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Subject: CHEMISTRY, MATHEMATICS & PHYSICS

Paper Code: JEE_Main_Sample Paper –I_Answer File

Part – A – Chemistry

1)

Ans: d

Exp: Sudden jump from $IE_2 \rightarrow IE_3$ is given by elements which attain stable configuration of noble gas after losing 2 electrons.

2)

Ans: c

Exp: NA

3)

Ans: c

Exp: It is nitrogen. It forms NCl_3, N_2O_5 and Mg_3N_2 but NCl_5 does not exist due to absence of vacant d-orbital. Nitrogen exhibits a covalency of three only.

4)

Ans: b

Exp: Factual question.

CH_4 = Tetrahedral Shape,

SeF_4 = Selenium in SeF_4 has an oxidation state of +4. Its shape in the gaseous phase is similar to that of SF_4, having a see-saw shape

5)

Ans: b

Exp: K.E. of emitted photoelectrons depend on frequency of radiation.

6)

Ans: c

Exp: Adding Catalyst increase or decrease the Activation Energy (Ea).

7)

Ans: c

Exp: NA

8)

Ans: a

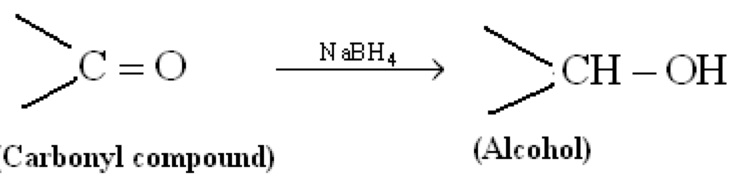
Exp: The number of chiral carbons = 5 (odd number).

It is symmetrical molecule so no. of meso forms

$= 2^{(n/2 - ½)} = 2^{(5/2 - ½)} = 2^2 = 4$

9)

Ans: b

Exp: 

10)

Ans: a

Exp: (A), (C) and (D) decreases down the group but (B) increases down the group with increasing metallic character.

11)

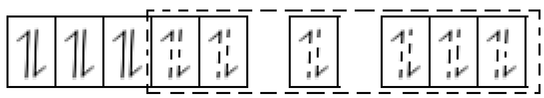
Ans: a

Exp: In $[CuCl]_4^{2-}$ orange compound there is one unpaired electron in d-orbital, therefore, its hybridization is sp^3 where as in $[CuCl_4]^{2-}$, yellow compound, unpaired electron jumps to 5s & thus dsp^2 hybridization.

12)

Ans: d

Exp: Complex is $[Fe(en)_3]^{2+}$; as 'en' is a strong field ligand pairing of electrons will take place



$[Fe(en)_3]^{2+}$: d^2sp^3 hybridization

Hence, hybridization is d^2sp^3 and complex is diamagnetic. As it has 3 bidentate symmetrical 'en' ligands so it will not show geometrical isomerism.

13)

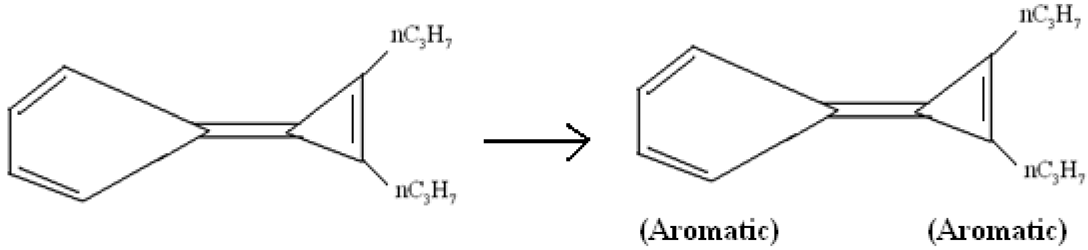
Ans: b

Exp: There are unpaired electrons, others have no unpaired electrons.

14)

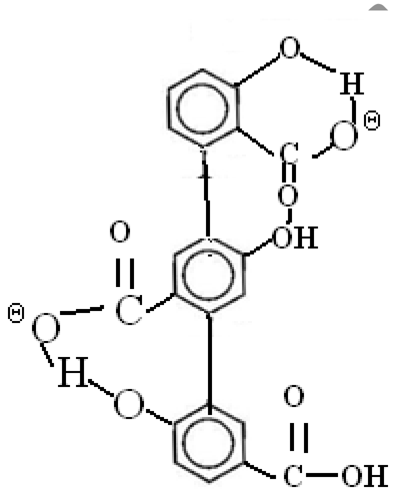
Ans: c

Exp:



15)

Ans: a



Exp:

16)

Ans: b

Exp: NA

17)

Ans: b

Exp: $\Delta H_f(CH_4) = (\Delta H_{comb.}$Of C) + (2 x $\Delta H_{comb.}$ Of H_2) – ($\Delta H_{comb.}$ Of CH_4)

18)

Ans: c

Exp: Increase in pressure favours the melting of ice.

19)

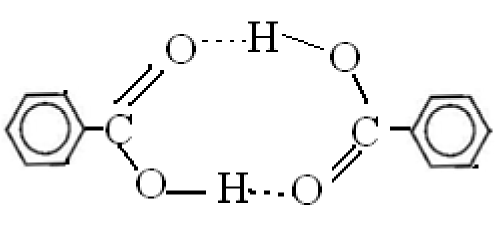
Ans: d

Exp: K_w increases with increase in temperature.

20)

Ans: c

Exp: The benzoic acids forms a dimer due to H-bonding as



21)

Ans: b

Exp: NH_3 is weakly ionising while NH4Cl will be converting into NH_4 + only.

22)

Ans: c

Exp: NA

23)

Ans: c

Exp: Rate is governed by slowest step

$$A + B_2 \xrightarrow{k_1} AB + B$$

$$r = k_1[A][B_2]..........(i)$$

$$\text{From } A_2 \underset{}{\overset{k_c}{\rightleftharpoons}} A + A$$

$$k_c = \frac{[A]^2}{[A_2]} ....(ii)$$

$$[A] = \sqrt{k_c}[A_2]^{1/2}$$

$$r = k_1\sqrt{k_c}[A_2]^{1/2}[B_2] \qquad \text{order is } = \frac{1}{2} + 1 = \frac{3}{2}$$

24)

Ans: d

Exp: When all particle along are body diagonal one removed, these 2X atoms from corner are removed, one Y particle removed & 2Z particle removed.

Hence new arrangement, X particle = 1/8 x 6 + ½ x 6 = 15/4; Y particle = 6: Z particle = 3

Hence formula = $X_{15/4}\ Y_3Z_6 = X_{5/4}YZ_2 = X_5Y_4Z_8$

25)

Ans: a

Exp: (A) $H_2S_2O_7$ = +6 Each, $Na_2S_4O_6$ = +5, 0 , $Na_2S_2O_3$ = +4,0

26)

Ans: c

Exp: The hydration energy of the ions of alkaline earth metals (M^{2+}) is nearly 4 or 5 times greater than that of alkali metals because of their small size and increased charge.

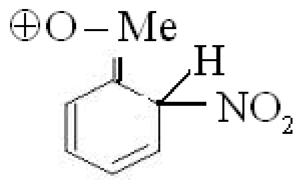
27)

Ans: a

Exp: It is believed that out of two hydrogen, one is associated with each oxygen atom at bond angle of 94.8^o. All the four atoms H-O-O-H do not lie in the same plane.

28)

Ans: b

Exp: 

: In this structure, every atom (except, of course, H) has a stable octet of elections. So it is the most stable.

29)

Ans: c

Exp: Ionization energy of hydrogen atom is about 1310kJmol^{-1}. Covalent single bond energies are of the order of a few hundred kJ mol^{-1}. Molecular translation energy of gasses = 3/2 RT, which is about 3.7 kJ mol^{-1} at 300K. Rotational barrier between eclipsed and staggered forms of ethane is about 12kJ mol^{-1}

30)

Ans: c

Exp: SF_6: sp^3d^2 hybridization; molecule is octahedral XeF_6:sp^3 hybridization; molecule is pentagonal [pyramid]

XeF_4: sp^3d^2 hybridization: molecule is square planar I^-_3: sp^3d hybridization: molecule is linear

Part – B - Physics

31) Ans: c

Exp: Initially, PV = nRT (for each vessel)

P = 1atm, T = 300k, V = vol of each vessel

Finally, $P_1V = n_1R\ 600$, $P_1V = n_2R300$

$n_1/n_2 \,.\, 2 = 1$

$2n_1 = n_2$

Total no of moles is constant

$\Rightarrow n_1 + n_2 = 2n$

$3n_1 = 2n \Rightarrow n = 3n_1/2$

We have PV = nR300 …………(1)

& $P_1V = n_1R600$ ………(2)

From 1 & 2,

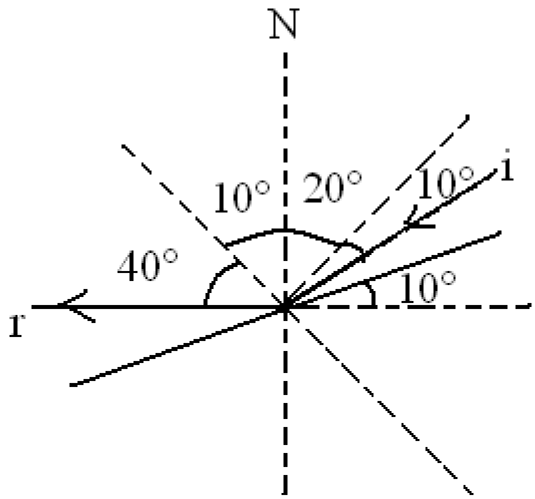
$P_1 x\ 3n_1/2.\ R \times 300 = n_1R\ 600$

$P_1 = 4/3$

32) Ans: c

Exp: Initial angle of reflected ray with the normal = 20^o

Final angle of the reflected ray with the same normal = 50^o



Angle through which reflected ray is rotated = $50^o - 20^o = 30^o$

33) Ans: c

Exp: T.E = KE + PE

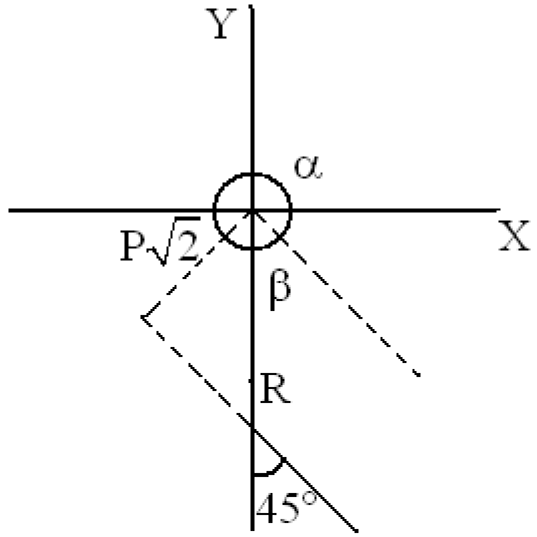
= KE – GMm/R (on surface of earth)

At infinity TE = 0, KE = GMm/R = mgR

($g = GM/R^2$)

34) Ans: c

Exp: B = M.I/4πd [sinα + sinβ]



$= M.I/4\pi(R/\sqrt{2})\ [\sin(2\pi-(\pi/4)) + \sin 90]$

$= M.I/4\pi R\ .\ (\sqrt{2} - 1)$

35) Ans: a

Exp: $P = \sigma\ \Delta T^4$

$T_1 = 288^o K$, $T_2 = 293K$

36) Ans: c

Exp: For movement of m_1, $kx = \mu m_1 g$

Now, by using work energy theorem

$F_{min}\ x = \mu m_2 g x + ½\ kx^2$

$\Rightarrow F_{min} = \mu m_2 g + ½\ kx$

$F_{min} = \mu g\ (m_2+m_1/2)$

37) Ans: d

Exp: $v_0 = 0 + \alpha \times 2n$

$\alpha = v_0/2n$

for the 2n sec s, displacement

$x_1 = ½\ v_0/2n\ (2n)^2$

for first n sec s,

$x_2 = ½\ v_0/2n\ (n)^2$

Displacement in the last n secs:

$x_1 – x_2 = ½\ v0/2n\ ((2n)^2 – (n)^2)$

$=3v_0n/4$

38) Ans: a

Exp: The distance travelled by the train in 20secs is

$½\ (0.5)\ (20)^2 = 100$

The distance between the two events μ & T is 100m

The observer has to move 100m in 20s in a direction opposite to that of train

39) Ans: b

Exp: $v_x = 4 \times 6 \cos 6t$

$v_y = 4 \times 6 \sin 6t$

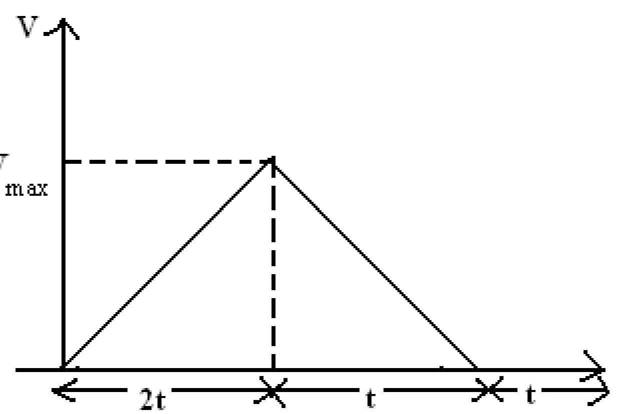
$|v| = \sqrt{(v_x^2 + v_y^2)} = 24 m/s$

Distance traversed = vt = 24 x ¼

= 6m

40) Ans: B

Exp: $v_{max}/2t = 10$



$V_{max} = 20t$

Area of graph

$\frac{1}{2} v_{max} (3t) = 200$

$t = \sqrt{(20/3)}$

total time = 3t = 2√(15) secs

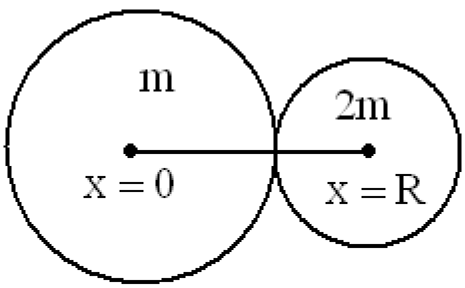
41) Ans: c

Exp: The x-coordinate of all the particles in this case cannot be of the same sign.

42)

Ans: c

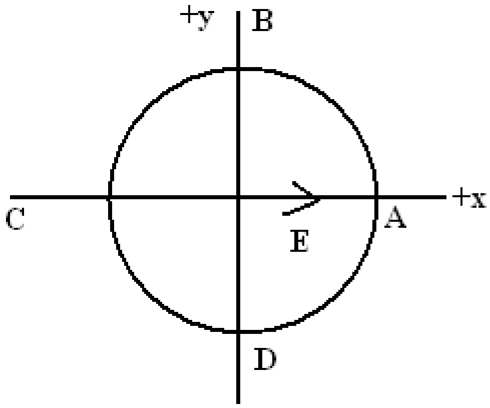
Exp:



Xcom = (2mx3R)/(m+2m) = 2R

COM lies at pt of contact

43) Ans: a



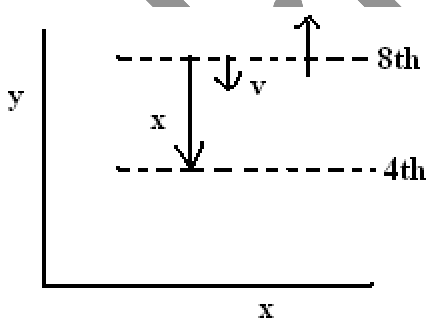
Exp:

The $\vec{E}$ is directed towards the +x axis, the potential has to be minimum at A,

$$V(r) = -\int_{\infty}^{r} \vec{E}.\vec{d}$$

44)

Ans: a



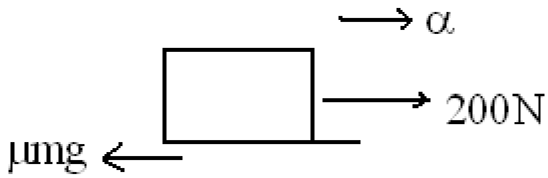
Exp:

The displacement & velocity are directed in the –ve of direction.

There would be retardation as lift stops at 4th floor, so acceleration is directed opposite to the velocity in the upward direction.

45)

Ans: c

Exp: 

$\alpha = 4/2 = 2 m/s^2$

$200 - \mu \times 30 \times 10 = 30 \times 2$

$\mu = 14/20 = 7/15 = 0.47$

46)

Ans: b

Exp: PR = d

∴ PO = d secθ

and CO = PO cos 2θ = d sec θ 2 θ

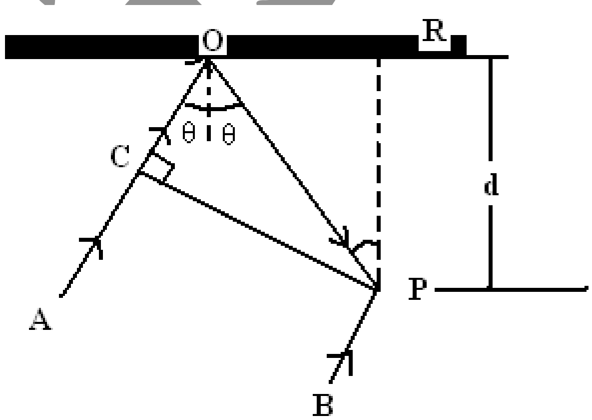
path difference between the two rays is,

$\Delta x = PO + OC = (d \sec\theta + d \sec\theta \cos 2\theta)$

Phase difference between the two rays is

$\Delta\phi = \pi$ (one is reflected, while another is direct)

Therefore, condition of construction interference should be



$\Delta x = \lambda/2, 3\lambda/2$ …..

Or $d \sec\theta (1 + \cos \theta) = \lambda/2$

Or $(d/\cos\theta) (2\cos^2\theta) = \lambda/2$

Or $\cos\theta = \lambda/4d$

47)

Ans: B

Exp: Heat released by 5kg of water when its temperature falls from $20^{o}C$ to $0^{o}C$ is,

$Q1 = mc\Delta\theta = (5)(103) (20-0) = 105cal$

When 2kg ice at -20oC comes to a temperature of $0^{o}C$, it takes energy

$Q2 = mc\Delta\theta = (2) (500) (20) = 0.2 \times 105cal$

The remaining heat

$Q = Q1 – Q2 = 0.8 \times 105cal$ will melt a mass m of the ice,

Where,

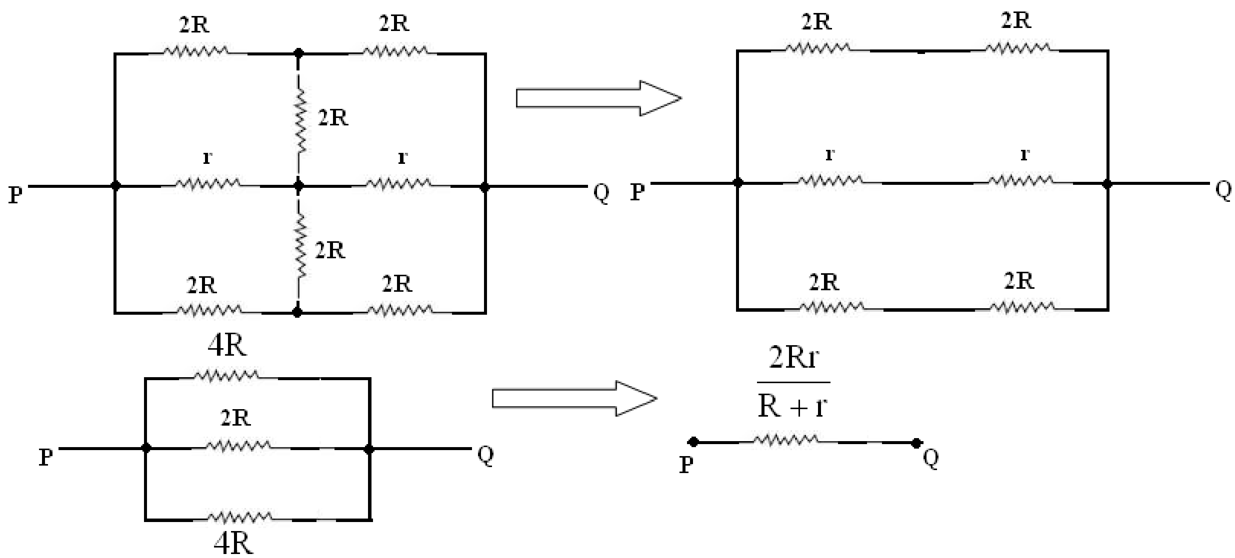
$m = Q/L = 0.8x105/80x103 = 1kg$

total mass of water=5+1=6kg

48)

Ans: a

Exp: The circuit can be redrawn as follows:



49)

Ans: b

Exp: All the three plates will produce electric field at P along negative z – axis.

Hence,

$$\vec{E}_p = \left[\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right]\left(-\hat{k}\right)$$

$$= -\frac{2\sigma}{\varepsilon_0}\hat{k}$$

∴Correct answer is (b).

50)

Ans: b

Exp: Net electrostatic energy of the configuration will be

$$U = K\left[\frac{q.q}{a} + \frac{Q.q}{\sqrt{2a}} + \frac{Q.q}{a}\right] \text{Here, } K = \frac{1}{4\pi\varepsilon_0}$$

$$\text{Putting } U = 0 \text{ we get, } Q = \frac{-2q}{2+\sqrt{2}}$$

51)

Ans: d

$$\frac{dN}{dt} = 50 - \frac{N}{0.5}$$

Exp: $$\int_0^N \frac{dN}{50-2N} = \int_0^t dt$$

$$N = \left(100\left(1 - e^{-t/2}\right)\right) = 25$$

$$t = 2\ln(4/3)$$

52) Ans: a

Exp: Path difference = (μ - 1) t = nλ;

For minimum t, n = 1;

∴ t = 2λ

53)

Ans : C

Exp: B.A'+B+A.B'+A

= B.(A'+1)+A.(B'+1)

=B.1+A.1=A+B

54)

Ans: d

$$eV_0 = \frac{hc}{\lambda_0} - W_0 \text{ and } eV' = \frac{hc}{2\lambda_0} - W_0$$

Exp: Subtracting them, we have

$$e(V_0 - V') = \frac{hc}{\lambda_0}\left[1 - \frac{1}{2}\right] = \frac{hc}{2\lambda_0} \text{ or } V' = V_0 - \frac{hc}{2e\lambda_0}$$

55)

Ans: D

Exp: Tension in the string b/w B and C=T,

Tension in the string attached to A and B=T_1

For block C: T=Mg, For block B: T-u100g-u(100+140)g-2T_1=0

For block C: T_1=u100g, hence,

Mg-100ug-240ug-200ug=0

M=u540=162kg

56)

Ans: D

Exp:

$$\int \vec{E}.\vec{dl} = \left|\frac{d\phi}{dt}\right|$$

$$= S\left|\frac{dB}{dt}\right|$$

or $\quad E(2\pi r) = \pi a^2\left|\frac{dB}{dt}\right|$ for $r \geq a$

$\therefore \quad E = \frac{a^2}{2r}\left|\frac{dB}{dt}\right|$

$\therefore \quad$ Induced electric field $\propto \frac{1}{r}$

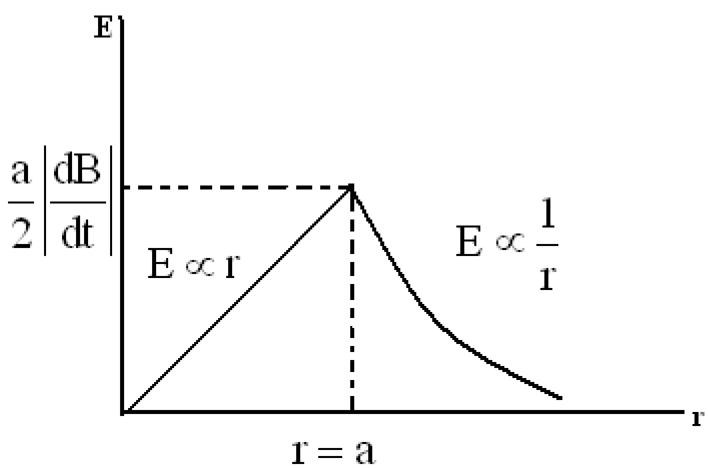
For $\quad r \leq a$

$$E(2\pi r) = \pi r^2\left|\frac{dB}{dt}\right|$$

or $\quad E = \frac{r}{2}\left|\frac{dB}{dt}\right|$ or $E \propto r$

At $\quad r = a, E = \frac{a}{2}\left|\frac{dB}{dt}\right|$

Therefore, variation of E with r (distance from centre) will be as follows:

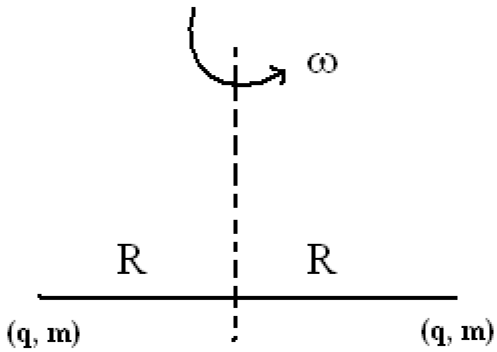


57)

Ans: a

Exp: Current, i = (frequency) (charge)

$$=\left(\frac{\omega}{2\pi}\right)(2q)=\frac{q\omega}{\pi}$$



Magnetic moment,

$$M=(i)(A)=\left(\frac{q\omega}{\pi}\right)(\pi R^2)=(q\omega R^2)$$

Angular momentum,

$$L=2I\omega=2(mR^2)\omega$$

$$\therefore\frac{M}{L}=\frac{q\omega R^2}{2(mR^2)\omega}=\frac{q}{2m}$$

58)

Ans: a

Exp:

$$U = eV = eV_0 \ln\left(\frac{r}{r_0}\right)$$

$$|F| = \left|-\frac{dU}{dr}\right| = \frac{eV_0}{r}$$

This force will provide the necessary centripetal force.

Hence, $\frac{mv^2}{r} = \frac{eV_0}{r}$

or $v = \sqrt{\frac{eV_0}{m}}$ ....(i)

Moreover, $mvr = \frac{nh}{2\pi}$ ....(ii)

Dividing Eq.(ii) by (i), we have

$$mr = \left(\frac{nh}{2\pi}\right)\sqrt{\frac{m}{eV_0}} \text{ or } r_n \propto n$$

59)

Ans: B

Exp: The apparent freq. of sound coming from the police car and from the stationary siren observed by the motorcyclist will be same.

(330-v)/(330-22) *176= (330+v)/330 *165

v is approx. 22m/s

60)

Ans: a

$$I_{remaining} = I_{whole} - I_{removed}$$

Exp: or $$I = \frac{1}{2}(9M)(R)^2 - \left[\frac{1}{2}m\left(\frac{R}{3}\right)^2 + m\left(\frac{2R}{3}\right)^2\right] \quad ....(i)$$

Here, $$m = \frac{9M}{\pi R^2} x\pi\left(\frac{R}{3}\right)^2 = M$$

Substituting in Eq. (i), we have I = $4MR^2$

∴correct answer is (a)

Part – C – Mathematics

61) Ans: c

Exp: Let $f(x) = x^{12} - x^9 + x^4 - x + 1$

$= (x^9 + x)(x^3 - 1) + 1 > 0, \forall x \geq 1$ ....(i)

Again, $f(x) = x^4 (x^8 + 1) – x (x^8 + 1) + 1$

$= x (x^8 + 1) (x^3 - 1) + 1 > 0, \forall x \leq 0$ ....(ii)

Next $f(x) = x^{12} + x^4(1 - x^5) + (1-x) > 0, \forall\ 0 < x < 1$ ....(iii)

Combining Eqs. (i), (ii), (iii) we get

$X \in (-\infty, \infty)$

62) Ans: b

Exp: $6! - 2!\,5! = 480$

63) Ans: b

Exp: $2x+y = 5$

$x + 3y = 5$

$x - 2y = 0$

Solving eqs. (i) and (ii), we get

$x = 2$ and $y = 1$

Which is satisfied Eq. (iii).

64) Ans: a

Exp: $i^i = (e^{i\pi/2})^i$

$= e^{-\pi/2}$

65) Ans: b

Exp: $\because |z_1 - z_2| = |z_1 - (z_2 - 3 - 4i) - (3+4i)|$

$\geq |z_1| - |z_2 - 3 - 4i| - |3+4i|$

$= 12-5-5=2$

$\therefore |z_1 - z_2| \geq 2$

66) Ans: d

Exp: In 10jumps = 10m

11^{th} jump = 10 + 2 = 12m (not slip)

67) Ans: c

Exp: $32 = 2^5$

$(32)^{32} = (2^5)^{32} = 2^{160}$

= $(3-1)^{160}$

= $3m+1, m \in I_+$

Now, $(32)^{32^{32}} = 32^{3^m+1} = 2^{5(3^m+1)} = 2^{15^m+5}$

∴ $32^{32^{32}} = 2^{3(5m+1)}.2^2$

$= 4.(8)^{5m+1}$

$= 4.(7+1)^{5m+1}$

$= 4.[7n + 1], n \in I_+$

$= 28n+4$

∴ Remainder = 4

68) Ans: d

Exp: Probability of showing even number in a throw = 3/6 = ½

∴Required probability

$= {}^{2n+1}C_1 \cdot ½ (1/2)^{2n} + {}^{2n+1}C_3 (1/2)^3(1/2)^{2n-2} + \ldots\ldots. + {}^{2n+1}C_{2n+1}(1/2)^{2n+1}$

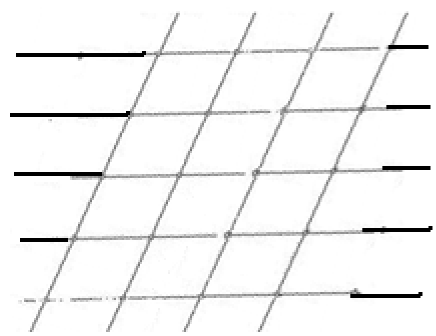
$=(1/2)^{2n+1}[{}^{2n+1}C_1 + {}^{2n+1}C_3 + \ldots\ldots\ldots + {}^{2n+1}C_{2n+1}]$

$= 1/2^{2n+1} \times 2^{2n+1-1} = ½$

69) Ans: b

Exp: Number of parallelograms = ${}^5C_2 x\, {}^4C_2$

= 60



70) Ans: b

Exp: Let A = $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$

$$\therefore A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This is nilponent matrix of index 2.

71) Ans: a

Exp: Total ways = $^{12}C_2 = 66$

E_3 = the even of the sum being 3

E_6 = the even of the sum being 6

E_9 = the even of the sum being 9

E_{12} = the even of the sum being 12

E_{15} = the even of the sum being 15

E_{18} = the even of the sum being 18

E_{21} = the even of the sum being 21

(sum maximum = 11 + 12 = 23)

$$\begin{bmatrix} (\text{Which is not divisible by } 3) \\ (\text{Heighest number divisible by 3 is } 21) \\ \therefore n(E_3) = 1,\ n(E_6) = 2, n(E_9) = 4, \end{bmatrix}$$

$$n(E_{12}) = 5, n(E_{15}) = 5, n(E_{18}) = 3, n(E_{21}) = 3$$

$$\therefore \quad \text{Required probability} = \frac{\sum_{i=1}^{7} n(E_{3i})}{66}$$

$$= \frac{22}{66} = \frac{1}{3}$$

72) Ans: b

Exp: $4\cos^2\theta - 2\sqrt{2}\cos\theta - 1 = 0$

$\cos\theta = 2\sqrt{2} \pm \sqrt{(8+16)}/8 = \sqrt{2} \pm \sqrt{6}/4$

$\cos\theta = \sqrt{6}+\sqrt{2}/4 \quad \Rightarrow \theta = \pi/12;\ 2\pi - \pi/12 = 23\pi/12$

$\cos\theta = -\sqrt{6} - \sqrt{2}/4$

$\cos\theta = \cos(\pi - 5\pi/12);\ \cos(\pi+5\pi/12)$

$\Theta = 7\pi/12;\ 17\pi/12$

73) Ans: c

Exp: $E = 4+5(\omega)^{334} + 3(\omega)^{365} = 4+5\omega + 3\omega^2$

$= 1+2\omega+3(1+\omega+\omega^2) = 1+(-1+i\sqrt{3}) = i\sqrt{3}$

74) Ans: b

Exp: Any line passing through the intersection of the given lines is

x+3y+4+λ(3x+y+4) = 0 ....(i)

Or, (1+3λ)x+(3+λ)y+4(1+λ) = 0

The slope of the line m = - 1+3λ/3+λ

As the line is equally inclined with the axes,

m = tan45° or tan135° = ±1

∴ - 1+3λ/3+λ = ±1, ⇒λ =±1

∴The required lines are (putting λ = - 1, 1 in (i))

x+ 3y + 4 ±(3x+y+4) = 0

Or, x + y + 2 = 0 and x – y = 0

75) Ans: b

Exp: Given equation is

$$\left(x^2+y^2\right)\sin^2\alpha=\left(x\cos\beta-y\sin\beta\right)^2$$

$$\Rightarrow x^2\left(\sin^2\alpha-\cos^2\beta\right)+2xy\sin\beta\cos\beta+y^2\left(\sin^2\alpha-\sin^2\alpha\right)=0 \quad ....(i)$$

Let the angle between the lines representing by (1) is θ

$$\therefore \tan\theta = 2\left|\frac{\sqrt{h^2-ab}}{a+b}\right|$$

$$= 2\frac{\sqrt{\sin^2\beta\cos^2\beta-\left(\sin^2\alpha-\cos^2\beta\right)\left(\sin^2\alpha-\sin^2\beta\right)}}{|\sin^2\alpha-\cos^2\beta+\sin^2\alpha-\sin^2\beta}$$

$$= 2\frac{\sqrt{\left\{\sin^2\beta\cos^2\beta-\sin^4\alpha+\sin^2\alpha\sin^2\beta+\sin^2\alpha\cos^2\beta-\sin^2\beta\cos^2\beta\right\}}}{|\left(2\sin^2\alpha-1\right)|}$$

$$= 2\frac{\sqrt{\sin\alpha\left(1-\sin^2\alpha\right)}}{|-\cos 2\alpha|} = \frac{2\sin\alpha\cos\alpha}{|-\cos 2\alpha|} = \tan 2\alpha$$

$$\Rightarrow \theta = 2\alpha$$

76) Ans: d

Exp: Equation of circle is $x^2+y^2+2gx+2fy+c=0$

$(1, t) \Rightarrow 1 + t^2+2g+2ft+c=0$ ....(i)

$(t, t) \Rightarrow t^2+t^2+2gt+2ft+c=0$..........(ii)

$(t, 1) \Rightarrow 1+t^2+2gt+2f+c=0$ ....(iii)

Subtracting (ii) from (i),

$1+2g - t^2-2gt = 0$

$\Rightarrow 1-t^2+2g(1-t)=0$

$\Rightarrow (1-t)(1+t+2g) = 0$

$\Rightarrow t = 1$

$\therefore$one point (t, t)

$\therefore$ passes through (1, 1)

77) Ans: c

Exp:

$$I\int \sec^2\theta(\sec\theta+\tan\theta)\bullet(\sec\theta+\tan\theta)d\theta$$

$$\text{Put } \sec\theta+\tan\theta = y \qquad \text{..........}(1)$$

$$\therefore \sec\theta(\tan\theta+\sec\theta)d\theta = dy$$

$$\text{Now}, \sec\theta-\tan\theta = \frac{1}{y} \qquad \text{.......}(2)$$

$$\text{From } (1) \text{ and } (2), 2\sec\theta = y+\frac{1}{y}$$

$$\therefore I = \frac{1}{2}\int y\left(y+\frac{1}{y}\right)dy = \frac{1}{2}\left[\frac{y^3}{3}+y\right]+C$$

$$=\frac{1}{2}\left[\frac{(\sec\theta+\tan\theta)^3}{3}+(\sec\theta+\tan\theta)\right]+C$$

$$=\frac{(\sec\theta+\tan\theta)}{6}\left[(\sec\theta+\tan\theta)^2+3\right]+C$$

78) Ans: d

Exp:

Given, $V = \pi r^2 h$

Differentiating both sides

$$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + 2r\frac{dr}{dt}h\right) = \pi r\left(r\frac{dh}{dt} + 2h\frac{dr}{dt}\right)$$

$$\frac{dr}{dt} = \frac{1}{10} \quad \text{and} \quad \frac{dh}{dt} = -\frac{2}{10}$$

$$\frac{dV}{dt} = \pi r\left(r\left(-\frac{2}{10}\right) + 2h\left(\frac{1}{10}\right)\right) = \frac{\pi r}{5}(-r+h)$$

Thus, when $r = 2$ and $h = 3$,

$$\frac{dV}{dt} = \frac{\pi(2)}{5}(-2+3) = \frac{2\pi}{5}$$

79) Ans: b

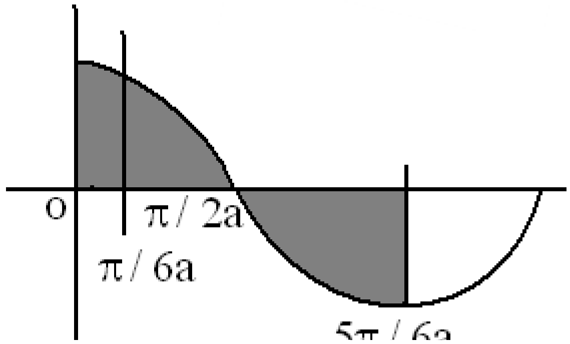
Exp:

$$\cos ax = 0 \text{ if } ax = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

$$x = \frac{\pi}{2a} \text{ or } \frac{3\pi}{2a}$$

$$A_1 = \int_{\frac{\pi}{6a}}^{\frac{\pi}{2a}} \cos ax\,dx$$

$$= \frac{1}{a}\int_{\pi/6}^{\pi/2} \cos t\,dt = \frac{1}{a}[\sin t]_{\pi/6}^{\pi/2} = \frac{1}{2a}$$



$$\text{Similarly } A_2 = \left| \int_{\frac{\pi}{2a}}^{\frac{\pi}{6a}} \cos ax\,dx \right| = \left| -\frac{1}{2a} \right| = \frac{1}{2a}$$

$$\therefore \text{Total area } = \frac{1}{a} > 3 \quad \therefore \quad 0 < a < \frac{1}{3}$$

80) Ans: b

Exp: $y = \sin^4 \pi x$ intersects the x-axis at

$x = 0, x = 1$

The curve, $y = \log x$ also passes through (1, 0)

Required area =

$$\left| \int_0^1 \sin^4 \pi x dx \right| + \left| \int_0^1 \log_e x dx \right| = \int_0^{\pi} \sin^4 \theta \frac{d\theta}{\pi} + \left| \left[x \log_e x - x \right]_0^1 \right|$$

$$= \frac{2}{\pi} \int_0^{\pi/2} \sin^4 \theta d\theta + 1 = \frac{2}{\pi} x \frac{3}{4} x \frac{1}{2} x \frac{\pi}{2} + 1 = \frac{11}{8}$$

81) Ans: b

Exp: $x^2 + (y-r)^2 = r^2$ ....(i)

$\therefore x + (y-r)\, dy/dx = 0, \therefore (r-y)\, dy/dx = x$

$\therefore r = y + x/(dy/dx)$

Put it in (i), we get $(x^2 – y^2)\, dy/dx – 2xy = 0$

82) Ans: c

Exp: $(1+y^2)\, dx + (1+x^2)dy = 0$

$\Rightarrow dx/1-x^2 + dy/1+y^2 = 0$

On integration, we get

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}C$

$\Rightarrow x+y/1-xy = C$ $\Rightarrow x + y = C(1-xy)$

83) Ans: b

Exp: Let a, ar, ar^2, ...

$a + ar = 12$ ....(1)

$ar^2 + ar^3 = 48$ ....(2)

Dividing (2) by (1), we have

$ar^2(1+r)/a(r) = 4$

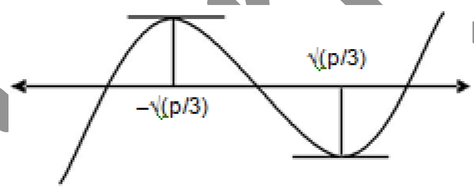
$\Rightarrow r^2 = 4$ if $r \neq -1$

$\therefore r = -2$

Also, a = - 12 (using (1)).

84) Ans: b

Exp: Let $f(x) = x^3 - px + q$



Now for

Maxima/minima $f'(x) = 0$

$\Rightarrow 3x^2 - p = 0$

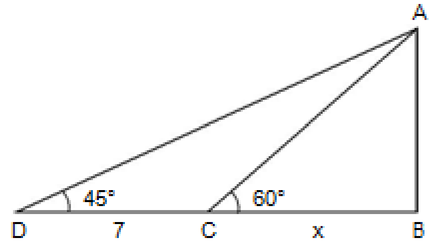
$\Rightarrow x^2 = P_3$

$\therefore x = \pm\sqrt{(p/3)}$

85) Ans: b

Exp: BD = AB = 7+x

Also AB = x tan 60° = $x\sqrt{3}$



∴x√3 = 7+x

X = 7/√3-1

AB = 7√3/2(√3+1)

86) Ans: c

Exp: A = {4, 5, 6}, B = {1, 2, 3, 4}.

Obviously P (AUB) = 1

87) Ans: c

Exp: Let (h, k) be the coordinates of the midpoint of a chord which subtends a right angle at the origin. Then equation of the chord is

$$kx + ky - 4 = h^2 + k^2 - 4 (u\sin g\ T = S')$$
$$or \quad hx + ky = h^2 + k^2$$

The combined equation of the pair of lines joining the origin to the points of intersection of $x^2 + y^2 = 4\ and\ hx + ky = h^2 + k^2\ is$

$$x^2 + y^2 - 4\left(\frac{hx+ky}{h^2+k^2}\right)^2 = 0$$

Lines given by the above equations of the pair of lines joining the origin to therefore coeff. of $x^2 + \text{coeff.of } y^2 = 0$

$$\Rightarrow 2(h^2+k^2) - (4h^2+4k^2) = 0 \Rightarrow h^2+k^2 = 2$$

$$\therefore Locus\ of\ (h,k)\ is\ x^2+y^2 = 2$$

88) Ans: d

Exp:

$$\text{Mean of } a, b, 8, 5, 10 \text{ is } 6$$

$$\Rightarrow \frac{a+b+8+5+10}{5} = 6$$

$$\Rightarrow a+b = 7 \qquad \ldots\ldots\ldots(1)$$

$$\therefore \text{Variance} = \sum \frac{(X-A)^2}{l}$$

$$= \frac{(a-6)^2 + (b-6)^2 + 4 + 1 + 16}{5} = 6.8$$

$$\Rightarrow a^2 + b^2 = 25$$

$$a^2 + (7-a)^2 = 25 \ (\text{Using } (1))$$

$$\Rightarrow a^2 - 7a + 12 = 0$$

$$\therefore a = 4, 3 \text{ and } b = 3, 4$$

89) Ans: a

Exp:

$$(x-h)^2+(y-2)^2=25 \qquad ....(1)$$

$$\Rightarrow 2(x-h)+2(y-2)\frac{dy}{dx}=0$$

$$\Rightarrow (x-h)=-(y-2)\frac{dy}{dx}$$

substituting in (1), we have

$$(y-2)^2\frac{dy}{dx}2+(y-2)^2=25$$

$$(y-2)2y'2=25-(y-2)^2$$

90) Ans: a

Exp:

$$S_1\equiv(6,5);S_2\equiv(-4,5),e=5/4$$

$$S_1S_2=10\Rightarrow 2ae=10\Rightarrow a=4$$

$$\text{and } b^2=a^2(e^2-1)=16\left(\frac{25}{16}-1\right)=9$$

Centre of the hyperbola is