Solutions of

of

Electronics Engineering

GATE-2016

Session 1 | Set-1

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GATE 2016 : Solutions

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Section - I (General Aptitude)

One Mark Questions

Q.1 Which of the following is **CORRECT** with respect to grammar and usage?
Mount Everest is _____.
(a) the highest peak in the world
(b) highest peak in the world
(c) one of highest peak in the world
(d) one of the highest peak in the world

Ans. **(a)**

End of Solution

Q.2 The policeman asked the victim of a theft, "What did you _____?
(a) loose
(b) lose
(c) loss
(d) louse

Ans. **(b)**

End of Solution

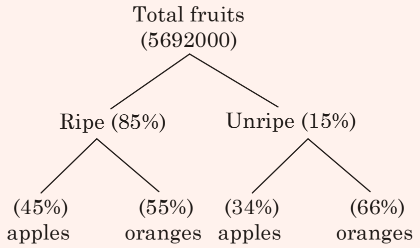
Q.3 Despite the new medicine's _____ in treating diabetes, it is not _____ widely.
(a) effectiveness - prescribed
(b) availability - used
(c) prescription - available
(d) acceptance - proscribed

Ans. **(a)**

End of Solution

Q.4 In a huge pile of apples and oranges, both ripe and unripe mixed together, 15% are unripe fruits. Of the unripe fruits, 45% are apples. Of the ripe ones, 66% are oranges. If the pile contains a total of 5692000 fruits, how many of them are apples?
(a) 2029198
(b) 2467482
(c) 2789080
(d) 3577422

Ans. **(a)**



Total number of apples = Ripe apples + Unripe apples
= (0.85 × 0.45 + 0.15 × 0.34)569200
= 2029198

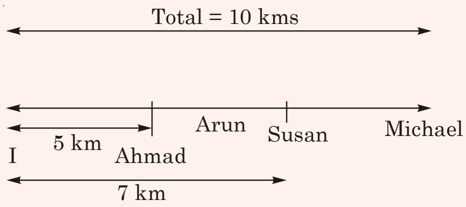
End of Solution

Q.5 Michael lives 10 km away from where I live. Ahmed lives 5 km away and Susan lives 7 km away from where I live. Arun is farther away than Ahmed but closer than Susan from where I live. From the information provided here, what is one possible distance (in km) at which I live from Arun's place?

(a) 3.00 (b) 4.99
(c) 6.02 (d) 7.01

Ans. **(c)**

Following line with respective distances can be drawn



Arun can reside anywhere between Ahmed and Susan i.e. between 5 km and 7 km from I.

$5 < 6.02 < 7$

End of Solution

Two Marks Questions

Q.6. A person moving through a tuberculosis prone zone has a 50% probability of becoming infected. However, only 30% of infected people develop the disease. What percentage of people moving through a tuberculosis prone zone remains infected but does not show symptoms of disease?

(a) 15 (b) 33
(c) 35 (d) 37

Ans. **(c)**

The required probability

$$0.5 \times 0.7 = 0.35 \approx \frac{35}{100} = 35\%$$

End of Solution

Q.7 In a world filled with uncertainty, he was glad to have many good friends. He has always assisted them in times of need and was confident that they would reciprocate. However, the events of the last week proved him wrong.

Which of the following inference(s) is/are logically valid and can be inferred from the above passage?

I. His friends were always asking him to help them.
II. He felt that when in need of help, his friends would let him down.
III. He was sure that his friends would help him when in need.
IV. His friends did not help him last week.

(a) I and II (b) III and IV
(c) III only (d) IV only

Ans. **(b)**

End of Solution

Q.8 Leela is older than her cousin Pavithra, Pavithra's brother Shiva is older than Leela. When Pavithra and Shiva are visiting Leela, all there like to play chess. Pavithra wins more often than Leela does.
Which one of the following statements must be TRUE based on the above?
(a) When Shiva plays chess with Leela and Pavithra, he often loses.
(b) Leela is the oldest of three.
(c) Shiva is better chess player than Pavithra.
(d) Pavithra is the youngest of the three.

Ans. **(d)**

L > P (Leela is older thean Pavithra)

S > L (Shiv is older than Leela)

So Pavithra is youngest

End of Solution

Q.9 If $q^{-a} = \frac{1}{r}$ and $r^{-b} = \frac{1}{s}$ and $s^{-c} = \frac{1}{q}$, the value of abc is _____.

(a) $(rqs)^{-1}$ (b) 0
(c) 1 (d) $r + q + s$

Ans. **(c)**

$$q^a = r,\ r^b = s,\ s^c = q$$

$$\Rightarrow \quad a \cdot \log q = \log r,$$
$$b \cdot \log r = \log s,$$
$$c \cdot \log s = \log q$$

So, $$a \times b \times c = \frac{\log r}{\log q} \times \frac{\log s}{\log r} \times \frac{\log q}{\log s} = 1$$

End of Solution

Q.10 P, Q, R and S are working on a project. Q can finish the task in 25 days, working alone for 12 hours a day. R can finish the task in 50 days, working alone for 12 hours per day. Q worked 12 hours a day but took sick leave in the beginning for two days. R worked 18 hours a day on all days. What is the ratio of work done by Q and R after 7 days from the start of the projects?
(a) 10:11 (b) 11:10
(c) 20:21 (d) 21:20

Ans. **(c)**

Q can do work in $25 \times 12 = 300$ hrs

R can do work in $50 \times 12 = 600$ hrs

So we can say Q is twice efficient as R

Now Q worked only for 5 days at a rate of 12 hrs/day. So for 60 units of his work (Total work for Q i.e. 300 hrs) he will do only $\frac{1}{5}$ of work $\left(\frac{60}{300} = \frac{1}{5}\right)$

While R worked for all 7 days at a rate of 18 hrs/day

So he will do $18 \times 7 = 126$ of his work (Total work for 600 hrs)

He will do $\left(\frac{126}{600} = 0.21\right)$ of his work

So required ratio $\left(\frac{1}{5} : \frac{126}{600}\right) = 120 : 126$

20 : 21

End of Solution

Section - II (Electronics Engineering)

One Mark Questions

Q.1 Let $M^4 = I$, (where I denotes the identity matrix) and $M \neq I$, $M^2 \neq I$ and $M^3 \neq I$. Then, for any natural number k, M^{-1} equals:

(a) M^{4k+1}
(b) M^{4k+2}
(c) M^{4k+3}
(d) M^{4k}

Ans. **(c)**

Given that $M^4 = I$ or $M^{4k} = I$ or $M^{4(k+1)} = I$

$\therefore \quad M^{-1} \times I = M^{4(k+1)} \times M^{-1}$

$\therefore \quad M^{-1} = M^{4k+3}$

End of Solution

Q.2 The second moment of a Poisson-distributed random variable is 2. The mean of the random variable is _____.

Ans. **(1)**

In Poisson distribution,

Mean = First moment = λ

second moment = $\lambda^2 + \lambda$

Given that second moment is 2

$\therefore \quad \lambda^2 + \lambda = 2$

$\lambda^2 + \lambda - 2 = 0$

$(\lambda + 2)(\lambda - 1) = 0$

$\lambda = 1$

End of Solution

Q.3 Given the following statements about a function $f: R \to R$, select the right option:

P: If $f(x)$ is continuous at $x = x_0$, then it is differential at $x = x_0$.
Q: If $f(x)$ is continuous at $x = x_0$, then it may not be differentiable at $x = x_0$.
R: If $f(x)$ is differentiable at $x = x_0$, then it is also continuous at $x = x_0$.

(a) P is true, Q is false, R is false
(b) P is false, Q is true, R is true
(c) P is false, Q is true, R is false
(d) P is true, Q is false, R is true

Ans. **(b)**

P : If $f(x)$ is continuous at $x = x_0$, then it is also differentiable at $x = x_0$
Q : If $f(x)$ is continuous at $x = x_0$, then it may or may not be derivable at $x = x_0$
R : If $f(x)$ is differentiable at $x = x_0$, then it is also continuous at $x = x_0$
P is false
Q is true
R is true Option (b) is correct

End of Solution

Q.4 Which one of the following is a property of the solutions to the Laplace equation: $\nabla^2 f = 0$?

(a) The solutions have neither maxima nor minima anywhere except at the boundaries.

(b) The solutions are not separable in the coordinates.

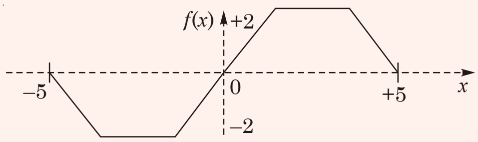
(c) The solutions are not continuous.

(d) The solutions are not dependent on the boundary conditions.

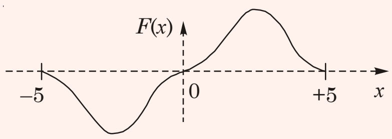
Ans. **(a)**

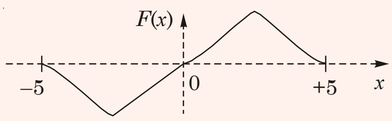
End of Solution

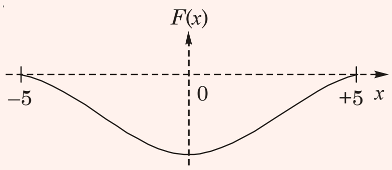
Q.5 Consider the plot $f(x)$ versus x as shown below.

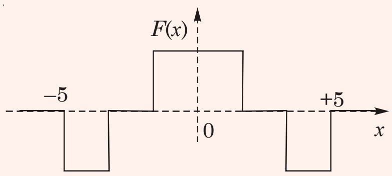


Suppose $F(x) = \int_{-5}^{x} f(y)dy$. Which one of the following is a graph of $F(x)$?

(a)


(b)


(c)


(d)


Ans. **(c)**

$$F'(x) = f(x) \text{ which is density function}$$

$$F'(x) = f(x) < 0 \text{ when } x < 0$$

$\therefore$ $F(x)$ is decreasing for $x < 0$

$$F'(x) = f(x) > 0 \text{ when } x > 0$$

$\therefore$ $F(x)$ is increasing for $x > 0$

End of Solution

Q.6 Which one of the following is an eigen function of the class of all continuous- time, linear, time-invariant systems ($u(t)$ denotes the unit-step function)?

(a) $e^{j\omega_0 t}u(t)$

(b) $\cos(\omega_0 t)$

(c) $e^{j\omega_0 t}$

(d) $\sin(\omega_0 t)$

Ans. **(c)**

If the input to the system is eigen signal output also the same eigen signal.

End of Solution

Q.7 A continuous time function $x(t)$ is periodic with period T. The function is sampled uniformly with a sampling period T_s. In which one of the following cases is the sampled signal periodic?

(a) $T = \sqrt{2}T_s$ (b) $T = 1.2\ T_s$

(c) Always (d) Never

Ans. **(b)**

A signal is said to be periodic if $\frac{T}{T_s}$ is a rational number.

Here, $T = 1.2\ T_s$

$$\Rightarrow \quad \frac{T}{T_s} = \frac{6}{5} \quad \text{Which is a rational number}$$

End of Solution

Q.8 Consider the sequence $x[n] = a^n u[n] + b^n u[n]$, where $u[n]$ denotes the unit-step sequence and $0 < |a| < |b| < 1$. The region of convergence (ROC) of the z-transform of $x[n]$ is

(a) $|z| > |a|$ (b) $|z| > |b|$

(c) $|z| < |a|$ (d) $|a| < |z| < |b|$

Ans. **(b)**

Given, $x[n] = a^n u[n] + b^n u[n]$

Also given, $0 < |a| < |b| < 1$

$$\text{ROC} = (|z| > |a|) \text{ and } (|z| > |b|)$$

$$\text{ROC} = |z| > |b|$$

End of Solution

Q.9 Consider a two-port network with the transmission matrix : $T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$.

If the network is reciprocal, then

(a) $T^{-1} = T$ (b) $T^2 = T$

(c) Determinant $(T) = 0$ (d) Determinant $(T) = 1$

Ans. **(d)**

For reciprocal network $AD - BC = 1$

$$|T| = 1$$

End of Solution

Q.10 A continuous-time sinusoid of frequency 33 Hz is multiplied with a periodic Dirac impulse train of frequency 46 Hz. The resulting signal is passed through an ideal analog low-pass filter with a cutoff frequency of 23 Hz. The fundamental frequency (in Hz) of the output is _____.

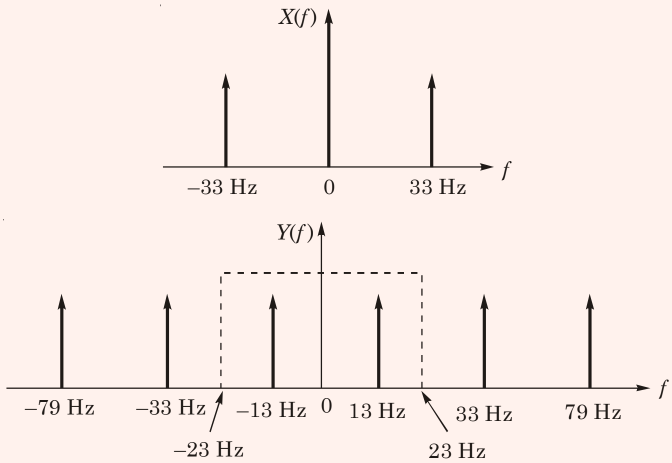
Ans. **(13)**

If $x(t)$ is a message signal and $y(t)$ is a sampled signal, then $y(t)$ is related to $x(t)$ as

$$y(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$Y(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

Spectrum of $X(f)$ and $Y(f)$ are as shown

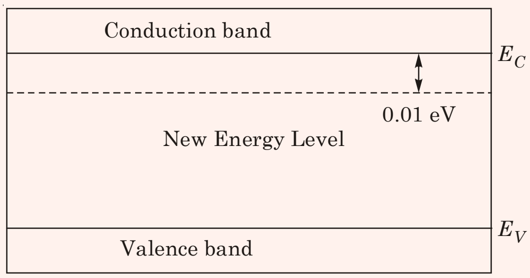


Cut off frequency of LPF = 23 Hz

Hence, frequency at the output is 13 Hz

End of Solution

Q.11 A small percentage of impurity is added to an intrinsic semiconductor at 300 K. Which one of the following statements is true for the energy band diagram shown in the following figure?



(a) Intrinsic semiconductor doped with pentavalent atoms to form n-type semiconductor
(b) Intrinsic semiconductor doped with trivalent atoms to form n-type semiconductor
(c) Intrinsic semiconductor doped with pentavalent atoms to form p-type semiconductor
(d) Intrinsic semiconductor doped with trivalent atoms to form p-type semiconductor

Ans. **(a)**

Pentavalent impurity when introduced in Intrinsic SC, a new discrete energy level called Donor energy level is created just below the conduction band.

End of Solution

Q.12 Consider the following statements for a metal oxide semiconductor field effect transistor (MOSFET):

P: As channel length reduces, OFF-state current increases.
Q: As channel length reduces, output resistance increases.
R: As channel length reduces, threshold voltage remains constant.
S: As channel length reduces, ON current increases.

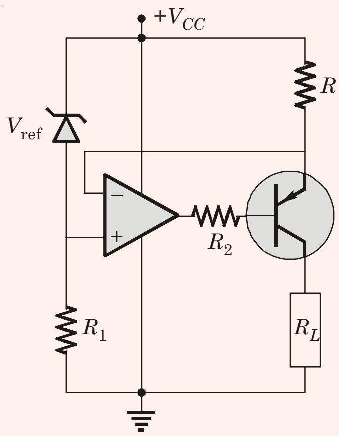
Which of the above statements are **INCORRECT**?

(a) P and Q
(b) P and S
(c) Q and R
(d) R and S

Ans. (c)

End of Solution

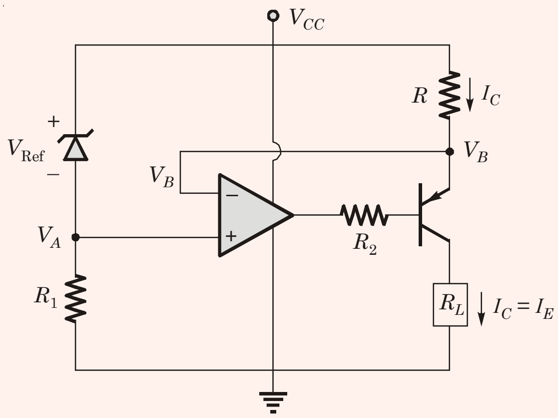
Q.13 Consider the constant current source shown in the figure below. Let β represent the current gain of the transistor.



The load current I_0 through R_L is

(a) $I_0 = \left(\frac{\beta+1}{\beta}\right)\frac{V_{ref}}{R}$
(b) $I_0 = \left(\frac{\beta}{\beta+1}\right)\frac{V_{ref}}{R}$
(c) $I_0 = \left(\frac{\beta+1}{\beta}\right)\frac{V_{ref}}{2R}$
(d) $I_0 = \left(\frac{\beta}{\beta+1}\right)\frac{V_{ref}}{2R}$

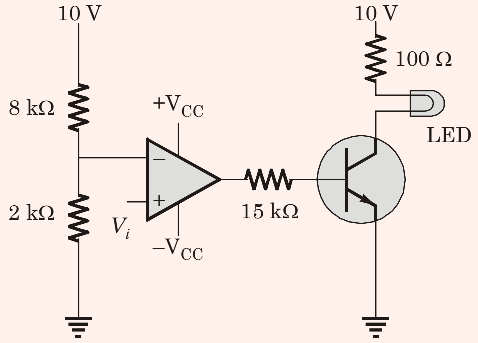
Ans. (b)

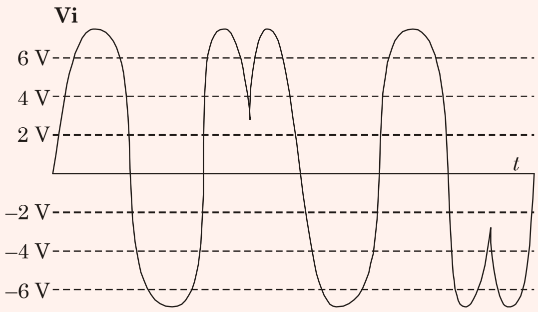


$$V_A = V_{CC} - V_{\text{ref}}$$
$$V_B = V_A \text{ (since virtual short)}$$
$$I_C = \frac{V_{CC} - V_B}{R} = \frac{V_{CC} - (V_{CC} - V_{\text{Ref}})}{R} = \frac{V_{\text{Ref}}}{R}$$
$$I_0 = I_E = \frac{I_C}{\alpha} = \left(\frac{\beta}{1+\beta}\right)\frac{V_{\text{Ref}}}{R}$$

End of Solution

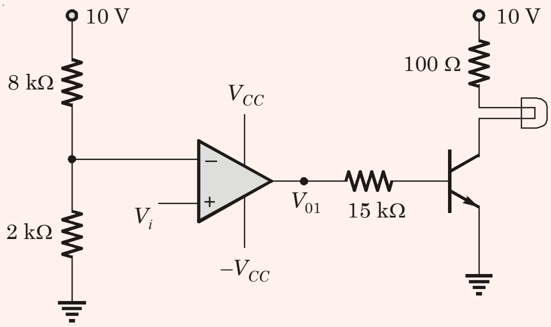
Q.14 The following signal V_i of peak voltage 8 V applied to the non-inverting terminal of an ideal opamp. The transistor has $V_{BE} = 0.7$ V, $\beta = 100$; $V_{LED} = 1.5$ V, $V_{CC} = 10$ V and $-V_{CC} = -10$ V





The number of times the LED glows is_____

Ans. **(3)**



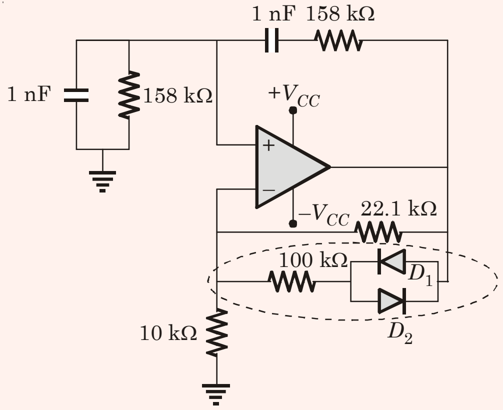
$$V_B = \frac{10\,\text{V} \times 2\,\text{K}}{8\,\text{K} + 2\,\text{K}} = 2\,\text{V}$$

When V_1 exceeds 2 V output of opamp V_{01} goes to V_{CC} and drives BJT into saturation shorted LED will glow,

In the given problem V_i exceeds $2V$ three times and hence output V_{01} of opamp goes to V_{CC} thrice so that LED glow three times.

End of Solution

Q.15 Consider the oscillator circuit shown in the figure. The function of the network (shown in dotted lines) consisting of the 100 kΩ resistor in series with the two diodes connected back-to-back is to:



(a) introduce amplitude stabilization by preventing the op amp from saturating and thus producing sinusoidal oscillations of fixed amplitude

(b) introduce amplitude stabilization by forcing the opamp to swing between positive and negative saturation and thus producing square wave oscillations of fixed amplitude

(c) introduce frequency stabilization by forcing the circuit to oscillate at a single frequency

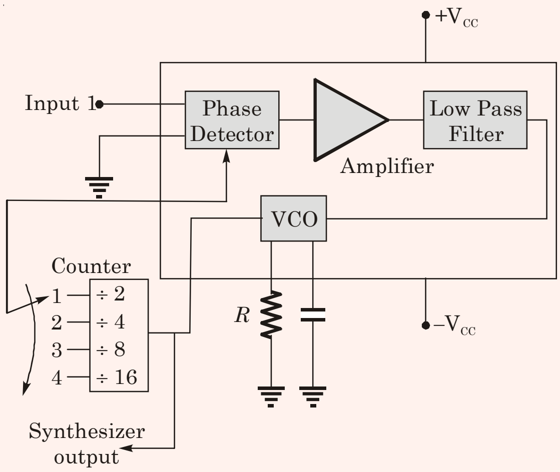
(d) enable the loop gain to take on a value that produces square wave oscillations

Ans. **(a)**

The given circuit is Wein-bridge oscillator which produced sinusoidal oscillations and the amplitude of output wave is decided by feedback through inverting input terminal of opamp.

End of Solution

Q.16 The block diagram of a frequency synthesizer consisting of a Phase Locked Loop (PLL) and a divide-by-N counter (comprising ÷ 2, ÷ 4, ÷ 8, ÷ 16 outputs) is sketched below. The synthesizer is excited with a 5 kHz signal (Input 1). The free-running frequency of the PLL is set to 20 kHz. Assume that the commutator switch makes contacts repeatedly in the order 1 - 2 - 3 - 4.



The corresponding frequency synthesized are:

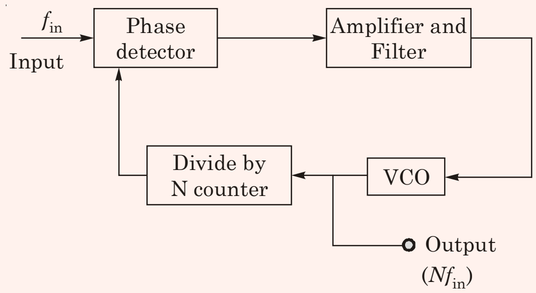
(a) 10 kHz, 20 kHz, 40 kHz, 80 kHz

(b) 20 kHz, 40 kHz, 80 kHz, 160 kHz

(c) 80 kHz, 40 kHz, 20 kHz, 10 kHz

(d) 160 kHz, 80 kHz, 40 kHz, 20 kHz

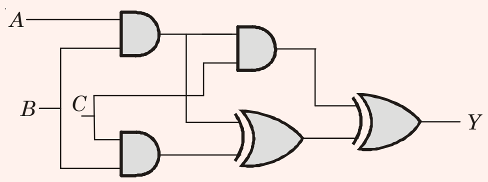
Ans. **(a)**



f_{in}	VCO output (Nf_{in})	Divide by N counter
5 kHz	10 kHz	2
5 kHz	20 kHz	4
5 kHz	40 kHz	8
5 kHz	80 kHz	16

End of Solution

Q.17 The output of the combinational circuit given below is



(a) $A + B + C$

(b) $A(B + C)$

(c) $B(C + A)$

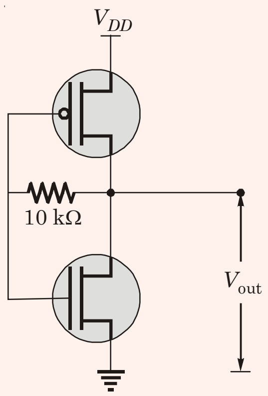
(d) $C(A + B)$

Ans. **(c)**

$$y = ABC \oplus AB \oplus BC$$
$$= \left[\overline{ABC}\cdot AB + ABC\cdot \overline{AB}\right] \oplus BC$$
$$= \left[\left(\overline{A}+\overline{B}+\overline{C}\right)\cdot AB + ABC\cdot\left(\overline{A}+\overline{B}\right)\right] \oplus BC$$
$$= (AB\overline{C}) \oplus (BC)$$
$$= \overline{AB\overline{C}}\cdot BC + AB\overline{C}\cdot\overline{BC}$$
$$= (\overline{A}+\overline{B}+C)\cdot BC + AB\overline{C}\cdot(\overline{B}+\overline{C})$$
$$= \overline{A}BC + BC + AB\overline{C}$$
$$= BC(\overline{A}+1) + AB\overline{C} = BC + AB\overline{C}$$
$$= B(C + A\overline{C}) = B(C + A)$$

End of Solution

Q.18 What is the voltage V_{out} in the following circuit?



(a) 0 V
(b) $(|V_T \text{ of PMOS}| + V_T \text{ of NMOS})/2$
(c) Switching threshold of inverter
(d) V_{DD}

Ans. **(c)**

End of Solution

Q.19 Match the inferences *X*, *Y* and *Z* about a system, to the corresponding properties of the elements of first column in Routh's Table of the system characteristic equation.

List - I	List - II
X. The system is stable..	**P.** .. when all elements are positive
Y. The system is unstable..	**Q.** ..when any one element is zero
Z. The test breaks down..	**R.** ..when there is a change in sign of coefficients

(a) X – P; Y – Q; Z – R
(b) X – Q; Y – P; Z – R
(c) X – R; Y – Q; Z – P
(d) X – P; Y – R; Z – Q

Ans. **(d)**

When all elements are positive, the system is stable. When any element is zero, the test breaks down. When there is change in sign of coefficients, the system is unstable.

End of Solution

Q.20 A closed-loop control system is stable if the Nyquist plot of the corresponding open-loop transfer function

(a) encircles the s-plane point $(-1 + j0)$ in the counterclockwise direction as many times as the number of right-half s-plane poles.

(b) encircles the s-plane point $(0 - j1)$ in the clockwise direction as many times as the number of right-half s-plane poles.

(c) encircles the s-plane point $(-1 + j0)$ in the counterclockwise direction as many times as the number of left-half s-plane poles.

(d) encircles the s-plane point $(-1 + j0)$ in the counterclockwise direction as many times as the number of right-half s-plane zeros.

Ans. **(a)**

$N = P - Z$

N = Number of encirclements of $(-1+j0)$. It ispositive if nyquist plot encircles the point $-1 + j0$ in counterclockwise direction.

Z = Number of closed loop poles lying in the right half of s-plane

P = Number of open loop poles lying in right half of s-plane

For stability $Z = 0 \Rightarrow N = P$

End of Solution

Q.21 Consider the binary data transmission at a rate of 56 kbps using baseband binary pulse amplitude modulation (PAM) that is designed to have a raised-cosine spectrum. The transmission bandwidth (in kHz) required of a roll-off factor of 0.25 is _____

Ans. **(35)**

Bit rate, $R_b = 56$ kbps

Roll-off factor, $\alpha = 0.25$

$$\text{Transmission BW} = \frac{R_b}{2}(1+\alpha)$$

$$= \frac{56}{2}(1.25) = 28 \times 1.25 = 35 \text{ kHz}$$

End of Solution

Q.22 A superheterodyne receiver operates in the frequency range of 58 MHz – 68 MHz. The intermediate frequency f_{IF} and local oscillator frequency f_{LO} are chosen such that $f_{IF} \le f_{LO}$. It is required that the image frequencies fall outside the 58 MHz – 68 MHz band. The minimum required f_{IF} (in MHz) is _____.

Ans. **(5)**

$$f_s = 58\text{ MHz} - 68\text{ MHz}$$

f_{si} should fall outside the range 58 MHz - 68 MHz

Hence $f_{s\min} = 58\text{ MHz}$

$$f_{si} = f_s + 2IF > 68\text{ MHz}$$

$$58\text{ MHz} + 2IF > 68\text{ MHz}$$

$$IF > 5\text{ MHz}$$

$\Rightarrow$ $(IF)_{\min} = 5\text{ MHz}$

End of Solution

Q.23 The amplitude of a sinusoidal carrier is modulated by a single sinusoid to obtain the amplitude modulated signal $s(t) = 5\cos 1600\pi t + 20\cos 1800\pi t + 5\cos 2000\pi t$. The value of the modulation index is_____.

Ans. **(0.5)**

$$s(t) = 5\cos 1600\pi t + 20\cos 1800\pi t + 5\cos 2000\pi t$$

$$s(t) = 20\cos 1800\pi t + 5\cos 1600\pi t + 5\cos 2000\pi t$$

$$s(t) = A_c\cos 2\pi f_c t + \frac{A_c\mu}{2}\cos 2\pi(f_c - f_m)t + \frac{A_c\mu}{2}\cos 2\pi(f_c + f_m)t$$

comparing, we get

$$A_c = 20\text{ V};\ \frac{A_c\mu}{2} = 5\text{ V}$$

$$\mu = \frac{10}{20} = 0.5$$

End of Solution

Q.24 Concentric spherical shells of radii 2 m, 4 m, and 8 m carry uniform surface charge densities of 20 nC/m^2, –4 nC/m^2 and ρ_s, respectively. The value of ρ_s(nC/m^2) required to ensure that the electric flux density $\vec{D} = \vec{0}$ at radius 10 m is _____.

Ans. **(–0.25)**

$$\oint D\cdot ds = Q \quad \text{(charge enclosed)}$$

$$Q_1 + Q_2 + Q_3 = 0$$

For $D = 0$

$$\rho_{s1}\,.\,4\pi 2^2 + \rho_{s2}\,.\,4\pi\,.\,4^2 + \rho_{s3}\,.\,4\pi\,.\,8^2 = Q = 0$$

$$20\cdot 4 - 4.4^2 + \rho_{s3}.\,8^2 = 0$$

$$80 - 64 + \rho_{s3}\,.\,8^2 = 0$$

$$\rho_{s3} = \frac{-16}{64} = -0.25\ \text{nC/m}^2$$

End of Solution

Q.25 The propagation constant of a lossy transmission line is $(2 + j5)\ m^{-1}$ and its characteristic impedance is $(50 + j0)\Omega$ at $\omega = 10^6$ rad s^{-1}. The value of the line constants L, C, R, G are respectively,

(a) $L = 200\ \mu H/m$, $C = 0.1\ \mu F/m$, $R = 50\ \Omega/m$, $G = 0.02$ S/m
(b) $L = 250\ \mu H/m$, $C = 0.1\ \mu F/m$, $R = 100\ \Omega/m$, $G = 0.04$ S/m
(c) $L = 200\ \mu H/m$, $C = 0.2\ \mu F/m$, $R = 100\ \Omega/m$, $G = 0.02$ S/m
(d) $L = 250\ \mu H/m$, $C = 0.2\ \mu F/m$, $R = 50\ \Omega/m$, $G = 0.04$ S/m

Ans. **(b)**

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\gamma \,.\, Z_0 = R + j\omega L = (2 + j5)\ (50 + j0) = 100 + j250$$

$$R = 100\ \Omega/m$$

$$L = \frac{250}{\omega} = \frac{250}{10^6} = 250\ \mu H/m$$

$$\frac{\gamma}{Z_0} = \frac{2 + j5}{50} = G + j\omega C = 0.04 + j\,0.1$$

$$G = 0.04\ S/m$$

$$C = \frac{0.1}{\omega} = \frac{0.1}{10^6} = 0.1\ \ \mu F/m$$

End of Solution

Two Marks Questions

Q.26 The integral $\frac{1}{2\pi}\iint_D (x + y + 10)dx,dy$, where D denotes the disc: $x^2 + y^2 \le 4$, evaluates to _____

Ans. **(20)**

Put
$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$dxdy = r dr d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\int_0^{2}(r(\cos\theta + \sin\theta) + 10)r\,dr d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\int_0^{2}(r^2(\cos\theta + \sin\theta) + 10r)\,dr d\theta$$

$$= \frac{1}{2\pi}\left(\int_0^{2\pi}(\cos\theta+\sin\theta)\left(\frac{r^3}{3}\right)\Bigg|_0^2 d\theta + 10\int_0^{2\pi}\left(\frac{r^2}{2}\right)\Bigg|_0^2 d\theta\right)$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\frac{8}{3}(\cos\theta+\sin\theta)d\theta + \frac{1}{2\pi}\int_0^{2\pi}5\cdot(4)d\theta$$

$$= \frac{1}{2\pi}\left[\frac{8}{3}(\sin\theta-\cos\theta)\right]_0^{2\pi} + \frac{1}{2\pi}.20(2\pi)$$

$$= \frac{1}{2\pi}\left(\frac{8}{3}(0-1)-(0-1)+20\right) = 0+20 = 20$$

End of Solution

Q.27 A sequence $x[n]$ is specified as

$$\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \text{ for } n \geq 2.$$

The initial conditions are $x[0] = 1$, $x[1] = 1$, and $x[n] = 0$ for $n < 0$. The value of $x[12]$ is _____

Ans. **(233)**

For
$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

equation
$$\begin{bmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{bmatrix} = 0$$

$$\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - \lambda - 1 = 0$$

By Cayley Hamilton Theorem

$$A^2 - A - I = 0$$

$$A^2 = A + I$$

$$A^4 = A^2 + 2A + I$$

$$= A + I + 2A + I = 3A + 2I$$

$$A^8 = 9A^2 + 12A + 4I$$

$$= 9(A + I) + 12A + 4I$$

$$= 21\,A + 13I$$

$$A^{12} = A^4 \cdot A^8 = 144A + 89\,I$$

$$= \begin{bmatrix} 233 & 144 \\ 144 & 89 \end{bmatrix}$$

$$\begin{bmatrix} x[12] \\ x[11] \end{bmatrix} = \begin{bmatrix} 233 & 144 \\ 144 & 89 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$x[12] = 233$$

End of Solution

Q.28 In the following integral, the contour C encloses the points $2\pi j$ and $-2\pi j$

$$-\frac{1}{2\pi}\oint_C \frac{\sin z}{(z-2\pi j)^3}dz$$

The value of the integral is _____

Ans. **(–133.87)**

$$I = -\frac{1}{2\pi}\int_c \frac{\sin z}{(z-2\pi j)^3}dz$$

$$= -\frac{1}{2\pi}\times\frac{2\pi j\, f''(2\pi j)}{2!}$$

$$f(z) = \sin z$$

$$f'(z) = \cos z$$

$$f''(z) = -\sin z$$

$$I = -\frac{1}{2\pi}\times 2\pi j\cdot\frac{-\sin(2\pi j)}{2}$$

$$= -\frac{1}{2}\sinh 2\pi = -133.87$$

End of Solution

Q.29 The region specified by $\{(\rho, \phi, z): 3 \le \rho \le 5, \frac{\pi}{8} \le \varphi \le \frac{\pi}{4}, 3 \le z \le 4.5\}$ in cylindrical coordinates has volume of _____.

Ans. **(4.712)**

$$V = \int_{\rho=3}^{5}\int_{\phi=\frac{\pi}{8}}^{\pi/4}\int_{z=3}^{4.5}\rho d\rho\, d\phi\, dz = \int_{3}^{4.5}\int_{\pi/8}^{\pi/4}\left(\frac{\rho^2}{2}\right)\Bigg|_3^5 d\phi\, dz$$

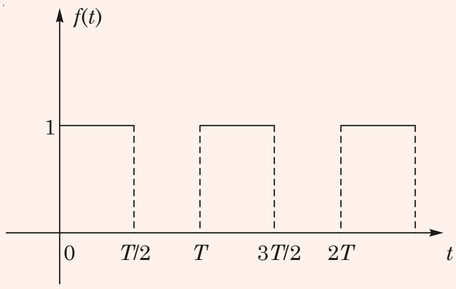
$$= \int_{3}^{4.5}\int_{\pi/8}^{\pi/4} 8\cdot d\phi dz = 8\phi\Big|_{\pi/8}^{\pi/4}\cdot z\Big|_3^{4.5}$$

$$= 8\left(\frac{\pi}{4}-\frac{\pi}{8}\right)(4.5-3) = 8\cdot\frac{\pi}{8}\cdot(1.5)$$

$$= 4.712$$

End of Solution

Q.30 The Laplace transform of the causal periodic square wave of period T shown in the figure below is



(a) $F(s) = \frac{1}{1+e^{-sT/2}}$

(b) $F(s) = \frac{1}{s\left(1+e^{-sT/2}\right)}$

(c) $F(s) = \frac{1}{s\left(1-e^{-sT/2}\right)}$

(d) $F(s) = \frac{1}{1-e^{-sT}}$

Ans. **(b)**

$$L(f(t)) = \frac{1}{1-e^{-sT}}\int_0^{T/2} e^{-st}dt = \frac{1}{1-e^{-sT}}\left(\frac{e^{-st}}{-s}\right)\Bigg|_0^{T/2}$$

$$= \frac{1}{s\left(1-e^{-sT}\right)}\cdot\left(1-e^{-sT/2}\right) = \frac{1}{s}\cdot\frac{1-e^{-sT/2}}{\left(1-e^{-sT/2}\right)\left(1+e^{-sT/2}\right)}$$

$$= \frac{1}{s}\cdot\frac{1}{1+e^{-sT/2}}$$

End of Solution

Q.31 A network consisting of a finite number of linear resistor (R), inducer (L), and capacitor (C) elements, connected all in series or all in parallel, is excited with a source of the form

$$\sum_{k=1}^{3} a_k \cos(k\omega_0 t), \text{ where } a_k \neq 0,\ \omega_0 \neq 0.$$

The source has nonzero impedance. Which one of the following is a possible form of the output measured across a resistor in the network?

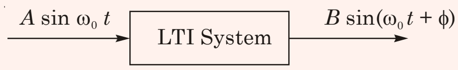
(a) $\sum_{k=1}^{3} b_k \cos(k\omega_0 t + \phi_k)$, where $b_k \neq a_k, \forall k$

(b) $\sum_{k=1}^{3} b_k \cos(k\omega_0 t + \phi_k)$, where $b_k \neq 0, \forall k$

(c) $\sum_{k=1}^{3} a_k \cos(k\omega_0 t + \phi_k)$

(d) $\sum_{k=1}^{2} a_k \cos(k\omega_0 t + \phi_k)$

Ans. **(a)**



When a sinusoidal input is given to LTI system, the output is also a sinusoid with change in magnitude and the phase shift offered by LTI system.

End of Solution

Q.32 A first-order low-pass filter of time constant T is excited with different input signals (with zero initial conditions up to $t = 0$). Match the excitation signals X, Y, Z with the corresponding time responses for $t \geq 0$:

List-I	**List-II**
X. Impulse	**P.** $1 - e^{-t/T}$
Y. Unit step	**Q.** $t - T(1 - e^{-t/T})$
Z. Ramp	**R.** $e^{-t/T}$

(a) X – R; Y – Q; Z – P
(b) X – Q; Y – P; Z – R
(c) X – R; Y – P; Z – Q
(d) X – P; Y – R; Z – Q

Ans. **(c)**

For 1st order system

$$G(s) = \frac{1}{sT};\ H(s) = 1$$

Impulse response $R(s) = 1$

$$Y(s) = \left(\frac{G(s)}{1 + G(s)H(s)} R(s)\right) = \left(\frac{1}{1 + sT}\right) = \frac{1}{T} e^{-t/T} \quad \text{for } t \geq 0$$

Unit step response $R(s) = \frac{1}{s}$

$$Y(s) = \frac{1}{s(1 + sT)} = \frac{(1 + sT) - (sT)}{s(1 + sT)} = \frac{1}{s} - \frac{T}{(1 + sT)}$$

$$= \frac{1}{s} - \frac{T}{T\left(s + \frac{1}{T}\right)}$$

$$y(t) = 1 - e^{-t/T} \quad \text{for } t \geq 0$$

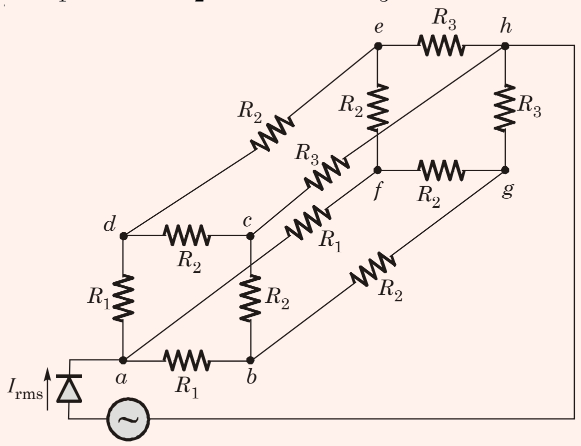
Ramp response $R(s) = \frac{1}{s^2}$

$$Y(s) = \frac{1}{s^2(1 + sT)} = \frac{1}{s^2} - \frac{T}{s} + \frac{T}{s + \frac{1}{T}}$$

$$y(t) = t - T(1 - e^{-t/T}) \quad \text{for } t \geq 0$$

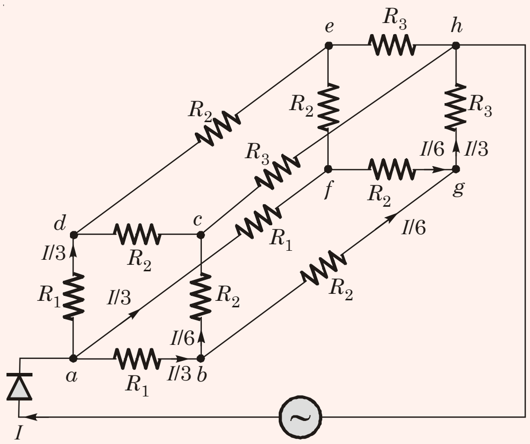
End of Solution

Q.33 An AC voltage source $V = 10\sin(t)$ volts is applied to the following network. Assume that $R_1 = 3$ kΩ, $R_2 = 6$ kΩ and $R_3 = 9$ kΩ, and that the diode is ideal.

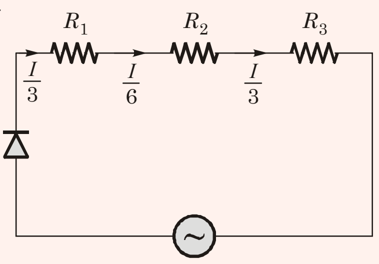


RMS current I_{rms} (in mA) through the diode is _____.

Ans. **(1)**



The equivalent resistance across terminal *ah* (outer loop) is



$$V = \frac{I}{3} \times 3 \text{ k}\Omega + \frac{I}{6} \times 6 \text{ k}\Omega + \frac{I}{3} \times 9 \text{ k}\Omega$$

$$V = 5I$$

or $$\frac{V}{I} = 5\text{ k}\Omega$$

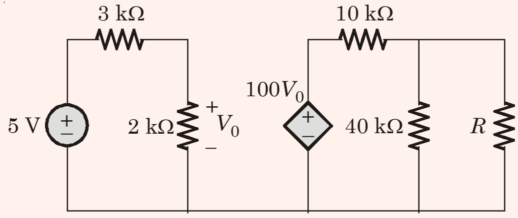
For half wave rectifier

$$I_{\text{rms}} = \frac{I_m}{(2)} = \frac{10\sin t}{5\text{ k}\Omega} = 2\text{ sint mA}$$

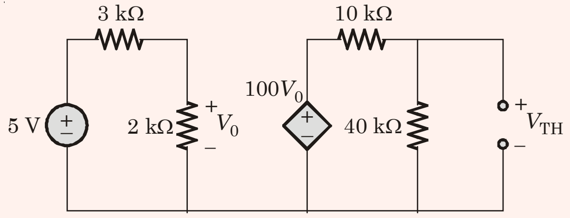
$$\therefore \quad I_{\text{rms}} = \frac{I_m}{2} = 1\text{ mA}$$

End of Solution

Q.34 In the circuit shown in the figure, the maximum power (in watt) delivered to the resistor R is _____



Ans. **(0.8)**



For maximum power transfer,

$$R = R_{TH}$$

$$V_0 = 5 \times \frac{2\text{ k}\Omega}{5\text{ k}\Omega} = 2\text{ V}$$

From output loop, $$V_{\text{TH}} = 100 \times 2 \times \frac{40\text{ k}\Omega}{50\text{ k}\Omega}$$

$$V_{\text{TH}} = 160\text{ V}$$

and $$R_{\text{TH}} = 10\text{ k}\Omega \,||\, 40\text{ k}\Omega = \frac{10 \times 40}{50} = 8\text{ k}\Omega$$

$$\therefore \quad \text{Maximum power} = \frac{V_{TH}^2}{4R_{TH}} = \frac{16 \times 16}{4 \times 8} = 0.8\text{ W}$$

End of Solution

Q.35 Consider the signal

$x[n] = 6\delta[n+2] + 3\delta[n+1] + 8\delta[n] + 7\delta[n-1] + 4\delta[n-2]$.

If $X(e^{j\omega})$ is the discrete-time Fourier transform of $x[n]$,

then $\frac{1}{\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\sin^2(2\omega)d\omega$ is equal to ___

Ans. **(8)**

From the definition of DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

$$x[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y(e^{j\omega})d\omega = \sum_{n=-\infty}^{\infty} x[0]y[0]$$

$$Y(e^{j\omega}) = \sin^2(2\omega)$$

$$= \frac{1-\cos 4\omega}{2} = \frac{1}{2} - \frac{1}{4}e^{4j\omega} - \frac{1}{4}e^{-4j\omega}$$

$$y[n] = \frac{1}{2}\delta[n] - \frac{1}{4}\delta[n+4] - \frac{1}{4}\delta[n-4]$$

$$y[n] = \left\{-\frac{1}{4}, 0, 0, 0, \underset{\uparrow}{\frac{1}{2}}, 0, 0, 0, -\frac{1}{4}\right\}$$

$$\Rightarrow \quad y[0] = \frac{1}{2}$$

$$x[n] = \{6, 3, \underset{\uparrow}{8}, 7, 4\}; \quad x[0] = 8$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y(e^{j\omega})d\omega = 2\sum_{n=-\infty}^{\infty} x[0]y[0] = 2 \times 8 \times \frac{1}{2} = 8$$

End of Solution

Q.36 Consider a silicon *p-n* junction with a uniform acceptor doping concentration of 10^{17} cm^{-3} on the p-side and a uniform donor doping concentration of 10^{16} cm^{-3} on the *n*-side. No external voltage is applied to the diode.

Given: kT/q = 26 mV, $n_i = 1.5 \times 10^{10}$ cm^{-3}, $\varepsilon_{si} = 12\ \varepsilon_0$, $\varepsilon_0 = 8.85 \times 10^{-14}$ F/m, and

$q = 1.6 \times 10^{-19}$ C.

The charge per unit junction area (nC cm^{-2}) in the depletion region on the *p*-side is ____

Ans. **(4.83)**

$$V_0 = V_T ln \frac{N_A N_D}{n_i^2}$$

$$= 26 \times 10^{-3}\ ln \frac{10^{16} \times 10^{17}}{(1.5 \times 10^{10})^2}$$

$$V_0 = 0.757\text{ V}$$

$$W = \sqrt{\frac{2\varepsilon}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0}$$

$$= \sqrt{\frac{2 \times 8.854 \times 10^{-16} \times 12}{1.6 \times 10^{-19}}\left[\frac{1}{10^{16}} + \frac{1}{10^{17}}\right]0.757}$$

$$W = 3.3255\ \mu\text{cm}$$

$$W_P = \frac{WN_D}{N_A + N_D} = \frac{3.3255 \times 10^{-6} \times 10^{16}}{10^{16} + 10^{17}}$$

$$= 0.3023\ \mu\text{cm}$$

Charge per unit junction area in the depletion layer on p side is

$$= qN_A W_P$$

$$= 1.6 \times 10^{-19} \times 10^{17} \times 0.3023 \times 10^{-6}$$

$$= 4.8368\text{ nc/cm}^2$$

End of Solution

Q.37 Consider an n-channel metal oxide semiconductor field effect transistor (MOSFET) with a gate-to-source voltage of 1.8 V. Assume that $\frac{W}{L} = 4$, $\mu_N C_{ox} = 70 \times 10^{-6}\ AV^{-2}$, the threshold voltage is 0.3 V, and the channel length modulation parameter is 0.09 V^{-1}. In the saturation region, the drain conductance (in micro seimens) is _____

Ans. **(28.35)**

In the saturation region

$$g_d = \lambda I_{DS}$$

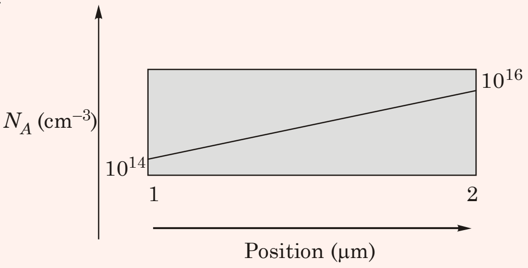
$$= \lambda\left[\frac{1}{2}\mu_n C_{ox}\frac{w}{L}(V_{GS} - V_T)^2\right]$$

$$= 0.09\left[\frac{1}{2} \times 70 \times 10^{-6} \times 4(1.8 - 0.3)^2\right]$$

$$g_d = 28.35\ \mu\text{s}$$

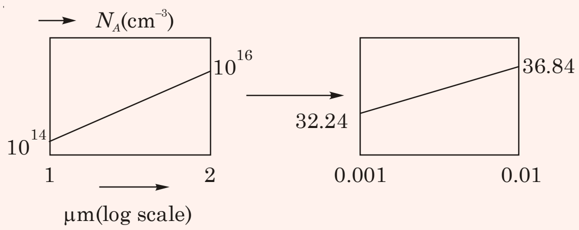
End of Solution

Q.38 The figure below shows the doping distribution in a p-type semiconductor in log scale.



The magnitude of the electric field (in kV/cm) in the semiconductor due to non uniform doping is _____

Ans. **(0.0133)**



Applying the current density equation

$$J = J_{\text{Drift}} + J_{\text{Diffusion}}$$

∴ There is no net flow of current

thus $$J = 0$$

hence, for holes we can write

$$0 = -qD_P \frac{dP}{dx} + q\mu_P PE$$

$$qD_P \frac{dP}{dx} = q\mu_P PE$$

$$\mu_P V_T = \mu_P PE$$

$$E = \frac{V_T}{P}\frac{dP}{dx} \qquad P \cong N_A$$

$$E = \frac{V_T}{N_A}\frac{dN_A}{dx}$$

$$\Rightarrow \qquad E = V_T \frac{d}{dx} ln[N_A(x)]$$

⇒ now since in the question it is mentioned that the units are in log scale, we can write.

$$\log_{10} x_1 = 1\mu\text{m}$$

$$\Rightarrow \qquad x_1 = 10^1\ \mu\text{m} = 0.001\ \text{cm}$$

$$\log_{10} x_2 = 2\ \mu\text{m}$$

$$\Rightarrow \quad x_2 = 10^{2}\ \mu\text{m} = 0.01\ \text{cm}$$

$$ln(10^{14}) = 32.23$$

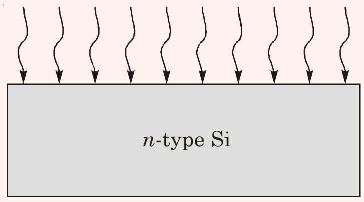
$$ln(10^{16}) = 36.84$$

$$E = 0.026\left[\frac{36.84-32.23}{0.01-0.001}\right]$$

$$E = 0.0133\ \text{kV/cm}$$

End of Solution

Q.39 Consider a silicon sample at T = 300 K, with a uniform donor density $N_d = 5 \times 10^{16}\ \text{cm}^{-3}$, illuminated uniformly such that the optical generation rate is $G_{opt} = 1.5 \times 10^{20}\ \text{cm}^{-3}\ \text{s}^{-1}$ throughout the sample. The incident radiation is turned off at $t = 0$. Assume low-level injection to be valid and ignore surface effects. The carrier lifetimes are τ_{p0} = 0.1 μs and τ_{n0} = 0.5 μs.



The hole concentration at t = 0 and the hole concentration at t = 0.3 μs, respectively, are

(a) $1.5 \times 10^{13}\ \text{cm}^{-3}$ and $7.47 \times 10^{11}\ \text{cm}^{-3}$
(b) $1.5 \times 10^{13}\ \text{cm}^{-3}$ and $8.23 \times 10^{11}\ \text{cm}^{-3}$
(c) $7.5 \times 10^{13}\ \text{cm}^{-3}$ and $3.73 \times 10^{11}\ \text{cm}^{-3}$
(d) $7.5 \times 10^{13}\ \text{cm}^{-3}$ and $4.12 \times 10^{11}\ \text{cm}^{-3}$

Ans. **(a)**

Given

$$G_{opt} = 1.5 \times 10^{20}/\ \text{cm}^3/\text{sec}$$

$$G_{opt} = R = \frac{N_A}{\tau_P} \Rightarrow 1.5 \times 10^{20} = \frac{N_A}{0.1 \times 10^{-6}}$$

$$\therefore \quad N_A = 1.5 \times 10^{13}/\text{cm}^3$$

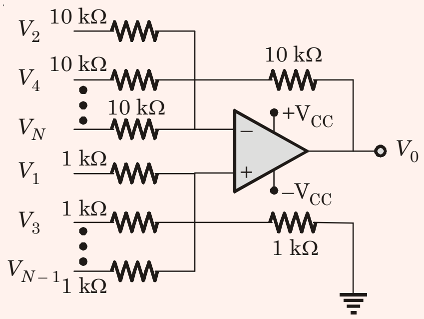
$$P(t) = P_{n0}e^{-t/\tau_p}$$

$$= 1.5 \times 10^{13}\ e^{\frac{-0.3}{0.1}}$$

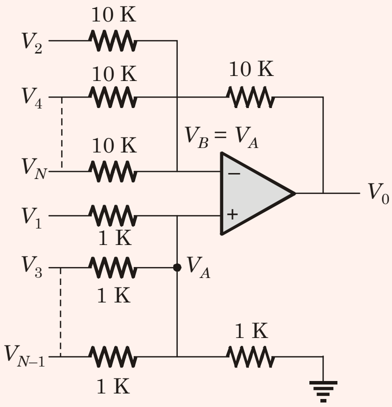
$$= 7.46 \times 10^{11}/\text{cm}^3$$

End of Solution

Q.40 An ideal opamp has voltage sources, V_1, V_3, V_5, ... V_{N-1} connected to the non-inverting input and V_2, V_4, V_6, ..., V_N connected to the inverting input as shown in the figure below ($+V_{CC}$ = 15 volt, $-V_{CC}$ = –15 volt). The voltages V_1, V_2, V_3, V_4, V_5, V_6, ... are 1, –1/2, 1/3, –1/4, 1/5, –1/6, ... volt, respectively. As N approaches infinity, the output voltage (in volt) is _____



Ans. **(15)**



Node A:

$$\frac{V_A - V_1}{1\text{ K}} + \frac{V_A - V_3}{1\text{ K}} + \ldots\ldots \frac{V_A - V_{N-1}}{1\text{ K}} + \frac{V_A}{1\text{ K}} = 0$$

$$V_A\left(\frac{N}{2}+1\right) = V_1 + V_3 + \ldots + V_{N-1}$$

$$V_B = V_A \qquad \because \text{ Virtual short}$$

Node B:

$$\frac{V_A - V_2}{10\text{ K}} + \frac{V_A - V_4}{10\text{ K}} + \ldots + \frac{V_A - V_N}{10\text{K}} + \frac{V_A - V_0}{10\text{K}} = 0$$

$$V_0 = V_A\left(\frac{N}{2}+1\right) - (V_2 + V_4 + V_6 + \ldots. + V_N)$$

$$= \left(\frac{N}{2}+1\right)\cdot\frac{(V_1 + V_3 + \ldots + V_{N-1})}{\left(\frac{N}{2}+1\right)} - (V_2 + V_4 + \ldots + V_N)$$

$$= V_1 - V_2 + V_3 - V_4 + \ldots.$$

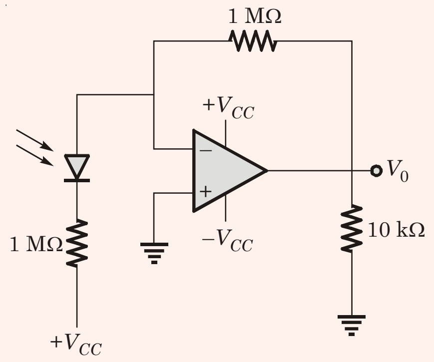
$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \ldots = \Sigma\frac{1}{N} = \infty$$

$\Rightarrow$ Output of opamp goes to saturation

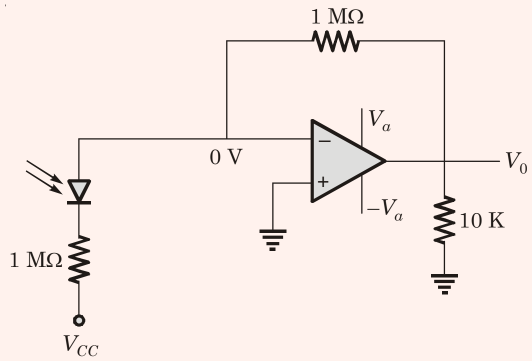
$$V_0 = V_{sat} = V_{CC}$$

End of Solution

Q.41 A *p-i-n* photodiode of responsivity 0.8 A/W is connected to the inverting input of an ideal opamp as shown in the figure, $+V_{CC}$ = 15 V, $-V_{CC}$ = –15 V, Load resistor R_L = 10 kΩ. If 10 μW of power is incident on the photodiode, then the value of the photocurrent (in μA) through the load is _____



Ans. **(800)**



$$\text{Responsivity} = \frac{\text{Generated photo current}}{\text{Incident light power}} = \frac{I_0}{P_i}$$

$$0.8 \text{ A/w} = \frac{I_0}{10 \times 10^{-6}}$$

$\Rightarrow$ $$I_0 = 8 \text{ MA}$$

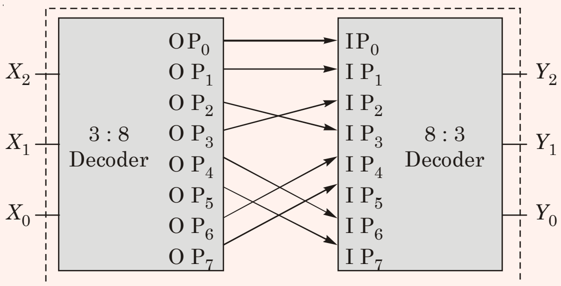
$$V_0 = I_0 \times 1 \text{ M}\Omega = 8 \times 10^{-6} \times 1 \times 10^6 = 8 \text{ V}$$

The photo current through load R_L = 10 kΩ is given by

$$I_L = \frac{V_0}{R_L} = \frac{8}{10 \times 10^3} = 800 \text{ μA} \quad \text{(in upward direction)}$$

End of Solution

Q.42 Identify the circuit below,



(a) Binary to Gray code converter
(b) Binary to XS3 converter
(c) Gray to Binary converter
(d) XS3 to Binary converter

Ans. **(*)**

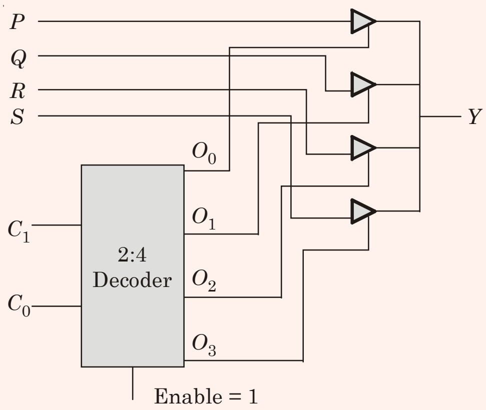
The truth table of the circuit is shown below,

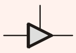
X_2	X_1	X_0	Y_2	y_1	y_0
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	1	1	1
1	1	0	1	0	0
1	1	1	1	0	1

As per the truth table, none of the options given in the question are correct. However, by making some (minor) changes in the circuit, the answer could be obtained as option (a)

End of Solution

Q.43 The functionality implemented by the circuit below is



 is a tristate buffer

(a) 2-to-1 multiplexer (b) 4-to-1 multiplexer
(c) 7-to-1 multiplexer (d) 6-to-1 multiplexer

Ans. **(b)**

When the outputs (O_0, O_1, O_2, O_3) of the decoder are at logic 1, the corresponding tristate buffer is activated. In that case, whatever data is applied at the input of a buffer, becomes its output.

Hence, when

$\Rightarrow \quad C_1C_0 = 00, \quad$ Then $O_0 = 1,$

$\therefore \quad Y = P$

$\Rightarrow \quad C_1C_0 = 01, \quad$ Then $O_1 = 1,$

$\therefore \quad Y = Q$

$\Rightarrow \quad C_1C_0 = 10, \quad$ Then $O_2 = 1,$

$\therefore \quad Y = R$

$\Rightarrow \quad C_1C_0 = 11, \quad$ Then $O_3 = 1,$

$\therefore \quad Y = S$

$\therefore$ the circuit effectively behaves as a 4 to 1 multiplexer.

End of Solution

Q.44 In an 8085 system, a PUSH operation requires more clock cycles than a POP operation. Which one of the following options is the correct reason for this?

(a) For POP, the data transceivers remain in the same direction as for instruction fetch (memory to processor), whereas for PUSH their direction has to be reversed.

(b) Memory write operations are slower than memory read operations in an 8085 based system.

(c) The stack pointer needs to be pre-decremented before writing registers in a PUSH, whereas a POP operation uses the address already in the stack pointer.

(d) Order of registers has to be interchanged for a PUSH operation, whereas POP uses their natural order.

Ans. **(c)**

For PUSH R_P instruction in 8085 machine cycles are Fetch(F), Write (W) and Write (W) i.e. 6 + 3 + 3 = 12 T -states/clock cycles. Stack pointer holds the address of previously stored temporary data, so to store new data SP is decremented by '1' after decoding on code, hence fetch has 6T-states unlike 4T - states for most of the instruction.

But for POP $R_P \rightarrow$ Fetch(F), Read (R) and Read (R)

i.e. $4 + 3 + 3 \rightarrow 10$ T - States

End of Solution

Q.45 The open-loop transfer function of a unity-feedback control system is

$$G(s) = \frac{K}{s^2 + 5s + 5}$$

The value of K at the breakaway point of the feedback control system's root-locus plot is _____.

Ans. **(1.25)**

Characteristic equation is $1 + G(s)\,H(s) = 0$

$$1 + \frac{K}{s^2 + 5s + 5} = 0$$

$$K = -s^2 - 5s - 5$$

For break away point $\frac{dK}{ds} = 0$

$$\frac{dK}{ds} = -2s - 5 = 0 \Rightarrow s = -2.5$$

Acc. to magnitude condition,

$$|G(s)\,H(s)|_{s=-2.5} = 1$$

$$|G(s)\,H(s)|_{s=-2.5} = \frac{K}{|(-2.5)^2 + 5 \times -2.5 + 5|} = 1$$

$$\Rightarrow \quad K = |(6.25 + 5 - 12.5)|$$

$$K = 1.25$$

End of Solution

Q.46 The open-loop transfer function of unity-feedback control system is given by

$$G(s) = \frac{K}{s(s+2)}$$

For the peak overshoot of the closed-loop system to a unit step input to be 10%, the value of K is _____

Ans. **(2.8)**

$$G(s) = \frac{K}{s(s+2)} \;;\; H(s) = 1$$

Characteristic equation $= 1 + G(s)\,H(s) = 0$

$$1 + \frac{K}{s(s+2)} = 0$$

$$s^2 + 2s + K = 0$$

$$\Rightarrow \quad \omega_n = \sqrt{K}$$

$$2\xi\omega_n = 2$$

$$\xi = \frac{1}{\sqrt{K}}$$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.1$$

$$-\frac{\pi\xi}{\sqrt{1-\xi^2}} = ln\,(0.1) \quad \Rightarrow \quad \frac{\pi\xi}{\sqrt{1-\xi^2}} = 2.3$$

$$\pi^2\xi^2 = (2.3)^2(1-\xi^2)$$

$$15.16\,\xi^2 = (2.3)^2$$

$$\Rightarrow \quad \xi = 0.59$$

$$\Rightarrow \quad \text{Also } K = \frac{1}{\xi^2}$$

$$\Rightarrow \quad K = 2.8$$

End of Solution

Q.47 The transfer function of a linear time invariant system is given by
$H(s) = 2s^4 - 5s^3 + 5s - 2$
The number of zeroes in the right half of the s-plane is ______

Ans. **(3)**

$$2s^4 - 5s^3 + 5s - 2 = 0$$

By Routh Array,

s^4	2	0	–2
s^3	–5	5	
s^2	2	–2	
s^1	0(2)		
s°	–2		

Number of sign changes = number of roots (zeros) in right half of s-plane = 3

End of Solution

Q.48 Consider a discrete memoryless source with alphabet $S = \{s_0, s_1, s_2, s_3, s_4, ...\}$ and respective probabilities of occurrence $P = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, ...\right\}$. The entropy of the source (in bits) is _____

Ans. **(2)**

Entropy of source is given as

$$H = \sum_{i=0}^{N} P_i \log_2 \frac{1}{P_i} \qquad \text{(Here, } N = \infty\text{)}$$

$$= \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \frac{1}{16}\log_2 16 + ...$$

$$H = \frac{1}{2} + 2\times\left(\frac{1}{2}\right)^2 + 3\times\left(\frac{1}{2}\right)^3 + 4\times\left(\frac{1}{2}\right)^4 + ... \qquad ...(i)$$

$$= \sum_{k=0}^{\infty} k\left(\frac{1}{2}\right)^k$$

$$\frac{H}{2} = \left(\frac{1}{2}\right)^2 + 2\times\left(\frac{1}{2}\right)^3 + 3\times\left(\frac{1}{2}\right)^4 + \ldots \qquad \ldots\text{(ii)}$$

Subtracting (ii) from (i)

$$\frac{H}{2} = \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots$$

$$\frac{H}{2} = \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)} = 1$$

$\Rightarrow$ $H = 2$ bits/symbol

End of Solution

Q.49 A digital communication system uses a repetition code for channel encoding/decoding. During transmission, each bit is repeated three times (0 is transmitted as 000, and 1 is transmitted as 111). It is assumed that the source puts out symbols independently and with equal probability. The decoder operates as follows: In a block of three received bits, if the number of zeros exceeds the number of ones, the decoder decides in favor of a 0, and if the number of ones exceeds the number of zeros, the decoder decides in favor of a 1. Assuming a binary symmetric channel with crossover probability p = 0.1. The average probability of error is _____.

Ans. **(0.028)**

Crossover probability, $P = 0.1$

Average probability of error $= 3p^2 - 3p^3$

$$= 3(0.1)^2 - 2(0.1)^3 = 0.028$$

End of Solution

Q.50 An analog pulse $s(t)$ is transmitted over an additive white Gaussian noise (A WGN) channel. The received signal is $r(t) = s(t) + n(t)$ where $n(t)$ is additive white Gaussian noise with power spectral density $\frac{N_0}{2}$. The received signal is .... $h(t)$. Let E_s and E_h denote the energies of the pulse $s(t)$ and the filter $h(t)$, respectively. When the signal to noise ratio (SNR) is maximized at the output of the filter (SNR_{max}), which of the following holds?

(a) $E_s = E_h;\ SNR_{max} = \frac{2E_s}{N_0}$

(b) $E_s = E_h;\ SNR_{max} = \frac{E_s}{2N_0}$

(c) $E_s > E_h;\ SNR_{max} > \frac{2E_s}{N_0}$

(d) $E_s < E_h;\ SNR_{max} = \frac{2E_h}{N_0}$

Ans. **(a)**

When the signal to Noise ratio is maximum

$$h(t) = s(T-t)$$

but shifting doesn't change the energy

$$\Rightarrow \quad E_h = E_s$$

and $$(\text{SNR})_{\max} = \frac{2E_s}{N_0}$$

End of Solution

Q.51 The current density in a medium is given by

$$\vec{J} = \frac{400\sin\theta}{2\pi(r^2+4)}\hat{a}_r \; Am^{-2}$$

The total current and the average current density flowing through the portion of a spherical surface $r = 0.8$ m, $\frac{\pi}{12} \le \theta \le \frac{\pi}{4}, 0 \le \phi \le 2\pi$ are given, respectively, by

(a) 15.09 A, 12.86 Am^{-2}
(b) 18.73 A, 13.65 Am^{-2}
(c) 12.86 A, 9.23 Am^{-2}
(d) 10.28 A, 7.56 Am^{-2}

Ans. **(d)**

$$I = \int J \cdot ds$$

$$= \int_{\theta=\frac{\pi}{12}}^{\pi/4} \int_{\phi=0}^{2\pi} \frac{400\sin\theta}{2\pi(r^2+4)} . r^2 \sin\theta d\theta d\phi$$

$$= \frac{400}{2\pi(r^2+4)} . r^2 . \phi\Big|_0^{2\pi} \int_{\pi/12}^{\pi/4} \sin^2\theta \, d\theta$$

$$= \frac{400r^2}{(r^2+4)} \int_{\pi/12}^{\pi/4} \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{400 \cdot r^2}{(r^2+4)} \left(\frac{\left(\frac{\pi}{4} - \frac{\pi}{12}\right)}{2} - \left(\frac{\sin 2\theta}{4}\right)_{\pi/12}^{\pi/4} \right)$$

$$= \frac{400 \cdot r^2}{(r^2+4)} \left(\frac{\pi}{12} - \left(\frac{1-1/2}{4}\right)\right)\Bigg|_{r=0.8}$$

$$= \frac{400 \times 0.8 \times 0.8}{4.64} \times 0.13 = 7.56 \text{ Amp}$$

$$\text{Total area} = \int ds = \iint r^2 \sin\theta d\theta d\phi$$

$$= r^2 \int_{\theta=\frac{\pi}{12}}^{\pi/4} \sin\theta d\theta . 2\pi = r^2 \cdot 2\pi \cdot 0.259\Big|_{r=0.8}$$

$$= 0.8^2 \times 0.5 \times 2\pi \times \frac{1}{4} = 1.041 \text{ m}^2$$

$$\text{Average current} = \frac{7.56}{1.041} = 7.56 \text{ A/m}^2$$

Note : Option (d) is the closest option

End of Solution

Q.52 An antenna pointing in a certain direction has a noise temperature of 50 K. The ambient temperature is 290 K. The antenna is connected to a pre-amplifier that has a noise figure of 2 dB and an available gain of 40 dB over an effective bandwidth of 12 MHz. The effective input noise temperature T_e for the amplifier and the noise power P_{a0} at the output of the preamplifier, respectively, are

(a) $T_e = 169.36$ K and $P_{a0} = 3.73 \times 10^{-10}$ W
(b) $T_e = 170.8$ K and $P_{a0} = 4.56 \times 10^{-10}$ W
(c) $T_e = 182.5$ K and $P_{a0} = 3.85 \times 10^{-10}$ W
(d) $T_e = 160.62$ K and $P_{a0} = 4.6 \times 10^{-10}$ W

Ans. **(a)**

(i) $$T_e = (F-1)T_0 = \left(10^{2/10} - 1\right)290$$
$$= 169.6 \text{ K}$$

(ii) $$N_i = k(T_{ant} + T_e)B$$
$$= 1.38 \times 10^{-23} \times (50 + 169.6) \times 12 \times 10^6$$
$$= 3.63 \times 10^{-14} \text{ W}$$
$$N_o = N_i \times \text{Gain}$$
$$= 3.63 \times 10^{-14} \times 10^4$$
$$= 3.63 \times 10^{-10} \text{ W}$$

End of Solution

Q.53 Two lossless X-band horn antennas are separated by a distance of 200λ. The amplitude reflection coefficients at the terminals of the transmitting and receiving antennas are 0.15 and 0.18, respectively. The maximum directivities of the transmitting and receiving antennas (over the isotropic antenna) are 18 dB and 22 dB, respectively. Assuming that the input power in the lossless transmission line connected to the antenna is 2 W, and that the antennas are perfectly aligned and polarization matched, the power (in mW) delivered to the load at the receiver is _____

Ans. **(3)**

$d = 200\,\lambda$

TX	RX
$\lvert\Gamma_t\rvert = 0.15$	$\lvert\Gamma_r\rvert = 0.18$
18 dB	22 dB
$P_t = 2$ W	$P_r = ?$

$$G_t = 10^{1.8},\ G_r = 10^{2.2}$$

$$P_r = \frac{\left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right)G_tG_r}{\left(\frac{4\pi d}{\lambda}\right)^2}\cdot P_t$$

$$= \frac{\left(1-|0.15|^2\right)\left(1-|0.18|^2\right)10^{1.8}\cdot 10^{2.2}}{\left(\frac{4\pi 200\lambda}{\lambda}\right)^2}\times 2$$

$$= 2.99476 \times 10^{-3}\text{ W} = 2.995\text{ mW} \approx 3\text{ mW}$$

End of Solution

Q.54 The electric field of a uniform plane wave travelling along the negative z direction is given by the following equation:

$$\vec{E}_w^i = (\hat{a}_x + j\hat{a}_y)E_0 e^{jkz}$$

This wave is incident upon a receiving antenna placed at the origin and whose radiated electric field towards the incident wave is given by the following equation:

$$\vec{E}_a = (\hat{a}_x + 2\hat{a}_y)E_I\frac{1}{r}e^{-jkr}$$

The polarization of the incident wave, the polarization of the antenna and losses due to the polarization mismatch are, respectively,

(a) Linear, Circular (clockwise), –5dB
(b) Circular (clockwise), Linear, –5dB
(c) Circular (clockwise), Linear, –3dB
(d) Circular (anticlockwise), Linear, –3dB

Ans. **(c)**

$$\vec{E}_w^i = (\hat{a}_x + j\hat{a}_y)E_0 e^{jkz}$$

⇒ Wave contains two orthogonal components and Y component leads X component leads by 90° and also wave is travelling in negative Z-direction.

⇒ Circular (clockwise) polarization

$$\vec{E}_a = (\hat{a}_x + 2\hat{a}_y)E_I\frac{1}{r}e^{-jkr}$$

⇒ Wave contains two orthogonal components with unequal amplitudes and both are in-phase.

⇒ Linear polarization.

$$\text{PLF} = |\hat{\rho}_{inc} \cdot \hat{\rho}_{ant}|^2$$

where
$$\hat{\rho}_{inc} = \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}$$

$$\hat{\rho}_{ant} = \frac{\hat{a}_x + 2\hat{a}_y}{\sqrt{5}}$$

$$\text{PLF} = \left|\frac{1+j2}{\sqrt{10}}\right|^2 = \frac{5}{10} = \frac{1}{2}$$

$\Rightarrow$
$$\text{PLF (dB)} = 10\log\frac{1}{2} = -3 \text{ dB}$$

End of Solution

Q.55 The far-zone power density radiated by a helical antenna is approximated as:

$$\vec{W}_{rad} = \vec{W}_{average} \approx \widehat{a_r} C_0 \frac{1}{r^2}\cos^4\theta$$

The radiated power density is symmetrical with respect to ϕ and exists only in the upper hemisphere $0 \le \theta \le \frac{\pi}{2}; 0 \le \phi \le 2\pi$; C_0 is a constant. The power radiated by the antenna (in watts) and the maximum directivity of antenna, respectively, are

(a) $1.5\ C_0$, 10 dB
(b) $1.256\ C_0$, 10 dB
(c) $1.256\ C_0$, 12 dB
(d) $1.5\ C_0$, 12 dB

Ans. **(b)**

$$\text{Power radiated} = \int W_{rad}.ds$$

$$= \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} C_0 \frac{1}{r^2}\cos^4\theta \cdot r^2 \sin\theta d\theta d\phi$$

$$= 2\pi \cdot \frac{C_0}{r^2} \cdot r^2 \int_{\theta=0}^{\pi/2} \cos^4\theta \cdot d(-\cos\theta)$$

$$= 2\pi.C_0\left(-\frac{\cos^5\theta}{5}\right)\Bigg|_0^{\pi/2} = \frac{2\pi}{5}C_0 = 1.256\ C_0$$

$$\text{Directivity} = \frac{4\pi \cdot U}{\int W_{rad} \cdot d\Omega}$$

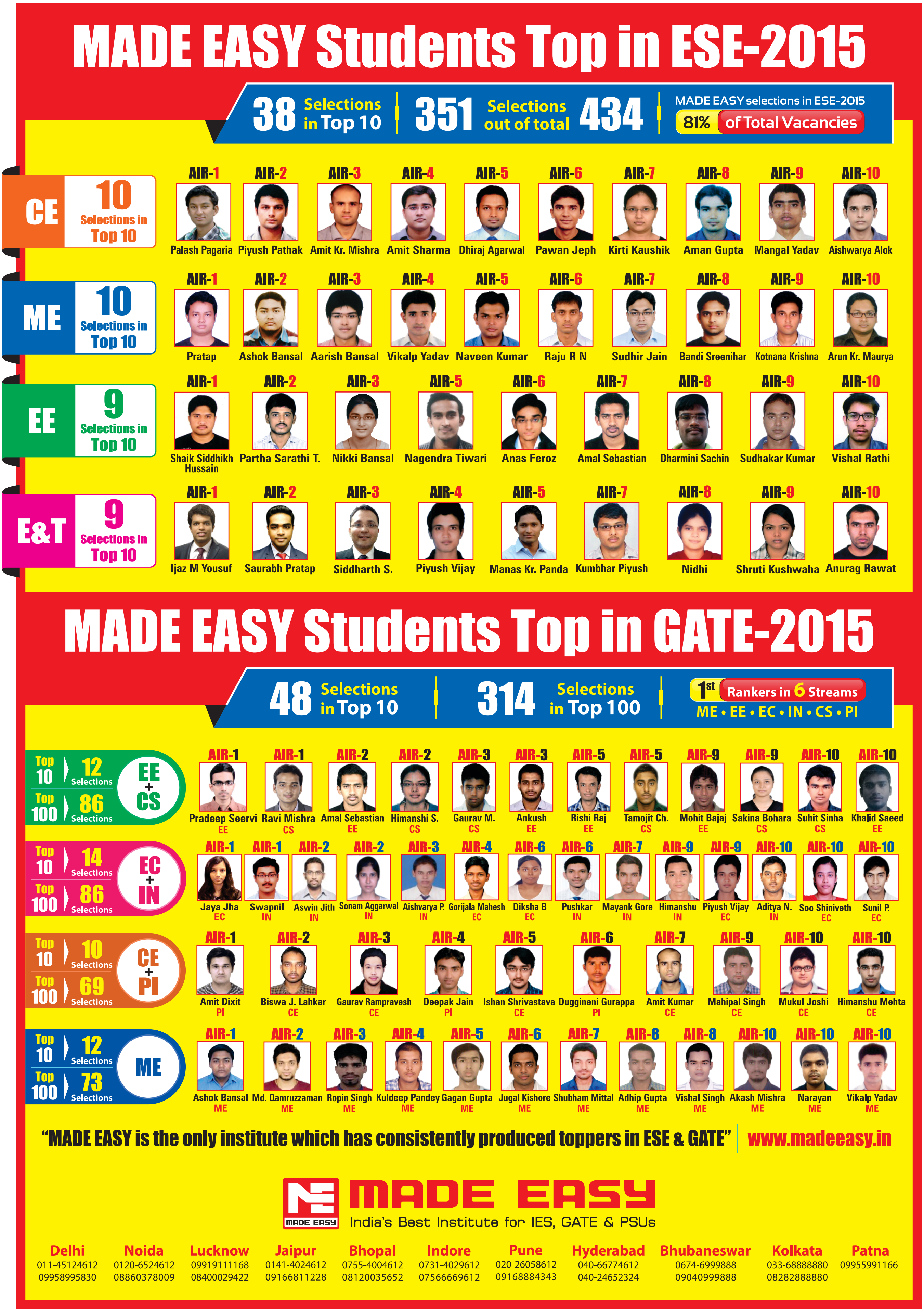
$$= \frac{4\pi.C_0.\cos^4\theta}{\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} C_0.\cos^4\theta.\sin\theta d\theta d\phi} = \frac{2\cos^4\theta}{1/5} = 10\cos^4\theta$$

$$\text{Max value} = 10$$

$\Rightarrow$
$$\text{max value (in dB)} = 10\log_{10} = 10 \text{ dB}$$

End of Solution

Advt.



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AIR-3 Ropin Singh ME
AIR-4 Kuldeep Pandey ME
AIR-5 Gagan Gupta ME
AIR-6 Jugal Kishore ME
AIR-7 Shubham Mittal ME
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