Solutions of

of

Electronics Engineering

GATE-2016

Session 3 | Set-2

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Section - I (General Aptitude)

One Mark Questions

Q.1 Based on the given statements, select the appropriate option with respect to grammar and usage. Statements

I. The height of Mr. X is 6 feet.

II. The height of Mr. Y is 5 feet.

(a) Mr. X is longer than Mr. Y.

(b) Mr. X is more elongated than Mr. Y

(c) Mr. X is taller than Mr. Y

(d) Mr. X is lengthier than Mr. Y

Ans. **(c)**

End of Solution

Q.2 The students _____ the teacher on teachers' day for twenty years of dedicated teaching.

(a) facilitated
(b) felicitated
(c) fantasized
(d) facillitated

Ans. **(b)**

End of Solution

Q.3 After India's cricket world cup victory in 1985, Shrotria who was playing both tennis and cricket till then, decided to concentrate only on cricket. And the rest is history.

What does the underlined phrase mean in this context?

(a) history will rest in peace
(b) rest is recorded in history books
(c) rest is well known
(d) rest is archaic

Ans. **(c)**

End of Solution

Q.4 Given (9 inches)$^{1/2}$ = (0.25 yards)$^{1/2}$, which one of the following statements is **TRUE**?

(a) 3 inches = 0.5 yards
(b) 9 inches = 1.5 yards
(c) 9 inches = 0.25 yards
(d) 81 inches = 0.0625 yards

Ans. **(c)**

End of Solution

Q.5 ***S***,***M***,***E*** and ***F*** are working in shifts in a team to finish a project. ***M*** works with twice the efficiency of others but for half as many days as ***E*** worked. ***S*** and ***M*** have 6 hour shifts in a day. whereas ***E*** and ***F*** have 12 hours shifts. What is the ratio of contribution of ***M*** to contribution of ***E*** in the project?

(a) 1 : 1 (b) 1 : 2
(c) 1 : 4 (d) 2 : 1

Ans. **(b)**

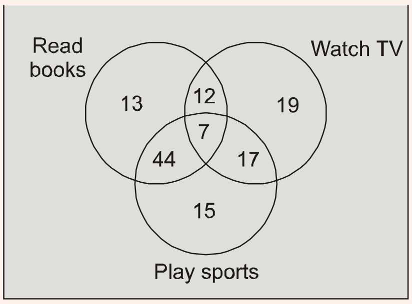
'M' works with twice efficiency as E but worked for half as many days. So in this repsect they will do equal work if their shifts would have been for same timings. But M's shift is for hrs, while E's shift for 12 hrs. Hence E will do twice the work as M.

Ratio of contribution of M : E in work, 1 : 2

End of Solution

Two Marks Questions

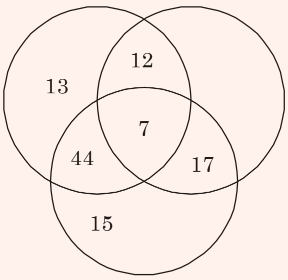
Q.6 The Venn diagram shows the preference of the student population for leisure activities.



From the data given, the number of students who like to read books or play sports is _____.

(a) 44 (b) 51
(c) 79 (d) 108

Ans. **(d)**



The number of student who like to read books or play sports have been shown

$$= 13 + 12 + 44 + 7 + 15 + 1$$
$$= 108$$

End of Solution

Q.7 Social science disciplines were in existence in an amorphous form until the colonial period when they were institutionalized. In varying degrees, they were intended to further the colonial interest. In the time of globalization and the economic rise of postcolonial countries like India, conventional ways of knowledge production have become obsolete.

Which of the following can be logically inferred from the above statements?

I. Social science disciplines have become obsolete.
II. Social science disciplines had a pre-colonial origin.
III. Social science disciplines always promote colonialism.
IV. Social science must maintain disciplinary boundaries.

(a) II only
(b) I and III only
(c) II and IV only
(d) III and IV only

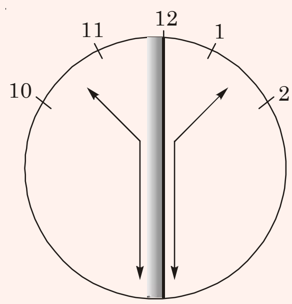
Ans. **(a)**

End of Solution

Q.8 Two and a quarter hours back, when seen in a mirror, the reflection of a wall clock without number markings seemed to show 1 : 30. What is the actual current time shown by the clock?

(a) 8:15
(b) 11:15
(c) 12:15
(d) 12:45

Ans. **(d)**



Mirror image of 1 : 20 is 10 : 30

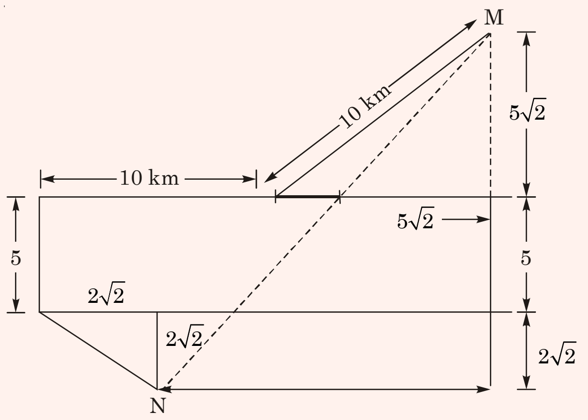
10 : 30 was the time two and quarter hour back so time now will be 12 : 45

End of Solution

Q.9 ***M*** and ***N*** start from the same location. ***M*** travels 10 km East and then 10 km North-East. ***N*** travels 5 km South and then 4 km south-East. What is the shortest distance (in km) between ***M*** and ***N*** at the end of their travel?

(a) 18.60
(b) 22.50
(c) 20.61
(d) 25.00

Ans. **(c)**



See the adjoining figure for solution

$$MM' = 5\sqrt{2} + 5 + 2\sqrt{2} = 5 + 7\sqrt{2}$$

$$NM' = 10 + 5\sqrt{2} - 2\sqrt{2} = 10 + 3\sqrt{2}$$

$$MN = \sqrt{(MM')^2 + (NM')^2}$$

$$M' = \sqrt{(5 + 7\sqrt{2}) + (10 + 3\sqrt{2})^2}$$

$$\approx 20.61$$

End of Solution

Q.10 A wire of length 340 mm is to be cut into two parts. One of the parts is to be made into a square and the other into a rectangle where sides are in the ratio of 1:2. What is the length of the side of the square (in mm) such that the combined area of the square and the rectangle is a **MINIMUM**?

(a) 30 (b) 40
(c) 120 (d) 180

Ans. **(b)**

End of Solution

Section - II (Electronics Engineering)

One Mark Questions

Q.1 The value of x for which the matrix

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 9 & 7 & 13 \\ -6 & -4 & -9+x \end{bmatrix} \text{ has zero as an eigenvalue is \_\_\_\_\_.}$$

Ans. **(1)**

A has an eigen value is zero

$$\therefore \qquad |A| = 0$$

$$\begin{vmatrix} 3 & 2 & 4 \\ 9 & 7 & 13 \\ -6 & -4 & -9+x \end{vmatrix} = 0$$

$3(-63 + 7x + 52) - 2(-81 + 9x + 78) + 4(-36 + 42) = 0$

$3(7x - 11) - 2(9x - 3) + 4(6) = 0$

$21x - 33 - 18x + 6 + 24 = 0$

$$3x - 3 = 0$$

$$x = 1$$

End of Solution

Q.2 Consider the complex valued function

$f(z) = 2z^3 + b|z|^3$ where z is a complex variable. The value of b for which the function $f(z)$ is analytic is _____

Ans. **(0)**

$$f(z) = 2z^3 + b_1|z|^3$$

Given that $f(z)$ is analytic.

which is possible only when $b = 0$

since $|z^3|$ is differentiable at the origin but not analytic.

$2z^3$ is analytic everywhere

$$\therefore \qquad f(z) = 2z^3 + b|z^3| \text{ is analytic}$$

$$\text{only when} \qquad b = 0$$

End of Solution

Q.3 As x varies from –1 to +3, which one of the following describes the behaviour of the function $f(x) = x^3 - 3x^2 + 1$?

(a) $f(x)$ increases monotonically.

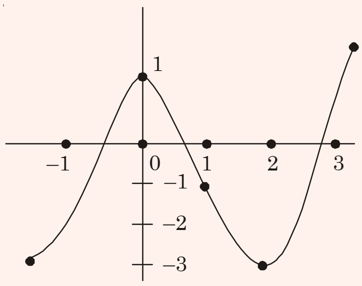
(b) $f(x)$ increases, then decreases and increases again.

(c) $f(x)$ decreases, then increases and decreases again.

(d) $f(x)$ increases and then decreases

Ans. **(b)**

$$f(x) = x^3 - 3x^2 + 1$$
$$f'(x) = 3x^2 - 6x$$
$$f'(x) = 0$$



$$3x^2 - 6x = 0$$
$$3x(x-2) = 0$$
$$x = 0, 2$$
$$f''(x) = 6x - 6$$

At $x = 0 \quad f''(0) = -6$ maxima

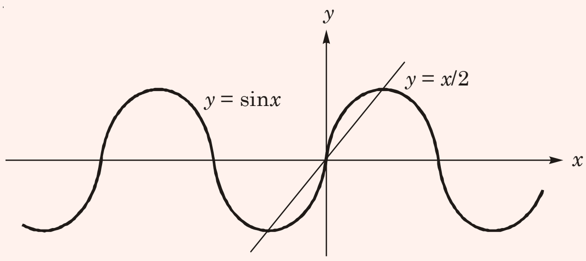
$x = 2 \quad f''(2) = 6$ minima

End of Solution

Q.4 How many distinct values of x satisfy the equation $\sin(x) = x/2$, where x is in radians?

(a) 1
(b) 2
(c) 3
(d) 4 or more

Ans. **(c)**



Hence 3 solutions.

End of Solution

Q.5 Consider the time-varying vector

$I = \hat{x}15\cos(\omega t) + \hat{y}5\sin(\omega t)$ in Cartesian coordinates, where $\omega > 0$ is a constant. When the vector magnitude $|I|$ is at its minimum value, the angle θ that I makes with the x axis (in degrees, such that $0 \le \theta \le 180$) is ______.

Ans. **(90)**

$$I = \hat{x}15\cos\omega t + \hat{y}5\sin\omega t$$

$$|I| = \sqrt{(15\cos\omega t)^2 + (5\sin\omega t)^2}$$

$$= \sqrt{225\cos^2\omega t + 25\sin^2\omega t}$$

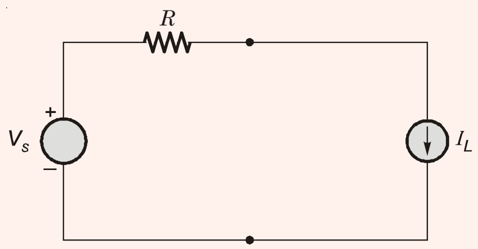
$$= \sqrt{25 + 200\cos^2\omega t}$$

$|I|$ is minimum when $\cos^2\omega t = 0$

or $\theta = \omega t = 90°$

End of Solution

Q.6 In the circuit shown below, V_s is constant voltage source and I_L is a constant current load.



The value of I_L that maximizes the power absorbed by the constant current load is

(a) $\frac{V_s}{4R}$
(b) $\frac{V_s}{2R}$
(c) $\frac{V_s}{R}$
(d) ∞

Ans. **(b)**

In maximum power transformation, half of the voltage drop across source resistance, remaining half across the load.

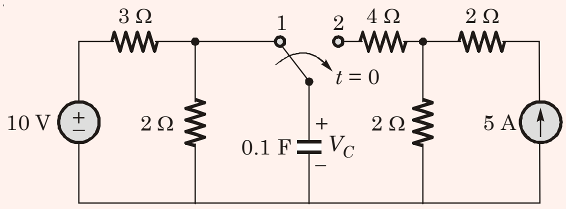
$\therefore$ voltage across source (R)

$$I_L R = \frac{V_s}{2}$$

$$I_L = \frac{V_s}{2R}$$

End of Solution

Q.7 The switch has been in position 1 for a long time and abruptly changes to position 2 at $t = 0$.



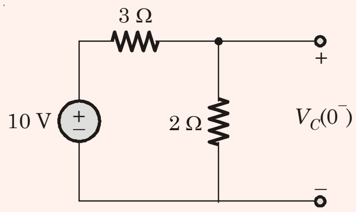
If time t is in seconds, the capacitor voltage V_C (in volts) for $t > 0$ is given by

(a) 4 (1 – exp (–t/0.5)) (b) 10 – 6 exp (–t/0.5)

(c) 4 (1 – exp (–t/0.6)) (d) 10 – 6 exp (–t/0.6)

Ans. **(d)**

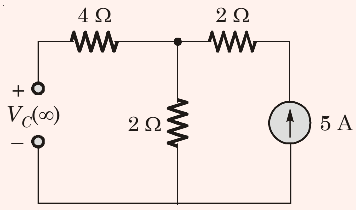
at $t = 0^-$, Switch is at position-1



where, $V_c(0^-) = \dfrac{10 \times 2}{2+3} = 4\text{ V}$ ...(1)

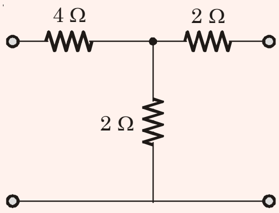
$\therefore$ $V_c(0^-) = V_c(0^+) = 4\text{ V}$

at $t = \infty$



$V_c(\infty) = 5 \times 2 = 10\text{ V}$ ...(2)

The time constant of the circuit is

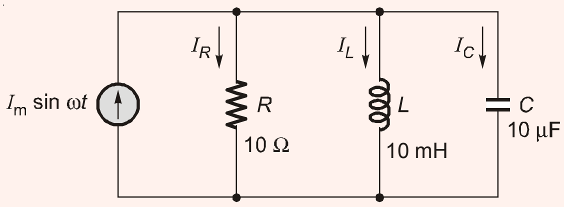


$$\tau = R_{eq}\,C_{eq}$$
$$= (4 + 2) \times 0.1 = 0.6\text{ sec}$$

Therefore
$$V_c(t) = V_c(\infty) + [V_c(0^+) - V_c(\infty)]e^{-t/\tau}$$
$$= 10 + (4 - 10)e^{-t/0.6}$$
$$V_c(t) = (10 - 6e^{-t/0.6})\text{ V}$$

End of Solution

Q.8 The figure shows an RLC circuit with a sinusoidal current source.



At resonance, the ratio $\frac{|I_L|}{|I_R|}$, i.e., the ratio of the magnitudes of the inductor current phasor and the resistor current phasor, is_____

Ans. **(0.316)**

At resonance (for parallel RLC circuit)

$$I_R = I$$
$$I_L = QI\angle{-90^\circ}$$
$$I_C = QI\angle 90^\circ$$

For parallel RLC circuit

$$\frac{|I_L|}{|I_R|} = \frac{IQ}{I} = Q = R\sqrt{\frac{C}{L}} = 10\sqrt{\frac{10\times10^{-6}}{10\times10^{-3}}}$$
$$= 0.316$$

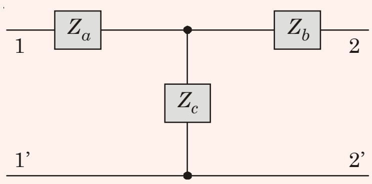
End of Solution

Q.9 The Z-parameter matrix for the two-port network shown is

$$\begin{bmatrix} 2j\omega & j\omega \\ j\omega & 3+2j\omega \end{bmatrix},$$

Where the entries are in Ω.

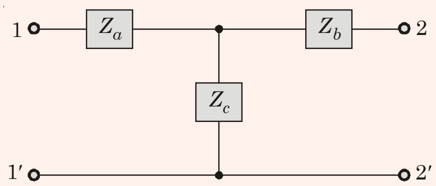
Suppose $Z_b(j\omega) = R_b + j\omega$



Then the value of R_b (in Ω) equals _____

Ans. **(3)**

For T-network



$$Z_{11} = Z_a + Z_c$$
$$Z_{22} = Z_b + Z_c$$

and $\quad Z_{12} = Z_{21} = Z_c$

Given $\quad [Z] = \begin{bmatrix} 2j\omega & j\omega \\ j\omega & 3+2j\omega \end{bmatrix}$

Therefore $\quad Z_{12} = j\omega$

and $\quad Z_{22} = 3+2j\omega$

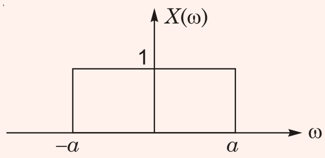
$$= 3 + j\omega + j\omega$$
$$= Z_b + Z_c$$
$$= R_b + j\omega + Z_c$$
$$\therefore \quad R_b = 3\ \Omega$$

End of Solution

Q.10 The energy of the signal $x(t) = \dfrac{\sin(4\pi t)}{4\pi t}$ is _____.

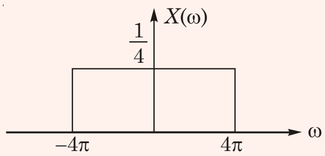
Ans. **(0.25)**

$$\frac{\sin at}{\pi t} \leftrightarrow$$



$$\frac{\sin 4\pi t}{4\pi t} =$$

$$\frac{1}{4} \times \frac{\sin 4\pi t}{\pi t} \leftrightarrow$$



$$\text{Energy, } E_{x(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-4\pi}^{4\pi}\left(\frac{1}{4}\right)^2 d\omega$$
$$= \frac{1}{2\pi} \times \frac{1}{16}[8\pi] = \frac{1}{4} = 0.25 \text{ J}$$

End of Solution

Q.11 The Ebers-Moll model of a BJT is valid
(a) only in a active mode
(b) only in active and saturation modes
(c) only in active and cut-off modes
(d) in active, saturation and cut-off modes

Ans. **(d)**

Ebers-Moll model is valid for all the region of operation.

End of Solution

Q.12 A long-channel NMOS transistor is biased in the linear region with V_{DS} = 50 mV and is used as a resistance. Which one of the following statements is NOT correct?
(a) If the device width W is increased, the resistance decreases.
(b) If the threshold voltage is reduced, the resistance decreases.
(c) If the device length L is increased, the resistance increase.
(d) If V_{GS} is increased, the resistance increases.

Ans. **(d)**

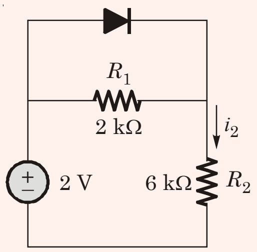
$$r_{ds} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_T)}$$

$$r_{ds} = \text{channel resistance}$$

$W \uparrow r \downarrow A \rightarrow$ correct
$V_T \downarrow r \downarrow B \rightarrow$ correct
$L \uparrow r \uparrow C \rightarrow$ correct
$V_{GS} \uparrow r \downarrow \therefore D \rightarrow$ Wrong statement

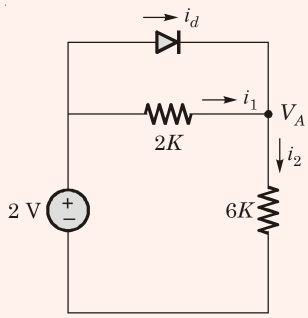
End of Solution

Q.13 Assume that the diode in the figures has V_{on} = 0.7 V, but is otherwise ideal.



The magnitude of the current i_2 (in mA) is equal to_____

Ans. **(0.25)**



Let diode : ON

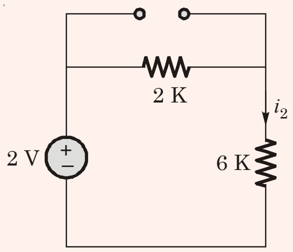
$\Rightarrow$ $$V_A = 2 - 0.7 = 1.3 \text{ V}$$

$$i_2 = \frac{1.3}{6K}; \qquad i_1 = \frac{0.7}{2K}$$

$$i_d = i_2 - i_1 = \frac{1.3}{6}\text{mA} - \frac{0.7}{2}\text{mA} = -\text{ve}$$

not possible

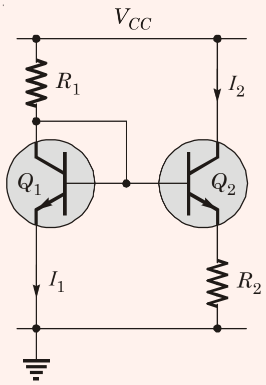
$\Rightarrow$ diode is OFF

⇒ 

$$i_2 = \frac{2\text{ V}}{2\text{K} + 6\text{K}} = 0.25\text{ mA}$$

End of Solution

Q.14 Resistor R_1 in the circuit below has been adjusted so that $I_1 = 1$ mA. The bipolar transistors Q_1 and Q_2 are perfectly matched and have very high current gain, so their base currents are negligible. The supply voltage V_{CC} is 6 V. The thermal voltage kT/q is 26 mV.



The value of R_2 (in Ω) for which $I_2 = 100$ μA is _____

Ans. **(598.67)**

$$R_2 = \frac{V_T}{I_2} ln\left(\frac{I_1}{I_2}\right) = \frac{26 \times 10^{-3}}{100 \times 10^{-6}} ln\left(\frac{1 \times 10^{-3}}{100 \times 10^{-6}}\right)$$

$$= 598.67\ \Omega$$

End of Solution

Q.15 Which one of the following statements is correct about an ac-coupled common-emitter amplifier operating in the mid-band region?

(a) The device parasitic capacitances behave like open circuits, whereas coupling and bypass capacitances behave like short circuits.

(b) The device parasitic capacitances, coupling capacitances and bypass capacitances behave like open circuits.

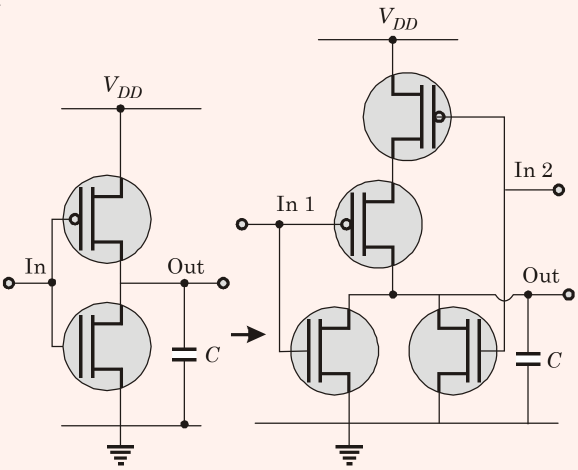
(c) The device parasitic capacitances, coupling capacitances and bypass capacitances behave like short circuits.

(d) The device parasitic capacitances behave like short circuits, whereas coupling and bypass capacitances behave like open circuits.

Ans. **(a)**

End of Solution

Q.16 Transistor geometries in a CMOS inverter have been adjusted to meet the requirement for worst case charge and discharge times for driving a load capacitor C. This design is to be converted to that of a NOR circuit in the same technology, so that its worst case charge and discharge times while driving the same capacitor are similar. The channel lengths of all transistors are to be kept unchanged. Which one of the following statements is correct?

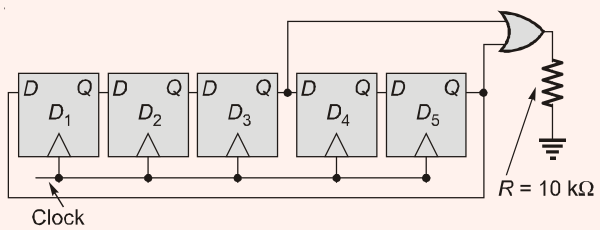


(a) Widths of PMOS transistors should be doubled, while widths of NMOS transistors should be halved.
(b) Widths of PMOS transistors should be doubled, while widths of NMOS transistors should not be changed.
(c) Widths of PMOS transistors should be halved, while widths of NMOS transistors should not be changed.
(d) Widths of PMOS transistors should be unchanged, while widths of NMOS transistors should be halved.

Ans. **(b)**

End of Solution

Q.17 Assume that all the digital gates in the circuit shown in the figure are ideal, the resistor R = 10 kΩ and the supply voltage is 5 V. The D flip-flops D_1, D_2, D_3, D_4 and D_5 are initialized with logic values 0,1,0,1 and 0, respectively. The clock has a 30% duty cycle.



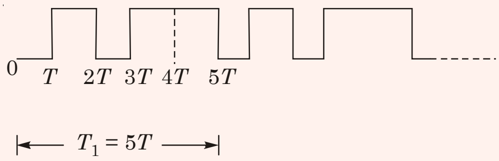
The average power dissipated (in mW) in the resistor R is _____.

Ans. **(1.5)**

CLK	Q_1	Q_2	Q_3	Q_4	Q_5	$Y = Q_3 + Q_5$
0	0	1	0	1	0	0
1	0	0	1	0	1	1
2	1	0	0	1	0	0
3	0	1	0	0	1	1
4	1	0	1	0	0	1
5	0	1	0	1	0	0

The waveform of the gate output

$$Y = Q_3 + Q_5$$



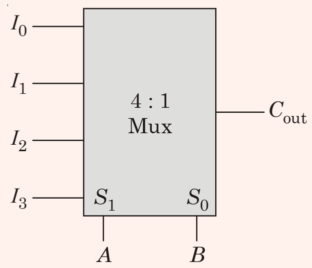
Average power dissipated

$$P = \frac{V^2}{R} \times \frac{T_{ON}}{T} = \frac{5^2}{10\text{ k}} \times \frac{3T}{5T}$$

$$= 1.5 \text{ mW}$$

End of Solution

Q.18 A 4:1 multiplexer is to be used for generating the output carry of a full adder. A and B are the bits to be added while C_{in} is the input carry and C_{out} is the output carry. A and B are to be used as the select bits with A being the more significant select bit.



Which one of the following statements correctly describes the choice of signals to be connected to the inputs I_0, I_1, I_2 and I_3 so that the output is C_{out}?

(a) $I_0 = 0$, $I_1 = C_{in}$, $I_2 = C_{in}$ and $I_3 = 1$
(b) $I_0 = 1$, $I_1 = C_{in}$, $I_2 = C_{in}$ and $I_3 = 1$
(c) $I_0 = C_{in}$, $I_1 = 0$, $I_2 = 1$ and $I_3 = C_{in}$
(d) $I_0 = 0$, $I_1 = C_{in}$, $I_2 = 1$ and $I_3 = C_{in}$

Ans. **(a)**

In case of a full adder,

$$C_{out}\,(\underbrace{A, B}_{\text{Applied at select input of MUX}}, \underbrace{C_{in}}_{\text{Applied at data input of MUX}}) = \Sigma\,(3, 5, 6, 7)$$

	I_0	I_1	I_2	I_3
$\overline{C}_{in}$	0	2	4	6
C_{in}	1	3	5	7
	0	C_{in}	C_{in}	1

$\therefore$

$$I_0 = 0$$
$$I_1 = C_{in}$$
$$I_2 = C_{in}$$
$$I_3 = 1$$

End of Solution

Q.19 The response of the system $G(s) = \dfrac{s-2}{(s+1)(s+3)}$ to the unit step input $u(t)$ is $y(t)$. The value of $\dfrac{dy}{dt}$ at $t = 0^+$ is ____

Ans. **(1)**

$$L\left(\frac{dy}{dt}\right) = (sY(s) - y(0))$$

$$Y(s) = G(s) \times \frac{1}{s} = \frac{s-2}{s(s+1)(s+3)}$$

$$y(0) = \underset{s\to\infty}{Lt}\; sY(s) \qquad \text{(Applying initial value theorem)}$$

$$= \underset{s\to\infty}{Lt} \frac{s-2}{(s+1)(s+3)} = \frac{\left(1-\frac{2}{s}\right)}{s\left(1+\frac{1}{s}\right)\left(1+\frac{3}{s}\right)}$$

$$y(0) = 0$$

$$L\left(\frac{dy}{dt}\right) = sY(s) = \frac{s\times(s-2)}{s(s+1)(s+3)} = \frac{s-2}{(s+1)(s+3)}$$

$$\left.\frac{dy}{dt}\right|_{t=0} = \underset{s\to\infty}{Lt}\; sL\left(\frac{dy}{dt}\right)$$

$$= \underset{s\to\infty}{Lt} \frac{s\times(s-2)}{(s+1)(s+3)} = \frac{\left(1-\frac{2}{s}\right)}{\left(1+\frac{1}{s}\right)\left(1+\frac{3}{s}\right)} = 1$$

End of Solution

Q.20 The number and direction of encirclements around the point $-1 + j0$ in the complex plane by the Nyquist plot of $G(s) = \dfrac{1-s}{4+2s}$ is

(a) zero.
(b) one, anti-clockwise.
(c) one, clockwise.
(d) two, clockwise.

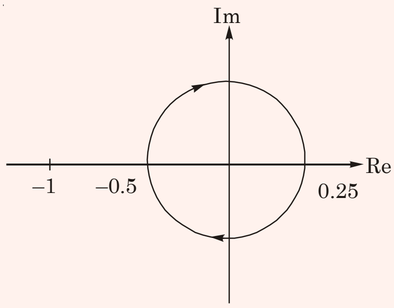
Ans. **(a)**

$$G(s) = \frac{1-s}{4+2s} \quad \text{and} \quad \angle G(s)| = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

at $s = 0,\ |G(s)| = \frac{1}{4} = 0.25$ and $\angle G(s)|_{\omega=0} = 0°$

at $s = \infty,\ |G(s)| = -\frac{1}{2} = -0.5$ and $\angle G(s)|_{\omega=\infty} = -180°$

Nyquist plot is



Hence number of encirclements of $(-1 + j0) = 0$

End of Solution

Q.21 A discrete memoryless source has an alphabet $\{a_1, a_2, a_3, a_4\}$ with corresponding probabilities $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right\}$. The minimum required average codeword length in bits to represent this source for error-free reconstruction is_____

Ans. **(1.75)**

Minimum required average codeword length in bits for error free reconstruction

$$L_{min} = H \quad \text{(Entropy)}$$

$$H = \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \frac{1}{8}\log_2 8$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{3}{8} = 1.75$$

$\Rightarrow$ $L_{min} = 1.75$ bits/word

End of Solution

Q.22 A speech signal is sampled at 8 kHz and encoded into PCM format using 8 bits/sample. The PCM data is transmitted through a baseband channel via 4-level PAM. The minimum bandwidth (in kHz) required for transmission is _____.

Ans. **(16)**

$$f_s = 8\text{ kHz}$$

$$n = 8\text{ bits/sample} \quad ; \quad M = 4$$

$$BW_{min} = \frac{R_b}{2\log_2 M} = \frac{R_b}{4} = \frac{8\times 8}{4} = 16\text{ kHz}$$

End of Solution

Q.23 A uniform and constant magnetic field $B = \hat{z}B$ exists in the $\hat{z}$ direction in vacuum. A particle of mass m with a small charge q is introduced into this region with an initial velocity $v = \hat{x}v_x + \hat{z}v_z$. Given that B, m, q, v_x and v_z are all non-zero, which one of the following describes the eventual trajectory of the particle?

(a) Helical motion in the $\hat{z}$ direction.
(b) Circular motion in the xy plane.
(c) Linear motion in the $\hat{z}$ direction.
(d) Linear motion in the $\hat{x}$ direction.

Ans. **(a)**

Ba_z magnetic field

$v_x a_x + v_z a_z$ velocity

$$F = Q\,(v \times B) \quad \text{by Lorent's law}$$

$$= Q\,(v_x a_x + v_z a_z) \times Ba_z$$

$$Fy = Q\,v_x\,.\,B\,(-a_y)$$

This results in a circular path in the XY plane with $v_z a_z$ component causing a linear path.

Both result in a helical path in Z axis.

End of Solution

Q.24 Let the electric field vector of a plane electromagnetic wave propagating in a homogenous medium

be expressed as $E = \hat{x}E_x e^{-j(\omega t - \beta z)}$, where the propagation constant β is a function of the angular frequency ω. Assume that $\beta(\omega)$ and E_x are known and are real. From the information available, which one of the following CANNOT be determined?

(a) The type of polarization of the wave.
(b) The group velocity of the wave.
(c) The phase velocity of the wave.
(d) The power flux through the $z = 0$ plane.

Ans. **(d)**

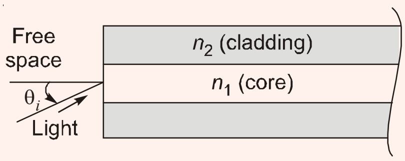
$v_p = \omega/\beta$ can be calculated. Polarization can be identified.

μ_r and ϵ_r cannot be found, due to which power flux cannot be calculated as power flux

$$P = \frac{1}{2}\frac{|E|^2}{\eta}, \text{ where } \eta = 120\pi \times \sqrt{\frac{\mu_r}{\epsilon_r}}$$

End of Solution

Q.25 Light from free space is incident at an angle θ_i to the normal of the facet of a step-index large core optical fibre. The core and cladding refractive indices are n_1=1.5 and n_2=1.4, respectively.



The maximum value of θ_i (in degrees) for which the incident light will be guided in the core of the fibre is ________.

Ans. **(32.58)**

$$\sin\alpha_{max} = \sqrt{n_1^2 - n_2^2} = \sqrt{1.5^2 - 1.4^2}$$

$$\Rightarrow \quad \alpha_{max} = \sin^{-1}(0.5385) = 32.58°$$

End of Solution

Two Marks Questions

Q.26 The ordinary differential equation

$$\frac{dx}{dt} = -3x + 2, \text{ with } x(0) = 1$$

is to be solved using the forward Euler method. The largest time step that can be used to solve the equation without making the numerical solution unstable is ________.

Ans. **(0.66)**

$$\frac{dy}{dx} = -3y + 2, \quad y(0) = 1$$

If $|1 - 3h| < 1$, then solution of differential equation is stable.

$$-1 < 1 - 3h < 1$$

$$-2 < -3h < 0$$

$$0 < h < \frac{2}{3}$$

$$h_{max} = \frac{2}{3} = 0.66$$

End of Solution

Q.27 Suppose C is the closed curve defined as the circle $x^2 + y^2 = 1$ with C oriented anti clockwise. The value of $\oint(xy^2dx + x^2ydy)$ over the curve C equals ________.

Ans. **(0)**

By Green's theorem

$$\int xy^2dx + x^2ydy = \iint_R \left(\frac{d}{dx}(x^2y) - \frac{d}{dy}(xy^2)\right)dxdy$$

$$= \iint_R (2xy - 2xy) = 0$$

End of Solution

Q.28 Two random variables X and Y are distributed according to

$$f_{X,Y}(x,y) = \begin{cases} (x+y) & 0 \le x \le 1, \quad 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

The probability $P(X + Y \le 1)$ is ________

Ans. **(0.33)**

$$P(X + Y \le 1) = \int_{x=0}^{1}\int_{y=0}^{(1-x)} f_{xy}(x,y)\,dx\,dy$$

$$= \int_{x=0}^{1}\int_{y=0}^{1-x} (x+y)\,dx\,dy = \int_{x=0}^{1}\left(xy + \frac{y^2}{2}\right)_0^{1-x}$$

$$= \int_{x=0}^{1}\left(x(1-x) + \frac{(1-x)^2}{2}\right)dx$$

$$= \int_{x=0}^{1}\left(\frac{1}{2} - \frac{x^2}{2}\right)dx = \left(\frac{x}{2} - \frac{x^3}{6}\right)_0^1$$

$$= \frac{1}{2} - \frac{1}{6} = \frac{1}{3} = 0.33$$

End of Solution

Q.29 The matrix $A = \begin{bmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{bmatrix}$ has det(A)=100 and trace (A)=14. The value of $|a - b|$ is ________

Ans. **(3)**

$$\text{Trace of } A = 14$$
$$a + 5 + 2 + b = 14$$
$$a + b = 7 \quad \text{...(i)}$$
$$\det(A) = 100$$
$$5\begin{vmatrix} a & 3 & 7 \\ 0 & 2 & 4 \\ 0 & 0 & b \end{vmatrix} = 100$$
$$5 \times 2 \times a \times b = 100$$
$$10\,ab = 100$$
$$ab = 10 \quad \text{... (ii)}$$

From equation (i) and (ii)

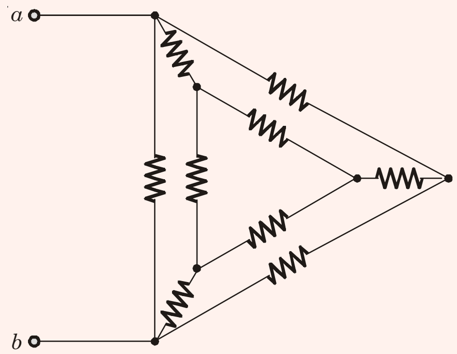
either $a = 5, \quad b = 2$

or $a = 2 \quad b = 5$

$$|a - b| = |5 - 2| = 3$$

End of Solution

Q.30 In the given circuit, each resistor has a value equal to 1 Ω.



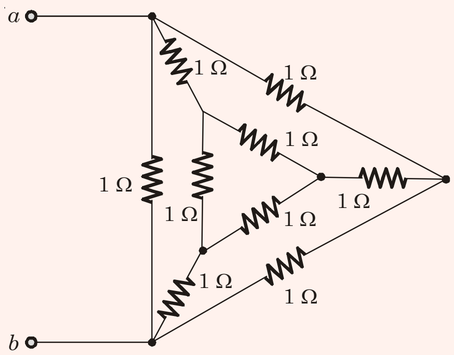
What is the equivalent resistance across the terminals a and b?

(a) 1/6 Ω (b) 1/3 Ω

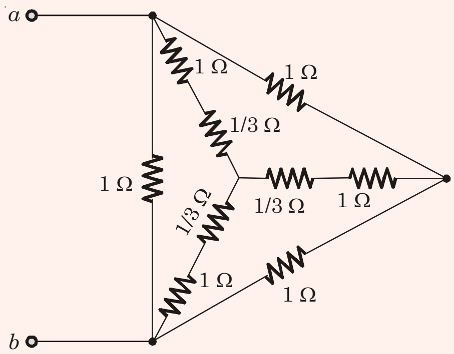
(c) 9/20 Ω (d) 8/15 Ω

Ans. (d)

$R_{eq} \Rightarrow$



By using delta to star conversion



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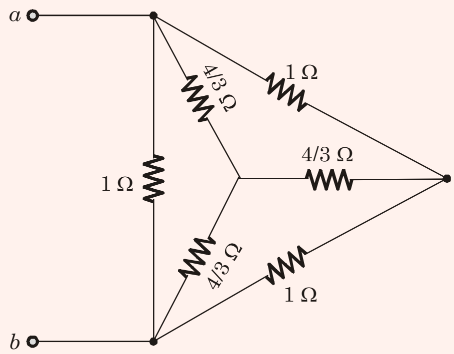
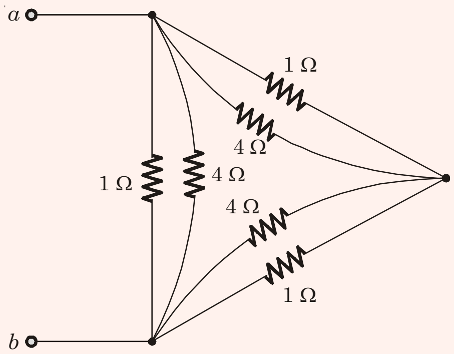


Again by star to delta conversion



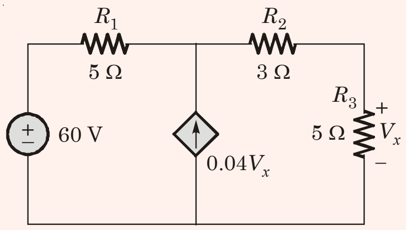
$$R_{ab} = \{4 \,||\, 1) + (4 \,||\, 1)\} \,||\, \{1 \,||\, 4)\}$$

$$= \left(\frac{4}{5} + \frac{4}{5}\right) \Bigg\| \frac{4}{5}$$

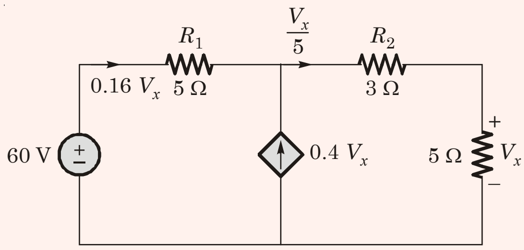
$$= \frac{8}{15}\ \Omega$$

End of Solution

Q.31 In the circuit shown in the figure, the magnitude of the current (in amperes) through R_2 is ___



Ans. **(5)**



Using KVL in the outer loop

$$60 - 5(0.16\ V_x) - \frac{V_x}{5} \times 3 - V_x = 0$$

or $$V_x = 25\text{ V}$$

$\therefore$ The current flowing through $R_2 = \dfrac{V_x}{5} = \dfrac{25}{5} = 5$ A

End of Solution

Q.32 A continuous-time filter with transfer function $H(s) = \dfrac{2s+6}{s^2+6s+8}$ is converted to a discrete time filter with transfer function $G(s) = \dfrac{2z^2 - 0.5032z}{z^2 - 0.5032z + k}$ so that the impulse response of the continuous-time filter, sampled at 2 Hz, is identical at the sampling instants to the impulse response of the discrete time filter. The value of k is _____.

Ans. **(0.049)**

Given, $$H(s) = \frac{2s+6}{s^2+6s+8} = \frac{1}{s+2} + \frac{1}{s+4}$$

$$h(t) = e^{-2t}\,u(t) + e^{-4t}\,u(t)$$

Given, $$f_s = 2\text{ Hz}$$

For discrete time, $$t = nT_s = \frac{n}{2}$$

$$h[n] = (e^{-n} + e^{-2n})\, u[n]$$

$$H(z) = \frac{1}{1-e^{-1}Z^{-1}} + \frac{1}{1-e^{-2}Z^{-1}}$$

$$= \frac{Z}{Z-e^{-1}} + \frac{Z}{Z-e^{-2}}$$

$$= \frac{2Z^2 - 0.5032Z}{Z^2 - 0.5032Z + 0.049}$$

$$\therefore \quad K = 0.049$$

End of Solution

Q.33 The Discrete Fourier Transform (DFT) of the 4-point sequence

$x[n] = \{x[0], x[1], x[2], x[3]\} = \{3, 2, 3, 4\}$ is

$X[k] = \{X[0], X[1], X[2], X[3]\} = \{12, 2j, 0, -2j\}$.

If $X_1[k]$ is the DFT of the 12-point sequence $x_1[n] = \{3, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0\}$, the value of $\left|\frac{X_1[8]}{X_1[11]}\right|$ is ________

Ans. **(6)**

From the given question

$$x_1[n] = x\left[\frac{n}{3}\right]$$

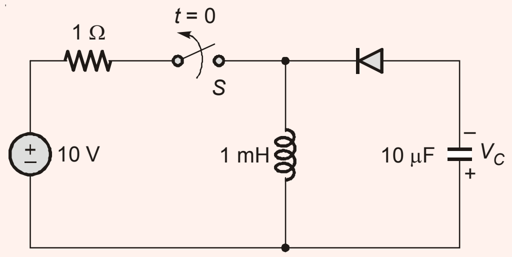
$$X_1[K] = \{12, 2j, 0, -2j, 12, 2j, 0, -2j, 12, 2j, 0, -2j\}$$

$$X_1[8] = 12; \quad X_1(11) = -2j$$

$$\therefore \quad \left|\frac{X_1(8)}{X_1(11)}\right| = \left|\frac{12}{-2j}\right| = 6$$

End of Solution

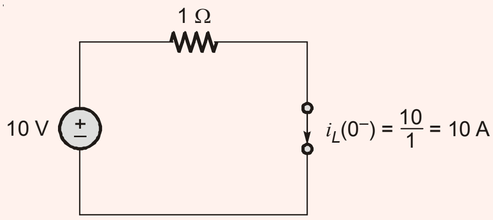
Q.34 The switch S in the circuit shown has been closed for a long time. It is opened at time t=0 and remains open after that. Assume that the diode has zero reverse current and zero forward voltage drop.



The steady state magnitude of the capacitor voltage V_C (in volts) is ______.

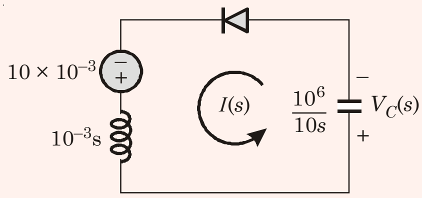
Ans. **(100)**

at $t = 0^-$



$$i_L(0^-) = \frac{10}{1} = 10 \text{ A}$$

for $t > 0$ (using Laplace transform)



$$I(s) = \frac{10 \times 10^{-3}}{10^{-3}s + \frac{10^6}{10s}}$$

$$V_c(s) = I(s) \times \frac{10^6}{10s}$$

$$V_c(s) = \frac{10^6}{s^2 + 10^8}$$

Taking inverse Laplace, we get

$$V_c(t) = 100 \sin 10^4 t \text{ V}$$

$\therefore$ steady state magnitude voltage across capacitor is 100 V.

End of Solution

Q.35 A voltage V_G is applied across a MOS capacitor with metal gate and p-type silicon substrate at $T = 300$ K. The inversion carrier density (in number of carriers per unit area) for $V_G = 0.8$ V is 2×10^{11} cm^{-2}. For $V_G = 1.3$ V, the inversion carrier density is 4×10^{11} cm^{-2}. What is the value of the inversion carrier density for $V_G = 1.8$ V?

(a) 4.5×10^{11} cm^{-2}
(b) 6.0×10^{11} cm^{-2}
(c) 7.2×10^{11} cm^{-2}
(d) 8.4×10^{11} cm^{-2}

Ans. **(b)**

In a MOS – Capacitor

$$(V_{G1} - V_T) \propto Q$$

$$\frac{(V_{G1} - V_T)}{(V_{G2} - V_T)} = \frac{Q_1}{Q_2}$$

$$\frac{0.8 - V_T}{1.3 - V_T} = \frac{2 \times 10^{11}}{4 \times 10^{11}}$$

On solving $V_T = 0.3$ V

Now Consider $\dfrac{V_{G2} - V_T}{V_{G3} - V_T} = \dfrac{Q_2}{Q_3}$

$$\frac{1.3 - 0.3}{1.8 - 0.3} = \frac{4 \times 10^{11}}{Q_3}$$

Inverse charge density with $V_G = 1.8$ V

$$Q_3 = 6 \times 10^{11}/\text{cm}^2$$

End of Solution

Q.36 Consider avalanche breakdown in a silicon p^+n junction. The n-region is uniformly doped with a donor density N_D. Assume that breakdown occurs when the magnitude of the electric field at any point in the device becomes equal to the critical field E_{crit}. Assume E_{crit} to be independent of N_D.

If the built-in voltage of the p^+n junction is much smaller than the breakdown voltage, V_{BR}, the relationship between V_{BR} and N_D is given by

(a) $V_{BR} \times \sqrt{N_D}$ = constant

(b) $N_D \times \sqrt{V_{BR}}$ = constant

(c) $N_D \times V_{BR}$ = constant

(d) N_D / V_{BR} = constant

Ans. **(c)**

In any type of PN junction

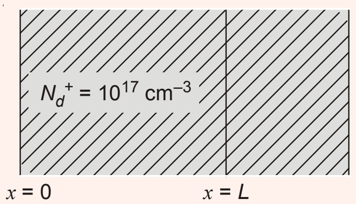
$$V_{Br} \propto \frac{1}{\text{Doping Concentration}}$$

i.e. $$V_{Br} \propto \frac{1}{N_D} \text{ (or) } V_{Br} = \frac{\varepsilon E^2}{2qN_D}$$

$\therefore$ $V_{Br} \times N_D \rightarrow$ is a constant

End of Solution

Q.37 Consider a region of silicon devoid of electrons and holes, with an ionized donor density of $N_d^+ = 10^{17} cm^{-3}$. The electric field at $x = 0$ is 0 V/cm and the electric field at $x = L$ is 50 kV/cm in the positive x direction. Assume that the electric field is zero in the y and z directions at all points.



Given $q = 1.6 \times 10^{-19}$ coulomb,

ϵ_0 = 8.85 × 10^{-14} F/cm, ϵ_r = 11.7 for silicon, the value of L in nm is _____

Ans. **(32.36)**

$$|\varepsilon| = \frac{q}{\varepsilon} N_D X_N$$

$$50 \times 10^3 = \frac{1.6 \times 10^{-19}}{8.85 \times 10^{-14} \times 11.7} \times 10^{17} \times X_N$$

On solving
$$X_N = 3.2356 \times 10^{-6} \text{ cm}$$
$$= 3.236 \times 10^{-8} \text{ m}$$
$$X_N = L = 32.36 \text{ nm}$$

End of Solution

Q.38 Consider a long-channel NMOS transistor with source and body connected together. Assume that the electron mobility is independent of V_{GS} and V_{DS}. Given,
g_m = 0.5 μA/V for V_{DS} = 50 mV and V_{GS} = 2 V,
g_d = 8 μA/V for V_{GS} = 2 V and V_{DS} = 0 V,

where $g_m = \frac{\partial I_D}{\partial V_{GS}}$ and $g_d = \frac{\partial I_D}{\partial V_{DS}}$

The threshold voltage (in volts) of the transistor is ________

Ans. **(1.2)**

Since $V_{GS} > V_{DS}$, MOSFET is in linear operation

$$I_D = K'_N [V_{GS} - V_T] V_{DS}$$

$$\frac{dI_D}{dV_{GS}} = K'_N V_{DS}$$

$$g_m = K'_N V_{DS}$$

$$0.5 \times 10^{-6} = K'_N [50 \times 10^{-3}] \qquad \therefore K'_N = 10^{-5}$$

$$I_D = K'_N [V_{GS} - V_T] V_{DS}$$

$$\frac{dI_D}{dV_{dS}} = K'_N [V_{GS} - V_T]$$

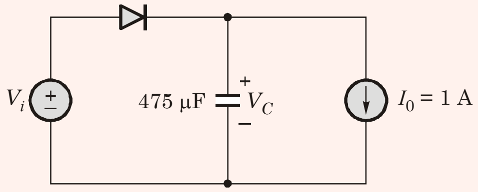
$$g_d = K'_N [V_{GS} - V_T]$$

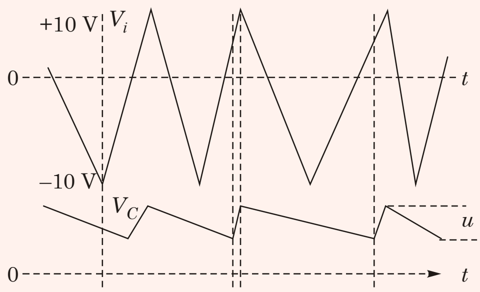
$$8 \times 10^{-6} = 10^{-5} [2 - V_T]$$

On solving
$$V_T = 1.2 \text{ V}$$

End of Solution

Q.39 The figure shows a half-wave rectifier with a 475 μF filter capacitor. The load draws a constant current I_0 =1 A from the rectifier. The figure also shows the input voltage V_i, the output voltage V_C and the peak-to-peak voltage ripple u on V_C. The input voltage V_i is a triangle-wave with an amplitude of 10 V and a period of 1 ms.





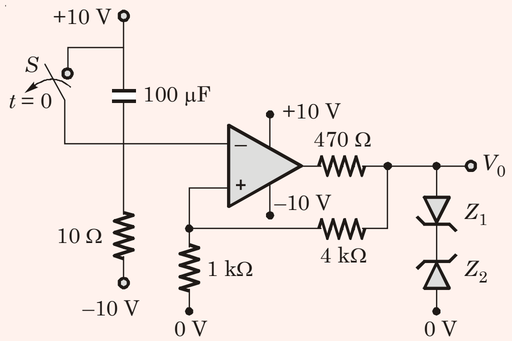
The value of the ripple u (in volts) is ______

Ans. **(2.105)**

$$V_{\text{ripple}} = \frac{I_{DC} \cdot T}{C} = \frac{1 \times 1 \times 10^{-3}}{475 \times 10^{-6}} = 2.105 \text{ Volts}$$

End of Solution

Q.40 In the opamp circuit shown, the Zener diodes Z_1 and Z_2 clamp the output voltage V_0 to + 5 V or –5 V. The switch S is initially closed and is opened at time $t = 0$.



The time $t = t_1$ (in seconds) at which V_0 changes state is ________

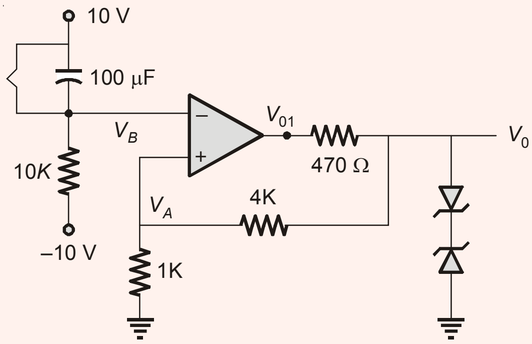
Ans. **(0.7985)**

Initially switch is closed and $V_B = 10$ V

$\Rightarrow \quad V_{01} = -10$ V

$\Rightarrow \quad V_0 = -V_2 = -5$ V

$$\Rightarrow \quad V_A = \frac{V_0}{4\text{K} + 1\text{K}} \times 1\text{K} = -1 \text{ V}$$



At $t = 0$;

The switch is opened and as $t \to \infty$, V_B approaches –10 V.

Let at $t = T_1$,

V_B exceeds V_A (–1 V) so that V_{01} changes from –10 V to 10 V

$\Rightarrow V_0$ charges from –5 V to 5 V

$$V_B = V_f + (V_i - V_f)e^{-t/\tau} = -10 + [10 - 1 - 10]\ e^{-t/RC}$$

At $t = T_1$ $\quad V_B = -1$

$$-1 \text{ V} = -10 + 20\ e^{-T1/RC}$$

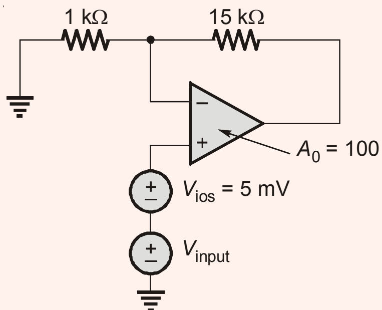
$$\Rightarrow \quad T_1 = RC\, ln\frac{20}{9}$$

$$= 10 \times 10^3 \times 100 \times 10^{-6} \times 0.798$$

$$= 0.798 \text{ sec}$$

End of Solution

Q.41 An opamp has a finite open loop voltage gain of 100. Its input offset voltage V_{ios} (= + 5 mV) is modeled as shown in the circuit below. The amplifier is ideal in all other respects. V_{input} is 25 mV.



The output voltage (in millivolts) is ________

Ans. **(413.8)**

$$\text{Overall input} = V_{ios} + V_{input}$$
$$= 5\text{ mV} + 25\text{ mV} = 30\text{ mV}$$

$$V_0 = \frac{\left(1+\frac{R_F}{R_1}\right)}{1+\frac{1}{A_{OL}}\left(1+\frac{R_F}{R_1}\right)} \times \text{Overall input}$$

$$= \frac{1+\frac{15K}{1K}}{1+\frac{1}{100}\left(1+\frac{15K}{1K}\right)} \times 30 \times 10^{-3} = 413.79\text{ mV}$$

End of Solution

Q.42 An 8 Kbyte ROM with an active low Chip Select input ($\overline{CS}$) is to be used in an 8085 microprocessor based system. The ROM should occupy the address range 1000 H to 2 FFFH. The address lines are designated as A_{15} to A_0, where A_{15} is the most significant address bit.

Which one of the following logic expressions will generate the correct $\overline{CS}$ signal for this ROM?

(a) $A_{15} + A_{14} + (A_{13} \cdot A_{12} + \overline{A_{13}} \cdot \overline{A_{12}})$

(b) $A_{15} \cdot A_{14} \cdot (A_{13} + A_{12})$

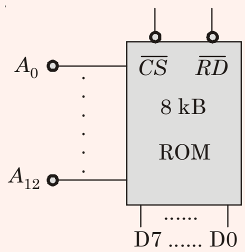
(c) $\overline{A_{15}} \cdot \overline{A_{14}} \cdot (A_{13} + A_{12} + \overline{A_{13}} \cdot A_{12})$

(d) $\overline{A_{15}} + \overline{A_{14}} + A_{13} \cdot A_{12}$

Ans. **(a)**

8 kB ROM is given

$\therefore\ 2^n = 8\text{ kB} = 2^3(2^{10}) = 2^{13}$



$\therefore$ 13 Address lines are required for memory chip.

But the address range given as 1000 H - 2FFF H

A_{15}	A_{14}	A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_8	A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1

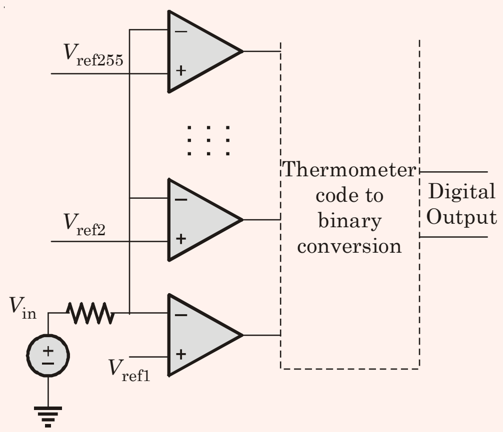
In order to get $\overline{CS}$ as low, the condition is

$A_{15} = A_{14} = 0$ and $A_{13} = 0/1$, $A_{12} = 1/0$.

End of Solution

Q.43 In an N bit flash ADC, the analog voltage is fed simultaneously to $2^N - 1$ comparators. The output of the comparators is then encoded to a binary format using digital circuits. Assume that the analog voltage source V_{in} (whose output is being converted to digital format) has a source resistance of 75 Ω as shown in the circuit diagram below and the input capacitance of each comparator is 8 pF.

The input must settle to an accuracy of 1/2 LSB even for a full scale input change for proper conversion. Assume that the time taken by the thermometer to binary encoder is negligible.

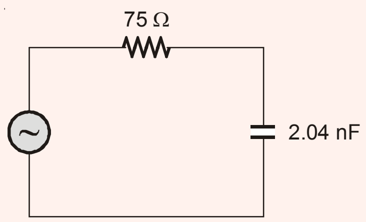


If the flash ADC has 8 bit resolution, which one of the following alternatives is closest to the maximum sampling rate ?

(a) 1 megasamples per second
(b) 6 megasamples per second
(c) 64 megasamples per second
(d) 256 megasamples per second

Ans. **(a)**

The total capacitance = $(2^n - 1) \times C = (2^8 - 1) \times$ 8 pF = 2.04 nF



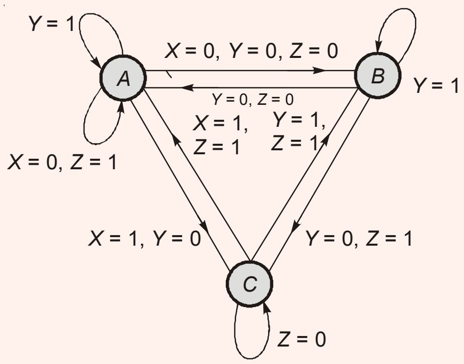
The time constant = τ = RC = 153 ns

$\because$ Settling time = 5τ = 5 RC = 765 ns

$\therefore$ Sampling rate = $\dfrac{1}{\text{Settling time}} \approx$ 1 M samples/sec

End of Solution

Q.44 The state transition diagram for a finite state machine with states A, B and C, and binary inputs X, Y and Z, is shown in the figure.
Which one of the following statements is correct?



(a) Transitions from State A are ambiguously defined.
(b) Transitions from State B are ambiguously defined.
(c) Transitions from State C are ambiguously defined.
(d) All of the state transitions are defined unambiguously.

Ans. **(c)**

For State *A*

X	*Y*	*Z*	Present State	Next State
0	0	0	*A*	*B*
0	0	1	*A*	*A*
0	1	0	*A*	*A*
0	1	1	*A*	*A*
1	0	0	*A*	*C*
1	0	1	*A*	*C*
1	1	0	*A*	*A*
1	1	1	*A*	*A*

For State *B*

X	*Y*	*Z*	Present State	Next State
0	0	0	*B*	*A*
0	0	1	*B*	*C*
0	1	0	*B*	*B*
0	1	1	*B*	*B*
1	0	0	*B*	*A*
1	0	1	*B*	*B*
1	1	0	*B*	*B*
1	1	1	*B*	*B*

For State C

X	Y	Z	Present State	Next State
0	0	0	C	C
0	0	1	C	C
0	1	0	C	C
0	1	1	C	B
1	0	0	C	C
1	0	1	C	A
1	1	0	C	C
1	1	1	C	AB

In state 'C' when XYZ = 111; the ambiguity occurs. Because from state 'C'

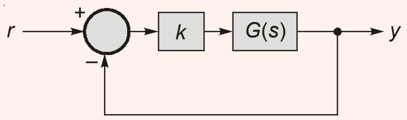
When, $X = 1$, $Z = 1 \Rightarrow$ Next state = A

When, $Y = 1$, $Z = 1 \Rightarrow$ Next state = B

End of Solution

Q.45 In the feedback system shown below $G(s) = \dfrac{1}{(s^2 + 2s)}$.

The step response of the closed-loop system should have minimum settling time and have no overshoot.



The required value of gain k to achieve this is ________

Ans. **(1)**

$$G(s) = \frac{1}{s^2 + 2s}$$

$$\frac{Y(s)}{R(s)} = \frac{\dfrac{K}{s^2+2s}}{1 + \dfrac{K}{s^2+2s}} = \frac{K}{s^2 + 2s + K}$$

Minimum settling time and no overshoot implies

$$\xi = 1$$

$$\omega_n = \sqrt{K}$$

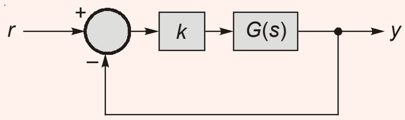
$$2 \times \xi \cdot \omega_n = 2$$

$$\omega_n = 1$$

$\Rightarrow$ $$\sqrt{K} = 1 \text{ or } K = 1$$

End of Solution

Q.46 In the feedback system shown below $G(s) = \dfrac{1}{(s+1)(s+2)(s+3)}$



The positive value of k for which the gain margin of the loop is exactly 0 dB and the phase margin of the loop is exactly zero degree is ________

Ans. **(60)**

$$1 + G(s)H(s) = 1 + \frac{K}{(s+1)(s+2)(s+3)} = 0$$

$(s + 1)\ (s^2 + 5s + 6) + K = 0$

$s^3 + 5s^2 + 6s + s^2 + 5s + 6 + K = 0$

$\Rightarrow s^3 + 6s^2 + 11s + 6 + K = 0$

Gain margin = 0 dB and phase margin = 0°

It implies marginal stable system

By Routh Array

$$\begin{array}{lcc} s^3 & 1 & 11 \\ s^2 & 6 & (6+K) \\ s & \dfrac{66-6-K}{6} & 0 \\ s^\circ & 6+K & \end{array}$$

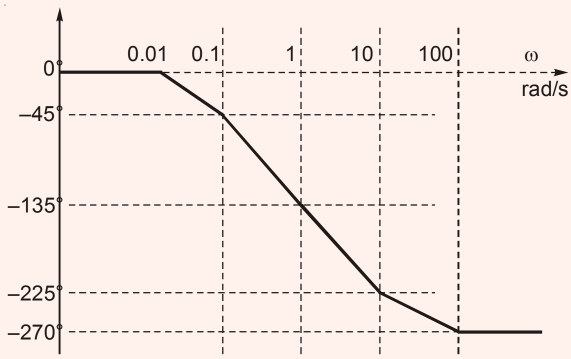
For marginal stable system,

$$60 - K = 0$$

$$\Rightarrow \quad K = 60$$

End of Solution

Q.47 The asymptotic Bode phase plot of $G(s) = \dfrac{k}{(s+0.1)(s+10)(s+p_1)}$, with k and p_1 both positive, is shown below.



The value of p_1 is ________

Ans. **(1)**

$$G(s) = \frac{K}{(s+0.1)(s+10)(s+p_1)}$$

$$\angle G(s) = -\tan^{-1}\frac{\omega}{0.1} - \tan^{-1}\frac{\omega}{10} - \tan^{-1}\frac{\omega}{p_1}$$

$$\angle G(s)\big|_{\omega=1} = -\tan^{-1}\frac{1}{0.1} - \tan^{-1}\frac{1}{10} - \tan^{-1}\frac{1}{p_1} = -135°$$

$$-\tan^{-1}10 - \tan^{-1}0.1 - \tan^{-1}\frac{1}{p_1} = -135°$$

$$-84.28° - 5.71° - \tan^{-1}\frac{1}{p_1} = -135°$$

$$-\tan^{-1}\frac{1}{p_1} = -135° + 90°$$

$$\tan^{-1}\frac{1}{p_1} = 45°$$

$$\frac{1}{p_1} = 1 \qquad \Rightarrow p_1 = 1$$

End of Solution

Q.48 An information source generates a binary sequence $\{\alpha_n\}$. α_n can take one of the two possible values −1 and +1 with equal probability and are statistically independent and identically distributed. This sequence is precoded to obtain another sequence $\{\beta_n\}$, as $\beta_n = \alpha_n + k\alpha_{n-3}$. The sequence $\{\beta_n\}$ is used to modulate a pulse $g(t)$ to generate the baseband signal

$$X(t) = \sum_{n=-\infty}^{\infty} \beta_n g(t - nT),$$

where $g(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$

If there is a null at $f = \frac{1}{3T}$ in the power spectral density of $X(t)$, then k is ________

Ans. **(–1)**

Power spectral density of $x(t) = S_X(f)$

$$S_X(f) = \frac{|G(f)|^2}{T} \sum_{n=-\infty}^{\infty} R_b(\tau) e^{j2\pi f n \tau}$$

$$R_b(\tau) = E[\beta_n \beta_{n-\tau}]$$

$$= E[(\alpha_n + k\alpha_{n-3})(\alpha_{n-\tau} + k\,\alpha_{n-\tau-3})]$$

$$= E[\alpha_n \alpha_{n-\tau}] + kE[\alpha_{n-3}\alpha_{n-\tau}] + kE[\alpha_n \alpha_{n-\tau-3}] + k^2E[\alpha_{n-3}\,\alpha_{n-\tau-3}]$$

$= E[\alpha_n\alpha_{n-\tau}] + kE[\alpha_{n-3}\,\alpha_{n-\tau-3+3}] + kE[\alpha_n\alpha_{n-\tau-3}] + k^2 E[\alpha_{n-3}\alpha_{n-\tau-3}]$

$= R(\tau) + kR(\tau-3) + kR(\tau+3) + k^2R(\tau)$

$= (1+k^2)R(\tau) + kR(\tau+3) + kR(\tau-3)$

Hence autocorrection function can be defined as :

$$R_b(\tau) = \begin{cases} 1+k^2 & \tau = 0 \\ k & \tau = \pm 3 \\ 0 & \text{otherwise} \end{cases}$$

Power spectral density

$$s_b(f) = 1 + k^2 + 2k\cos(2\pi f 3T)$$

Null will occur at $f = \dfrac{1}{3T}$

at $\quad f = \dfrac{1}{3T}$

$$s_b(f) = 1 + k^2 + 2k\cos(2\pi) = 0$$

$\Rightarrow \quad 1 + k^2 + 2k = 0$

$\Rightarrow \quad (k+1)^2 = 0$

$\therefore \quad k = -1$

End of Solution

Q.49 An ideal band-pass channel 500 Hz- 2000 Hz is deployed for communication. A modem is designed to transmit bits at the rate of 4800 bits/s using 16-QAM. The roll-off factor of a pulse with a raised cosine spectrum that utilizes the entire frequency band is ________

Ans. **(0.25)**

Channal spectrum 500 Hz – 2000 Hz

hence BW = 1500 Hz

$$\text{BW} = \frac{R_b}{\log_2 M}(1+\alpha)$$

$$1500 = \frac{R_b}{\log_2 16}(1+\alpha)$$

$$1500 = \frac{4800}{\log_2 16}(1+\alpha)$$

$$1500 = 1200\,(1+\alpha)$$

$$\Rightarrow \quad \alpha = 0.25$$

End of Solution

Q.50 Consider a random process $X(t) = 3V(t) - 8$, where $V(t)$ is a zero mean stationary random process with autocorrelation $R_v(\tau) = 4e^{-5|\tau|}$. the power in $X(t)$is ________

Ans. **(100)**

$$X(t) = 3V(t) - 8 \quad \text{and} \quad E[V(t)] = 0$$

$$R_v(\tau) = 4e^{-5|\tau|}$$

$$\text{Power of } X(t) = E[X^2(t)]$$

$$= E[9V^2(t)] + 64 - 48\,E[V(t)]$$

$$= 9E[V^2(t)] + 64 - 48\,E[V(t)]$$

$$E[V^2(t)] = R_v(0) = 4$$

$$\text{Power of } X(t) = ((9 \times 4) + 64)$$

$$= 100$$

End of Solution

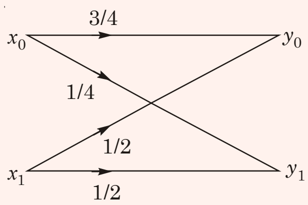
Q.51 A binary communication system makes use of the symbols "zero" and "one". There are channel errors. Consider the following events:

x_0 : a "zero" is transmitted
x_1 : *a* "one" is transmitted
y_0 : *a* "zero" is received
y_1 : *a* "one" is received

The following probabilities are given: $P(x_0) = \frac{1}{2}, P(y_0|x_0) = \frac{3}{4}$, and $P(y_0|x_1) = \frac{1}{2}$.

The information in bits that you obtain when you learn which symbol has been received (while you know that a "zero" has been transmitted) is ________

Ans. **(0.405)**



$$P(x_0) = \frac{1}{2}; \qquad P(x_1) = \frac{1}{2}$$

$$P\left(\frac{y_0}{x_0}\right) = \frac{3}{4}; \qquad P\left(\frac{y_0}{x_1}\right) = \frac{1}{2}$$

$$[P(x, y)] = [P(x)]_d\,[P(y|x)]$$

$$= \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$$

$$[P(x, y)] = \begin{bmatrix} 3/8 & 1/8 \\ 1/4 & 1/4 \end{bmatrix}$$

$$P(y|x_0) = -\sum_{k=0}^{1} P(x_0, y_k) \log_2 P\left(\frac{y_k}{x_0}\right)$$

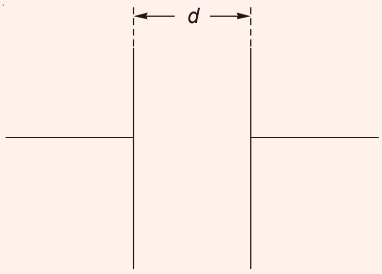
$$= -\{P(x_0, y_0)\log_2 P(y_0 \mid x_0) + P(x_0, y_1)\log_2 P(y_1 \mid x_0)\}$$

$$= -\left\{\frac{3}{8}\log_2\frac{3}{4} + \frac{1}{8}\log_2\frac{1}{4}\right\}$$

$$= 0.405$$

End of Solution

Q.52 The parallel-plate capacitor shown in the figure has movable plates. The capacitor is charged so that the energy stored in it is E when the plate separation is d. The capacitor is then isolated electrically and the plates are moved such that the plate separation becomes $2d$.



At this new plate separation, what is the energy stored in the capacitor, neglecting fringing effects?

(a) $2E$ (b) $\sqrt{2}E$

(c) E (d) $E/2$

Ans. **(a)**

Let $E = E_1$, Energy $E_1 = \dfrac{Q_1^2}{2C_1}$

Electrically isolated $\Rightarrow$ $Q_2 = Q_1$

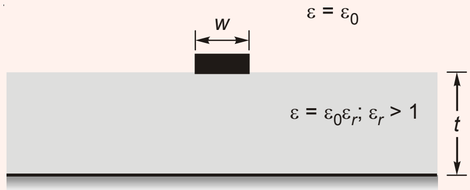
$d_2 = 2d_1$ $\Rightarrow$ $C_2 = \dfrac{C_1}{2}$

$$E_2 = \frac{Q_2^2}{2C_2} = \frac{Q_1^2}{\frac{2C_1}{2}} = 2\left(\frac{Q_1^2}{2C_1}\right)$$

$$= 2E_1 = 2E$$

End of Solution

Q.53 A lossless microstrip transmission line consists of a trace of width w. It is drawn over a practically infinite ground plane and is separated by a dielectric slab of thickness t and relative permittivity $\varepsilon_r > 1$. The inductance per unit length and the characteristic impedance of this line are L and Z_0, respectively.



Which one of the following inequalities is always satisfied?

(a) $Z_0 > \sqrt{\dfrac{Lt}{\varepsilon_0\varepsilon_r w}}$ (b) $Z_0 < \sqrt{\dfrac{Lt}{\varepsilon_0\varepsilon_r w}}$

(c) $Z_0 > \sqrt{\dfrac{Lw}{\varepsilon_0\varepsilon_r t}}$ (d) $Z_0 < \sqrt{\dfrac{Lw}{\varepsilon_0\varepsilon_r t}}$

Ans. **(a)**

$$Z_0 = \sqrt{\frac{L}{C}}$$

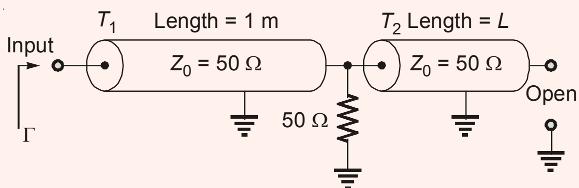
$\Rightarrow$ $$Z_0 = \sqrt{\frac{Lt}{\epsilon_{eff} W}}$$

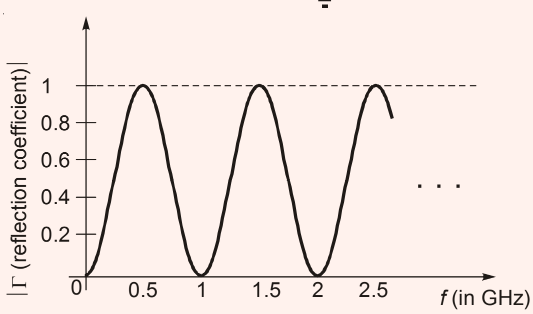
as $$\epsilon_{\text{eff}} < \epsilon_0\epsilon_r$$

$$Z_0 > \sqrt{\frac{Lt}{\epsilon_0\ \epsilon_r\ W}}$$

End of Solution

Q.54 A microwave circuit consisting of lossless transmission lines T_1 and T_2 is shown in the figure. The plot shows the magnitude of the input reflection coefficient Γ as a function of frequency f. The phase velocity of the signal in the transmission ^{8} m/s.





The length L (in meters) of T_2 is ________

Ans. **(0.1)**

At frequency of 1 GHz the line behaves as $\frac{\lambda}{2}$ length there by giving open circuit at the input of T_2 line.

With 50 Ω termination at T_1 the load is matched and $|\Gamma| = 0$ as seen in the graph.

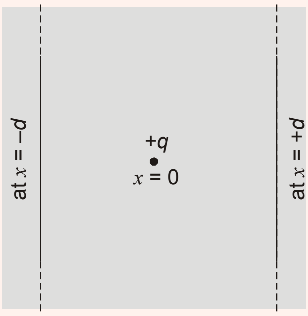
$$f = 1 \text{ GHz}$$

$$\lambda = \frac{2 \times 10^{10}}{1 \times 10^{9}} \text{ cm}$$

$$\frac{\lambda}{2} = L = 10 \text{ cm } = 0.1 \text{ m}$$

End of Solution

Q.55 A positive charge q is placed at $x = 0$ between two infinite metal plates placed at $x = -d$ and at $x = +d$ respectively. The metal plates lie in the yz plane.



The charge is at rest at $t = 0$, when a voltage +V is applied to the plate at –d and voltage –V is applied to the plate at $x = +d$. Assume that the quantity of the charge q is small enough that it does not perturb the field set up by the metal plates. The time that the charge q takes to reach the right plate is proportional to

(a) d/V (b) $\sqrt{d}/V$

(c) $d/\sqrt{V}$ (d) $\sqrt{d/V}$

Ans. **(c)**

Velocity being free velocity,

$$\frac{1}{2}\text{mv}^2 = \text{qV}$$

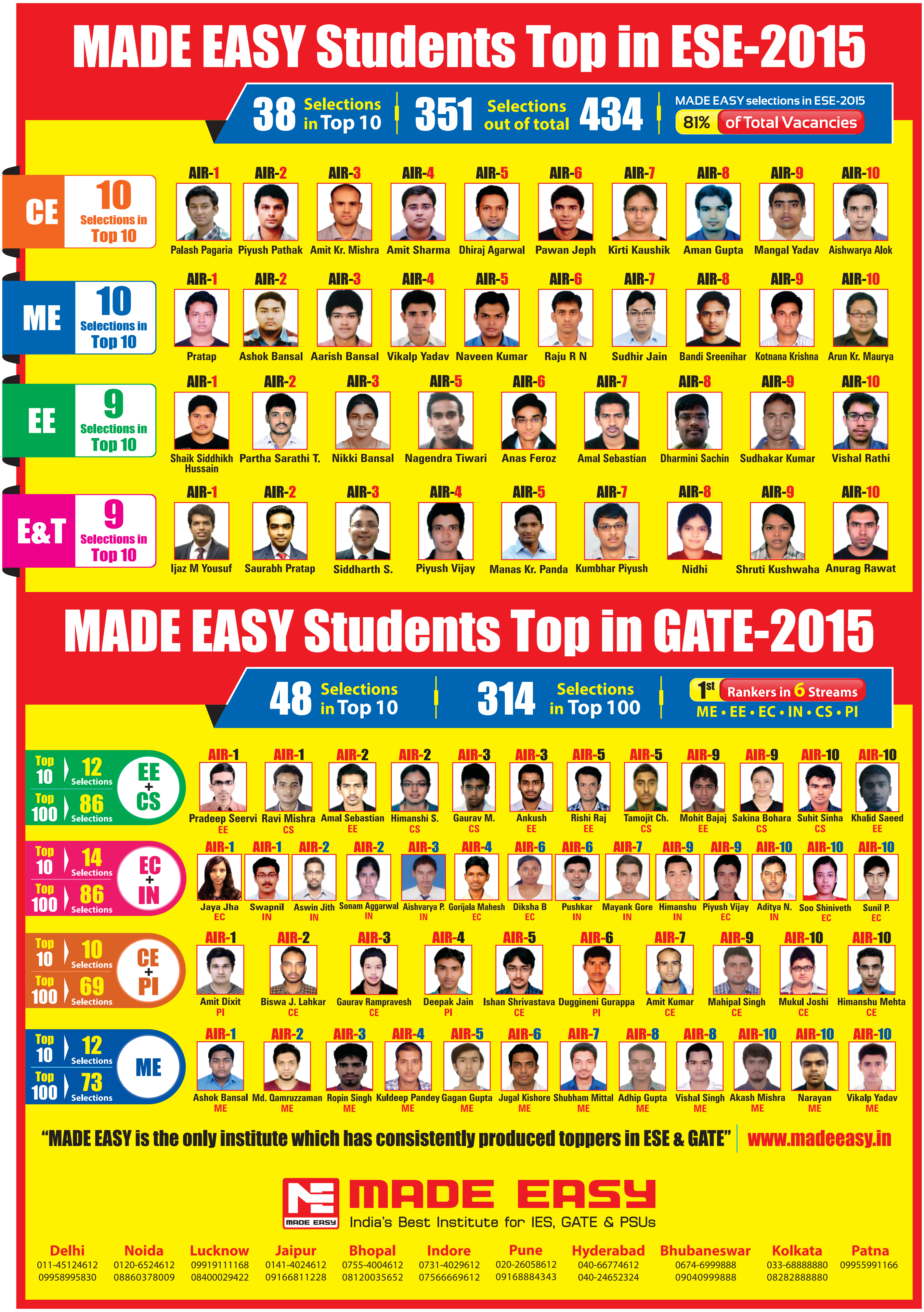
$$\text{v} = \frac{d}{t} = \sqrt{\frac{2qV}{m}}$$

$$\Rightarrow \quad t \propto \frac{d}{\sqrt{V}}$$

End of Solution

❍❍❍❍

Advt.



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AIR-3 Amit Kr. Mishra
AIR-4 Amit Sharma
AIR-5 Dhiraj Agarwal
AIR-6 Pawan Jeph
AIR-7 Kirti Kaushik
AIR-8 Aman Gupta
AIR-9 Mangal Yadav
AIR-10 Aishwarya Alok
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Top 100 86 Selections
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AIR-10 Soo Shiniveth EC
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Top 10 10 Selections
Top 100 69 Selections
CE + PI
AIR-1 Amit Dixit PI
AIR-2 Biswa J. Lahkar CE
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AIR-4 Deepak Jain PI
AIR-5 Ishan Shrivastava CE
AIR-6 Duggineni Gurappa PI
AIR-7 Amit Kumar CE
AIR-9 Mahipal Singh CE
AIR-10 Mukul Joshi CE
AIR-10 Himanshu Mehta CE
Top 10 12 Selections
Top 100 73 Selections
ME
AIR-1 Ashok Bansal ME
AIR-2 Md. Qamruzzaman ME
AIR-3 Ropin Singh ME
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