MATHEMATICS

(Three hours)

*(Candidates are allowed additional 15 minutes for **only** reading the paper. They must NOT start writing during this time.)*

Section A - *Answer* **Question 1** *(compulsory) and* **five** *other questions.*

Section B and Section C - *Answer* **two** *questions from* **either** *Section B* **or** *Section C.*

All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

Slide rule may be used.

SECTION A

Question 1 **[10×3]**

(i) Find the value of k if $M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $M^2 - kM - I_2 = 0$

(ii) Find the equation of an ellipse whose latus rectum is 8 and eccentricity is $\frac{1}{3}$.

(iii) Solve: $\cos^{-1}(\sin \cos^{-1} x) = \frac{\pi}{6}$

(iv) Using L'Hospital's rule, evaluate: $\lim_{x \to 0} \frac{x - \sin x}{x^2 \sin x}$

(v) Evaluate: $\int \frac{2y^2}{y^2 + 4}\, dy$

(vi) Evaluate: $\int_0^3 f(x)dx$, where $f(x) = \begin{cases} \cos 2x, & 0 \le x \le \frac{\pi}{2} \\ 3, & \frac{\pi}{2} \le x \le 3 \end{cases}$

(vii) The two lines of regressions are $4x + 2y - 3 = 0$ and $3x + 6y + 5 = 0$. Find the correlation co-efficient between x and y.

(viii) A card is drawn from a well shuffled pack of playing cards. What is the probability that it is either a spade or an ace or both?

(ix) If 1, ω and ω^2 are the cube roots of unity, prove that $\frac{a + b\omega + c\omega^2}{c + a\omega + b\omega^2} = \omega^2$

(x) Solve the differential equation: $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

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Turn over

Question 2

(a) Using properties of determinants, prove that: [5]

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

(b) Given two matrices A and B [5]

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 4 & 1 \\ 1 & -3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix},$$

find AB and use this result to solve the following system of equations:

$$x-2y+3z=6,\ x+4y+z=12,\ x-3y+2z=1$$

Question 3

(a) Solve the equation for x: $sin^{-1}\frac{5}{x} + sin^{-1}\frac{12}{x} = \frac{\pi}{2},\ \ x \neq 0$ [5]

(b) A, B and C represent switches in 'on' position and A', B' and C' represent them in 'off' position. Construct a switching circuit representing the polynomial ABC + AB′C + A′B′C. Using Boolean Algebra, prove that the given polynomial can be simplified to C(A + B′). Construct an equivalent switching circuit. [5]

Question 4

(a) Verify Lagrange's Mean Value Theorem for the following function: [5]

$f(x) = 2\sin x + \sin 2x$ on $[0, \pi]$

(b) Find the equation of the hyperbola whose foci are $(0, \pm\sqrt{10})$ and passing through the point (2, 3). [5]

Question 5

(a) If $y = e^{m\cos^{-1}x}$, prove that: [5]

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$$

(b) Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius 10 cm is a square of side $10\sqrt{2}$ cm. [5]

Question 6

(a) Evaluate: [5]

$$\int \frac{\sec x}{1+\operatorname{cosec} x}dx$$

(b) Find the smaller area enclosed by the circle $x^2 + y^2$ and the line $x + y = 2$. [5]

Question 7

(a) Given that the observations are: **[5]**

(9, −4), (10, −3), (11, −1), (12, 0), (13, 1), (14, 3), (15, 5), (16, 8).

Find the two lines of regression and estimate the value of y when $x = 13\cdot5$.

(b) In a contest the competitors are awarded marks out of 20 by two judges. The scores of the 10 competitors are given below. Calculate Spearman's rank correlation. **[5]**

Competitors	A	B	C	D	E	F	G	H	I	J
Judge A	2	11	11	18	6	5	8	16	13	15
Judge B	6	11	16	9	14	20	4	3	13	17

Question 8

(a) An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white, it is not replaced into the urn. Otherwise, it is replaced with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black. **[5]**

(b) Three persons A, B and C shoot to hit a target. If A hits the target four times in five trials, B hits it three times in four trials and C hits it two times in three trials, find the probability that: **[5]**

(i) Exactly two persons hit the target.

(ii) At least two persons hit the target.

(iii) None hit the target.

Question 9

(a) If $z = x + iy$, $w = \frac{2-iz}{2z-i}$ and $|w|=1$, find the locus of z and illustrate it in the Argand Plane. **[5]**

(b) Solve the differential equation: **[5]**

$$e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)+\left(1+e^{\frac{x}{y}}\right)\frac{dx}{dy}=0 \text{ when } x=0;\ y=1$$

SECTION B

Question 10

(a) Using vectors, prove that angle in a semicircle is a right angle. **[5]**

(b) Find the volume of a parallelopiped whose edges are represented by the vectors: **[5]**

$\vec{a} = 2\hat{i} - 3\hat{j} - 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, and\ \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$.

Question 11

(a) Find the equation of the plane passing through the intersection of the planes: **[5]**

$x + y + z + 1 = 0$ and $2x - 3y + 5z - 2 = 0$ and the point $(-1, 2, 1)$.

(b) Find the shortest distance between the lines $\vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ and $\vec{r} = 2\hat{\imath} + 4\hat{\jmath} + 5\hat{k} + \mu(4\hat{\imath} + 6\hat{\jmath} + 8\hat{k})$ **[5]**

Question 12

(a) Box I contains two white and three black balls. Box II contains four white and one black balls and box III contains three white and four black balls. A dice having three red, two yellow and one green face, is thrown to select the box. If red face turns up, we pick up box I, if a yellow face turns up we pick up box II, otherwise, we pick up box III. Then, we draw a ball from the selected box. If the ball drawn is white, what is the probability that the dice had turned up with a red face? **[5]**

(b) Five dice are thrown simultaneously. If the occurrence of an odd number in a single dice is considered a success, find the probability of maximum three successes. **[5]**

SECTION C

Question 13

(a) Mr. Nirav borrowed ₹ 50,000 from the bank for 5 years. The rate of interest is 9% per annum compounded monthly. Find the payment he makes monthly if he pays back at the beginning of each month. **[5]**

(b) A dietician wishes to mix two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below: **[5]**

Food	Vitamin A	Vitamin B	Vitamin C
X	1unit	2 units	3 units
Y	2 units	2 units	1 unit

One kg of food X costs ₹ 24 and one kg of food Y costs ₹ 36. Using Linear Programming, find the least cost of the total mixture which will contain the required vitamins.

Question 14

(a) A bill for ₹ 7,650 was drawn on 8^{th} March, 2013, at 7 months. It was discounted on 18^{th} May, 2013 and the holder of the bill received ₹ 7,497. What is the rate of interest charged by the bank? **[5]**

(b) The average cost function, AC for a commodity is given by $AC = x + 5 + \frac{36}{x}$, in terms of output x. Find: **[5]**

(i) The total cost, C and marginal cost, MC as a function of x.

(ii) The outputs for which AC increases.

Question 15

(a) Calculate the index number for the year 2014, with 2010 as the base year by the weighted aggregate method from the following data: **[5]**

Commodity	**Price in ₹**		**Weight**
	2010	**2014**	
A	2	4	8
B	5	6	10
C	4	5	14
D	2	2	19

(b) The quarterly profits of a small scale industry (in thousands of rupees) is as follows : **[5]**

Year	**Quarter 1**	**Quarter 2**	**Quarter 3**	**Quarter 4**
2012	39	47	20	56
2013	68	59	66	72
2014	88	60	60	67

Calculate four quarterly moving averages. Display these and the original figures graphically on the same graph sheet.