WARNING : Any malpractice or any attempt to commit any kind of malpractice in the Examination will DISQUALIFY THE CANDIDATE.

PAPER – II MATHEMATICS – 2014

Version Code	B3	Question Booklet Serial Number :	
Time : 150 Minutes		Number of Questions : 120	Maximum Marks : 480
Name of Candidate			
Roll Number			
Signature of Candidate			

INSTRUCTIONS TO THE CANDIDATE

1. Please ensure that the **VERSION CODE shown at the top of this Question Booklet is the same as that shown in the OMR Answer Sheet issued to you.** If you have received a Question Booklet with a different **VERSION CODE**, please get it replaced with a Question Booklet with the same VERSION CODE as that of the OMR Answer Sheet from the invigilator. **THIS IS VERY IMPORTANT.**
2. Please fill in the items such as name, signature and roll number of the candidate in the columns given above. Please also write the Question Booklet Sl. No. given at the top of this page against item 5 in the OMR Answer Sheet.
3. Please read the instructions given in the OMR Answer Sheet for marking answers. Candidates are advised to strictly follow the instructions contained in the OMR Answer Sheet.
4. This Question Booklet contains 120 questions. For each question, five answers are suggested and given against (A), (B), (C), (D) and (E) of which, only one will be the **Most Appropriate Answer**. Mark the bubble containing the letter corresponding to the 'Most Appropriate Answer' in the OMR Answer Sheet, by using either **Blue or Black ball-point pen only.**
5. Negative Marking: In order to discourage wild guessing, the score will be subject to penalization formula based on the number of right answers actually marked and the number of wrong answers marked. Each correct answer will be awarded FOUR marks. One mark will be deducted for each incorrect answer. More than one answer marked against a question will be deemed as incorrect answer and will be negatively marked.

IMMEDIATELY AFTER OPENING THIS QUESTION BOOKLET, THE CANDIDATE SHOULD VERIFY WHETHER THE QUESTION BOOKLET ISSUED CONTAINS ALL THE 120 QUESTIONS IN SERIAL ORDER. IF NOT, REQUEST FOR REPLACEMENT.

DO NOT OPEN THE SEAL UNTIL THE INVIGILATOR ASKS YOU TO DO SO.

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PLEASE ENSURE THAT THIS BOOKLET CONTAINS 120 QUESTIONS SERIALLY NUMBERED FROM 1 TO 120.
(Printed Pages : 32)

1. If $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$, $|\vec{b}| = 5$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then the area of the triangle formed by these two vectors as two sides is

(A) $\frac{15}{4}$ (B) $\frac{15}{2}$ (C) 15 (D) $\frac{15\sqrt{3}}{2}$ (E) $15\sqrt{3}$

2. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} + \vec{b}$ makes an angle of 60° with $\vec{a}$, then

(A) $|\vec{a}| = 2|\vec{b}|$ (B) $2|\vec{a}| = |\vec{b}|$ (C) $|\vec{a}| = \sqrt{3}|\vec{b}|$
(D) $|\vec{a}| = |\vec{b}|$ (E) $\sqrt{3}|\vec{a}| = |\vec{b}|$

3. If $\hat{i} + \hat{j}$, $\hat{j} + \hat{k}$, $\hat{i} + \hat{k}$ are the position vectors of the vertices of a triangle ABC taken in order, then $\angle A$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{5}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

4. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a} \cdot \vec{b} = |\vec{b}|^2$ and $|\vec{a} - \vec{b}| = \sqrt{7}$, then $|\vec{b}| =$

(A) $\sqrt{7}$ (B) $\sqrt{3}$ (C) 7 (D) 3 (E) $7\sqrt{3}$

5. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero vectors such that each one of them being perpendicular to the sum of the other two vectors, then the value of $|\vec{a} + \vec{b} + \vec{c}|^2$ is

(A) $|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$ (B) $|\vec{a}| + |\vec{b}| + |\vec{c}|$ (C) $2\left(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2\right)$
(D) $\frac{1}{2}\left(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2\right)$ (E) 0

Space for rough work

6. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors such that $\vec{u}+\vec{v}+\vec{w}=\vec{0}$. If $|\vec{u}|=3, |\vec{v}|=4$ and $|\vec{w}|=5$ then

$\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{w}+\vec{w}\cdot\vec{u}=$

(A) 0 (B) −25 (C) 25 (D) 50 (E) 47

7. If $\lambda\left(3\hat{i}+2\hat{j}-6\hat{k}\right)$ is a unit vector, then the values of λ are

(A) $\pm\frac{1}{7}$ (B) ± 7 (C) $\pm\sqrt{43}$

(D) $\pm\frac{1}{\sqrt{43}}$ (E) $\pm\frac{1}{\sqrt{7}}$

8. If the direction cosines of a vector of magnitude 3 are $\frac{2}{3}, \frac{-a}{3}, \frac{2}{3}$, $a>0$, then the vector is

(A) $2\hat{i}+\hat{j}+2\hat{k}$ (B) $2\hat{i}-\hat{j}+2\hat{k}$ (C) $\hat{i}-2\hat{j}+2\hat{k}$

(D) $\hat{i}+2\hat{j}+2\hat{k}$ (E) $\hat{i}+2\hat{j}-2\hat{k}$

9. Equation of the plane through the mid-point of the line segment joining the points $P(4,5,-10)$ and $Q(-1,2,1)$ and perpendicular to PQ is

(A) $\vec{r}\cdot\left(\frac{3}{2}\hat{i}+\frac{7}{2}\hat{j}-\frac{9}{2}\hat{k}\right)=45$ (B) $\vec{r}\cdot\left(-\hat{i}+2\hat{j}+\hat{k}\right)=\frac{135}{2}$ (C) $\vec{r}\cdot\left(5\hat{i}+3\hat{j}-11\hat{k}\right)+\frac{135}{2}=0$

(D) $\vec{r}\cdot\left(4\hat{i}+5\hat{j}-10\hat{k}\right)=85$ (E) $\vec{r}\cdot\left(5\hat{i}+3\hat{j}-11\hat{k}\right)=\frac{135}{2}$

Space for rough work

10. The angle between the straight lines $x-1=\frac{2y+3}{3}=\frac{z+5}{2}$ and $x=3r+2;\ y=-2r-1;$ $z=2$, where r is a parameter, is

(A) $\frac{\pi}{4}$ (B) $\cos^{-1}\left(\frac{-3}{\sqrt{182}}\right)$ (C) $\sin^{-1}\left(\frac{-3}{\sqrt{182}}\right)$

(D) $\frac{\pi}{2}$ (E) 0

11. Equation of the line through the point (2,3,1) and parallel to the line of intersection of the planes $x-2y-z+5=0$ and $x+y+3z=6$ is

(A) $\frac{x-2}{-5}=\frac{y-3}{-4}=\frac{z-1}{3}$ (B) $\frac{x-2}{5}=\frac{y-3}{-4}=\frac{z-1}{3}$ (C) $\frac{x-2}{5}=\frac{y-3}{4}=\frac{z-1}{3}$

(D) $\frac{x-2}{4}=\frac{y-3}{3}=\frac{z-1}{2}$ (E) $\frac{x-2}{-4}=\frac{y-3}{-3}=\frac{z-1}{2}$

12. A unit vector parallel to the straight line $\frac{x-2}{3}=\frac{3+y}{-1}=\frac{z-2}{-4}$ is

(A) $\frac{1}{\sqrt{26}}(3\hat{i}-\hat{j}+4\hat{k})$ (B) $\frac{1}{\sqrt{26}}(\hat{i}+3\hat{j}-\hat{k})$ (C) $\frac{1}{\sqrt{26}}(3\hat{i}-\hat{j}-4\hat{k})$

(D) $\frac{1}{\sqrt{26}}(3\hat{i}+\hat{j}+4\hat{k})$ (E) $\frac{1}{\sqrt{26}}(\hat{i}-3\hat{j}+4\hat{k})$

Space for rough work

13. The angle between a normal to the plane $2x-y+2z-1=0$ and the z-axis is

(A) $\cos^{-1}\left(\frac{1}{3}\right)$ (B) $\sin^{-1}\left(\frac{2}{3}\right)$ (C) $\cos^{-1}\left(\frac{2}{3}\right)$

(D) $\sin^{-1}\left(\frac{1}{3}\right)$ (E) $\sin^{-1}\left(\frac{3}{5}\right)$

14. Foot of the perpendicular drawn from the origin to the plane $2x-3y+4z=29$ is

(A) $(5,-1,4)$ (B) $(7,-1,3)$ (C) $(5,-2,3)$

(D) $(2,-3,4)$ (E) $(1,-3,4)$

15. The distance between the x-axis and the point (3, 12, 5) is

(A) 3 (B) 13 (C) 14 (D) 12 (E) 5

16. If $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$, then the standard deviation of the 9 items $x_1, x_2, \cdots, x_9$ is

(A) 9 (B) 4 (C) 3 (D) 2 (E) 1

17. If two dice are thrown simultaneously, then the probability that the sum of the numbers which come up on the dice to be more than 5 is

(A) $\frac{5}{36}$ (B) $\frac{1}{6}$ (C) $\frac{5}{18}$ (D) $\frac{7}{18}$ (E) $\frac{13}{18}$

Space for rough work

18. Let A and B be two events such that $P(A \cup B) = P(A) + P(B) - P(A)P(B)$.
If $0 < P(A) < 1$ and $0 < P(B) < 1$, then $P(A \cup B)' =$

(A) $1 - P(A)$ (B) $1 - P(A')$ (C) $1 - P(A)P(B)$
(D) $[1 - P(A)]P(B')$ (E) 1

19. The standard deviation of 9, 16, 23, 30, 37, 44, 51 is

(A) 7 (B) 9 (C) 12 (D) 14 (E) 16

20. The value of $\lim_{x \to 3} \frac{x^5 - 3^5}{x^8 - 3^8}$ is equal to

(A) $\frac{5}{8}$ (B) $\frac{5}{64}$ (C) $\frac{5}{216}$ (D) $\frac{1}{27}$ (E) $\frac{1}{63}$

21. Let $f(x) = (x^5 - 1)(x^3 + 1)$, $g(x) = (x^2 - 1)(x^2 - x + 1)$ and let $h(x)$ be such that $f(x) = g(x)h(x)$. Then $\lim_{x \to 1} h(x)$ is

(A) 0 (B) 1 (C) 3 (D) 4 (E) 5

22. $\lim_{x \to 0} \frac{\log(1 + 3x^2)}{x(e^{5x} - 1)} =$

(A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) $\frac{-3}{5}$ (D) $\frac{-5}{3}$ (E) 1

Space for rough work

23. If $f(x)=\dfrac{x+2}{3x-1}$, then $f(f(x))$ is

(A) x (B) $-x$ (C) $\dfrac{1}{x}$ (D) $-\dfrac{1}{x}$ (E) 0

24. Let $f(x)=\begin{cases} ax+3, & x\le 2 \\ a^2x-1, & x>2 \end{cases}$. Then the values of a for which f is continuous for all x are

(A) 1 and -2 (B) 1 and 2 (C) -1 and 2
(D) -1 and -2 (E) 0 and 3

25. Let R be the set of all real numbers. Let $f:R\to R$ be a function such that $|f(x)-f(y)|^2 \le |x-y|^3$, $\forall\, x, y\in R$. Then $f'(x)=$

(A) $f(x)$ (B) 1 (C) 0 (D) x^2 (E) x

26. Let $f(x)=\int_1^x \sin^2\left(\dfrac{t}{2}\right)dt$. Then the value of $\lim\limits_{x\to 0}\dfrac{f(\pi+x)-f(\pi)}{x}$ is equal to

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{3}{4}$ (D) 1 (E) 0

Space for rough work

27. If $y = f\left(x^2+2\right)$ and $f'(3)=5$, then $\dfrac{dy}{dx}$ at $x=1$ is

(A) 5 (B) 25 (C) 15 (D) 20 (E) 10

28. Let $f(x)=x^2+bx+7$. If $f'(5)=2f'\left(\dfrac{7}{2}\right)$, then the value of b is

(A) 4 (B) 3 (C) -4 (D) -3 (E) 2

29. If $x=\sin t$ and $y=\tan t$, then $\dfrac{dy}{dx}=$

(A) $\cos^3 t$ (B) $\dfrac{1}{\cos^3 t}$ (C) $\dfrac{1}{\cos^2 t}$ (D) $\sin^2 t$ (E) $\dfrac{1}{\sin^2 t}$

Space for rough work

30. If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, then $1 + \left(\frac{dy}{dx}\right)^2$ is

(A) $\tan\theta$ (B) $\tan^2\theta$ (C) 1 (D) $\sec^2\theta$ (E) $\sec\theta$

31. If $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$, $-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{x}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{2}{\sqrt{1-x^2}}$

(D) $\frac{2x}{\sqrt{1-x^2}}$ (E) $\frac{-2x}{\sqrt{1-x^2}}$

32. A straight line parallel to the line $2x - y + 5 = 0$ is also a tangent to the curve $y^2 = 4x + 5$. Then the point of contact is

(A) (2, 1) (B) (−1,1) (C) (1, 3)

(D) (3, 4) (E) (−1, 2)

Space for rough work

33. The function $f(x)=2x^3-15x^2+36x+6$ is strictly decreasing in the interval

(A) $(2,3)$ (B) $(-\infty,2)$ (C) $(3,4)$

(D) $(-\infty,3)\cup(4,\infty)$ (E) $(-\infty,2)\cup(3,\infty)$

34. The slope of the tangent to the curve $y^2e^{xy}=9e^{-3}x^2$ at $(-1, 3)$ is

(A) $\frac{-15}{2}$ (B) $\frac{-9}{2}$ (C) 15 (D) $\frac{15}{2}$ (E) $\frac{9}{2}$

35. The radius of a cylinder is increasing at the rate of 5 cm/min so that its volume is constant. When its radius is 5 cm and height is 3 cm the rate of decreasing of its height is

(A) 6 cm/min (B) 3 cm/min (C) 4 cm/min (D) 5 cm/min (E) 2 cm/min

36. The function $f(x)=\begin{cases} 2x^2-1 & \text{if } 1\le x\le 4 \\ 151-30x & \text{if } 4<x\le 5 \end{cases}$ is not suitable to apply Rolle's theorem since

(A) $f(x)$ is not continuous on $[1,5]$ (B) $f(1)\neq f(5)$

(C) $f(x)$ is continuous only at $x=4$ (D) $f(x)$ is not differentiable in (4, 5)

(E) $f(x)$ is not differentiable at $x=4$

Space for rough work

37. The slope of the normal to the curve $y = x^2 - \frac{1}{x^2}$ at $(-1, 0)$ is

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) 4 (D) -4 (E) 0

38. The minimum value of $\sin x + \cos x$ is

(A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $-\frac{1}{\sqrt{2}}$ (E) 1

39. $\int \frac{1}{x^2 \left(x^4+1\right)^{\frac{3}{4}}} dx$ is equal to

(A) $-\frac{(1+x^4)^{\frac{3}{4}}}{x} + C$ (B) $-\frac{(1+x^4)^{\frac{1}{4}}}{2x} + C$ (C) $-\frac{(1+x^4)^{\frac{1}{4}}}{x} + C$

(D) $-\frac{(1+x^4)^{\frac{1}{4}}}{x^2} + C$ (E) $-\frac{(1+x^4)^{\frac{1}{2}}}{x} + C$

40. $\int \frac{(1+x)e^x}{\sin^2\left(xe^x\right)} dx$ is equal to

(A) $-\cot(e^x) + C$ (B) $\tan(xe^x) + C$ (C) $\tan(e^x) + C$

(D) $\cot(xe^x) + C$ (E) $-\cot(xe^x) + C$

Space for rough work

41. $\int \frac{xe^x}{(1+x)^2}\,dx$ is equal to

(A) $\frac{-e^x}{x+1}+C$ (B) $\frac{e^x}{x+1}+C$ (C) $\frac{xe^x}{x+1}+C$ (D) $\frac{-xe^x}{x+1}+C$ (E) $\frac{e^x}{(x+1)^2}+C$

42. $\int e^x(\sin x+2\cos x)\sin x\,dx$ is equal to

(A) $e^x\cos x+C$ (B) $e^x\sin x+C$ (C) $e^x\sin^2 x+C$

(D) $e^x\sin 2x+C$ (E) $e^x(\cos x+\sin x)+C$

43. $\int\sqrt{1+\cos x}\,dx$ is equal to

(A) $2\sin\left(\frac{x}{2}\right)+C$ (B) $\sqrt{2}\sin\left(\frac{x}{2}\right)+C$ (C) $\frac{1}{2}\sin\left(\frac{x}{2}\right)+C$

(D) $\frac{\sqrt{2}}{2}\sin\left(\frac{x}{2}\right)+C$ (E) $2\sqrt{2}\sin\left(\frac{x}{2}\right)+C$

Space for rough work

44. $\int \frac{\sqrt{x^2-1}}{x}\,dx$ is equal to

(A) $\sqrt{x^2-1} - \sec^{-1} x + C$ (B) $\sqrt{x^2-1} + \tan^{-1} x + C$ (C) $\sqrt{x^2-1} + \sec^{-1} x + C$

(D) $\sqrt{x^2-1} - \tan x + C$ (E) $\sqrt{x^2-1} + \sec x + C$

45. $\int \frac{\sqrt{5+x^2}}{x^4}\,dx$ is equal to

(A) $\frac{1}{15}\left(1+\frac{5}{x^2}\right)^{3/2} + C$ (B) $\frac{-1}{15}\left(1+\frac{1}{x^2}\right)^{3/2} + C$ (C) $\frac{-1}{15}\left(1+\frac{5}{x^2}\right)^{3/2} + C$

(D) $\frac{1}{15}\left(1+\frac{1}{x^2}\right)^{3/2} + C$ (E) $\frac{-1}{10}\left(1+\frac{1}{x^2}\right)^{3/2} + C$

46. The value of $\int_0^1 \frac{dx}{e^x+e}$ is equal to

(A) $\frac{1}{e}\log\left(\frac{1+e}{2}\right)$ (B) $\log\left(\frac{1+e}{2}\right)$ (C) $\frac{1}{e}\log(1+e)$

(D) $\log\left(\frac{2}{1+e}\right)$ (E) $\frac{1}{e}\log\left(\frac{2}{1+e}\right)$

Space for rough work

47. Area bounded by the curves $y=e^x, y=e^{-x}$ and the straight line $x=1$ is (in sq. units)

(A) $e+\frac{1}{e}$ (B) $e+\frac{1}{e}+2$ (C) $e+\frac{1}{e}-2$ (D) $e-\frac{1}{e}+2$ (E) $e-\frac{1}{e}$

48. The value of the integral $\int_1^e \frac{1+\log x}{3x}\,dx$ is equal to

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) e (E) $\frac{1}{e}$

49. The value of the integral $\int_0^1 \frac{x^3}{1+x^8}\,dx$ is equal to

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{16}$ (D) $\frac{\pi}{6}$ (E) $\frac{\pi}{12}$

50. The value of the integral $\int_2^4 \left(\frac{\log t}{t}\right) dt$ is equal to

(A) $\frac{1}{2}(\log 2)^2$ (B) $\frac{5}{2}(\log 2)^2$ (C) $\frac{3}{2}(\log 2)^2$ (D) $(\log 2)^2$ (E) $\frac{3}{2}(\log 2)$

Space for rough work

51. The solution of the differential equation $(kx-y^2)dy=\left(x^2-ky\right)dx$ is

(A) $x^3-y^3=3kxy+C$
(B) $x^3+y^3=3kxy+C$
(C) $x^2-y^2=2kxy+C$
(D) $x^2+y^2=2kxy+C$
(E) $x^3-y^2=3kxy+C$

52. The solution of the differential equation $\frac{dy}{dx}=e^x+1$ is

(A) $y=e^x+C$
(B) $y=x+e^x+C$
(C) $y=xe^x+C$
(D) $y=x\left(e^x+1\right)+C$
(E) $y=e^x+Cx$

53. The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{\frac{3}{2}}=y$ are respectively

(A) 1, 1
(B) 1, 2
(C) 1, 3
(D) 2, 1
(E) 2, 2

54. An integrating factor of the differential equation $\sin x\frac{dy}{dx}+2y\cos x=1$ is

(A) $\sin^2 x$
(B) $\frac{2}{\sin x}$
(C) $\log|\sin x|$
(D) $\frac{1}{\sin^2 x}$
(E) $2\sin x$

Space for rough work

55. If the operation $\oplus$ is defined by $a \oplus b = a^2 + b^2$ for all real numbers a and b, then $(2 \oplus 3) \oplus 4 =$

(A) 120 (B) 185 (C) 175 (D) 129 (E) 312

56. The number of students who take both the subjects mathematics and chemistry is 30. This represents 10% of the enrolment in mathematics and 12% of the enrolment in chemistry. How many students take at least one of these two subjects?

(A) 520 (B) 490 (C) 560 (D) 480 (E) 540

57. Let $f(x) = |x-2|$, where x is a real number. Which one of the following is true?

(A) f is periodic

(B) $f(x+y) = f(x) + f(y)$

(C) f is an odd function

(D) f is not a 1-1 function

(E) f is an even function

58. If $A = \{1, 3, 5, 7\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$, then the number of one-to-one functions from A into B is

(A) 1340 (B) 1860 (C) 1430 (D) 1880 (E) 1680

Space for rough work

59. The range of the function $f(x)=x^2+2x+2$ is

(A) $(1,\infty)$ (B) $(2,\infty)$ (C) $(0,\infty)$ (D) $(1,\infty)$ (E) $(-\infty,\infty)$

60. If $f(x)=\sqrt{x}$ and $g(x)=2x-3$, then $(f\,o\,g)(x)$ is

(A) $(-\infty,-3)$ (B) $\left(-\infty,-\frac{3}{2}\right)$ (C) $\left[-\frac{3}{2},0\right]$

(D) $\left[0,\frac{3}{2}\right]$ (E) $\left[\frac{3}{2},\infty\right)$

61. If $z=\dfrac{\left(\sqrt{3}+i\right)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to

(A) 8 (B) 2 (C) 5 (D) 4 (E) 10

62. Let $w\neq\pm1$ be a complex number. If $|w|=1$ and $z=\dfrac{w-1}{w+1}$, then $\operatorname{Re}(z)$ is equal to

(A) 1 (B) $\dfrac{1}{|w+1|}$ (C) $\operatorname{Re}(w)$ (D) 0 (E) $w+\overline{w}$

63. If $z=e^{2\pi i/3}$, then $1+z+3z^2+2z^3+2z^4+3z^5$ is equal to

(A) $-3e^{\pi i/3}$ (B) $3e^{\pi i/3}$ (C) $3e^{2\pi i/3}$ (D) $-3e^{2\pi i/3}$ (E) 0

Space for rough work

64. If $z_1 = 2\sqrt{2}(1+i)$ and $z_2 = 1+i\sqrt{3}$, then $z_1^2 z_2^3$ is equal to

(A) $128\,i$ (B) $64\,i$ (C) $-64\,i$ (D) $-128\,i$ (E) 256

65. If the complex numbers z_1, z_2 and z_3 denote the vertices of an isosceles triangle, right angled at z_1, then $(z_1 - z_2)^2 + (z_1 - z_3)^2$ is equal to

(A) 0 (B) $(z_2 + z_3)^2$ (C) 2 (D) 3 (E) $(z_2 - z_3)^2$

66. If the roots of $x^2 - ax + b = 0$ are two consecutive odd integers, then $a^2 - 4b$ is

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

67. If α and β are the roots of $x^2 - ax + b^2 = 0$, then $\alpha^2 + \beta^2$ is equal to

(A) $a^2 + 2b^2$ (B) $a^2 - 2b^2$ (C) $a^2 - 2b$ (D) $a^2 + 2b$ (E) $a^2 - b^2$

68. If α and β are the roots of the equation $x^2 + 3x - 4 = 0$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to

(A) $\frac{-3}{4}$ (B) $\frac{3}{4}$ (C) $\frac{-4}{3}$ (D) $\frac{4}{3}$ (E) $\frac{3}{2}$

Space for rough work

69. The value of x such that $3^{2x}-2(3^{x+2})+81=0$ is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

70. If the roots of the equation $x^2+2bx+c=0$ are α and β, then $b^2-c=$

(A) $\frac{(\alpha-\beta)^2}{4}$ (B) $(\alpha+\beta)^2-\alpha\beta$ (C) $(\alpha+\beta)^2+\alpha\beta$

(D) $\frac{(\alpha-\beta)^2}{2}+\alpha\beta$ (E) $\frac{(\alpha+\beta)^2}{2}+\alpha\beta$

71. The equation whose roots are the squares of the roots of the equation $2x^2+3x+1=0$ is

(A) $4x^2+5x+1=0$ (B) $4x^2-x+1=0$ (C) $4x^2-5x-1=0$

(D) $4x^2-5x+1=0$ (E) $4x^2+5x-1=0$

72. The sum of the series $\sum_{n=8}^{17}\frac{1}{(n+2)(n+3)}$ is equal to

(A) $\frac{1}{17}$ (B) $\frac{1}{18}$ (C) $\frac{1}{19}$ (D) $\frac{1}{20}$ (E) $\frac{1}{21}$

Space for rough work

73. If two positive numbers are in the ratio $3+2\sqrt{2} : 3-2\sqrt{2}$, then the ratio between their A.M. and G.M. is

(A) 6 : 1 (B) 3 : 2 (C) 2 : 1 (D) 3 : 1 (E) 1 : 6

74. Let $x_1, x_2, \cdots, x_n$ be in an A.P. If $x_1 + x_4 + x_9 + x_{11} + x_{20} + x_{22} + x_{27} + x_{30} = 272$, then $x_1 + x_2 + x_3 + \cdots + x_{30}$ is equal to

(A) 1020 (B) 1200 (C) 716 (D) 2720 (E) 2072

75. If the second and fifth terms of a G.P. are 24 and 3 respectively, then the sum of first six terms is

(A) 181 (B) $\frac{181}{2}$ (C) 189 (D) $\frac{189}{2}$ (E) 191

76. If the sum of first 75 terms of an A.P. is 2625, then the 38th term of the A.P. is

(A) 39 (B) 37 (C) 36 (D) 38 (E) 35

Space for rough work

77. If $-5, k, -1$ are in A.P., then the value of k is equal to

(A) –5 (B) –3 (C) –1 (D) 3 (E) 5

78. Let T_n denote the number of triangles which can be formed by using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 36$, then n is equal to

(A) 2 (B) 5 (C) 6 (D) 8 (E) 9

79. The middle term in the expansion of $\left(\frac{10}{x} + \frac{x}{10}\right)^{10}$ is

(A) ${}^{10}C_5$ (B) ${}^{10}C_6$ (C) ${}^{10}C_5 \frac{1}{x^{10}}$ (D) ${}^{10}C_5 x^{10}$ (E) ${}^{10}C_5 10^{10}$

80. The coefficient of x^{49} in the product $(x-1)(x-2)(x-3) \cdots (x-50)$ is

(A) −2250 (B) −1275 (C) 1275 (D) 2250 (E) −49

Space for rough work

81. The sum of the coefficients in the binomial expansion of $\left(\frac{1}{x}+2x\right)^6$ is equal to

(A) 1024 (B) 729 (C) 243 (D) 512 (E) 64

82. The value of ${}^2P_1 + {}^3P_1 + \ldots + {}^nP_1$ is equal to

(A) $\frac{n^2-n+2}{2}$ (B) $\frac{n^2+n+2}{2}$ (C) $\frac{n^2+n-1}{2}$

(D) $\frac{n^2-n-1}{2}$ (E) $\frac{n^2+n-2}{2}$

83. How many four digit numbers $abcd$ exist such that a is odd, b is divisible by 3, c is even and d is prime?

(A) 380 (B) 360 (C) 400 (D) 520 (E) 480

84. If $a_1, a_2, a_3, \ldots$ are in A.P., then the value of $\begin{vmatrix} a_1 & a_2 & 1 \\ a_2 & a_3 & 1 \\ a_3 & a_4 & 1 \end{vmatrix}$ is equal to

(A) $a_4 - a_1$ (B) $\frac{a_1+a_4}{2}$ (C) 1 (D) $\frac{a_2+a_3}{2}$ (E) 0

Space for rough work

85. If $\begin{vmatrix} 2a & x_1 & y_1 \\ 2b & x_2 & y_2 \\ 2c & x_3 & y_3 \end{vmatrix} = \frac{abc}{2} \neq 0$, then the area of the triangle whose vertices are $\left(\frac{x_1}{a}, \frac{y_1}{a}\right), \left(\frac{x_2}{b}, \frac{y_2}{b}\right)$ and $\left(\frac{x_3}{c}, \frac{y_3}{c}\right)$ is

(A) $\frac{1}{4}abc$ (B) $\frac{1}{8}abc$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$ (E) $\frac{1}{12}$

86. The system of linear equations $3x + y - z = 2$, $x - z = 1$ and $2x + 2y + az = 5$ has unique solution when

(A) $a \neq 3$ (B) $a \neq 4$ (C) $a \neq 5$ (D) $a \neq 2$ (E) $a \neq 1$

87. If A = $\begin{bmatrix} 2-k & 2 \\ 1 & 3-k \end{bmatrix}$ is a singular matrix, then the value of $5k - k^2$ is equal to

(A) 0 (B) 6 (C) −6 (D) −4 (E) 4

88. If a, b, c are non-zero and different from 1, then the value of $\begin{vmatrix} \log_a 1 & \log_a b & \log_a c \\ \log_a\left(\frac{1}{b}\right) & \log_b 1 & \log_a\left(\frac{1}{c}\right) \\ \log_a\left(\frac{1}{c}\right) & \log_a c & \log_c 1 \end{vmatrix}$ is

(A) 0 (B) $1 + \log_a(a+b+c)$ (C) $\log_a(ab+bc+ca)$

(D) 1 (E) $\log_a(a+b+c)$

Space for rough work

89. The number of solutions for the system of equations $2x+y=4,\ 3x+2y=2,$ and $x+y=-2$ is

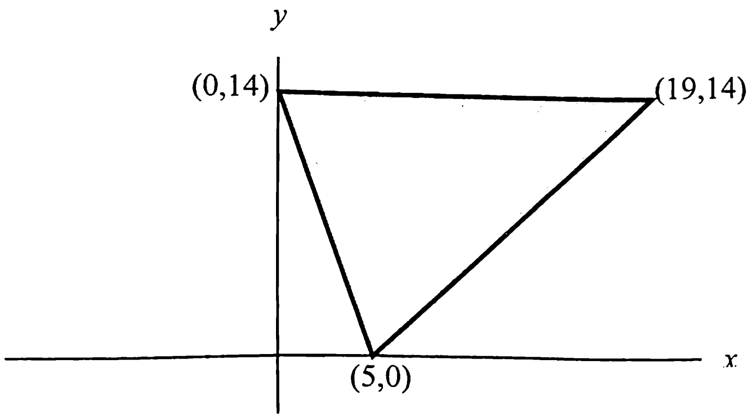
(A) 1 (B) 2 (C) 3

(D) infinitely many (E) 0

90. The number of solutions of the inequation $|x-2|+|x+2|<4$ is

(A) 1 (B) 2 (C) 4 (D) 0 (E) infinite

91. The shaded region shown in the figure is given by the inequations



(A) $14x+5y\ge 70,\ y\le 14$ and $x-y\ge 5$
(B) $14x+5y\le 70,\ y\le 14$ and $x-y\ge 5$
(C) $14x+5y\ge 70,\ y\ge 14$ and $x-y\ge 5$
(D) $14x+5y\ge 70,\ y\ge 14$ and $x-y\le 5$
(E) $14x+5y\ge 70,\ y\le 14$ and $x-y\le 5$

Space for rough work

92. Let p, q and r be any three logical statements. Which one of the following is true?

(A) $\sim[p \wedge (\sim q)] \equiv (\sim p) \wedge q$

(B) $\sim(p \vee q) \wedge (\sim r) \equiv (\sim p) \vee (\sim q) \vee (\sim r)$

(C) $\sim[p \vee (\sim q)] \equiv (\sim p) \wedge q$

(D) $\sim[p \wedge (\sim q)] \equiv (\sim p) \wedge \sim q$

(E) $\sim[p \wedge (\sim q)] \equiv p \wedge q$

93. The truth values of p, q and r for which $(p \wedge q) \vee (\sim r)$ has truth value F are respectively

(A) F, T, F (B) F, F, F (C) T, T, T (D) T, F, F (E) F, F, T

94. $\sim[(\sim p) \wedge q]$ is logically equivalent to

(A) $\sim(p \vee q)$ (B) $\sim[p \wedge (\sim q)]$ (C) $p \wedge (\sim q)$

(D) $p \vee (\sim q)$ (E) $(\sim p) \vee (\sim q)$

95. Let $\theta \in \left[0, \frac{\pi}{2}\right]$. Which one of the following is true?

(A) $\sin^2\theta > \cos^2\theta$ (B) $\sin^2\theta < \cos^2\theta$ (C) $\sin\theta > \cos\theta$

(D) $\cos\theta > \sin\theta$ (E) $\sin\theta + \cos\theta \le \sqrt{2}$

96. The value of $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) + \sin^{-1}\left(\frac{1}{3}\right)$ is equal to

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$ (E) 0

Space for rough work

97. If $ab<1$ and $\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)+\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)=2\tan^{-1}x$, then x is equal to

(A) $\frac{a}{1+ab}$ (B) $\frac{a}{1-ab}$ (C) $\frac{a-b}{1+ab}$ (D) $\frac{a+b}{1+ab}$ (E) $\frac{a+b}{1-ab}$

98. The value of $\tan(1°)+\tan(89°)$ is equal to

(A) $\frac{1}{\sin 1^\circ}$ (B) $\frac{2}{\sin 2^\circ}$ (C) $\frac{2}{\sin 1^\circ}$ (D) $\frac{1}{\sin 2^\circ}$ (E) $\frac{\sin 2^\circ}{2}$

99. Let $s_n=\cos\left(\frac{n\pi}{10}\right)$, $n=1,2,3,\ldots$ Then the value of $\frac{s_1 s_2 \ldots s_{10}}{s_1+s_2+\ldots+s_{10}}$ is equal to

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $2\sqrt{2}$ (D) 0 (E) $\frac{1}{2}$

100. $\cos^{-1}\left(\cos\left(\frac{7\pi}{5}\right)\right)=$

(A) $\frac{3\pi}{5}$ (B) $\frac{2\pi}{5}$ (C) $\frac{-7\pi}{5}$ (D) $\frac{7\pi}{5}$ (E) $\frac{-2\pi}{5}$

101. The value of $\sec^2\left(\tan^{-1}3\right)+\text{cosec}^2\left(\cot^{-1}2\right)$ is equal to

(A) 5 (B) 13 (C) 15 (D) 23 (E) 25

102. If $\sin\theta+\text{cosec}\,\theta=2$, then the value of $\sin^6\theta+\text{cosec}^6\theta$ is equal to

(A) 0 (B) 1 (C) 2 (D) 2^3 (E) 2^6

Space for rough work

103. If $0 < x < \pi$, then $\frac{\sin 8x + 7\sin 6x + 18\sin 4x + 12\sin 2x}{\sin 7x + 6\sin 5x + 12\sin 3x} =$

(A) $2\sin x$ (B) $\sin x$ (C) $\sin 2x$ (D) $2\cos x$ (E) $\cos x$

104. The points (2,5) and (5,1) are the two opposite vertices of a rectangle. If the other two vertices are points on the straight line $y = 2x + k$, then the value of k is

(A) 4 (B) 3 (C) –4 (D) –3 (E) 1

105. The circumcentre of the triangle with vertices $(8,6),(8,-2)$ and $(2,-2)$ is at the point

(A) $(2, -1)$ (B) $(1,-2)$ (C) $(5,2)$ (D) $(2,5)$ (E) $(4, 5)$

106. The ratio by which the line $2x + 5y - 7 = 0$ divides the straight line joining the points $(-4, 7)$ and $(6,-5)$ is

(A) 1 : 4 (B) 1 : 2 (C) 1 : 1 (D) 2 : 3 (E) 1 : 3

107. The number of points (a, b), where a and b are positive integers, lying on the hyperbola $x^2 - y^2 = 512$ is

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Space for rough work

108. If p is the length of the perpendicular from the origin to the line whose intercepts with the coordinate axes are $\frac{1}{3}$ and $\frac{1}{4}$ then the value of p is

(A) $\frac{3}{4}$ (B) $\frac{1}{12}$ (C) 5 (D) 12 (E) $\frac{1}{5}$

109. The slope of the straight line joining the centre of the circle $x^2+y^2-8x+2y=0$ and the vertex of the parabola $y=x^2-4x+10$ is

(A) $\frac{-5}{2}$ (B) $\frac{-7}{2}$ (C) $\frac{-3}{2}$ (D) $\frac{5}{2}$ (E) $\frac{7}{2}$

110. A straight line perpendicular to the line $2x+y=3$ is passing through $(1,1)$. Its y-intercept is

(A) 1 (B) 2 (C) 3 (D) $\frac{1}{2}$ (E) $\frac{1}{3}$

111. If p and q are respectively the perpendiculars from the origin upon the straight lines whose equations are $x\sec\theta+y\,\text{cosec}\,\theta=a$ and $x\cos\theta-y\sin\theta=a\cos 2\theta$, then $4p^2+q^2$ is equal to

(A) $5a^2$ (B) $4a^2$ (C) $3a^2$ (D) $2a^2$ (E) a^2

112. The shortest distance between the circles $(x-1)^2+(y+2)^2=1$ and $(x+2)^2+(y-2)^2=4$ is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Space for rough work

113. The centre of the circle whose radius is 5 and which touches the circle $x^2+y^2-2x-4y-20=0$ at (5, 5) is

(A) (10 , 5) (B) (5 , 8) (C) (5 , 10) (D) (8 , 9) (E) (9 , 8)

114. A circle passes through the points (0,0) and (0,1) and also touches the circle $x^2+y^2=16$. The radius of the circle is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

115. A circle of radius $\sqrt{8}$ is passing through origin and the point (4, 0). If the centre lies on the line $y=x$, then the equation of the circle is

(A) $(x-2)^2+(y-2)^2=8$ (B) $(x+2)^2+(y+2)^2=8$ (C) $(x-3)^2+(y-3)^2=8$

(D) $(x+3)^2+(y+3)^2=8$ (E) $(x-4)^2+(y-4)^2=8$

116. The parametric form of the ellipse $4(x+1)^2+(y-1)^2=4$ is

(A) $x=\cos\theta-1,\ \ y=2\sin\theta-1$ (B) $x=2\cos\theta-1,\ \ y=\sin\theta+1$

(C) $x=\cos\theta-1,\ \ y=2\sin\theta+1$ (D) $x=\cos\theta+1,\ \ y=2\sin\theta+1$

(E) $x=\cos\theta+1,\ \ y=2\sin\theta-1$

Space for rough work

117. A point P on an ellipse is at a distance 6 units from a focus. If the eccentricity of the ellipse is $\frac{3}{5}$, then the distance of P from the corresponding directrix is

(A) $\frac{8}{5}$ (B) $\frac{5}{8}$ (C) 10 (D) 12 (E) 15

118. If the length of the latus rectum and the length of transverse axis of a hyperbola are $4\sqrt{3}$ and $2\sqrt{3}$ respectively, then the equation of the hyperbola is

(A) $\frac{x^2}{3}-\frac{y^2}{4}=1$ (B) $\frac{x^2}{3}-\frac{y^2}{9}=1$ (C) $\frac{x^2}{6}-\frac{y^2}{9}=1$

(D) $\frac{x^2}{6}-\frac{y^2}{3}=1$ (E) $\frac{x^2}{3}-\frac{y^2}{6}=1$

119. If the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{5}{4}$ and $2x+3y-6=0$ is a focal chord of the hyperbola, then the length of transverse axis is equal to

(A) $\frac{12}{5}$ (B) 6 (C) $\frac{24}{7}$ (D) $\frac{24}{5}$ (E) $\frac{12}{7}$

120. The length of the transverse axis of a hyperbola is $2\cos\alpha$. The foci of the hyperbola are the same as that of the ellipse $9x^2+16y^2=144$. The equation of the hyperbola is

(A) $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{7-\cos^2\alpha}=1$ (B) $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{7+\cos^2\alpha}=1$

(C) $\frac{x^2}{1+\cos^2\alpha}-\frac{y^2}{7-\cos^2\alpha}=1$ (D) $\frac{x^2}{1+\cos^2\alpha}-\frac{y^2}{7+\cos^2\alpha}=1$

(E) $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{5-\cos^2\alpha}=1$

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