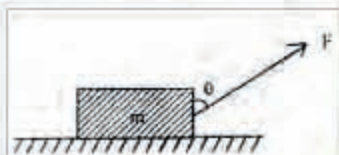
Which of the above are correct ?

- a. 1 and 2
- b. 1 and 4
- c. 2 and 3
- d. 3 and 4

78. Let  $f(x) = (x - a)(x - b)(x - c)$  be a real-valued function where  $a < b < c$  ( $a, b, c \in \mathbb{R}$ ) such that  $f''(\alpha) = 0$ . Then if  $\alpha \in (c_1, c_2)$ , which one of the following is correct?

- a.  $a < c_1 < b$ , and  $b < c_2 < c$
- b.  $a < c_1, c_2 < b$
- c.  $b < c_1, c_2 < c$
- d. None of the above (Here  $c_1 \neq c_2$ )

79.



A block of mass  $m$  rests on a rough horizontal surface as shown in the figure given above. The coefficient of friction between the block and the surface is  $\mu$ . A force  $F = mg \cos \theta$  with the vertical side of the block pulls it. Under what condition can the block be pulled along the surface?

- a.  $\tan \theta \geq \mu$
- b.  $\cos \theta \geq \mu$
- c.  $\tan \theta \leq \mu$
- d.  $\sin \theta \geq \mu$

80. Where is the centre of gravity of the arc of the astroid  $x^{2/3} + y^{2/3} = a^{2/3}$  lying in the first quadrant?

- a.  $(0, 0)$
- b.  $\left(\frac{a}{2}, \frac{a}{2}\right)$
- c.  $\left(\frac{5a}{2}, \frac{5a}{2}\right)$
- d.  $\left(\frac{2a}{5}, \frac{2a}{5}\right)$

81. Forces  $\vec{AB}, \vec{DC}$  act along the two sides  $AB$  and  $DC$  of a square  $ABCD$ , respectively and forces  $\vec{CA}, \vec{BD}$  act along the diagonals  $CA, BD$  of the square  $ABCD$ , respectively. Which one of the following is correct?

This system of forces

- a. reduces to a couple
- b. reduces to a single force passing through the centre of the square
- c. is in equilibrium
- d. reduces to a single force parallel to  $\vec{AB}$

82.  $\hat{i}$  and  $\hat{j}$  are unit vectors along the  $x$  and  $y$  axes respectively. Coplanar forces  $3\hat{i} + 4\hat{j}$ ,  $7\hat{i} - 8\hat{j}$ ,  $18\hat{i} + 7\hat{j}$ , and  $X\hat{i} + Y\hat{j}$ , act at the points  $(2, 3)$ ,  $(3, 4)$ ,  $(5, 6)$  and  $(1, 2)$  respectively in the  $xy$ -plane. If the system is equivalent to a couple, then what are the force components  $X$  and  $Y$ ?

- a.  $X = 10, Y = 5$
- b.  $X = 8, Y = 3$
- c.  $X = 8, Y = -3$
- d.  $X = 5, Y = 3$

83.



A uniform bar ( $AB$ ) of weight  $W$  is hinged at  $A$  to a vertical wall and held in horizontal position by a cord ( $BC$ ) as shown in the figure above. What is the tension in the cord?

- a.  $W$
- b.  $2W/3$
- c.  $\sqrt{3}W$
- d.  $W/\sqrt{3}$

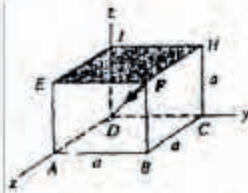
84. The resultant of two forces inclined to each other at  $120^\circ$  is at right angle to the smaller force. If the greater force is  $200$  N then what are the magnitudes of the smaller force and the resultant respectively?

- a.  $100$  N,  $100$  N

- b.  $100\sqrt{3}\text{ N}, 100\sqrt{3}\text{ N}$
- c.  $100\text{ N}, 100\sqrt{3}\text{ N}$
- d.  $100\sqrt{3}\text{ N}, 100\text{ N}$

85. What is the work done by the force  $\vec{F} = \frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j}$ , Where  $V = \tan^{-1}(x/y)$ , in taking a particle from a point (0, 1) to a point (1, 0)?
- a.  $\pi$
  - b.  $\pi/2$
  - c.  $\pi/4$
  - d.  $\pi/6$

86.



What is the moment of 50 N force directed along the diagonal of a cube (shown above) about the corner A of the cube? (Side of the cube is 'a' m)

- a.  $50a(\hat{k} - \hat{j})\text{ N-m}$
- b.  $50a(\hat{j} - \hat{k})\text{ N-m}$
- c.  $\frac{50}{\sqrt{3}}a(\hat{k} - \hat{j})\text{ N-m}$
- d.  $\frac{50}{\sqrt{3}}a(\hat{j} - \hat{k})\text{ N-m}$

87. If a particle is moving according to the law  $v^2 = 2(x \sin x + \cos x)$ , where  $v$  is the velocity and  $x$  is the distance described, what is its acceleration?
- a.  $x \cos x$
  - b.  $\sin x$
  - c.  $x \cos x$
  - d.  $\frac{x \sin x}{v}$

88. By what percent approximately should be velocity of moon in its orbit be increased so that it escapes?
- a. 20.7%
  - b. 41.4%
  - c. 82.8%
  - d. 100%

89. A particle is moving on x-y plane with velocity at any instant  $t$  given  $\vec{v} = v_0 \hat{i} + (a \omega \cos \omega t) \hat{j}$ . What is the equation to trajectory of the particle if the particle starts from origin when time  $t = 0$ ?
- a.  $y = a \cos (\omega x/v_0)$
  - b.  $y = a \sin (\omega x/v_0)$
  - c.  $y = x + a \cos (\omega x/v_0)$
  - d.  $y = x + a \sin (\omega x/v_0)$

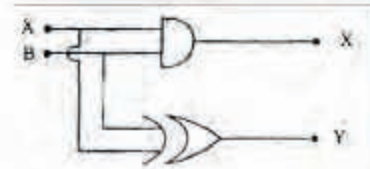
90. Set 1  
 Comment: n is a natural number  
 Step 2:  $x \leftarrow 1$   
 Step 3:  $p \leftarrow 1$   
 Step 4:  $v \leftarrow (2 * x - 1)$   
 Step 5:  $p \leftarrow p / y$   
 Step 6:  $x \leftarrow x + 1$   
 Step 7: If  $x \leq n$   
 then go to step 4  
 else go to step 8  
 Step 8: write p  
 Step 9: stop

What is the output of the algorithm given above?

- a.  $\frac{|2n|}{|n|}$
- b.  $\frac{|2n|}{|(n)^2|}$
- c.  $2^n |n|$
- d.  $2^n |2n|$

91. What is the cube root of  $(1000)_2^{(11)_2} + (111)_2^{(11)_2} + (10101000)_2 \times (1111)_2^2$ ?
- a.  $(1110)_2$
  - b.  $(10010)_2$
  - c.  $(11011)_2$
  - d.  $(1111)_2$

92.



If only NAND gates are to be used to realize the circuit given above, what is their minimum number among the following?

- a. 8

- b. 11  
c. 13  
d. 15
93. A ROM chip has a capacity  $1024 \times 4$ . Consider the following:  
1. 4 data inputs are required  
2. 4 data outputs are required  
3. 1024 address inputs are required  
Which of the above is/are correct?  
a. 1 only  
b. 2 only  
c. 1 and 2 only  
d. 1, 2 and 3
94. What is the capacity of 5-M-byte memory chip?  
a.  $5 \times 2^6$  bits  
b.  $5 \times 2^9$  bits  
c.  $5 \times 2^{20}$  bits  
d.  $5 \times 2^{23}$  bits
95. Assertion (A): Two projectiles are projected with same speed making complementary angles with each other. The sum of the heights attained by them is independent of angle of projection.  
Reason (R): If  $H_1$  and  $H_2$  are the heights attained by the projectiles projected with same speed  $u$  making complementary angles with each other then  $H_1 + H_2 = \frac{u^2}{g}$   
a. Both A and R are individually true and R is the correct explanation of A  
b. Both A and R are individually true but R is not the correct explanation of A  
c. A is true but R is false  
d. A is false but R is true
96. Assertion (A): A system of equations  $AX = b$ , where  $A$  is the matrix of a linear transformation  $T$ , non-singular from a vector space,  $V$  into vector space  $V'$ , has a unique solution.  
Reason (R): Any  $T: V \rightarrow V'$  is either 1-1 or onto.  
a. Both A and R are individually true and R is the correct explanation of A  
b. Both A and R are individually true but R is not the correct explanation of A  
c. A is true but R is false  
d. A is false but R is true
97. Assertion (A): In the hypothesis of Rolles' theorem, the statement that the function is continuous on a closed interval  $[a, b]$ ,  $-\infty < a$ ,  $b < \infty$  is needed to be assumed.  
Reason (R): Differentiability on  $(a, b)$  does not imply continuity on  $[a, b]$ .  
a. Both A and R are individually true and R is the correct explanation of A  
b. Both A and R are individually true but R is not the correct explanation of A  
c. A is true but R is false  
d. A is false but R is true
98. Assertion (A):  $\lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0$   
Reason (R):  $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = \lim_{x \rightarrow \infty} \sin x \lim_{x \rightarrow \infty} \frac{1}{x}$   
a. Both A and R are individually true and R is the correct explanation of A  
b. Both A and R are individually true but R is not the correct explanation of A  
c. A is true but R is false  
d. A is false but R is true
99. Assertion (A): At the mid position between mean position and extreme position of a particle executing SHM, the kinetic energy is equal to the potential energy.  
Reason (R): Both potential energy and kinetic energy depend upon position of the particle from its mean position.  
a. Both A and R are individually true and R is the correct explanation of A  
b. Both A and R are individually true but R is not the correct explanation of A  
c. A is true but R is false  
d. A is false but R is true
100. Assertion (A): The multiplicative group  $G = \{1, -1, i, -i\}$   
Reason (R):  $i$  is the generator of  $G$ .  
a. Both A and R are individually true and R is the correct explanation of A  
b. Both A and R are individually true but R is not the correct explanation of A  
c. A is true but R is false  
d. A is false but R is true