Solutions of

of

Electronics Engineering

GATE-2016

Session 4 | Set-3

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GATE 2016 : Solutions

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B. Singh (Ex. IES), *Chairman & Managing Director, MADE EASY Group*

Section - I (General Aptitude)

One Mark Questions

Q.1 An apple costs ₹ 10. An onion costs ₹ 8.
Select the most suitable sentence with respect to grammar and usage.
(a) The price of an apple is greater than an onion.
(b) The price of an apple is more than onion.
(c) The price of an apple is greater than of an onion.
(d) Apples are more costlier than onions.

Ans. **(c)**

End of Solution

Q.2 The Buddha said, "Holding on to anger is like grasping a hot coal with the intent of throwing it at someone else; you are the one who gets burnt."
Select the word below which is closest in meaning to the word underlined above.
(a) burning (b) igniting
(c) clutching (d) flinging

Ans. **(c)**

End of Solution

Q.3 M has a son Q and a daughter R. He has no other children. E is the mother of P and daughter-in-law of M. How is P related to M?
(a) P is the son-in-law of M
(b) P is the grandchild of M
(c) P is the daughter-in law of M
(d) P is the grandfather of M

Ans. **(b)**

End of Solution

Q.4 The number that least fits this set:
(324, 441, 97 and 64) is ________.
(a) 324 (b) 441
(c) 97 (d) 64

Ans. **(c)**
97 is not a perfect square.

End of Solution

Q.5 It takes 10s and 15s, respectively, for two trains travelling at different constant speeds to completely pass a telegraph post. The length of the first train is 120 m and that of the second train is 150 m. The magnitude of the difference in the speeds of the two trains (in m/s) is ________.

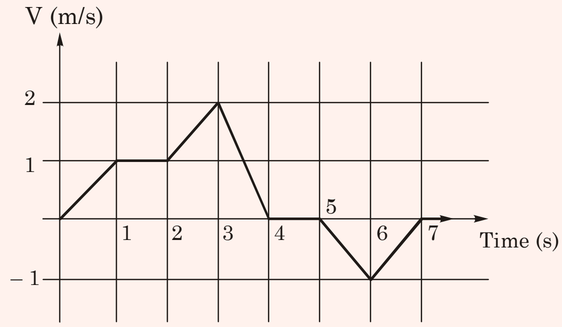
(a) 2.0 (b) 10.0
(c) 12.0 (d) 22.0

Ans. **(a)**

End of Solution

Two Marks Questions

Q.6 The velocity V of a vehicle along a straight line is measured in m/s and plotted as shown with respect to time in seconds. At the end of the 7 seconds, how much will the odometer reading increase by (in m)?



(a) 0 (b) 3
(c) 4 (d) 5

Ans. **(d)**

The odometer reading increases from starting point to end point
Magnitude of area of the given diagram = Odometer reading
Magnitude of area of the velocity and time graph per second

1^{st} sec $\Rightarrow$ triangle = $\frac{1}{2}\times 1\times 1=\frac{1}{2}$

2^{nd} sec $\Rightarrow$ square = $1 \times 1 = 1$

3^{rd} sec $\Rightarrow$ square + triangle = $1+1+\frac{1}{2}\times 1\times 1=1\frac{1}{2}$

4^{th} sec $\Rightarrow$ triangle = $\frac{1}{2}\times 1\times 2=1$

5^{th} sec $\Rightarrow$ straight line = 0

6^{th} sec $\Rightarrow$ triangle = $\frac{1}{2}\times 1\times 1=\frac{1}{2}$

7^{th} sec $\Rightarrow$ triangle = $\frac{1}{2}\times 1\times 1=\frac{1}{2}$

Total Odometer reading at 7 seconds = $\frac{1}{2}+1+1\frac{1}{2}+1+0+\frac{1}{2}+\frac{1}{2}=5$

End of Solution

Q.7 The overwhelming number of people infected with rabies in India has been flagged by the World Health Organization as a source of concern. It is estimated that inoculating 70% of pets and stray dogs against rabies can lead to a significant reduction in the number of people infected with rabies.

Which of the following can be logically inferred from the above sentences?

(a) The number of people in India infected with rabies is high.

(b) The number of people in other parts of the world who are infected with rabies is low

(c) Rabies can be eradicated in India by vaccinating 70% of stray dogs

(d) Stray dogs are the main source of rabies worldwide

Ans. (a)

End of Solution

Q.8 A flat is shared by four first year undergraduate students. They agreed to allow the oldest of them to enjoy some extra space in the flat. Manu is two months older than Sravan, who is three months younger than Trideep. Pavan is one month older than Sravan. Who should occupy the extra space in the flat?

(a) Manu (b) Sravan

(c) Trideep (d) Pavan

Ans. (c)

$$\text{Manu's age} = \text{Sravan's age} + 2 \text{ months}$$
$$\text{Manu's age} = \text{Trideep's age} - 3 \text{ months}$$
$$\text{Pavan's age} = \text{Sravan's age} + 1 \text{ month}$$

From this Trideep's age > Manu > Pavan > Sravan

$\therefore$ Trideep can occupy the extra spane in the flat.

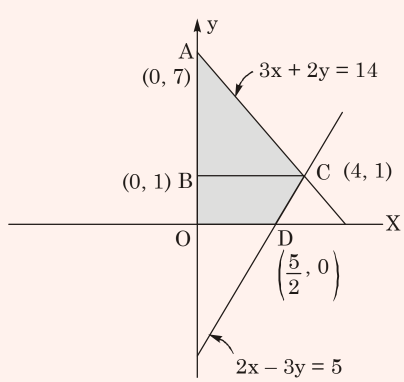
End of Solution

Q.9 Find the area bounded by the lines $3x + 2y = 14$, $2x - 3y = 5$ in the first quadrant.

(a) 14.95 (b) 15.25

(c) 15.70 (d) 20.35

Ans. (b)



$$\text{Required area} = \text{Area of } \Delta ABC + \text{Area of trapezoid BCDO}$$
$$= \frac{1}{2} \times 4 \times 6 + \frac{1}{2} \times (4 + 2.5) \times 1$$
$$= 12 + 3.25 = 15.25 \text{ sq. units}$$

End of Solution

Q.10 A straight line is fit to a data set ($ln\ x$, y). This line intercepts the abscissa at ln x = 0.1 and has a slope of – 0.02. What is the value of y at x = 5 from the fit?

(a) – 0.030 (b) – 0.014
(c) 0.014 (d) 0.030

Ans. **(a)**

$$\text{Straight line equation } y = mx + c$$
$$m = \text{slope} = -0.02$$

(ln x, y)

$$\text{If } ln\ x = X, \text{ then set } (\underset{0.1}{\underset{\downarrow}{x}}, \underset{0}{\underset{\downarrow}{y}})$$

$$y = mX + C$$
$$o = -0.02 \times 0.1 + C$$
$$\therefore \quad C = 0.002$$
$$y = mX + C$$
$$y = -0.02 \times \log x + C$$
$$\text{at } x = 5$$
$$y = -0.02 \times \log 5 + 0.002$$
$$= -0.030$$

End of Solution

Section - II (Electronics Engineering)

One Mark Questions

Q.1 Consider a 2 × 2 square matrix

$$A = \begin{bmatrix} \sigma & x \\ \omega & \sigma \end{bmatrix}$$

where x is unknown. If the eigenvalues of the matrix A are $(\sigma + j\omega)$ and $(\sigma - j\omega)$, then x is equal to

(a) $+j\omega$
(b) $-j\omega$
(c) $+\omega$
(d) $-\omega$

Ans. **(d)**

$$A = \begin{bmatrix} \sigma & x \\ \omega & \sigma \end{bmatrix}$$

$$\text{Trace} = \text{sum of eigen values}$$

$$2\sigma = \sigma + j\omega + \sigma - j\omega$$

$$|A| = \text{product of eigens}$$

$$\sigma^2 - x\omega = (\sigma + j\omega)(\sigma - j\omega) = \sigma^2 + \omega^2$$

which is possible only when $x = -\omega$

End of Solution

Q.2 For $f(z) = \dfrac{\sin(z)}{z^2}$, the residue of the pole at $z = 0$ is ________.

Ans. **(1)**

$$\text{Residue of } \frac{\sin z}{z^2} = \text{coefficient of } \frac{1}{z} \text{ in } \left\{\frac{z - \frac{z^3}{3!} + \frac{z^5}{5!} - - -}{z^2}\right\}$$

$$= \text{coefficient of } \frac{1}{z} \text{ in } \left\{\frac{1}{z} - \frac{z}{3!} + \frac{z^3}{5!} - - -\right\}$$

$$= 1$$

End of Solution

Q.3 T he probability of getting a "head" in a single toss of a biased coin is 0.3. The coin is tossed repeatedly till a "head" is obtained. If the tosses are independent, then the probability of getting "head" for the first time in the fifth toss is _______.

Ans. **(0.072)**

$$P(H) = 0.3$$

$$P(T) = 0.7$$

since all tosses are independent

so, probability of getting head for the first time in 5th toss is

$$= P(T)\,P(T)\,P(T)\,P(T)\,P(H)$$

$$= 0.7 \times 0.7 \times 0.7 \times 0.7 \times 0.3$$
$$= 0.072$$

End of Solution

Q.4 The integral $\int_0^1 \frac{dx}{\sqrt{(1-x)}}$ is equal to _____.

Ans. **(2)**

$$\int_0^1 \frac{1}{\sqrt{1-x}} dx = -2\int_0^1 \frac{1}{2\sqrt{1-x}} dx = -2\left(\sqrt{1-x}\right)\Big|_0^1$$
$$= -2(0-1) = 2$$

End of Solution

Q.5 Consider the first order initial value problem

$y' = y + 2x - x^2,\ y(0) = 1, (0 \le x < \infty$

with exact solution $y(x) = x^2 + e^x$. For $x = 0.1$, the percentage difference between the exact solution and the solution obtained using a single iteration of the second-order Runge-Kutta method with step-size $h = 0.1$ is ___.

Ans. **(0.044)**

$$y' = y + 2x - x^2$$

Given that

Exact solution is given by $y(x) = x^2 + e^x$

$$y(0.1) = (0.1)^2 + e^{0.1} = 1.115$$
$$\frac{dy}{dx} = y + 2x - x^2$$
$$y(0) = 1, \quad h = 0.1$$
$$f(x, y) = y + 2x - x^2, \quad y_0 = 1, \quad x_0 = 0$$

(i) $$k_1 = h\,f(x_0, y_0) = h\,(y_0 + 2x_0 - x_0^2)$$
$$= 0.1\,(1 + 0 + 0) = 0.1$$

(ii) $$k_2 = h\,(f\,(x_0 + h, y_0 + k_1)$$
$$= h\,(y_0 + k_1 + 2\,(x_0 + h) - (x_0 + h)^2)$$
$$= 0.1\,(1 + 0.1 + 2(0 + 0.1) - (0 + 0.1)^2)$$
$$= 0.1\,(1 + 0.1 + 0.2 - 0.01) = 0.129$$

(iii) $$k = \frac{k_1 + k_2}{2} = \frac{0.1 + 0.129}{2} = 0.1145$$

(iv) $$y_1 = y_0 + k = 1 + 0.1145 = 1.1145$$
$$x_1 = x_0 + h = 0 + 0.1 = 0.1$$
$$\text{error} = \text{exact value} - \text{approximate value}$$
$$= 1.115 - 1.1145 = 0.0005$$

The percentage difference

$$= \frac{0.0005}{1.115} \times 100 = 0.044\%$$

End of Solution

Q.6 Consider the signal $x(t) = \cos(6\pi t) + \sin(8\pi t)$, where t is in seconds. The Nyquist sampling rate (in samples/second) for the signal $y(t) = x(2t + 5)$ is

(a) 8 (b) 12

(c) 16 (d) 32

Ans. **(c)**

$$X(t) = \cos(6\pi t) + \sin(8\pi t)$$
$$Y(t) = x(2t+5)$$
$$Y(t) = \cos[6\pi(2t+5) + \sin(8\pi(2t+5)]$$
$$= \cos(12\pi t + 30\pi) + \sin(16\pi t + 40\pi)$$
$$f_{m_1} = 6 \text{ Hz}; \qquad f_{m_2} = 8 \text{ Hz}$$
Nyquist sampling rate, $f_s = 2f_{\max}$
$$= 16 \text{ samples/second}$$

End of Solution

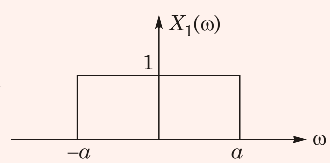
Q.7 If the signal $x(t) = \dfrac{\sin(t)}{\pi t} * \dfrac{\sin(t)}{\pi t}$ with * denoting the convolution operation, then $x(t)$ is equal to

(a) $\dfrac{\sin(t)}{\pi t}$ (b) $\dfrac{\sin(2t)}{2\pi t}$

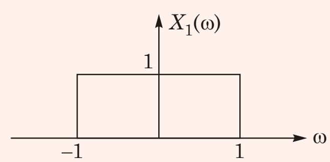
(c) $\dfrac{2\sin(t)}{\pi t}$ (d) $\left(\dfrac{\sin(t)}{\pi t}\right)^2$

Ans. **(a)**

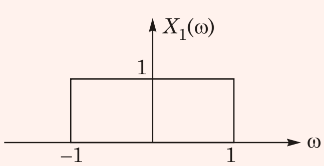
$$x_1(t) = \frac{\sin at}{\pi t} \leftrightarrow$$



$$x_1(t) = \frac{\sin t}{\pi t} \leftrightarrow$$



$$x(t) = x_1(t) * x_1(t)$$
$$X(\omega) = X_1(\omega) \cdot X_1(\omega)$$



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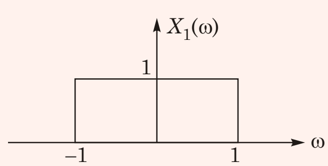

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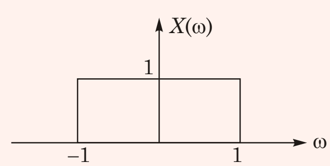


$$\therefore \qquad x(t) = \frac{\sin t}{\pi t}$$

End of Solution

Q.8 A discrete-time signal $x[n] = \delta[n-3] + 2\,\delta[n-5]$ has z-transform $X(z)$. If $Y(z) = X(-z)$ is the z-transform of another signal $y[n]$, then

(a) $y[n] = x[n]$
(b) $y[n] = x[-n]$
(c) $y[n] = -x[n]$
(d) $y[n] = -x[-n]$

Ans. **(c)**

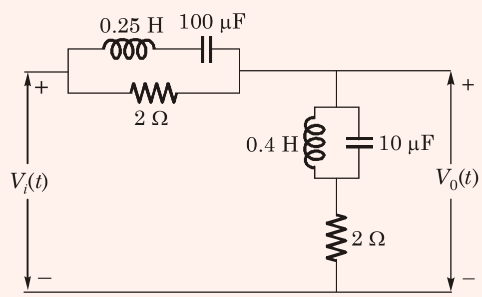
Given,

$$x[n] = \delta[n-3] + 2\delta[n-5]$$
$$X(z) = z^{-3} + 2z^{-5}$$
$$X(-z) = (-z)^{-3} + 2(-z)^{-5}$$
$$Y(z) = X(-z) = -z^{-3} - 2z^{-5} = -[z^{-3} + 2z^{-5}]$$
$$y[n] = -x[n]$$

End of Solution

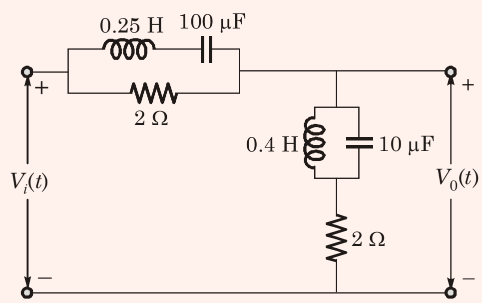
Q.9 In the RLC circuit shown in the figure, the input voltage is given by $V_i(t) = 2\cos(200\,t) + 4\sin(500\,t)$

The output voltage $V_0(t)$ is



(a) $\cos(200t) + 2\sin(500t)$
(b) $2\cos(200t) + 4\sin(500t)$
(c) $\sin(200t) + 2\cos(500t)$
(d) $2\sin(200t) + 4\cos(500t)$

Ans. **(b)**

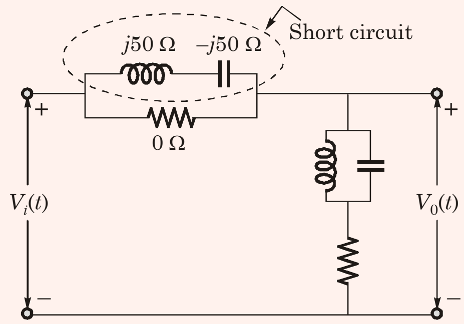


Where, $V_i(t) = 2\cos(200t) + 4\sin(500t)$

As different frequencies are operating, using superposition theorem, we get

for $\omega = 200$

$$X_L = \omega L = (200)\,(0.25) = 50\ \Omega\,;$$
$$X_C = \frac{1}{\omega C} = \frac{1}{200 \times 100 \times 10^{-6}} = 50\ \Omega$$

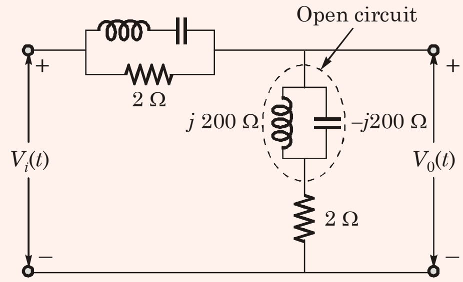


$$V_0(t) = V_i(t)$$

for ω = 500 rad/sec

$$X_L = 0.4 \times 500 = 200\ \Omega$$

$$X_C = \frac{1}{10 \times 10^{-6} \times 500} = 200\ \Omega$$

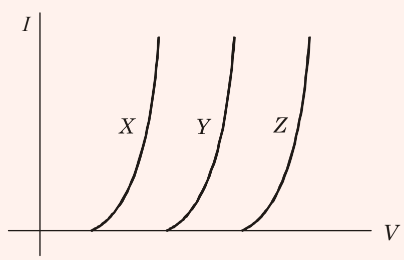


∴ $V_0(t) = V_i(t)$

Therefore $V_0(t) = 2\cos(200t) + 4\sin(500t)$

End of Solution

Q.10 The I-V characteristics of three types of diodes at the room temperature, made of semiconductors X, Y and Z, are shown in the figure. Assume that the diodes are uniformly doped and identical in all respects except their materials. If E_{gX}, E_{gY}, and E_{gZ} are the band gaps of X, Y and Z, respectively, then



(a) $E_{gX} > E_{gY} > E_{gZ}$
(b) $E_{gX} = E_{gY} = E_{gZ}$
(c) $E_{gX} < E_{gY} < E_{gZ}$
(d) no relationship among these band gaps exists

Ans. **(c)**

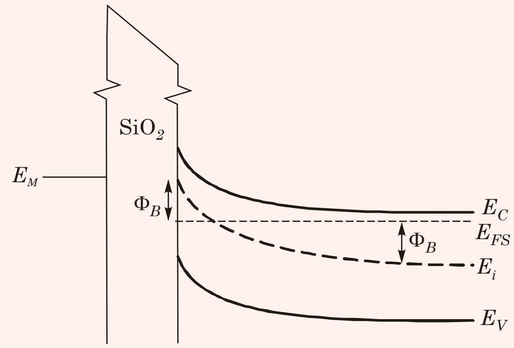
$X \rightarrow$ Ge Diode ....$E_G = 0.2$ V

$Y \rightarrow$ Si Diode ....$E_G = 0.7$ V

$Z \rightarrow$ GaAs Diode (or) LED ....$E_G = 1.3$ V

End of Solution

Q.11 The figure shows the band diagram of a Metal Oxide Semiconductor (MOS). The surface region of this MOS is in



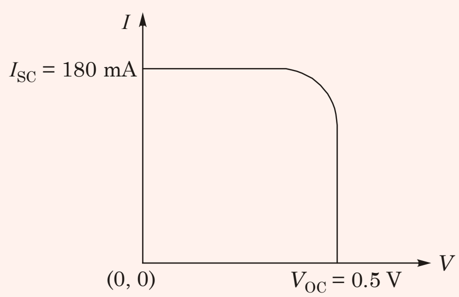
(a) inversion
(b) accumulation
(c) depletion
(d) flat band

Ans. **(a)**

Substrate is N-type SC.

End of Solution

Q.12 The figure shows the I-V characteristics of a solar cell illuminated uniformly with solar light of power 100 mW/cm^2. The solar cell has an area of 3 cm^2 and a fill factor of 0.7. The maximum efficiency (in %) of the device is ______.



Ans. **(21)**

$$\text{Fill factor} = \frac{\text{Maximum obtained power}}{V_{OC} \cdot I_{SC}}$$

$$0.7 = \frac{\text{Maximum obtained power}}{0.5 \times 180 \times 10^{-3}}$$

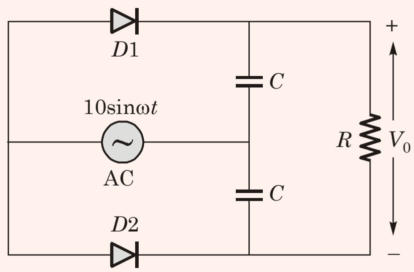
$\therefore$ Maximum obtained power, $P_m = 0.063$ W

$$\text{Efficiency } \eta = \frac{P_m}{G \cdot A} = \frac{0.063}{100 \times 10^{-3} \times 3} = 21\%$$

$G \rightarrow$ input light in W/cm^2

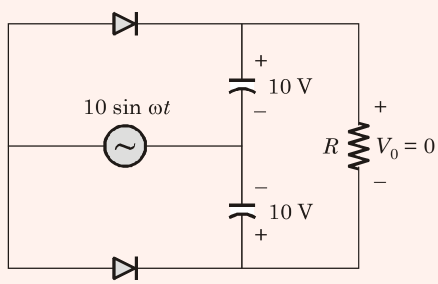
End of Solution

Q.13 The diodes $D1$ and $D2$ in the figure are ideal and the capacitors are identical. The product RC is very large compared to the time period of the ac voltage. Assuming that the diodes do not breakdown in the reverse bias, the output voltage V_O (in volt) at the steady state is ________.



Ans. **(0)**

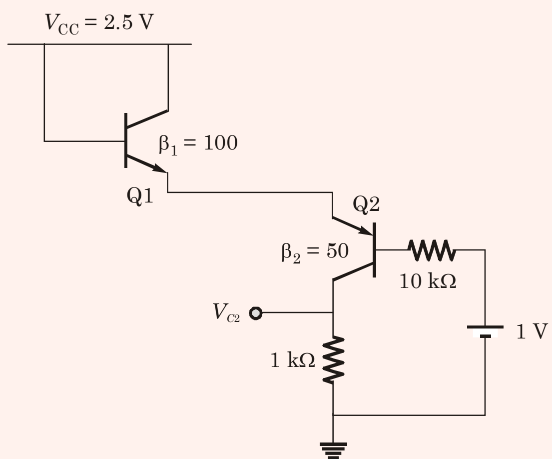
During first positive quarter cycle both diodes $D1$ and $D2$ conduct and at $t = \frac{T}{4}$ both will be OFF.



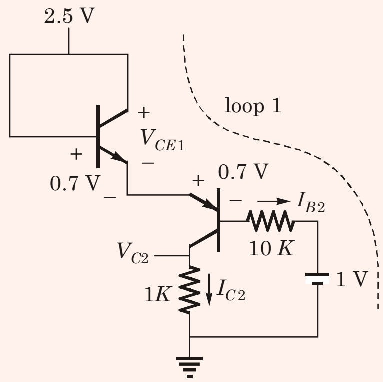
Both the capacitors will charge to 10 V as shown is above figure and $V_0 = 0$ in steady state.

End of Solution

Q.14 Consider the circuit shown in the figure. Assuming $V_{BE1} = V_{EB2} = 0.7$ volt, the value of the dc voltage V_{C2} (in volt) is ____.



Ans. **(0.5)**



Q_1 : $V_{CB} = 0$

We know $V_{CB1} + V_{BE1} = V_{CE1}$

$\Rightarrow$ $V_{CE1} = V_{BE1} = 0.7\text{ V}$

KVL to loop 1

$-2.5 + V_{CE1} + 0.7 + I_{B2} \times 10\text{ K} + 1\text{ V} = 0$

$\rightarrow$ $$I_{B2} = \frac{2.5 - 1.4 - 1}{10 K} = 0.01\text{ mA}$$

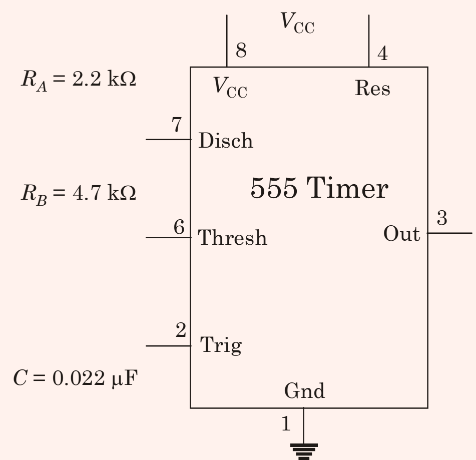
$$I_{C2} = \beta_2 I_{B2} = 0.5\text{ mA}$$

$$V_{C2} = I_{C2} \times 1\,K = 0.5 \times 10^{-3} \times 1 \times 10^3$$

$$= 0.5\text{ V}$$

End of Solution

Q.15 In the astable multivibrator circuit shown in the figure, the frequency of oscillation (in kHz) at the output pin 3 is _______.



Ans. **(5.6818)**

$$f = \frac{1}{0.69(R_A + 2R_B)C}$$

$$= \frac{1}{0.69(2.2 \times 10^3 + 2 \times 4.7 \times 10^3) \times 0.022 \times 10^{-6}}$$

$$= 5.6818 \text{ kHz}$$

End of Solution

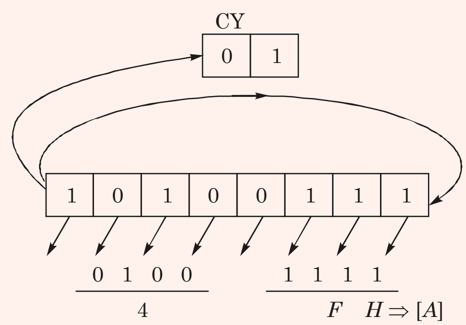
Q.16 In an 8085 microprocessor, the contents of the accumulator and the carry flag are A7 (in hex) and 0, respectively. If the instruction RLC is executed, then the contents of the accumulator (in hex) and the carry flag, respectively, will be

(a) 4E and 0
(b) 4E and 1
(c) 4F and 0
(d) 4F and 1

Ans. **(d)**

Given, $CY = 0$: $[A] = \text{A7H}$

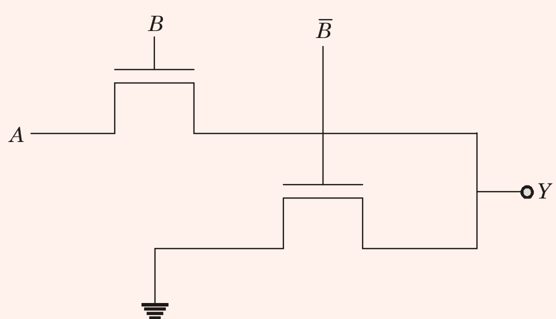
RLC - Rotate accumulator content 1 bit left without carry



$\therefore$ $[A] = 4\text{F}$ and $CY = 1$

End of Solution

Q.17 The logic functionality realized by the circuit shown below is



(a) OR
(b) XOR
(c) NAND
(d) AND

Ans. **(d)**

The output Y will be logic 1, when

$A = 1$, $B = 1$ and $\overline{B} = 0$

$\Rightarrow \quad Y = A \cdot B \cdot \overline{\overline{B}} = A \cdot B$

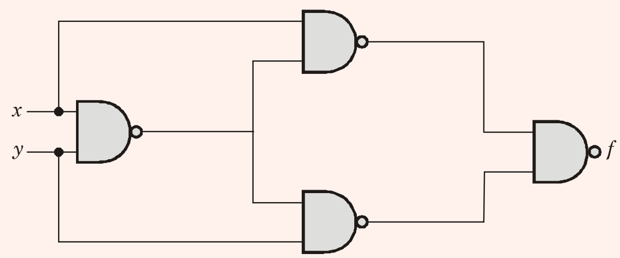
$\Rightarrow$ AND operation

End of Solution

Q.18 The minimum number of 2-input NAND gates required to implement a 2-input XOR gate is

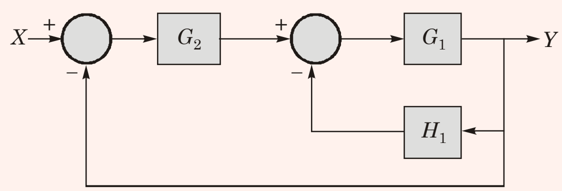
(a) 4
(b) 5
(c) 6
(d) 7

Ans. **(a)**



End of Solution

Q.19 The block diagram of a feedback control system is shown in the figure. The overall closed-loop gain G of the system is



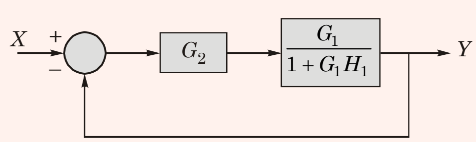
(a) $G = \dfrac{G_1G_2}{1 + G_1H_1}$

(b) $G = \dfrac{G_1G_2}{1 + G_1G_2 + G_1H_1}$

(c) $G = \dfrac{G_1G_2}{1 + G_1G_2H_1}$

(d) $G = \dfrac{G_1G_2}{1 + G_1G_2 + G_1G_2H_1}$

Ans. **(b)**



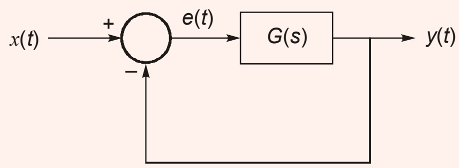
$$\frac{Y(s)}{X(s)} = \frac{\dfrac{G_2G_1}{1+G_1H_1}}{1 + \dfrac{G_2G_1}{1+G_1H_1}} = \frac{G_2G_1}{1 + G_1G_2 + G_1H_1}$$

End of Solution

Q.20 For the unity feedback control system shown in the figure, the open-loop transfer function $G(s)$ is given as

$$G(s) = \frac{2}{s(s+1)}$$

The steady state error e_{ss} due to a unit step input is



(a) 0 (b) 0.5
(c) 1.0 (d) ∞

Ans. **(a)**

$$E(s) = \frac{R(s)}{1+G(s)H(s)}$$

$$R(s) = \frac{1}{s};\ G(s) = \frac{2}{s(s+1)};\ H(s) = 1$$

$$E(s) = \frac{\frac{1}{s}}{1+\frac{2}{s(s+1)}} = \frac{s+1}{s^2+s+2}$$

$$e_{ss} = \underset{s\to 0}{Lt}\, sE(s) = \underset{s\to 0}{Lt} \frac{s(s+1)}{s^2+s+2}$$

$$e_{ss} = 0$$

End of Solution

Q.21 For a superheterodyne receiver, the intermediate frequency is 15 MHz and the local oscillator frequency is 3.5 GHz. If the frequency of the received signal is greater than the local oscillator frequency, then the image frequency (in MHz) is ________.

Ans. **(3485)**

$$\text{IF} = 15\text{ MHz}$$

$$f_l = 3.5\text{ GHz}$$

$$\Rightarrow \quad f_{si} = f_l - \text{IF}$$

$$= 3500 - 15\text{ MHz} = 3485\text{ MHz}$$

End of Solution

Q.22 An analog baseband signal, bandlimited to 100 Hz, is sampled at the Nyquist rate. The samples are quantized into four message symbols that occur independently with probabilities $p_1 = p_4 = 0.125$ and $p_2 = p_3$. The information rate (bits/sec) of the message source is ________.

Ans. **(362.2)**

$$f_m = 100 \text{ Hz}$$
$$f_s = 2f_m = 200 \text{ samples/sec}$$
$$P_1 = P_4 = \frac{1}{8}$$
$$P_1 + P_2 + P_3 + P_4 = 1$$
$$\Rightarrow \quad 2P_2 = 2P_3 = 1 - \frac{1}{4} = \frac{3}{4}$$
$$\Rightarrow \quad P_2 = P_3 = \frac{3}{8}$$
$$H = \sum_{i=1}^{N} P_i \log \frac{1}{P_i}$$
$$= \frac{1}{8}\log_2 8 + \frac{1}{8}\log_2 8 + \frac{3}{8}\log_2 \frac{8}{3} + \frac{3}{8}\log_2 \frac{8}{3}$$
$$H = 1.811 \text{ bits/sample}$$

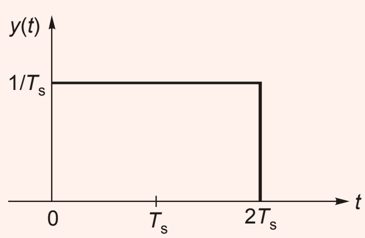
Information rate $R = H(\text{bits/sample}) \times r\,(\text{samples/sec})$
$$= 1.811 \times 200$$
$$= 362.2 \text{ bits/sec}$$

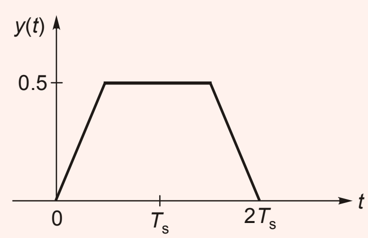
End of Solution

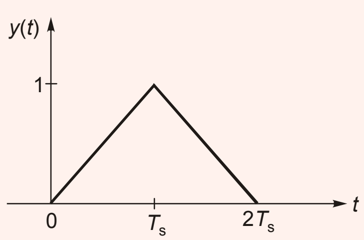
Q.23 A binary baseband digital communication system employs the signal

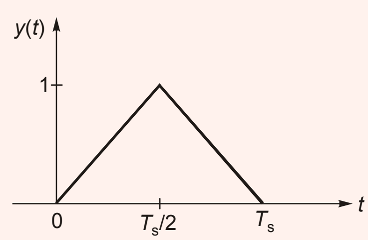
$$p(t) = \begin{cases} \dfrac{1}{\sqrt{T_s}}, & 0 \le t \le T_s \\ 0, & \text{otherwise} \end{cases}$$

for transmission of bits. The graphical representation of the matched filter output $y(t)$ for this signal will be

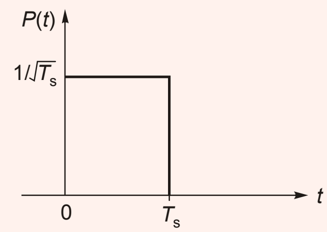
(a)


(b)


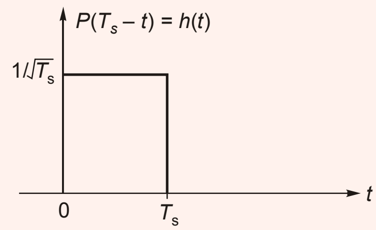
(c)


(d)


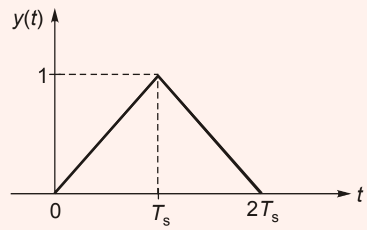
Ans. **(c)**



Impulse response of matched filter, $h(t) = P(T_s - t)$



Output of matched filter, $y(t) = P(t) * h(\mathrm{t})$



End of Solution

Q.24 If a right-handed circularly polarized wave is incident normally on a plane perfect conductor, then the reflected wave will be

(a) right-handed circularly polarized
(b) left-handed circularly polarized
(c) elliptically polarized with a tilt angle of 45°
(d) horizontally polarized

Ans. **(b)**

Left circularly polarized

- due to 180° phase difference between reflected and incident wave.
- due to direction change after the reflection from conductor.

End of Solution

Q.25 Faraday's law of electromagnetic induction is mathematically described by which one of the following equations?

(a) $\nabla \cdot \vec{B} = 0$
(b) $\nabla \cdot \vec{D} = \rho_v$
(c) $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$
(d) $\nabla \times \vec{H} = \sigma \vec{E} + \dfrac{\partial \vec{D}}{\partial t}$

Ans. **(c)**

Rate of change of magnetic field results in induced voltage

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

End of Solution

Two Marks Questions

Q.26 The particular solution of the initial value problem given below is

$$\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 36y = 0 \text{ with } y(0) = 3 \text{ and } \left.\frac{dy}{dx}\right|_{x=0} = -36$$

(a) $(3 - 18x)e^{-6x}$
(b) $(3 + 25x)e^{-6x}$
(c) $(3 + 20x)e^{-6x}$
(d) $(3 - 12x)e^{-6x}$

Ans. **(a)**

$$\begin{aligned}
(D^2 + 12D + 36)^4 &= 0 \\
(D+6)^2 y &= 0 \\
D &= -6, -6 \\
y &= C_1 + xC_2)e^{-6x} \\
y &= C_1 e^{-6x} + C_2 x e^{-6x} \\
y(0) &= 3 \qquad 3 = C_1 + 0 \qquad \Rightarrow \qquad C_1 = 3 \\
y' &= -6C_1 e^{-6x} + C_2 e^{-6x} - 6C_2 x e^{-6x} \\
y'(0) &= -36 \\
-36 &= -6C_1 + C_2 \\
-36 &= -18 + C_2 \qquad C_2 = -18 \\
\therefore \quad y &= 3e^{-6x} - 18x e^{-6x} \\
y &= (3 - 18x)e^{-6x}
\end{aligned}$$

End of Solution

Q.27 If the vectors $e_1 = (1, 0, 2)$, $e_2 = (0, 1, 0)$ and $e_3 = (-2, 0, 1)$ form an orthogonal basis of the three-dimensional real space R^3, then the vector $u = (4, 3, -3) \in R^3$ can be expressed as

(a) $u = -\frac{2}{5}e_1 - 3e_2 - \frac{11}{5}e_3$
(b) $u = -\frac{2}{5}e_1 - 3e_2 + \frac{11}{5}e_3$
(c) $u = -\frac{2}{5}e_1 + 3e_2 + \frac{11}{5}e_3$
(d) $u = -\frac{2}{5}e_1 + 3e_2 - \frac{11}{5}e_3$

Ans. **(d)**

$$\begin{bmatrix} 4 \\ 3 \\ -3 \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{aligned}
a - 2c &= 4 \\
b &= 3 \\
2a + c &= -3
\end{aligned}$$

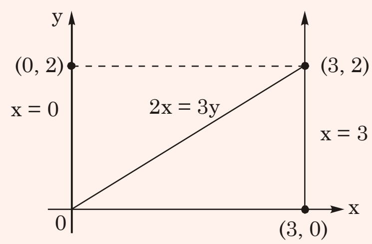
from here

$$a = -\frac{2}{5}$$
$$b = 3$$
$$c = -\frac{11}{5}$$
$$u = -\frac{2}{5}e_1 + 3e_2 - \frac{11}{5}e_3$$

End of Solution

Q.28 A triangle in the xy-plane is bounded by the straight lines $2x = 3y$, $y = 0$ and $x = 3$. The volume above the triangle and under the plane $x + y + z = 6$ is ________.

Ans. **(10)**



$$\text{Volume} = \iiint dz dx dy = \iint z dy dx$$

$$= \int_0^3 \int_0^{2/3x} (6 - x - y)\, dy dx = \int_0^3 \left(6y - xy - \frac{y^2}{2}\right)\Bigg|_0^{2/3x} dx$$

$$= \int_0^3 \left(4x - \frac{8}{9}x^2\right) dx = \left(4\frac{x^2}{2} - \frac{8}{9}.\frac{x^3}{3}\right)\Bigg|_0^3$$

$$= \left[2x^2 - \frac{8}{9}\left(\frac{x^3}{3}\right)\right]_0^3$$

$$= 18 - 8 = 10$$

End of Solution

Q.29 The values of the integral $\frac{1}{2\pi j}\oint_c \frac{e^z}{z-2}dz$ along a closed contour c in anti-clockwise direction for

(i) the point $z_0 = 2$ inside the contour c, and

(ii) the point $z_0 = 2$ outside the contour c, respectively, are

(a) (i) 2.72, (ii) 0
(b) (i) 7.39, (ii) 0
(c) (i) 0, (ii) 2.72
(d) (i) 0, (ii) 7.39

Ans. **(b)**

(i) $$Z_0 = 2 \text{ -lies inside } C,$$

so $$\text{Res} f(z) = \lim_{z\to 2}(z-2).\frac{e^z}{z-2} = e^2 = 7.39$$

$$\frac{1}{2\pi i}\int_c \frac{e^z}{z-2}dz = 2\pi i.\frac{1}{2\pi i}(7.39) = 7.39$$

(ii) $$Z_0 = -2 \text{ lies out side } C \text{ then}$$

$$\text{Res} f(z) = 0$$

so $$\int_c \frac{e^z}{z-2}dz = 2\pi i.\frac{1}{2\pi i}(0) = 0$$

End of Solution

Q.30 A signal $2\cos\left(\frac{2\pi}{3}t\right) - \cos(\pi t)$ is the input to an LTI system with the transfer function

$$H(s) = e^s + e^{-s}$$

If C_k denote the k^{th} coefficient in the exponential Fourier series of the output signal, then c_3 is equal to

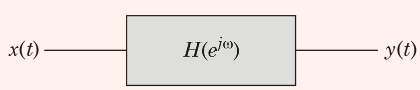
(a) 0 (b) 1

(c) 2 (d) 3

Ans. **(b)**

Given, $$H(s) = e^s + e^{-s}$$

$$H(e^{j\omega}) = e^{j\omega} + e^{-j\omega} = 2\cos\omega$$



If $$x(t) = 2\cos\left(\frac{2\pi}{3}t\right)$$

$$\omega_0 = \frac{2\pi}{3}$$

$$H(j\omega_0) = 2\cos\left(\frac{2\pi}{3}\right) = 2\left(-\frac{1}{2}\right) = -1$$

$$y(t) = 2\cos\left(\frac{2\pi}{3}t + 180°\right)$$

$$x(t) = \cos \pi t$$

if $$x(t) = \cos \pi t$$

$$\omega_0 = \pi$$

$$H(e^{j\omega_0}) = 2\cos(\pi) = -2$$

$$y(t) = 2\cos(\pi t + 180°)$$

$$y(t) = 2\cos\left(\frac{2\pi}{3}t + \pi\right) - 2\cos(\pi t + \pi)$$

$$\omega_1 = \frac{2\pi}{3}, \quad \omega_2 = \pi$$

$$T_1 = 3 \qquad T_2 = 2$$

$$\therefore \quad T_0 = 6$$

$$\Rightarrow \quad \omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{3}$$

$$y(t) = 2\cos(2\omega_0 t + \pi) - 2\cos(3\omega_0 t + \pi)$$

$$y(t) = e^{j(2\omega_0 t+\pi)} + e^{-j(2\omega_0 t+\pi)} - e^{j(3\omega_0 t+\pi)} - e^{-j(3\omega_0 t+\pi)}$$

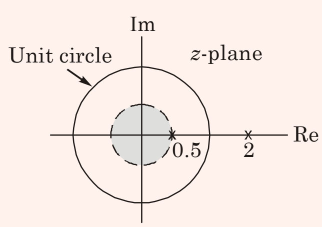
$$y(t) = -e^{j(2\omega_0 t)} - e^{-j(2\omega_0 t)} + e^{j(3\omega_0 t)} + e^{-j(3\omega_0 t)}$$

$$\therefore \quad C_3 = 1$$

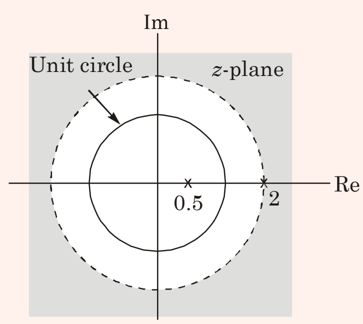
End of Solution

Q.31 The ROC (region of convergence) of the z-transform of a discrete-time signal is represented by the shaded region in the z-plane. If the signal $x[n] = (2.0)^{|n|}, -\infty < n < +\infty$, then the ROC of its z-transform is represented by

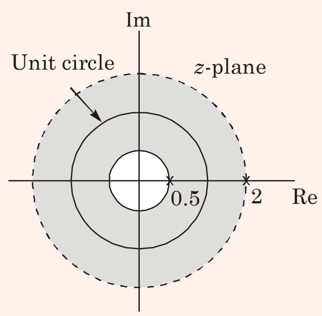
(a)



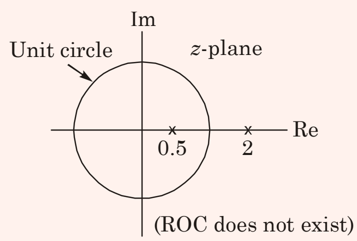
(b)



(c)



(d)



Ans. **(d)**

Given,

$$x[n] = (2.0)^{|n|}, -\infty < n < +\infty$$

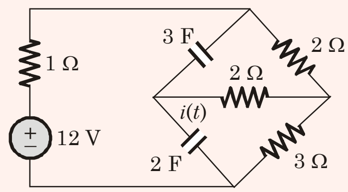
$$= 2^n u[n] + \left(\frac{1}{2}\right)^n u[-n-1]$$

$$\text{ROC} : |Z| > 2 \qquad \text{ROC} : |Z| < \frac{1}{2}$$

$\therefore$ No ROC.

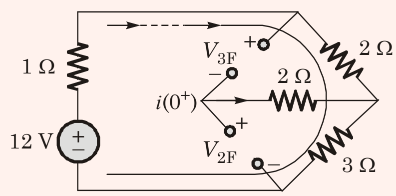
End of Solution

Q.32 Assume that the circuit in the figure has reached the steady state before time $t = 0$ when the 3 Ω resistor suddenly burns out, resulting in an open circuit. The current $i(t)$ (in ampere) at $t = 0^+$ is ________.



Ans. **(–1)**

at $t = 0^-$

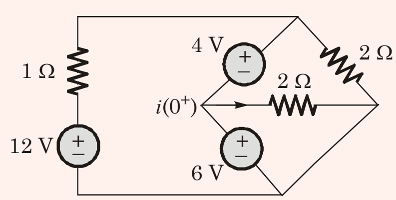


$$I = \frac{12}{6} = 2 \text{ A}$$

$$V_{3F} = 10 \times \frac{2}{5} = 4 \text{ V}$$

$$V_{2F} = 10 \times \frac{3}{5} = 6 \text{ V}$$

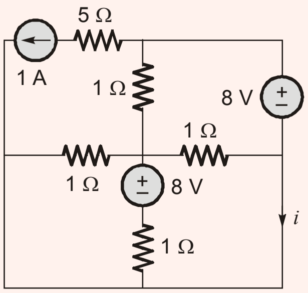
at $t = 0^+$



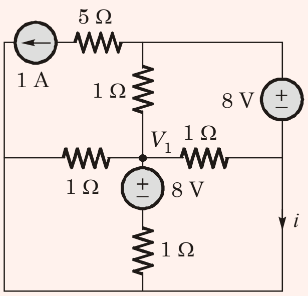
$$i(0^+) = \frac{-4}{2+2} = -1 \text{ A}$$

End of Solution

Q.33 In the figure shown, the current i (in ampere) is ________.



Ans. **(–1)**

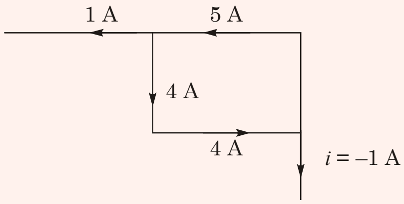


Using KCL at V_1

$$\frac{V_1}{1} + \frac{V_1 - 8}{1} + \frac{V_1 - 8}{1} + \frac{V_1}{1} = 0$$

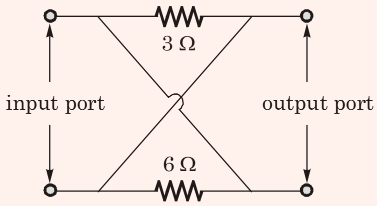
or $V_1 = 4$ V

considering KVL, we get



End of Solution

Q.34 The Z-parameter matrix $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$ for the two-port network shown is

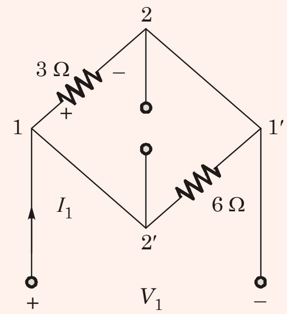


(a) $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 9 & -3 \\ 6 & 9 \end{bmatrix}$ (d) $\begin{bmatrix} 9 & 3 \\ 6 & 9 \end{bmatrix}$

Ans. **(a)**

Redrawing the circuit

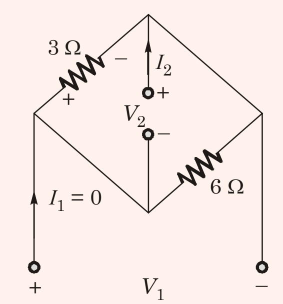


$$Z_{11} = \frac{V_1}{I_1} = 3 \| 6 = 2\ \Omega$$

$$V_2 = -3 \times I_{3\,\Omega} = -3 \times I_1 \times \frac{6}{9} = -2I_1$$

$\therefore$ $$Z_{21} = \frac{V_2}{I_1} = -2$$

now, $$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0} = 3 \| 6 = 2\ \Omega$$



$$V_1 = -6 \times I_{6\,\Omega} = -6 \times I_2 \times \frac{3}{9} = -2$$

$\therefore$ $$[Z] = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

End of Solution

Q.35 A continuous-time speech signal $x_a(t)$ is sampled at a rate of 8 kHz and the samples are subsequently grouped in blocks, each of size N. The DFT of each block is to be computed in real time using the radix-2 decimation-in-frequency FFT algorithm. If the processor performs all operations sequentially, and takes 20 μs for computing each complex multiplication (including multiplications by 1 and -1) and the time required for addition/subtraction is negligible, then the maximum value of N is ________.

Ans. **(4096)**

$$f_s = 8000 \text{ samples/sec}$$

$$\Rightarrow \quad T_s = \frac{1}{f_s} = \frac{1}{8000} \text{ sec.}$$

$$\text{Time for each multiplication} = T_m = 20\ \mu\text{sec.}$$

$$T_m = 20 \times 10^{-6} \text{ sec}$$

Suppose block is of size N, then time taken to generate each block $= N \times T_s = N \times \frac{1}{8000}$. No. of multiplications that can be performed by processor in the time taken for each block $= \frac{N \times T_s}{T_m}$

No. of multiplications required to compute DFT by Radix – 2 FFT algorithm

$$= \frac{N}{2}\log_2 N$$

No. of multiplication required by FFT≤ No. of multiplications that can be performed by the processor in the time taken in each block

[Reason : real time]

$$\frac{N}{2}\log_2 N \le \frac{N \times T_s}{T_m}$$

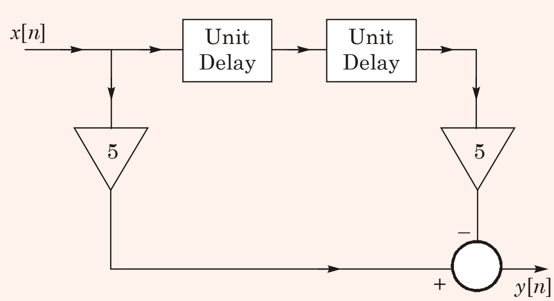
$$\log_2 N \le \frac{2 \times \frac{1}{8000}}{20 \times 10^{-6}} = 12.5$$

$$N \le 2^{12.5}$$

$$\Rightarrow \quad N \le 2^{12} = 4096$$

End of Solution

Q.36 The direct form structure of an FIR (finite impulse response) filter is shown in the figure.



The filter can be used to approximate a

(a) low-pass filter
(b) high-pass filter
(c) band-pass filter
(d) band-stop filter

Ans. **(c)**

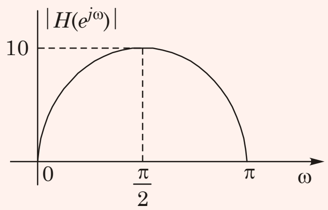
$$y[n] = 5[x[n] - x[n-2]]$$

$$H(z) = \frac{Y(z)}{X(z)} = 5\,[1 - z^{-2}]$$

But,

$$z = e^{j\omega}$$

$$H(e^{j\omega}) = 5\,[1 - e^{-j2\omega}]$$



$$\omega = 0 \qquad |H(e^{j\omega})| = 0$$

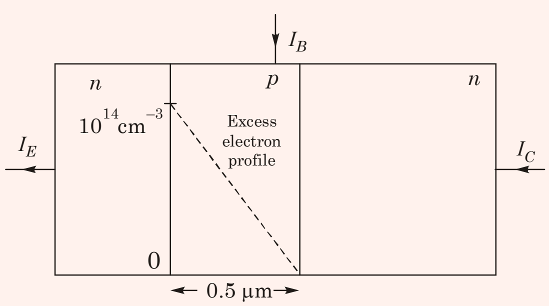
$$\omega = \frac{\pi}{2} \qquad |H(e^{j\omega})| = 10$$

$$\omega = \pi \qquad |H(e^{j\omega})| = 0$$

$\therefore$ The given one is bandpass filter.

End of Solution

Q.37 The injected excess electron concentration profile in the base region of an *npn* BJT, biased in the active region, is linear, as shown in the figure. If the area of the emitter-base junction is 0.001 cm^2, μ_n = 800 cm^2/(V-s) in the base region and depletion layer widths are negligible, then the collector current I_C (in mA) at room temperature is
(Given: thermal voltage V_T = 26 mV at room temperature, electronic charge $q = 1.6 \times 10^{-19}$ C)



Ans. **(6.656)**

$$I_c = AqD_n \frac{dn}{dx} \text{ where } D_n = \mu_n V_T$$

$$= 0.001\,(1.6 \times 10^{-19})\,800 \times 26 \times 10^{-3} \left[\frac{10^{14}}{0.5 \times 10^{-4}}\right]$$

$$= 66.56 \times 10^{-4}\text{ A} = 6.656\text{ mA}$$

End of Solution

Q.38 Figures I and II show two MOS capacitors of unit area. The capacitor in Figure I has insulator materials X (of thickness t_1 = 1 nm and dielectric constant ε_1 = 4) and Y (of thickness t_2 = 3 nm and dielectric constant ε_2 = 20). The capacitor in Figure II has only insulator material X of thickness t_{Eq}. If the capacitors are of equal capacitance, then the value of t_{Eq} (in nm) is ________.

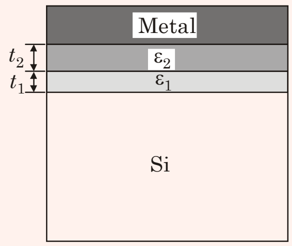


Figure I

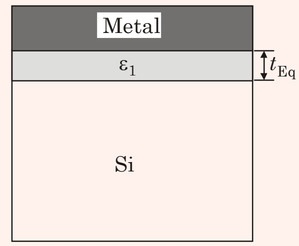


Figure II

Ans. **(1.6)**

From the figure I

Both the capacitors C_1 and C_2 are in series

Total Capacitance $$C = \frac{C_1 C_2}{C_1 + C_2}$$

$$C = \frac{\frac{A\varepsilon_1}{t_1} \cdot \frac{A\varepsilon_2}{t_2}}{\frac{A\varepsilon_1}{t_1} + \frac{A\varepsilon_2}{t_2}} = \frac{\frac{A \times 4}{10^{-9}} \cdot \frac{A \times 20}{3 \times 10^{-9}}}{\frac{A \times 4}{10^{-9}} + \frac{A \times 20}{3 \times 10^{-9}}}$$

$$C = 2.5 \times 10^9 \times A$$

From figure II

$$C = \frac{\varepsilon A}{t_{eq}}$$

Since both capacitors must be equal in figures I and II

$$2.5 \times 10^9 \times A = \frac{A \times 4}{t_{eq}}$$

$\therefore$ $$t_{eq} = 1.6 \text{ nm}$$

End of Solution

Q.39 The I-V characteristics of the zener diodes $D1$ and $D2$ are shown in Figure I. These diodes are used in the circuit given in Figure II. If the supply voltage is varied from 0 to 100 V, then breakdown occurs in

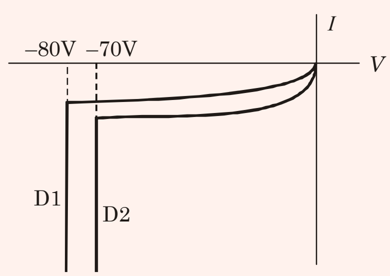


Figure I

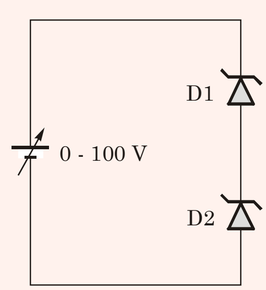


Figure II

(a) $D1$ only
(b) $D2$ only
(c) both $D1$ and $D2$
(d) none of $D1$ and $D2$

Ans. **(d)**

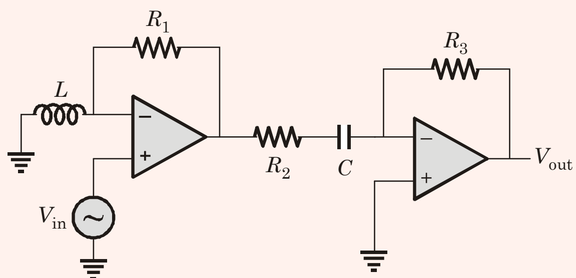
Zener Diode D_1 has $V_{Br} = 80$ V
Zener Diode D_2 has $V_{Br} = 70$ V
In series connection of ZD, V_{Br} are added total

$$V_{Br} = 150 \text{ V}$$

Since applied voltage is variable from 0 – 100 V.
so, None of the ZD will enter into breakdown.

End of Solution

Q.40 For the circuit shown in the figure, $R_1 = R_2 = R_3 = 1\ \Omega$, $L = 1\ \mu$H and $C = 1\ \mu$F. $V_{in} = \cos(10^6 t)$, then the overall voltage gain (V_{out}/V_{in}) of the circuit is ________.



Ans. **(–1)**

$$H_1(s) = \frac{V_{01}}{V_i}$$

$$= 1 + \frac{R_1}{sL} = 1 + \frac{1}{\frac{sL}{R_1}} = 1 + \frac{1}{s \times 10^{-6}}$$

$$H_2(s) = \frac{V_0}{V_{01}} = -\frac{R_3}{\frac{R_2 Cs + 1}{Cs}}$$

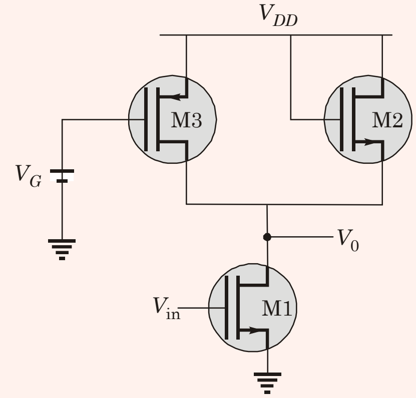
$$= \frac{-sR_3C}{1 + sR_3C} = \frac{-Cs}{1 + Cs} = \frac{-1}{1 + \frac{1}{Cs}}$$

$$H(s) = H_1(s)\, H_2(s)$$

$$= \left(1 + \frac{1}{s \times 10^{-6}}\right)\left(\frac{-s \times 1 \times 10^{-6}}{1 + s \times 10^{-6}}\right) = -1$$

End of Solution

Q.41 In the circuit shown in the figure, the channel length modulation of all transistors is non-zero ($\lambda \neq 0$). Also, all transistors operate in saturation and have negligible body effect. The ac small signal voltage gain (V_o/V_{in}) of the circuit is



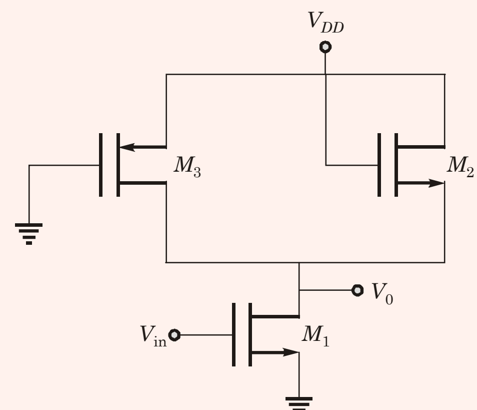
(a) $-g_{m1}\left(r_{01}\|r_{02}\|r_{03}\right)$

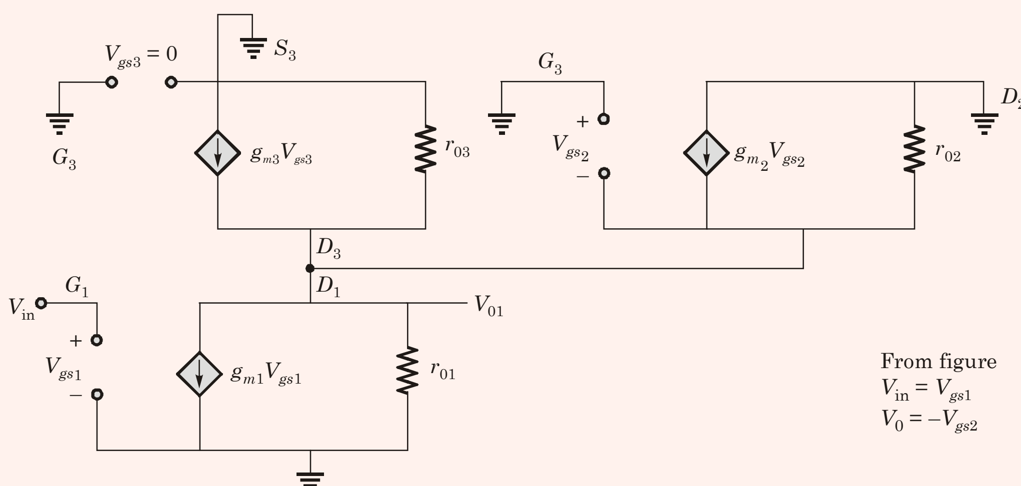
(b) $-g_{m1}\left(r_{01}\left\|\frac{1}{g_{m3}}\right\|r_{03}\right)$

(c) $-g_{m1}\left(r_{01}\left\|\left(\frac{1}{g_{m2}}\|r_{02}\right)\right\|r_{03}\right)$

(d) $-g_{m1}\left(r_{01}\left\|\left(\frac{1}{g_{m3}}\|r_{03}\right)\right\|r_{02}\right)$

Ans. **(c)**





From figure
$V_{in} = V_{gs1}$
$V_0 = -V_{gs2}$

Node equation at P_1 :

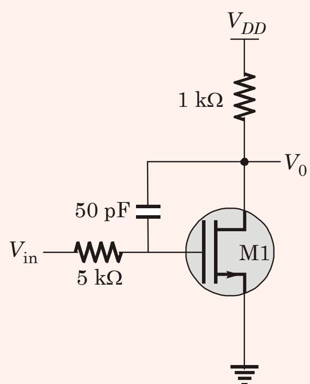
$$g_{m1}V_{gs1} + \frac{V_0}{r_{01}} + \frac{V_0}{r_{03}} + \frac{V_0}{r_{02}} - g_{m_2}V_{gs_2} = 0$$

$$V_0\left(\frac{1}{r_{01}} + \frac{1}{r_{02}} + g_{m2} + \frac{1}{r_{03}}\right) = -g_{m1}\,V_{in}$$

$$\Rightarrow \qquad A_v = \frac{V_0}{V_{in}} = -g_{m1}\left[r_{01} \parallel \left[r_{02} \parallel \frac{1}{g_{m2}}\right] \parallel r_{03}\right]$$

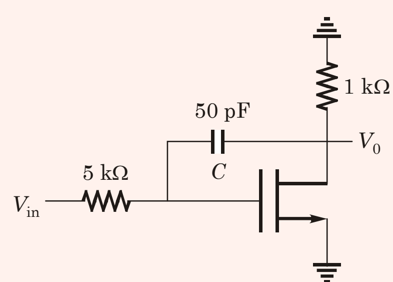
End of Solution

Q.42 In the circuit shown in the figure, transistor M1 is in saturation and has transconductance $g_m = 0.01$ siemens. Ignoring internal parasitic capacitances and assuming the channel length modulation λ to be zero, the small signal input pole frequency (in kHz) is _______.

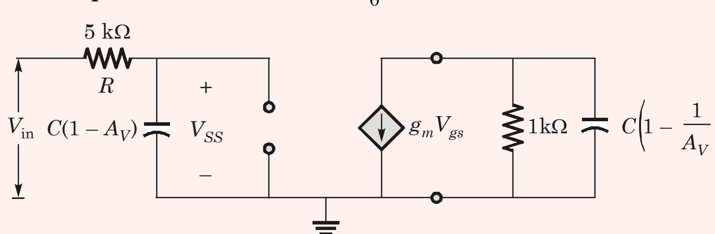


Ans. **(57.85)**

With respect to A.C



Taking miller's equivalent and assume $r_0 = \propto$



$$A_V = -g_m\,R_D = -0.01 \times 10^3 = -10$$

small signal input pole frequency

$$= \frac{1}{2\pi \times 5 \times 10^3 \times 50 \times 10^{-12}(1+10)} = 57.87 \text{ kHz}$$

End of Solution

Q.43 Following is the K-map of a Boolean function of five variables P, Q, R, S and X. The minimum sum-of-product (SOP) expression for the function is

$X = 0$

RS \ PQ	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

$X = 1$

RS \ PQ	00	01	11	10
00	0	1	1	0
01	0	0	0	0
11	0	0	0	0
10	0	1	1	0

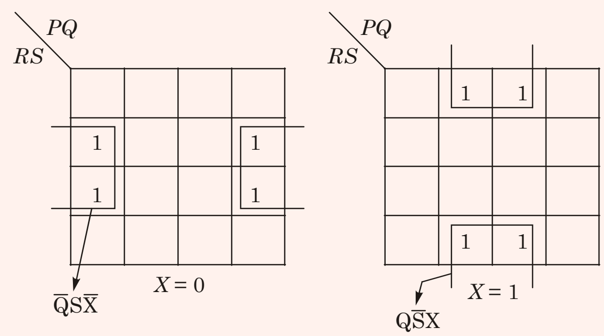
(a) $\bar{P}\bar{Q}S\bar{X} + P\bar{Q}S\bar{X} + Q\bar{R}\bar{S}X + QR\bar{S}X$

(b) $\bar{Q}S\bar{X} + Q\bar{S}X$

(c) $\bar{Q}SX + Q\bar{S}\bar{X}$

(d) $\bar{Q}S + Q\bar{S}$

Ans. **(b)**

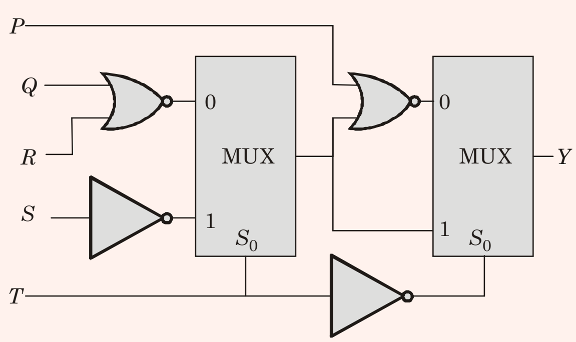


$\therefore$ minimum sum of product expression of the function is

$$= \bar{Q}S\bar{X} + Q\bar{S}X$$

End of Solution

Q.44 For the circuit shown in the figure, the delays of NOR gates, multiplexers and inverters are 2 ns, 1.5 ns and 1 ns, respectively. If all the inputs P, Q, R, S and T are applied at the same time instant, the maximum propagation delay (in ns) of the circuit is _________.



Ans. **(6)**

When, T = logic 0, the path followed by the circuit would be,

NOR gate → MUX 1 → MUX 2

⇒ 2 ns → 1.5 ns → 1.5 ns

⇒ 5 ns

When, T = logic 1, the path followed by the circuit would be,

NOR gate → MUX 1 → NOR gate → MUX 2

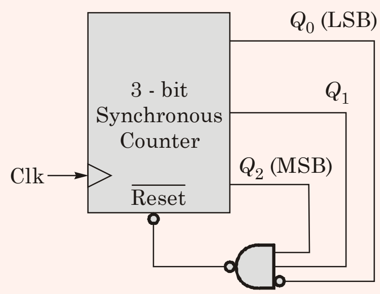
⇒ 1 ns → 1.5 ns → 2 ns → 1.5 ns

⇒ 6 ns

∴ Maximum propagation delay is 6 ns

End of Solution

Q.45 For the circuit shown in the figure, the delay of the bubbled NAND gate is 2 ns and that of the counter is assumed to be zero.



If the clock (Clk) frequency is 1 GHz, then the counter behaves as a

(a) mod-5 counter
(b) mod-6 counter
(c) mod-7 counter
(d) mod-8 counter

Ans. **(d)**

Clock frequency = 1 GHz

⇒ Clock time period = 1 ns

If the propagation delay of the NAND gate were 0 ns, the circuit would have behaved as MOD 6 counter.

However, the delay of NAND gate is 2 ns. During this time, two more clock pulses would reach the counter, and therefore it would count two more states. Hence, it acts as MOD 8 counter.

End of Solution

Q.46 The first two rows in the Routh table for the characteristic equation of a certain closed-loop control system are given as

s^3	1	$(2K + 3)$
s^2	$2K$	4

The range of K for which the system is stable is

(a) $-2.0 < K < 0.5$
(b) $0 < K < 0.5$
(c) $0 < K < \infty$
(d) $0.5 < K < \infty$

Ans. **(d)**

s^3	1	$(2K+3)$
s^2	$2K$	4
s	$\dfrac{4K^2+6K-4}{2K}$	0
s°	4	

For stability, $K > 0$ ...(i)

$$4K^2 + 6K - 4 > 0$$
$$2K^2 + 3K - 2 > 0$$
$$(2K-1)(K+2) > 0$$

$$\Rightarrow \quad K > \frac{1}{2} \text{ or, } K < -2 \quad \text{...(ii)}$$

Hence, $0.5 < K < \infty$

End of Solution

Q.47 A second-order linear time-invariant system is described by the following state equations

$$\frac{d}{dt}x_1(t) + 2x_1(t) = 3u(t)$$
$$\frac{d}{dt}x_2(t) + x_2(t) = u(t)$$

where $x_1(t)$ and $x_2(t)$ are the two state variables and $u(t)$ denotes the input. If the output $c(t) = x_1(t)$, then the system is

(a) controllable but not observable
(b) observable but not controllable
(c) both controllable and observable
(d) neither controllable nor observable

Ans. **(a)**

$$\dot{x}_1(t) + 2x_1(t) = 3u(t)$$
$$\dot{x}_2(t) + x_2(t) = u(t)$$
$$\dot{x}_1(t) = -2x_1(t) + 3u(t)$$
$$\dot{x}_2(t) = -x_2(t) + u(t)$$
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-2 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}3\\ 1\end{bmatrix}U$$
$$c(t) = x_1(t)$$
$$= \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$
$$A = \begin{bmatrix}-2 & 0\\ 0 & -1\end{bmatrix}; \quad B = \begin{bmatrix}3\\ 1\end{bmatrix}; \quad C = \begin{bmatrix}1 & 0\end{bmatrix}$$

For controllability $Q_C = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 1 & -1 \end{bmatrix}$

$|Q_C| \neq 0 \rightarrow$ Hence system is controllable.

For observability

$$Q_0 = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}$$

$|Q_0| = 0 \qquad \rightarrow$ Hence system is not observable

End of Solution

Q.48 The forward-path transfer function and the feedback-path transfer function of a single loop negative feedback control system are given as

$$G(s) = \frac{K(s+2)}{s^2+2s+2} \text{ and } H(s) = 1,$$

respectively. If the variable parameter K is real positive, then the location of the breakaway point on the root locus diagram of the system is ________.

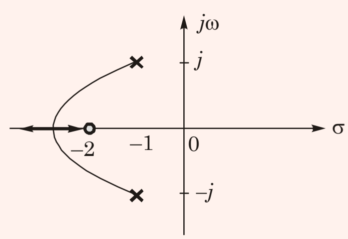
Ans. **(–3.41)**

$$G(s) = \frac{K(s+2)}{s^2+2s+2}; \; H(s) = 1$$

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{(s^2)+2s+2}(s+2) = 0$$

$$K = -\frac{(s^2+2s+2)}{(s+2)}$$



$$\frac{dK}{ds} = -\left(\frac{(2s+2)(s+2)-(s^2+2s+2)}{(s+2)^2}\right)$$

For break away points, $\dfrac{dK}{ds} = 0$

$(2s + 2)\,(s + 2) – (s^2 + 2s + 2) = 0$

$2s^2 + 6s + 4 – s^2 – 2s – 2 = 0$

$s^2 + 4s + 2 = 0$

$$s = \frac{-4 \pm \sqrt{16-8}}{2}$$

$$= \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

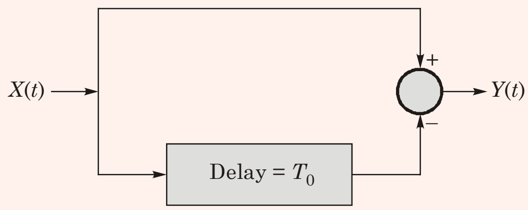
$$s = -0.58 \ ; \ s = -3.41$$

But, $s = -3.41$ lies on root locus

Hence $s = -3.41$

End of Solution

Q.49 A wide sense stationary random process $X(t)$ passes through the LTI system shown in the figure. If the autocorrelation function of $X(t)$ is $R_X(\tau)$, then the autocorrelation function $R_Y(\tau)$ of the output $Y(t)$ is equal to



(a) $2R_X(\tau) + R_X(\tau - T_0) + R_X(\tau + T_0)$ (b) $2R_X(\tau) - R_X(\tau - T_0) - R_X(\tau + T_0)$

(c) $2R_X(\tau) + 2R_X(\tau - 2T_0)$ (d) $2R_X(\tau) - 2R_X(\tau - 2T_0)$

Ans. **(b)**

$$y(t) = x(t) - x(t - T_0)$$

$$R_y(\tau) = E(y(\tau)\, y(t-\tau)]$$

$$y(\tau) = x(\tau) - x(\tau - T_0)$$

$$y(t-\tau) = x(t-\tau) - x(t-\tau-T_0)$$

$$R_y(\tau) = E((x(\tau) - x(\tau - T_0))\,(x(t-\tau) - x(t-\tau-T_0))$$

$$= E(x(\tau)\,x(t-\tau) - x(\tau)\,x(t-\tau-T_0) - x(\tau-T_0)\,x(t-\tau) + x(\tau-T_0)\,x(t-\tau-T_0))$$

$$= E[x(\tau)x(t-\tau)] - E[x(\tau)x(t-\tau-T_0)] - E[x(\tau-T_0)\,x(t-\tau)] + E[x(\tau-T_0)x(t-\tau-T_0)]$$

$$= R_x(\tau) - R_x(\tau + T_0) - R_x(\tau - T_0) + R_x(\tau)$$

$$R_y(\tau) = 2R_x(\tau) - R_x(\tau + T_0) - R_x(\tau - T_0)$$

End of Solution

Q.50 A voice-grade AWGN (additive white Gaussian noise) telephone channel has a bandwidth of 4.0 kHz and two-sided noise power spectral density $\frac{\eta}{2} = 2.5 \times 10^{-5}$ Watt per Hz. If information at the rate of 52 kbps is to be transmitted over this channel with arbitrarily small bit error rate, then the minimum bit-energy E_b (in mJ/bit) necessary is _________.

Ans. **(31.5)**

$$C = B\log_2\left(1+\frac{S}{N}\right)$$

$$S = E_b \cdot R_b \qquad (E_b = \text{bit energy}, R_b = \text{information rate bits/sec})$$

$$C = B\log_2\left(1+\frac{E_b\,R_b}{N_o\,B}\right)$$

For distortionless transmission, C should be atleast of R_b

$$\frac{R_b}{B} = \log_2\left(1+\frac{E_b\,R_b}{N_o\,B}\right)$$

$$E_b = (2^{Rb/B}-1)\,\frac{N_o\,B}{R_b}$$

$$= \frac{(2^{52/13}-1)\times 2\times 2.5\times 10^{-5}\times 4}{52}$$

$$E_b \approx 31.504 \text{ mJ/bit}$$

End of Solution

Q.51 The bit error probability of a memoryless binary symmetric channel is 10^{-5}. If 10^5 bits are sent over this channel, then the probability that not more than one bit will be in error is ________.

Ans. **(0.7357)**

bit error probability, $p = 10^{-5}$

Number of bits transmitted,

$$n = 10^5$$

If n-bits are transmitted, probability of getting crror in 'r' bits is given by

$$p_e = {}^nC_r\,p^r(1-p)^{n-r}$$

The probability that not more than one bit will be in crror is

$$= {}^nC_0\,p^0(1-p)^{n-0} + {}^nC_1(p)^1(1-p)^{n-1}$$

$$= (1-p)^n + np(1-p)^{n-1}$$

$$= (1-10^{-5})^{10^5} + 10^5\times 10^{-5}(1-10^{-5})^{10^5-1}$$

$$= 0.7357$$

End of Solution

Q.52 Consider an air-filled rectangular waveguide with dimensions a = 2.286 cm and b = 1.016 cm. At 10 GHz operating frequency, the value of the propagation constant (per meter) of the corresponding propagating mode is __________.

Ans. **(158)**

Operation frequency = 10 GHz

Cut off frequency for TE_{10} $\quad f_c = \dfrac{c}{2a}$

$$f_c = \frac{3\times10^8}{2\times2.286\times10^{-2}} = 6.57\text{ GHz}$$

Next mode TE_{20} has $f_c = 13$ GHz

10 GHz operates only in dominant mode.

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\in}$$

$$= \sqrt{\left(\frac{\pi}{a}\right)^2 - \frac{(2\pi f)^2}{(3\times10^8)^2}}$$

$$= \pi\sqrt{\left(\frac{1}{2.286\times10^{-2}}\right)^2 - \left(\frac{2\times10\times10^9}{3\times10^8}\right)^2}$$

$$= j158\ (\text{m}^{-1})$$

End of Solution

Q.53 Consider an air-filled rectangular waveguide with dimensions $a = 2.286$ cm and $b = 1.016$ cm. The increasing order of the cut-off frequencies for different modes is

(a) $TE_{01} < TE_{10} < TE_{11} < TE_{20}$
(b) $TE_{20} < TE_{11} < TE_{10} < TE_{01}$
(c) $TE_{10} < TE_{20} < TE_{01} < TE_{11}$
(d) $TE_{10} < TE_{11} < TE_{20} < TE_{01}$

Ans. **(c)**

$$a = 2.286\text{ cm}$$
$$b = 1.016\text{ cm}$$

$$f_{c_{TE_{10}}} = \frac{c}{2a} \quad ; \quad f_{c_{TE_{01}}} = \frac{c}{2b}$$

$$f_{c_{TE_{20}}} = \frac{c}{a} \quad ; \quad f_{c_{TE_{11}}} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

As $a > 2b \Rightarrow f_{c_{TE_{10}}} < f_{c_{TE_{20}}} < f_{c_{TE_{01}}} < f_{c_{TE_{11}}}$

End of Solution

Q.54 A radar operating at 5 GHz uses a common antenna for transmission and reception. The antenna has a gain of 150 and is aligned for maximum directional radiation and reception to a target 1 km away having radar cross-section of 3 m^2. If it transmits 100 kW, then the received power (in μW) is __________.

Ans. **(0.012)**

$f = 5$ GHz, $G = 150$, $R = 1$ km, $P_t = 100$ kW, $\sigma = 3$ m^2

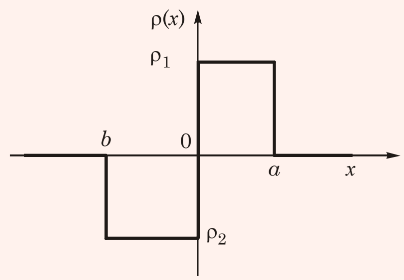
$$P_r = \frac{(G\lambda)^2 \cdot \sigma}{(4\pi)^3 R^4} \times P_t$$

$$= \frac{\left(150 \times \dfrac{3\times 10^8}{5\times 10^9}\right)^2 \cdot 3}{(4\pi)^3 \cdot 10^{12}} \times 10^5 = \frac{81 \times 3 \times 10^5}{(4\pi)^3 \times 10^{12}}$$

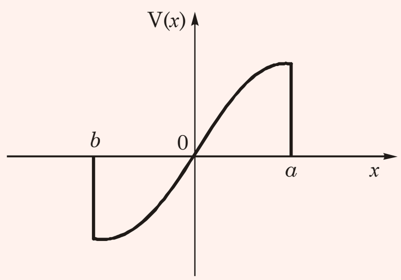
$$= 1.224 \times 10^{-8} = 0.012\ \mu\text{W}$$

End of Solution

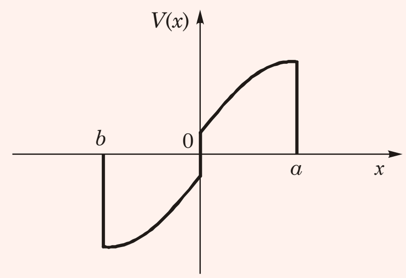
Q.55 Consider the charge profile shown in the figure. The resultant potential distribution is best described by



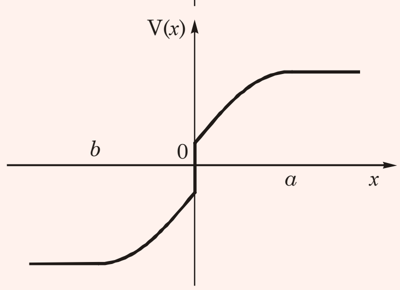
(a)



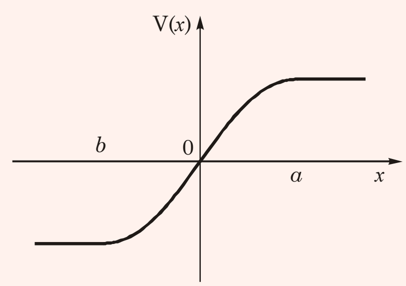
(b)



(c)



(d)



Ans. **(c)**

As ρ is given constant (solid charge) E is a linear function of x

$V = \int E \cdot dx$ = Non-linear function increasing with x.

with some discontinuity due to different charge on either side, the voltage has a discontinuity as in option (c).

End of Solution

❍❍❍❍