ANSWER KEY

CHEMISTRY

1. a	2. c	3. b	4. a	5. b	6. a	7. c	8. d	9. b	10. c	11. c	12. b	13. d
14. d	15. a	16. a	17. b	18. b	19. d	20. b	21. b	22. a	23. b	24. a	25. c	26. b
27. b	28. a	29. a	30. d									

PHYSICS

1. a	2. d	3. b	4. b	5. b	6. d	7. d	8. c	9. a	10. d	11. b	12. a	13. d
14. b	15. b	16. b	17. c	18. b	19. d	20. c	21. b	22. b	23. a	24. a	25. d	26. a
27. a	28. a	29. c	30. c									

MATHEMATICS

1. a	2. b	3. b	4. d	5. a	6. b	7. b	8. a	9. b	10. d	11. c	12. c	13. b
14. c	15. c	16. b	17. a	18. a	19. c	20. a	21. b	22. b	23. a	24. a	25. a	26. b
27. d	28. d	29. a	30. b									

CHEMISTRY

Sol 1.

In a face centered cubic arrangement each ion is surrounded by six oppositely charged ions, i.e.,co-ordination number is six. Radius ratio for coordination number 6 is 0.414 – 0.732.

when $\frac{r_+}{r_-} = 0.414$; $\frac{r_+}{241.5} = 0.414$

$r_+ = 0.414 \times 241.5 = 99.98 = 100$

when $\frac{r_+}{r_-} = 0.732$; $\frac{r_+}{241.5} = 0.732$

$r_+ = 0.732 \times 241.5 = 176.77$

Radius of cation should be between 100 – 176.7 pm

Sol2.

$\log\frac{k_2}{k_1} = \frac{e_a}{2.303\ R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$, Since energy of activation is zero, rate constant should be the same

Sol 3.

$CH_4(g) + 4CuO(s) \rightarrow CO_2(g) + 4Cu(s) + 2H_2O(l)$

-74.9   - 4 x 157.3   -393.5 - x 285.9

-74.9   - 629.2 = - 704.1   -393.5 -5718 = 965.3

Enthalpy change of the reaction = - 965.3 – (- 704.1) = -261.2

Sol 4.

The addition of chlorine to the reaction mixture shifts the equilibrium backwards, thereby releasing more heat.

Sol 5.

Mole fraction of naphthalene = 0.1

mole fraction of benzene = 0.9;

Amount of benzene = 0.9 x 78 = 70.2

Molality of the solution = $0.1 \times \frac{1000}{70.2} = 1.42$ m

Sol 6.

Polari ability of halide ions increases with the increase in size of the anion.

Sol 7.

The gold number of a protective colloid is its minimum amount in milligrams which is just sufficient to prevent the coagulation of the 10 ml of a gold sol on the addition of 1 mL. of 10% sodium chloride solution. Amount in milligram of 0.025 g. of starch is 25, so gold number is 25.

Sol 8.

The total energy of a Bohr's orbit is proportional to $1/n^2$

Sol 9.

$K_p = K_c (RT)^{\Delta n}$

$\Delta n = 2 - 0 = 2$, therefore, $K_p = K_c (RT)^2$

Sol 10.

EMF of cell = 1.229 V

Sol 11.

$0.00046100 = 4.6100 \times 10^{-4}$

; number of significant figures = 5

Sol 12.

For one nitrogen in the compound required of N = 14

10.5 g nitrogen is present in 100g of compound 14 g nitrogen will be present in $\frac{100}{10.5} \times 14 = 133.3$

The minimum molecular weight of the compound would be 133.3

Sol 13.

$\Delta T_b = 354.11 - 353.23 = 0.88$

$\Delta T_b = K_b . \frac{w_2}{m_2} \, x \frac{1000}{w_1}$

$0.88 = 2.53 . \frac{18}{m_2} \, x \frac{1000}{90}$

$m_2 = 2.53 . \frac{1.8}{0.88} \, x \frac{1000}{90} = 57.5$

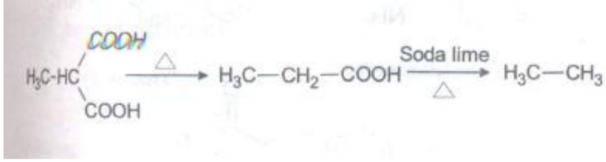
Sol 14.

Option 4 is correct.

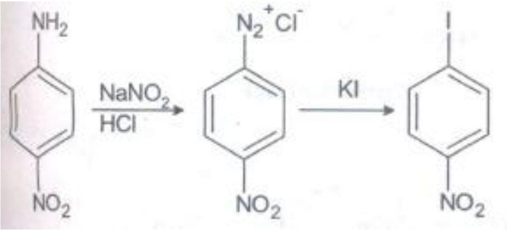
Sol 15.

The formula for compound A suggests that it is a dicarboxylic acid. A ß- keto acid or a ß – carboxyl acid readily undergoes decarboxylation on heating.

Therefore, A is $CH_3CH(COOH)_2$.



Sol 16.



Sol 17.

$3Fe^{2+} + 4H^{+} + NO_3^{-} \rightarrow 3Fe^{3+} + NO + 2H_2O$

$[Fe(H_2O)_6]^{2+} + NO \rightarrow [Fe\ (H_2O)_5\ (NO)]^{2+}\ H_2O$

Sol 18.

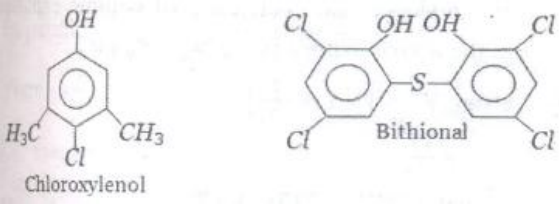
It is a Crossed Cannizzarro reaction.

$C_6H_5CHO + HCHO + NaOH \rightarrow C_6H_5CH_2OH +$

HCOONa

Sol 19.

Dettol is an antiseptic. It is a mixture of chloroxylenol and terpeneol in a suitable solvent. Chloroxylenol has both antiseptic and disinfectant properties. Bithional is antiseptic which is generally added to medicated soaps to reduce the odour produced by bacterial decomposition of organic matter o the skin. Iodine is powerful antiseptic. It is used as a tincture of iodine which is 2 – 3 % iodine solution of alcohol- water.

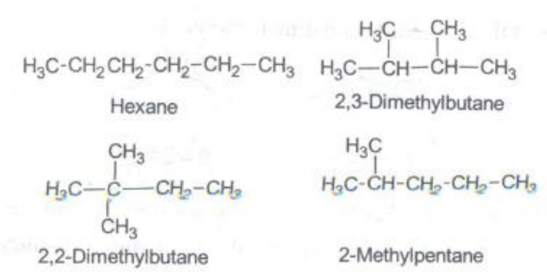


Sol 20.

HgS is not soluble in nitric acid

Sol 21.

Hexane which contains only 2 different types of hydrogens will give two monochlorinatedcompounds. Out the given choices only 2, 3 – Dimethylbutane contains two types of hydrogens; primary and secondary hydrogens.

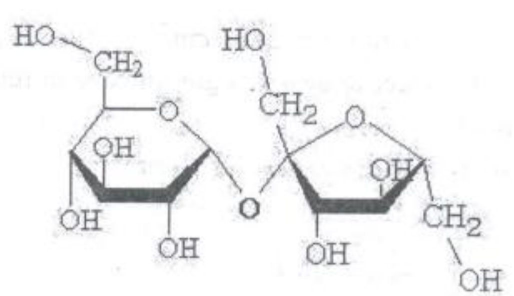


Sol 22.

Schiff's reagent give pink colour with aldehydes only. It is a chemical test for the detection of aldehydes. On adding an aldehyde to the decolorized Schiff reagent a characteristic magenta or purple colour is developed which indicates the presence of an aldehyde. The Schiff reagent is the reaction product of Fuchsine and sodium bisulfate.

Sol 23.

In sucross, C – 1 of glucose is linked to C – 2 of fructose by ß – glycosidic linkage.



Sol 24.

Peptization is a process of passing of a precipitate into colloidal perticles on adding suitable electrolyte. The electrolyte added is known as peptizing agent. A reddish brown coloured colloidal solution is obtained by adding small quantity of ferric chloride solution to the freshly precipitate ferric hydroxide.

Sol 25.

1.0 g. of H_2 and 1.0 g. of O_2 contain different number of molecules at STP because their molecular weights are different.

Sol 26.

Pb salts give gives a shining metallic bead in charcoal cavity test that markes paper.

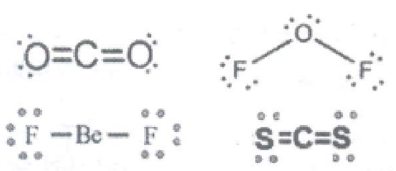
Sol 27.

The roasting of mercury sulphide in the air in metallurgical process produces mercury.

$HgS + O_2 \rightarrow Hg + SO_2$

Sol 28.

OF2 is a bent, non-linear molecule.

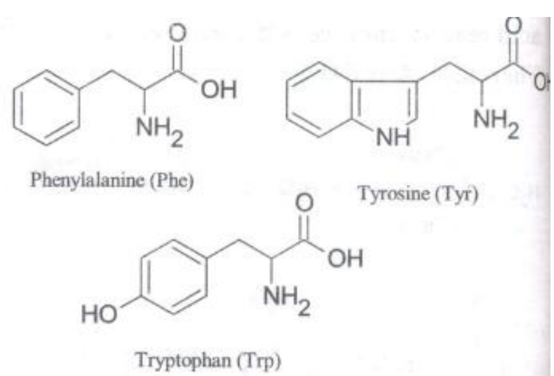


Sol 29.

A Ziegler – Natta catalyst a catalyst used in the synthesis of polymers of 1 – alkenes (α - olefins) It uses organoaluminium, $Al(C_2H_5)_3$.

Sol 30.

When heated with conc. HNO_2 acid, proteins containing an aromatic ring give yellow ppt. On adding NaOH, the ppt turns orange, This is known as Xanthoproties test



PHYSICS

Sol 1.

Total surface area = 6 x 7.203 x 7.203 = 311.169 cm^2

Volume = $(7.203)^3$ = 373.71475 cm^3

As there are 4 significant figures in 7.203 Total surface = 373.7 cm^2 And Total volume = 373.7 cm^3

Sol 2.

It will appear to be a straight line due to relative motion.

Sol 3.

Force = Mass x acceleration

= Mass x $\frac{velocity}{Time}$

= 0.05 x 400 = 20 N

Sol 4.

Let M be the mass of the gun, m be the mass of the bullet and V be the velocity of bullet then

MV = $\frac{mv}{M}$

⇒ V is the velocity of recoil of gun when held loosely. But when the gun is held tightly with shoulder, the mass of the man supports the gun thus reducing the speed of gun. Hence the gun experiences intensive recoil when fired with the bullet held loosely with the shoulder.

Sol 5.

Centre of mass will be given by intersection point of medians.

Sol 6.

Using V = $-\frac{gm}{r}$

$V_{net} = -\frac{gm}{1} - \frac{gm}{2} - \frac{gm}{4} - \frac{gm}{8}$................Which shows that it is a G.P. with r = $\frac{1}{2}$

Assuming G.P. to be infinite

$\therefore V_{net} = -Gm\left[\frac{1}{1-\frac{1}{2}}\right] = -2\,Gm$

Sol 7.

Here $\Delta V = 0.01\ \%$ of V

$= \frac{0\ 0.1}{100}$ x V = 10^{-14} V

Since P = hpg = 10^3 x 10^3 x 9.8

= 9.8 x 10^6 Nm^{-2}

and $K = \frac{pv}{\Delta v} = \frac{9.8\ x\ 10^6\ x\ V}{10^{-4} V}$ = 9.8 x 10^{10} Nm^{-2}

Sol 8.

Let $l_x = l_0\ (1 + \propto_2 \Delta T)$

$l_y = l_0\ (1 + \propto_y \Delta T)$

and $l_z = l_0\ (1 + \propto_2 \Delta T)$

Since $V = l_x l_y l_z$

$\Rightarrow V = l_0^3\ (1 + \propto_x \Delta T)\ (1 + \propto_y \Delta T)\ (1 + \propto_2 \Delta T)$

$V = V_0\ [1 + (\propto_x + \propto_y + \propto_z)\ \Delta T]$

Which shows that coefficient of volume expansion in the above expression is $\propto_x + \propto_y + \propto_z$

Sol 9.

Using $V' = V\left(1 + \frac{T}{273}\right)$

$= V + \frac{VT}{273}$

we get 273 V' = 273 V + VT

$\Rightarrow T = -\ 273 + \left(\frac{273}{V}\right) V'$

$T = \frac{273}{V} V' -\ 273$

comparing with equation of straight line y = mx + C, we can say that above equation is a straight line with temperature along y – axis and volume along x – axis.

Sol 10.

Time period on earth

$T_e = 2\pi \sqrt{\frac{l}{g_e}}$

$T_e = 2\pi \sqrt{\frac{l}{9.8}}$

Time period on moon

$T_m = 2\alpha \sqrt{\frac{l}{gm}} = 2\pi \sqrt{\frac{l}{1.5}}$

Dividing

$\frac{T_m}{T_e} = \sqrt{\frac{9.8}{1.5}}$

$\Rightarrow T_m = T_e\sqrt{\frac{9.8}{1.5}} = 2.56\ T_e$

Sol 11.

In case of damped vibration, there is an exponential decrease in amplitude with time

i.e. $A = A_0\ e^{-bt} \Rightarrow \frac{A}{A_0} = e^{-bt}$

or $\frac{1}{2} = e^{-b \times 2}$ and $\frac{A_1}{A_0} = e^{-b\ x\ 6}$

or $\frac{A_1}{A_0} = \left(e^{-\,2b}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

Sol 12.

Net electrostatics energy

$= \frac{kQq}{l} + \frac{kQq}{l} + \frac{kQq}{l} = 0$

or $2Q = -q^2 \Rightarrow Q = -\frac{q^2}{2}$

Sol 13.

Here $\frac{v^2}{R_1}t_1 = \frac{V^2}{R_2}t_2$

i.e. $\frac{v^2\ x\ 10}{R_1} = \frac{v^2\ x\ 15}{R_2} \Rightarrow R_2 = \frac{3}{2}R_1$

In parallel, resistance becomes $\frac{3R_1}{5}$ Then $\frac{V^2 t}{3R_{1/5}} = \frac{V^2 t_1}{R_1}$

i.e. $t = \frac{3t_1}{5} = \frac{3\ x\ 10}{5} = 6\ minutes$

Sol 14.

Here $I - I_g = \frac{l_g X\ G}{S} = \frac{36 l_g}{4} = 9I_g \Rightarrow I = 10I_g$

∴ Percentage of total current that passes through the galvanometer $= \frac{I_g}{I}$ x 100

$= \frac{1}{10}\ x\ 100 = 10\%$

Sol 15.

Using $r = \frac{mV}{qB} = \frac{\sqrt{2mK}}{qB}$

Here K is the kinetic energy

$\frac{r_p}{r_d} = \frac{q_d}{q_p}\sqrt{\frac{M_p}{M_d}} = \frac{a}{q}\sqrt{\frac{M_p}{2M_p}} = \frac{1}{\sqrt{2}}$

Sol 16.

Given $I = 50\%\ I_0$

or $I = \frac{50}{100} I_0 = I_0 = \frac{l_0}{2}$ But $I_0 = \frac{E}{R}\frac{200}{50} = 4A$

Using $I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$ we get $\frac{l_2}{2} = I_0\left(1 - e^{-\frac{50}{80}t}\right)$

or $e^{-\frac{5}{8}t} = \frac{1}{2} \Rightarrow e^{\frac{5}{8}t} = 2$

or $\frac{5}{8}t = \log_e 2 = 0.6932$

$\Rightarrow$ time, $t\ \frac{0.6932\ x\ 8}{5} = 1.11\ seconds$

Sol 17.

Maximum energy stored

$= \frac{1}{2} L I_0^2$

$= \frac{1}{2} x\ 80\ x\ 16 = 640\ J$

Sol 18.

Displacement current is givn by

$I_d = \epsilon_0 A \frac{dE}{dt}$

$= \epsilon_0 A \frac{1}{\epsilon_0 A} \frac{dq}{dt} = \frac{dq}{dt}$

Using $I_d = 2.5 \times 10^{-8}$ c / s = 2.5×10^{-8} A

⇒ I_d has the same magnitude as the conduction current but opposite direction. The total current, I = $I_c + I_d = 0$

From Ampere's law

$\Phi\ \vec{B}.d\vec{l} = ⍰_0 I = ⍰_0 (I_c + I_d) = 0$

∴ Magnetic field within plates is zero at all points

Sol 19.

Focal length of a spherical mirror = $\frac{1}{2} x$ Radius of curvature of mirror

Sol 20.

According to Cauchy's formula

$⍰ = A + \frac{B}{\lambda^2}$

Sol 21.

As $⍰ = \frac{1}{\sin C}$

Also $⍰ = \frac{v_2}{V_1} = \frac{1480}{340}$

∴ Sin C = $\frac{340}{1480}$ ⇒ C = $\sin^{-1}\left(\frac{340}{1480}\right) = 13.28°$

or C = 13.3°

Sol 22.

Using Moseley's Law

$v \propto (2 - b)^2$

For $K_\propto$ - x ray, b = 1

$\Rightarrow v \propto (2 - 1)^2$

For z = 31, $v \propto (31 - 1)^2 \propto (30)^2$

For z = 51, $v \propto (51 - 1)^2 \propto (50)^2$

$\therefore \frac{v_1}{v_2} = \left(\frac{50}{30}\right)^2 = \frac{25}{9}$

$\Rightarrow v_1 = \frac{28}{9} v_2$

Sol 23.

Here $\frac{1}{64} = \frac{1}{2^6} \Rightarrow 6\ Half\ lives$

i.e. 6 x 2 = 12 Hours

Sol 24.

The kinetic energy of election in Bohr's orbit,

$K.E. = \frac{1}{2}\frac{Ke^2}{r}$

and Potential Energy of election in Bohr's orbit,

P.E. : E = 1 : - 1

Sol 25.

The length of base will be least

$\therefore$Ans is (4)

Sol 26.

Line of Sight Principal is essential for MW communication because microwaves in a straight line.

Sol 27.

Radius of circular path is given by

$r = \frac{mv}{Bq} = \sqrt{\frac{3km}{Bq}}$

$\Rightarrow r \propto \frac{\sqrt{m}}{q}$ if k and b are same

$\Rightarrow r_p : r_d : r_\alpha = \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2}$

$= 1 : \sqrt{2} : 1$

Sol 28.

The force on satellite always acts towards earth, thereby employing that acceleration of satellite is always directed towards the centre of the earth. Since force F is conservative in nature, the mechanical energy of satellite remains constant. Speed of satellite is maximum when it is nearest to earth and minimum when it is farthest.

Sol.29

Power $= \vec{F}.\vec{v} = Fv$

And $F = V\left(\frac{dm}{dt}\right) = V\left[\frac{d}{dt}(\rho \times volume)\right]$

$= \rho V\left[\frac{d}{dt}(volume)\right] = \rho V\ AV = \rho AV^2$

$\therefore Power\ P = (\rho AV^2)V = \rho AV^3$ or $P \propto V^3$

Sol.30

Change in momentum of two particles

$= |(m_1 \overrightarrow{V_1} + m_2 \overrightarrow{V_2}) - (m_1\vec{V_1} + m_2\vec{V_2})|$

$= External\ Force\ on\ system)$? $\times\ time\ interval$

$= m_1 + m_2)\ g\ (2t_0) = 2(m_1 + m_2)gt_0$

MATHEMATICS

Sol.1

Given that $f(x) = \sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$

For domain of f(x)

$\log(\frac{5x-x^2}{4}) \geq 0 \Rightarrow \frac{5X-X^2}{4} \geq 1 \Rightarrow 5x - x^2 \geq 4 \Rightarrow x^2 - 5x + 4 \leq 0 \Rightarrow (x-1)(x-4) \leq 0 \Rightarrow x - 1 \geq 0$ and $x - 4 \leq 0 \Rightarrow \leq x \leq 4 \Rightarrow x \in [1, 4]$

Sol.2

$\mathrm{Sin}^2\theta = \frac{1-\cos 2\theta}{2} = \frac{1}{2} - \frac{1}{2}\cos 2\theta$

$\therefore$ period of $\sin^2\theta = \frac{2\pi}{2} = \pi$

Sol.3

Given that $3^{2x^2-7x+7} = 3^2$

$\therefore 2x^2 - 7x + 7 = 2, \Rightarrow 2x^2 - 7x + 5 = 0$

$D = b^2 - 4ac = (-7)^2 - 4 \times 2 \times 5 = 49 - 40 = 9 > 0$

Hence, it has two real roots.

Sol.4

$(1 + w - w^2)^7 = (-w^2 - w^2)^7 \qquad [\because 1 + w + w^2 = 0]$

$= (-2w^2)^7 = -2^7 w^{14} = -128(w^3)^4 w^2 = 128(1)w^2 = -128w^2 (\because w^2 = 1)$

Sol.5

Given $\begin{vmatrix} 1 & w + w^2 & w^2 - 1 \\ 1 - i & -1 & w^2 - 1 \\ -1 & -1 + w - i & -1 \end{vmatrix}$

$= \begin{vmatrix} 1 - i & -1 & w^2 - 1 \\ 1 - i & -1 & w^2 - 1 \\ -1 & -1 + w + i & -1 \end{vmatrix} [\because w + w^2 = -1] = 0$

Sol.6

Given that A $= \begin{bmatrix} a & b \\ b & a \end{bmatrix}, A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$

$A^2 = AA = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix}$

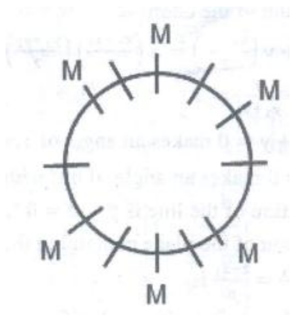
$A^2 = \begin{bmatrix} a^2+b^2 & ab+ba \\ ab+ba & b^2+a^2 \end{bmatrix}$ Therefore $\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{bmatrix}$

$\Rightarrow \alpha = a^2 + b^2, \beta = 2ab$

Sol.7

${}^nc_{r+1} + {}^nc_{r-1} + 2\,{}^nc_r = {}^nc_{r+1} + {}^nc_r + {}^nc_{r-1} + {}^nc_r = {}^{n+1}C_{r+1} + {}^{n+1}c_r = {}^{n+2}c_{r+1}$

Sol.8



First we fix the position of men, the number of ways to sit men = 5! and the number of ways to sit women = ${}^6P_5$

Total number of ways to dine at a round table = $5! \times {}^6P_5 = 5! \times 6!$

Sol.9

$S(k) = 1 + 3 + 5 + \dots\dots\dots + (2k-1) = 3 + k^2$ Put k = 1, we get

L.H.S. = 1 and R.H.S = 4

L.H.S. ≠ R.H.S Let S(k) is true

$\therefore 1 + 3 + 5 + \dots\dots\dots + (2k-1) = 3 + k^2 \Rightarrow 1 + 3 + 5 + \dots\dots\dots + (2k-1) + (2k+1) = 3 + k^2 + 2k + 1$

$\Rightarrow S(k+1) = 1 + 3 + 5 + \dots\dots\dots + (2k-1) + (2k+1) = 3 + (k+1)^2$

$\therefore S(k+1)$ is true .

Hence $S(k) \Rightarrow s(k+1)$

Sol.10

$(1 + x + x^2 + x^3 + \dots)^2 = [(1-x)^{-1}]^2 = (1-x)^{-2}$

Coefficient of x^n in $(1 + x + x^2 + x^3 + \dots)^2$ = Coefficient of x^nin $(1-x)^{-2} = {}^{n+2-1}C_{2-1} = {}^{n+1}C_1 = (n+1)$

Sol.11

$(r+1)^{th}$ term in the expansion of $(1+x)^{27/5} = \frac{\frac{27}{5}\left(\frac{27}{5}-1\right)\ldots\ldots\ldots\left(\frac{27}{5}-r+1\right)}{r!}x^r$

This term will be negative if

$\frac{27}{5} - r + 1 < 0 \Rightarrow \frac{32}{5} < r \Rightarrow 6.4 < r$

∴ Least value of r is 7

∴ First negative term will be 8^{th}

Sol.12

$e^{x-\frac{x^2}{2}+\frac{x}{3}-\frac{x^4}{4}+\cdots} = e^{\log(1+x)=1+x}$

Sol.13

Given 5^{th} term of G.P. = 2

Let the first term of G.P. is a and common ratio is r. Then $ar^4 = 2$. The product of 9 terms

$a \times ar \times ar^2 \times ar^3 \times ar^4 \times ar^5 \times ar^6 \times ar^7 \times ar^8 = a^9r^{36} = (ar^4)^9 = 2^9 = 512$

Sol.14

$Lt_{x\to 2}\frac{xf(2)-2f(x)}{x-2} = Lt_{x\to 2}\frac{xf(2)-2f(2)+2f(2)-2f(x)}{x-2} = Lt_{x\to 2}\frac{(x-2)f(2)-2(f(x)-f(2)}{x-2} = f(2) - 2Lt_{x\to 2}\frac{f(x)-f(2)}{x-2}$

$= f(2) - 2f'(2) = 4 - 2\times 4 = -4$

Sol.15

$Lt_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x = Lt_{x\to\infty}\left[1-\frac{5}{x+2}\right]^x$

$$= Lt_{x\to\infty}\left[\left[1+\left(-\frac{5}{x+2}\right)\right]^{\frac{1}{\left(-\frac{5}{x+2}\right)}}\right]^{\left(\frac{5x}{x+2}\right)}$$

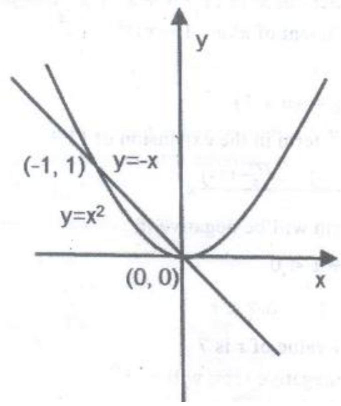
$= Lt_{x\to\infty} e^{\left(\frac{5X}{X+2}\right)}$

$= Lt_{x\to\infty} e^{\left(\frac{5}{1+\frac{2}{x}}\right)}$

$= e^{-5}$

Sol.16

The points of intersection of $y = x^2$ and $y = -x$ are



$x^2 = -x$

$x^2 + x = 0$

$x(x + 1) = 0$

$x = 0, x = -1$

$x = 0 \Rightarrow y = 0$

$x = -1 \Rightarrow y = 1$

Points of intersection are (0, 0), (−1, 1)

Area $= \int_{-1}^{0} \int_{x^2}^{-x} dydx$

$= \int_{-1}^{0} [y \quad]_{x^2}^{-x} dx = \int_{-1}^{0} (-x - x^2)\, dx$

$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^{0}$

$= (-0 - 0) - \left(-\frac{1}{2} + \frac{1}{3}\right) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

Sol.17

$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ (i)

$I = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx$

$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ (ii)

Adding (i) & (ii) we get

$21 = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

$21 = \int_0^{\pi/2} dx$

$21 = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

Sol.18

The general equation of all non-vertical lines in a plane is $ax + by = 1$, where $b \neq 0$

Differentiating , we get

$a + b\frac{dy}{dx} = 0$

again differentiating $b\frac{d^2y}{dx^2} = 0 \Rightarrow \frac{d^2y}{dx^2} = 0$

Sol.19

Given $\frac{d^2y}{dx^2} = e^{3x} + 4x$ integrating $\frac{dy}{dx} = \frac{e^{3x}}{3} + 2x^2 + c$

Again integrating with x, we get $y = \frac{e^{3x}}{9} + \frac{2x^3}{3} + cx + d$

Sol.20

Mid point of the chord is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

The equation of the chord is $x\left(\frac{y_1+y_2}{2}\right) + y\left(\frac{x_1+x_2}{2}\right) = 2\left(\frac{x_1+x_2}{2}\right)\left(\frac{y_1+y_2}{2}\right)$

$\Rightarrow \frac{x}{x_1+x_2} + \frac{y}{y_1+y_2} = 1$

Sol.21

Line $\sqrt{3}x + y = 0$ makes an angle of 120^0 and $\sqrt{3}x - y = 0$ makes an angle of 60^0 with x-axis.

$\therefore$ The equation of the line is $y - 2 = 0$

Sol.22

The equation of the plane containing the line $\frac{x-x_1}{1} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

It is given that the normal of the plane is perpendicular to the given line. Therefore

$al + bm + cn = 0$

Sol.23

The equation of plane and sphere are $12x + 4y + 3x = 327$ and $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ respectively.

Centre of sphere is $(-2, 1, 3)$ and

Radius of sphere $= \sqrt{4 + 1 + 9 + 155} = 13$

Length of $\perp$ from centre to the plane $= \left|\frac{(-2)(12)+(1)(4)+(3)(3)-327}{\sqrt{(12)^2+(4)^2+(3)^2}}\right| = \frac{338}{13} = 26$

$\therefore$ Shortest distance between the plane and sphere $= 26 -$ radius of sphere $= 26 - 13 = 13$

Sol.24

Given that the vector $\hat{i} + x\hat{j} + 3\hat{k}$ is doubled in magnitude and it become $4\hat{i} + (4x - 2)\hat{j} + 2\hat{k}$

$\therefore 2\left|\hat{i} + x\hat{j} + 3\hat{k}\right| = \left|\hat{4i} + (4x - 2)\hat{j} + 2\hat{k}\right|$

$\Rightarrow 2\sqrt{1 + x^2 + 9} = \sqrt{16 + (4x - 2)^2 + 4}$

$\Rightarrow 4(x^2 + 10) = (4x - 2)^2 + 20$

$\Rightarrow 3x^2 - 4x - 4 = 0$

$\Rightarrow (x - 2)(3x + 2) = 0$

$\Rightarrow x = 2, -\frac{2}{3}$

Sol.25

The given vectors $\hat{\imath} - \hat{\jmath}$ and $\hat{\imath} + 2\hat{\jmath}$ lies in xy plane

$\therefore$ The vector coplanar with them is $\vec{b} = x\hat{\imath} + y\hat{\jmath}$

Since $\vec{b} \perp (\hat{\imath} + \hat{\jmath})$

$\Rightarrow (x\hat{\imath} + y\hat{\jmath}).(\hat{\imath} - \hat{\jmath}) = 0$

$\Rightarrow x - y = 0$

$\Rightarrow x = y$

$\therefore \vec{b} = x\hat{\imath} + y\hat{\jmath}$ and $|\vec{a}| = \sqrt{x^2 + x^2} = x\sqrt{2}$ Required unit vector $= \frac{\vec{a}}{|\vec{a}|} = \frac{x(\hat{\imath}+\hat{\jmath})}{x\sqrt{2}} = \frac{1}{\sqrt{2}}(\hat{\imath} + \hat{\jmath})$

Sol.26

$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= (1+abc) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= 1 + abc = 0 \qquad \left[\therefore \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \right]$$

$\therefore abc = -1$

Sol.27

Given that $\vec{u} = \hat{\imath} + \hat{\jmath}, \vec{v} = \hat{\imath} - \hat{\jmath},$

$\vec{w} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\vec{u}.\hat{n} = 0$ and $\vec{v}.\hat{n} = 0$

$\therefore \hat{n} = \frac{|\vec{w}.\vec{u}\times\vec{v}|}{\vec{u}\times\vec{v}}$

$$\vec{u} \times \vec{V} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = -2\hat{k}\backslash$$

$\vec{w}.\vec{u} \times \vec{V} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}).(-2\hat{k}) = -6$

$|\vec{w}.\hat{n}| = \frac{|-6|}{|-2\hat{k}|} = 3$

Sol.28

Given that A and B are two mutually exclusive events.

$\therefore A \cap B = \emptyset$

$\therefore$ either A is subset of $\overline{B}$or B is subset of $\overline{A}$

$\therefore P(A) \leq P(\overline{B})$ or $P(B) \leq P(\overline{A})$

Sol.29

The probability of solving the problem by A, B and C are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively.

Probability that the problem is not solved $= \mathrm{P(A)}(\overline{\mathrm{B}})\mathrm{P}(\overline{\mathrm{C}}) = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$

Hence, the probability that problem is solved $= 1 - \frac{1}{4} = \frac{3}{4}$

Sol.30

$$\tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left[\frac{\frac{1}{2}-\frac{1}{2}}{1+\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}\right]$$

$$= \tan^{-1}\left[\frac{\frac{1}{6}}{\frac{7}{6}}\right] = \tan^{-1}\left(\frac{1}{7}\right)$$