ANSWERS KEY

CHEMISTRY

1.c	2.d	3.d	4.b	5.c	6.b	7.d	8.c	9.d	10.d
11. c	12.b	13.b	14.b	15.c	16.a	17.a	18.b	19.c	20.b
21.b	22.c	23.c	24.a	25.a	26.c	27.c	28.b	29.d	30.b

PHYSICS

1.b	2.c	3.b	4.a	5.b	6.a	7.c	8.d	9.b	10.d
11.d	12.b	13.d	14.b	15.b	16.c	17.c	18.a	19.a	20.a
21.b	22.a	23.c	24.a	25.a	26.b	27.b	28.a	29.c	30.b

MATHEMATICS

1.c	2.c	3.c	4.a	5.d	6.b	7.c	8.a	9.b	10.d
11.a	12.a	13.c	14.b	15.b	16.a	17.a	18.b	19.a	20.a
21.a	22.b	23.d	24.c	25.b	26.a	27.a	28.b	29.c	30.c

HINTS AND EXPLANATIONS

CHEMISTRY

Sol. 1

In a face cantered cubic arrangement each ion is surrounded by six oppositely charged ions. i.e., coordination number is six. Radius ratio for coordination number 6 is 0.414-0.732.

When$\frac{r+}{r-} = 0.414$; $\frac{r+}{241.5}$=0.414

r_+ = 0.414×241.5=99.98=100

when$\frac{r+}{r-} = 0.732$; $\frac{r+}{241.5}$=0.732

r_+ = 0.732×241.5 =176.77

Sol.2

$log\frac{k_2}{k_1} = \frac{E_a}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$, since energy of activation is zero, rate constant should be the same.

Sol.3

CH_4 (g) + 4CuO(s) →CO_2 (g) + 4Cu(s) + $2H_2O$ (I)

-7.49 -4×157.3 -393.5 -2×285.9

-74.9 -629.2 =-704.1 -393.5 -571.8 =965.3

Enthalpy change of the reaction

= -965.3- (-704.1) =-261.2

Sol.4

The addition of chlorine to the reaction mixture shifts the equilibrium backwards, thereby releasing more heat.

Sol. 5

Mole fraction of naphthalene =0.1

Mole fraction of benzene = 0.9; amount of benzene = 0.9 × 78 =70.2

Molality of the solution = 0.1 $x\frac{1000}{70.2} = 1.42m$

Sol. 6

Polarisability of halide ions increases with the increase in size of the anion.

Sol.7

The gold number of a protective colloid is its minimum amount in milligrams which is just sufficient to prevent the coagulation of 10 ml of a gold sol on the addition of 1 mL of 10% sodium chloride solution. Amount in milligrams of 0.025g of starch is 25, so gold number is 25.

Sol.8

The total energy of a Bohr's orbit is proportional to $1/n^2$.

Sol. 9

$K_p = K_c(RT)^{\Delta n}$

Δn = 2-0=2, there, $K_p = k_c\ (RT)^2$

Sol.10

EMF of cell = 1.229V

Sol. 11

$0.00046100 = 4.6100 \times 10^{-4}$

; number of significant figures =5

Sol. 12

For one nitrogen in the compound required % age of N = 14 10.5 g nitrogen is present in 100 g of compound 14 g nitrogen will be present in $\frac{100}{10.5} \times 14 = 133.3$

The minimum number molecular weight of the compound would be 133.3

Sol.13

$\Delta T_b = 354.11 - 353.23 = 0.88$

$\Delta T_b = K_b . \frac{w_2}{w_1} x \frac{1000}{w_1}$

$0.88 = 2.53. \frac{1.8}{0.88} x \frac{1000}{90} = 57.5$

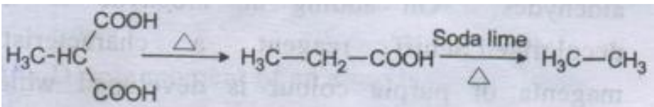
Sol.14

Option 4 is correct.

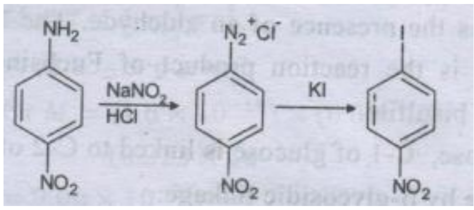
Sol.15

The formula for compound A suggests that it is a dicarboxylic acid. A ß-= keto acid or ß- carboxyl acid readily undergoes decarboxylation on heating.

Therefore, A is $CH_3CH(COOH)_2$



Sol.16



Sol.17

$3Fe^{2+} + 4H^{+} + NO_3 \rightarrow 3Fe^{3+} + NO + 2H_2O$

$[Fe\ (H2O)_6\ ]^{2+} + NO \rightarrow [Fe(H2O)5(NO)]^{2+}H_2O$

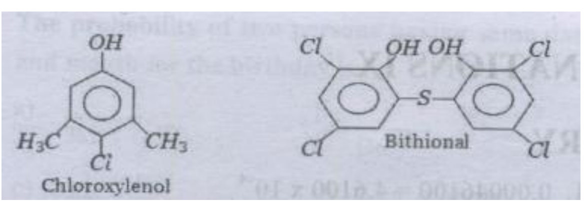
Sol. 18

It is a Crossed Cannizzarro reaction.

$C_6H_5CHO + HCHO + NaOH \rightarrow C_6H_5CH_2OH + HCOONa$

Sol. 19

Dettol is an antiseptic. It is a mixture of chloroxylenol and terpeneol in a suitable solvent. Chloroxylenol has both antiseptic and disinfectant properties. **Bithional** is antiseptic which is generally added to medicated soaps to reduce the odour produced by bacterial decomposition of organic matter on the skin. **Iodine** is powerful antiseptic. It is used as a tincture of iodine which is 2-3% iodine solution of alcohol-water.

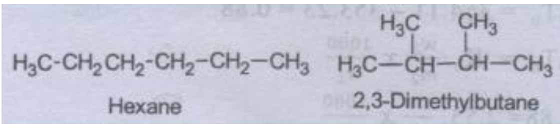


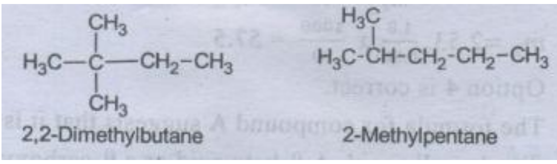
Sol. 20

HgS is not soluble in nitric acid

Sol.21

Hexane which contains only 2 different types of hydrogens will give two monochlorinated compounds. Out of the given choices only 2, 3- Dimethylbutane contains two types of hydrogens; Primary and secondary hydrogens.



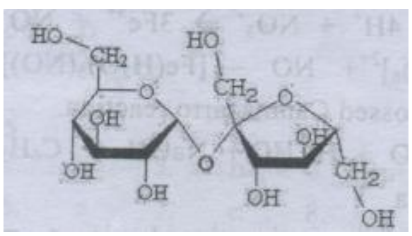


Sol.22

Schiff's reagent give pink colour with aldehydes only. It is a chemical test for the detection of aldehydes. On adding an aldehyde to the decolorized Schiff reagent a characteristic magenta or purple colour is developed which indicates the presence of an aldehyde. The Schiff reagent is the reaction product of Fuchsine and sodium bisulfite.

Sol.23

In sucrose, C-1 of glucose is linked to C-2 of fructose by ß-glycosidic linkage.



Sol.24

Peptization is a process of passing of a precipitate into colloidal particles on adding suitable electrolyte. The electrolyte added is known as peptizing agent. A reddish brown coloured colloidal solution is obtained by adding small quantity of ferric chloride solution to the freshly precipitated ferric hydroxide.

Sol.25

1.0 g of H_2 and 1.0 g of $O_{2,}$ contain different number of molecules at STP because their molecular weights are different.

Sol.26

Pb salts give gives a shining metallic bead in charcoal cavity test that marks paper.

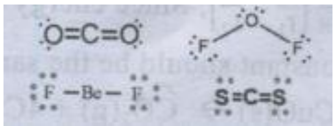
Sol. 27

The roasting of mercury sulphide in the air in metallurgical process produces mercury.

$HgS + O_2 \rightarrow Hg + SO_2$

Sol. 28

OF_2 is a bent, non-linear molecule.

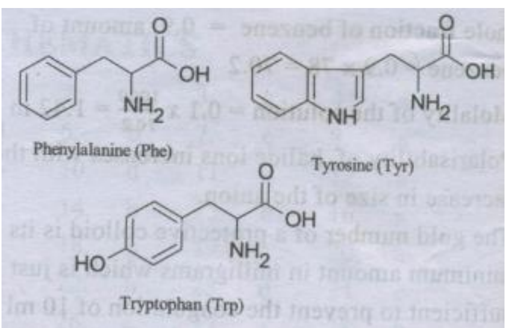


Sol. 29

A Ziegler- natta catalyst is a catalyst used in the synthesis of polymers of 1-alkenes (α-olefins). It uses organoaluminum compounds such as triethlaluminium $Al(C_2H_5)_3$.

Sol.30

When heated with conc. HNO_3 acid, proteins containing an aromatic ring give yellow ppt. On adding NaOH, the ppt turns orange. This is known as Xanthoprotein test.



PHYSICS

Sol.1

As magnetic moment = current × area ⇒ [Magnetic Moment] = $[AL^2]$

Sol. 2

If v is the velocity of projection then kinetic energy= $\frac{1}{2}mv^2$ velocity of body at the highest point, $v_x = vsin\Theta$

∴K.E. of the body at the highest point is $\frac{1}{2}m(vsin\,\Theta)^2 = \frac{1}{2}mv^2\sin^2\Theta$ = (K.E.)$\sin^2\Theta$ For $\Theta=45^0$

K.E. = (K.E.)$\sin^2 45^0 = \frac{K.E.}{2}$

Sol.3

As $\frac{mv^2}{r} = \mu R = \mu mg$

$\Rightarrow v^2 = \mu rg = 0.6 \times 150 \times 10 = 900$

$\Rightarrow v = \frac{30m}{s}$

Sol.4

The momentum of third fragment is given by

$P_3 = \sqrt{p_1^2 + P_2^2}$

$or p_3 = \sqrt{(mv)^2 + (mv)^2} = \sqrt{2}mv$

Final kinetic energy of system

$= \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{p_3^2}{2(2m)}$

$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{3}{2}mv^2$

Since initial kinetic energy is zero, therefore energy released = $\frac{3}{2}mv^2$

Sol.5

Given $X_{cm} = \frac{1}{M}\int_0^1 x\, dm = \frac{1}{M}\int_0^1 x\, (\mu dx)$

$= \frac{1}{M}\int_0^1 x\, (ax)dx = \frac{a}{M}\int_0^1 x^2\, dx = \frac{al^3}{3M}$

Also $M = \int_0^1 \mu\, dx \int_0^1 ax\, dx = \frac{al^2}{2}$

$\Rightarrow X_{cm} = \frac{al^3 \times 2}{3(al^2)} = \frac{2}{3}l$

Sol.6

Both the statements are self explanatory.

Sol.7

As stress = $\frac{F}{A}$ ∴ Ultimate shear stress < F/A

⇒ F > shear stress × Area as Area = $2\pi\frac{d}{2}yt$ Or F= $\pi d\sigma t$

Sol. 8

Here $\frac{\Delta Q}{\Delta t} \in \sigma A T^4$, also $\Delta Q = m\, C\Delta T$

Dividing

$\therefore \frac{\Delta T}{\Delta t} = \frac{\in \sigma A T^4}{mC}$ Now Area, A = πr^2 =$\pi\left(\frac{3}{4}\frac{m}{\pi\delta}\right)^{2/3}$

$\because m = \frac{4}{3}\,\pi r^3 \delta$

$$\frac{\Delta T}{\Delta t} = \frac{\in \sigma T^4}{mC}\left[\pi\left(\frac{3m}{4\pi\delta}\right)^{\frac{2}{3}}\right]$$

= $k\left(\frac{1}{3}\right)^{1/3}$ where k is a constant

$\Rightarrow \frac{\frac{\Delta T_1}{\Delta t_1}}{\frac{\Delta T_2}{\Delta t_2}} = \left(\frac{m_2}{m_1}\right)^{\frac{1}{3}} = \left(\frac{1}{3}\right)^{\frac{1}{3}}$

Sol.9

Using $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$V_{rms}\,(at\ 120^0) = V = \sqrt{\frac{3R}{M} \times 120}$$

$$V_{rms}\,(at\ 480^0) = \sqrt{\frac{3R}{M}480}$$

$$V_{rms} = \sqrt{\frac{480}{120}} = 2$$

$\Rightarrow V_{rms} = 2V$

Sol.10

As $f' = \frac{300+V}{300} f$

And $f'' = \frac{300-V}{300} f \Rightarrow f' - f'' = \frac{2}{100} f$

i.e. $\frac{300+V}{300} - \frac{300-V}{300} = \frac{2}{100}$ or 2V = 6 ⇒V=3m/s

Sol.11

As $I = \frac{1}{2}\rho A^2\omega^2 c^2$

$I = \frac{1}{2} \times (1.3) \times (10^{-8})^2 (2\pi \times 2000)^2 (340)$

$=3.5\times10^{-6}$w/m^2

Sol.12

$\vec{E}$ at all point on the y-axis is along $\hat{i}$.$\vec{E}$ at all points on x-axis cannot have the same direction.

Sol.13

Resistance of wire becomes x^2 times and not x times.

Sol.14

Dipole moment of an atom is

$M = I \times A = \frac{e}{T}\pi r^2$

$\Rightarrow M = e \times v \times \pi r^2$

Or $M=(1.6 \times 10^{-19}) \times (6.6 \times 10^{15}) \times 3.142 \times (0.523 \times 10^{-10})^2$

$=9.06 \times 10^{-24}$ Am^2

Sol.15

Using τ=n/AB sinΘ, here Θ is angle between magnetic induction and normal to the surface to loop.

$A = \frac{1}{2} \times base \times height$

$= \frac{1}{2} \times 0.02 \times 0.02 \sin 60 = 1.732 \times 10^{-4}$

$\Rightarrow \tau = 1 \times 0.1 \times 1.732 \times 10^{-4} \times 5 \times 10^{-2} \sin 90$

$=8.66 \times 10^{-7}$ Nm

Sol.16

As per Lenz's law induced emf opposes the cause producing it.

Sol.17

Using E α N

$\frac{E_s}{E_p} = \frac{N_s}{N_p} \Rightarrow E_s = \frac{N_s}{N_p} E_p$

$\Rightarrow E_s = \frac{5}{1} \times 200 = 1000V$

Sol.18

As $B = \frac{E}{c} = \frac{9.3}{3\times10^8} = 3.1 \times 10^{-8} T$

Sol.19

As normal to mirror 2 is parallel to mirror 1,the reflected ray is parallel to the ray incident on mirror 1 and is in opposite direction.

Sol.20

Using $\boldsymbol{\beta}=\frac{\lambda D}{d}$ and $\theta = \frac{\beta}{D}$

i.e. $\boldsymbol{\beta} = D\theta \Rightarrow D\theta = \frac{\lambda D}{d}\ or \theta = \frac{\lambda}{d}$

with lens $\theta = \frac{\beta'}{f}$ or $\frac{\beta'}{f} = \frac{\lambda}{d}$

$\Rightarrow \boldsymbol{\beta}' = \frac{\lambda f}{d} = \frac{5890\times10^{-10}\times1}{0.2\times10^{-3}} = 3mm$

Sol.21

For coherence to be observed, same frequency is the essential requirement. Different wavelengths give different frequencies. The lights of different intensities can give coherence, however the contrast of fringes may be poor.

Sol. 22

Both the statements are self explanatory.

Sol.23

Given that Energy in a day = (1×10^6) $(24 \times 60 \times 60)$J

Energy per fission = 3.2×10^{-11}J Mass of uranium required in a day

$= \frac{1\times10^6(24\times60\times60)(235\times1.67\times10^{-27})}{3.2\times10^{-11}}$

$=1.05 \times 10^{-3}$kg = 1.05g

Sol.24

$_{28}R^{222} \xrightarrow{a} {}_{86}R^{218} \xrightarrow{\vec{\beta}} {}_{86}Fr^{218} \xrightarrow{a} {}_{85}At^{214} \xrightarrow{\vec{\beta}} {}_{86}Rn^{214}$

There are 4α decays and 2 **β** decays

Sol.25

The width of depletion region increases with increase in reverse voltage in a diode.

Sol. 26

Using $\mu=\frac{Am}{A_c} \Rightarrow A_m = \mu \times A_c = \frac{75}{100} \times 12=9V$

Sol. 27

If (b−a) ≥ r, radius of the circular path of particle. The particle cannot enter the region $x > b$

To enter in the region $x > b$

$r> (b-a) \Rightarrow \frac{mV}{Bq} > (b - a) \Rightarrow V > \frac{q(b-a)}{m} B$

Sol. 28

As $y = kt^2, \frac{d^2y}{dt^2} = 2k\ as\ k = 1\frac{m}{s^2}$

Or y = 2 m/s^2

$T_1 = 2\pi\sqrt{\frac{l}{g}}$ and $T_2 = 2\pi\sqrt{\frac{1}{g+ay}} \Rightarrow \frac{T_1^2}{T_2^2} = \frac{g+ay}{g} = \frac{10+2}{10} = \frac{6}{5}$

Sol.29

$W_{apparent} = W_{actual} - U_{pthrust}$

$U_{pthrust,\ F = Vs\delta L'g} \Rightarrow \frac{F'}{F} = \frac{V_s^1}{V_s}\frac{\delta_L'}{\delta_L} = \frac{(1+\gamma s\Delta\theta)}{(1+\gamma L\Delta\theta)}$

$As \gamma < \gamma_L \Rightarrow F' < F$ or $W_{apparent} > W_{actual}$

Sol. 30

Using Wien's Displacement Law $\lambda_m T$ = constant

$$T = \frac{1}{\lambda_m}$$

$\therefore \frac{T_{sun}}{T_{nort\ h\ star}} = \frac{(\lambda_m)_{nort\ h star}}{(\lambda_m)_{sun}} = \frac{350}{510} = 069$

MATHEMATICS

Sol. 1

$\propto +\beta + \gamma = \frac{\pi}{2} \Rightarrow \propto +\gamma = \frac{\pi}{2} - \beta$

$\cot(\propto +\gamma) = \cot\left(\frac{\pi}{2} - \beta\right)$

$\frac{\cot\alpha \cot\gamma - 1}{\cot\alpha + \cot\gamma} = \tan\beta$

$\Rightarrow \cot\alpha \cot\gamma - 1 = \tan\beta \; (\cot\alpha + \cot\gamma)$

Given that cotα, cotβ, cotγ are in A.P. therefore, $\cot\alpha \cot\gamma - 1 = 2 \tan\beta \cot\beta \Rightarrow \cot\alpha \cot\gamma = 3$

Sol.2

Given that $\cos 2x + a\sin x = 2a - 7$ posses a solution

$1 - 2\sin^2 x - a\sin x = 2a - 7$

$\Rightarrow 2\sin^2 x - a\sin x + (2a - 8) = 0$

$\Rightarrow \sin x = \frac{a \pm \sqrt{a^2 - 8(2a-8)}}{4}$

$\Rightarrow \sin x = \frac{a-2}{2}, 2$ [$\because -1 \leq \sin x \leq 1$]

$\Rightarrow -1 \leq \frac{a-2}{2} \leq 1$ $\Rightarrow 0 \leq a \leq 4$

Number of integral values of a are 5

Sol.3

Given $\sin^{-1} x + \tan^{-1} = \frac{\pi}{2}$

$\Rightarrow \tan^{-1} x = \cos^{-1} x = \tan^{-1} \frac{\sqrt{1-x^2}}{x}$

$\Rightarrow x = \frac{\sqrt{1-x^2}}{x}$

$\Rightarrow x^2 = \sqrt{1 - x^2} \Rightarrow x^4 = (1 - x^2)$

$\Rightarrow x^4 + x^2 - 1 = 0$ $\Rightarrow x^2 = -\frac{1 \pm \sqrt{5}}{2}$

$x^2 = \frac{\sqrt{5} - 1}{2}$ [because $x^2 > 0$]

$\therefore 2x^2 + 1 = \sqrt{5}$

Sol.4

The point P is on the line parallel to $x - y = 3$ and passing through A(2, 1).

$\frac{x-2}{1/\sqrt{2}} = \frac{y-1}{1/\sqrt{2}} \pm 4$ (because distance AP = 4)

∴P is either (2√2 +2, 2√2+1) or (−2√2 + 2, −2√2 +1)

∵ P lies in third quadrant

∴ P is (−2√2 + 2, −2√2+1)

Sol.5

Centre of circle is (0, 0) and radius = 3. If line $x + y \tan\theta = \cos\theta$ touches the circle, then perpendicular distance from (0, 0) to the line in equal to radius of circle $\frac{|0+0-\cos\theta|}{\sqrt{1^2+\tan^2\theta}} = a$

∴$\cos^2\theta = a$ which is impossible

Sol .6

Let PQ be a focal chord and $P = (at^2, 2at)Q = \left(\frac{a}{t^2}, -\frac{2a}{t}\right)$

∴$PQ = a\left(t + \frac{1}{t}\right)^2$ and $\tan\alpha = \frac{2}{t-\frac{1}{t}}$

∴$\cot\alpha = \frac{1}{4}\left(t - \frac{1}{t}\right)$

⇒$\cot^2\alpha + 1 = \frac{1}{4}\left(t + \frac{1}{t}\right)^2$

⇒$\frac{PQ}{4a} = \frac{a\left(t+\frac{1}{t}\right)^2}{4a}$

$= \frac{1}{4}\left(t + \frac{1}{t}\right)^2 = cosec^2\alpha$

Sol.7

Given $f(x) = e^x$

∵x_1, x_2, x_3 from G.P. Therefore

$x_1 + x_3 = 2x_2 \Rightarrow e^{x_1+x_3} = e^{2x_2}$

⇒$e^{x_1}.e^{x_3} = (e^{x_2})^2$ ⇒$f(x_1), f(x_3) = \left(f(x_2)\right)^2$

Therefore $f(x_1), f(x_2), (fx_3)$ are in G.P.

Sol.8

$$\text{Lt}_{x\to 0}\frac{(cos\ x)^{\frac{1}{2}}-(cos\ x)^{\frac{1}{3}}}{sin^2x}=\underset{x\to 0}{\text{Lt}}\frac{\left(\frac{1}{2}\right)(\cos\ x)^{-\frac{1}{2}}(-\sin\ x)-\left(\frac{1}{3}\right)(\cos\ x)^{-\frac{2}{3}}(-\sin\ x)}{2\sin\ x\cos\ x}$$

$$=\underset{x\to 0}{\text{Lt}}\frac{-\frac{1}{2}(\cos\ x)^{-\frac{1}{2}}+\frac{1}{3}(\cos\ x)^{-\frac{2}{3}}}{2\cos\ x}=\frac{1}{2}\left(\frac{-1}{2}+\frac{1}{3}\right)=-\frac{1}{12}$$

Sol.9

Given that $f(x)=|\cos 2x|$

$$f'\left(\frac{\pi}{4}\right)=\underset{h\to 0}{\text{Lt}}\frac{f\left(\frac{\pi}{4}+h\right)-f\left(\frac{\pi}{4}\right)}{h}$$

$$=\underset{h\to 0}{\text{Lt}}\frac{\left|cos\,2\left(\frac{\pi}{4}+h\right)\right|-\left|cos\,2\left(\frac{\pi}{4}\right)\right|}{h}=\underset{h\to 0}{\text{Lt}}\frac{\left|\cos\left(\frac{\pi}{4}+2h\right)\right|-\left|\cos\frac{\pi}{2}\right|}{h}$$

$$=\underset{h\to 0}{\text{Lt}}\frac{\sin\ 2h}{h}=2$$

Sol.10

Given that $y=\tan^{-1}\sqrt{1-x^2}$

At $x=4,\sqrt{1-x^2}$ becomes imaginary

∴y does not exist.

Sol.11

Given $y^2=x(2-x)^2$

Differentiating both sides w. r. t. x, we get

$2y\frac{dy}{dx}=3x^2-8x+4\Rightarrow\left(\frac{dy}{dx}\right)_{(1,1)}=-\frac{1}{2}$ Equation to tangent at (1,1) is

$y-1=\left(-\frac{1}{2}\right)(x-1)\Rightarrow y=-\frac{x}{2}+\frac{3}{2}$

$\Rightarrow y^2=\frac{1}{4}(3-x)^2$

$\therefore x(2-x)^2=\frac{1}{4}(3-x)^2\Rightarrow(x-1)^2(4x-9)=0$

$\Rightarrow x=\frac{9}{4}$

$x=\frac{9}{4}\Rightarrow y=\frac{3}{8}$

Sol.12

$I = \int \frac{\sin x}{\sin 4x} dx = \int \frac{\sin x}{2 \sin 2x cos\ 2x}\ dx$

$= \int \frac{\sin\ x}{4 \sin\ x \cos\ x \cos\ 2x} dx$

$= \frac{1}{4} \int \frac{\cos\ x}{(1-\sin^2 x)(1-2\sin^2 x)} dx$ Put $\sin x = t \Rightarrow$ $\cos x dx = dt$

$= \frac{1}{4} \int \frac{dt}{(1-t^2)(1-2t^2)} = \frac{1}{4} \int \left(\frac{2}{1-2t^2} - \frac{1}{1-t^2}\right) dt$

$= \frac{1}{4} \left\{\frac{2}{2\sqrt{2}} \log \left|\frac{1+\sqrt{2}t}{1-\sqrt{2}t}\right| - \frac{1}{2} \log \left|\frac{1+t}{1-t}\right|\right\} + c$

$= \frac{1}{4\sqrt{2}} \log \left|\frac{1+\sqrt{2}\sin\ x}{1-\sqrt{2}\sin\ x}\right| - \frac{1}{8} \log \left|\frac{1+\sin\ x}{1-\sin\ x}\right| + c$

$= \frac{1}{8} \log \left|\frac{\sin\ x-1}{\sin\ x+1}\right| - \frac{1}{4\sqrt{2}} \log \left|\frac{\sqrt{2}\sin\ x-1}{\sqrt{2}\sin\ x+1}\right| + c$

Sol.13

Let $I_1 = \int_0^1 \frac{e^x}{x+1} dx = a\ and\ I_2 = \int_{b-1}^{b} \frac{e^{-x}}{x-b-1} dx$

Put $x = b - y \Rightarrow$ dx=-dy

$I_2 = -\int_1^0 \frac{e^{-b+y}}{-y-1} dy$

$= -e^{-b} \int \frac{e^y dy}{y+1} = -e^{-b}$a

Sol.14

Let $\lim_{n\to\infty} \frac{(n!)^{\frac{1}{n}}}{n} = a \Rightarrow a = \lim_{n\to\infty} \left(\frac{n!}{n^n}\right)^{\frac{1}{n}} \Rightarrow$a=$\text{Lt}_{n\to\infty} \left(\frac{1}{n}.\frac{2}{n}.\frac{3}{n}.\frac{4}{n}\ldots.\frac{n}{n}\right)^{\frac{1}{n}}$ Taking log on both sides, we get

$\log a = \text{Lt}_{n\to\infty} \frac{1}{n} \left[\log\frac{1}{n} + \log\frac{2}{n} + \frac{3}{n} + \cdots + \log\frac{n}{n}\right]$

$= \text{Lt}_{n\to\infty} \sum_{r=1}^{n} \log\left(\frac{r}{n}\right) \frac{1}{n}$

$= \int_0^1 \log x\ dx = [x \log x - x]_0^1$

$= -1 \therefore a = e^{-1}$

Sol.15

Given that $\vec{a}, \vec{b}, \vec{c}$ are linear dependent vectors.

$\therefore |\vec{a}\vec{b}\vec{c}|=0$

$$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{bmatrix} = 0$$

Using $c_2 \rightarrow c_1$, $c_3 \rightarrow c_3 - c_1$ transformation

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 1 & \alpha - 1 & \beta - 1 \end{bmatrix} = 0$$

$\Rightarrow -(\beta - 1) = 0$ $\quad\quad$ $\Rightarrow \beta = 1$

Given $|c| = \sqrt{6}$

$\therefore \sqrt{1 + \alpha^2 + \beta^2} = \sqrt{6}$

$\Rightarrow 1+\alpha^2 + \beta^2 = 6$

$\Rightarrow 1+\alpha^2 + 1 = 6$

$\Rightarrow \alpha^2 = 4$

$\Rightarrow \alpha = \pm 2$

Sol.16

Given that $(a + ib)(c + id) = (a^2 + b^2)\,\hat{\imath}$

$ac + iad + ibc - bd = (a^2 + b^2)\hat{\imath}$

$(ac - bd) + i(ad + bc) = (a^2 + b^2)\hat{\imath}$

$\Rightarrow ac - bd = 0$ & $ad + bc = a^2 + b^2$

From $ad + bc = a^2 + b^2$, it is obvious that $a = d$ and $b = c$

Sol. 17

Given that

$f(x, y, z) = x^2 + 4y^2 + 3z^2 - 2x^2 - 12y - 6z + 15 = (x - 1)^2 + (2y - 3)^2 + 3(z - 1)^2 + 2$

$\therefore$ Least value of $f(x, y, z)$ is obtained at $x=1, y=3/2, z=1$

$\therefore$ Least value of $f(x, y, z) = 2$

Sol. 18

1 + (1 + 3) + (1+ 3 + 5)……. To n terms

= 1 + 4 + 9 +……. To n terms

= $1^2 + 2^2 + 3^2 + \ldots$ to n terms

$= \frac{n(n+1)(2n+1)}{6}$

Sol. 19

Let a, ar, ar^2 be in G. P.

Let $T_1 = a + ar$, $T_2 = ar + ar$, $T_3 = ar^2 + ar$

$$\frac{2T_1T_3}{T_1 + T_3} = \frac{2(a + ar)(ar^2 + ar)}{(a + ar) + (ar^2 + ar)}$$

$= \frac{2a^2(1+r)ar(1+r)}{a(1+r+r+r^2)}$

$= \frac{2a^2r(1+r)^2}{a(1+r)^2} = 2ar = T_2$

$\therefore T_1, T_2, T_3$ are in H.P.

Sol.20

$180 = 2^2. \; 3^2. \; 5^1$

Sum of division of 180

$= (1 + +2 + 2^2)(1 + 3 + 3^2)(1 + 5)$

= 7 × 13 × 6 = 546

Sol. 21

The required number of ways are

$$3!\left\{1 - \frac{1}{1!} + \frac{2}{2!} + \frac{3}{3!}\right\}$$

$= 3!\left\{\frac{2}{2!} - \frac{3}{3!}\right\} = 3 - 1 = 2$

Sol.22

The required number of four digits number are

$= 3 \times 4 \times 4 \times 4 = 192$

Sol.23

$$\left(x^3 + 1 + \frac{1}{x^3}\right) = \left[1 + \left(x^3 + \frac{1}{x^3}\right)\right]^n$$

$$= {}^nC_0 + {}^nC_1\left(x^3 + \frac{1}{x^3}\right) + {}^nC_2\left(x^3 + \frac{1}{x^3}\right)^2 \ldots\ldots + {}^nC_n\left(x^3 + \frac{1}{x^3}\right)^n$$

Here, all the terms all positive and will contain powers

$$(x^3)^0, (x^3)^1, (x^3)^2 \ldots\ldots, (x^3)^n, (x^3)^{-1}, (x^3)^{-2}, \ldots\ldots\ldots\ldots, (x^3)^{-n}$$

Hence number of terms = 2n +1

Sol.24

$$\Delta = \begin{vmatrix} al & bm & cn \\ cm & an & bl \\ bn & cl & am \end{vmatrix}$$

$= lmn(a^3 + b^3 + c^3) - abc(l^3 + m^3 + n^3)$

Given that $a + b + c = 0 = 1 + m + n$

$\therefore a^3 + b^3 + c^3 = 3abc$

Therefore Δ = lmn (3abc) − abc (3lmn)=0

Sol. 25

The required no. of ways = $2^{n-1} - 1$

Sol.26

Total number of cases = $(365)^2$ Favourable number of cases = ${}^{361}C_1 = 365$

Required probability = $\frac{365}{(365)^2} = \frac{1}{365}$

Sol.27

Required probability = $\frac{2!6!6!}{12!} = \frac{1}{462}$

Sol. 28

Given $A \cup \{2, 4, 6\} = \{2,4,6,8,10\} \therefore$ smallest $A = \{8,10\}$

Largest $A = \{2,4,6,8,10\}$

n (A) = m $\Rightarrow 2 \le m \le 5$

Sol.29

a), (b), (d) are correct (c) is incorrect

Sol.30

$$S = \frac{1}{2}\left(\frac{1}{6}\right)^2 + \left(\frac{2}{3}\right)\left(\frac{1}{6}\right)^3 + \left(\frac{3}{4}\right)\left(\frac{1}{6}\right)^4 + \cdots$$

$$=\left(1 - \frac{1}{2}\right)\left(\frac{1}{6}\right)^2 + \left(1 - \frac{1}{3}\right)\left(\frac{1}{6}\right)^3 + \left(1 - \frac{1}{4}\right)\left(\frac{1}{6}\right)^4 + \cdots$$

$$\Rightarrow S = \left[\left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^3 + \left(\frac{1}{6}\right)^4 + \cdots\right]$$

$$- \left[\frac{1}{2}\right]\left(\frac{1}{6}\right)^2 + \frac{1}{3}\left(\frac{1}{6}\right)^3 + \frac{1}{4}\left(\frac{1}{6}\right)^4 + \cdots$$

$$\Rightarrow S = \left[\left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^3 + \left(\frac{1}{6}\right)^4\right]$$

$$-\left[\left\{\frac{1}{6} + \frac{1}{2}\left(\frac{1}{6}\right)^2 + \frac{1}{3}\left(\frac{1}{6}\right)^3 + \cdots\right\} - \frac{1}{6}\right]$$

$$\Rightarrow S = \frac{\left(\frac{1}{6}\right)^2}{1-\frac{1}{6}} + \log\left[1 - \frac{1}{6}\right] + \frac{1}{6}$$

$$\Rightarrow S = \frac{1}{30} + \frac{1}{6} + \log\frac{5}{6}$$

$\Rightarrow S = \frac{6}{30} + \log\frac{5}{6}$.mxx