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Subject: CHEMISTRY, MATHEMATICS & PHYSICS

Paper Code: JEE\_ Main\_ Sample Paper - III

## Part – A – Chemistry

1) b

Exp: In an fcc system, number of atoms in the unit cell is,

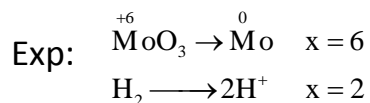
At the Corners  $8 \times \frac{1}{8} = 1$

At the face centers  $6 \times \frac{1}{2} = 3$

Total = 4

For each atom there is one octahedral void and two tetrahedral voids.

2) b



$\therefore$  1 moles  $\text{MoO}_3$  will react with 3 moles of  $\text{H}_2$

125/144 moles  $\text{MoO}_3$  will react with  $3 \times 125/144 \times 22.4$  lit. of  $\text{H}_2$  at NTP = 58.33lit. at NTP.

3) c

Exp: (b) and (d) are ruled out on the basis that at the initial point of 273K, 1 atm, for 1.0 mole volume must be 22.4L, and it should increase with rise in temperature.

4) c

Exp: Rate of effusion  $\propto p_i$ ;  $p_i$  = Particle pressure of  $i^{\text{th}}$  component

$$\propto \sqrt{1/M}$$

5) a

Exp: Temperature and volume are related for adiabatic process as:  $T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$

$$\therefore T_2 = T_1 V_1^{\gamma-1} = T_1 (V_1/V_2)^{\gamma-1} = 300(1/8)^{\gamma-1}$$

$$= 300 \times \frac{1}{2} = 150\text{K}$$

$$C_p/C_v = \gamma = 4/3 \quad \therefore C_v = 3R \ \& \ C_p = 4R$$

Adiabatic work done

$$\Delta w = - C_v \Delta T = - 3 \times 2 \times (150-300) = 900\text{cal}$$

6) c

Exp:  $PV = RT$  at temp T for one mol

$$P(V + \Delta V) = R(T+1) \text{ at temp.}(T + 1) \text{ for one mol}$$

$$\therefore P \Delta V = R$$

7) a

Exp: The energy of an electron in a Bohr atom is expressed as

$$E_n = - kZ^2/n^2 \quad \text{where, } k = \text{constant,}$$

Z = atomic number,

n = orbit number

$$= - 13.6\text{eV for H (n = 1)}$$

When n = 2,

$$E_2 = -13.6/2^2\text{eV}$$

$$= - 3.40\text{eV}$$

(n can have only integral value 1, 2, 3, .....  $\infty$ )

8) d

Exp:  $\text{Mg}^{2+}$ :  $1s^2 2s^2 2p^6$ : no unpaired electron.

$\text{Ti}^{3+}$ :  $1s^2 2s^2 2p^6 3s^2 3p^6 3d^1$ : one unpaired electron

$\text{V}^{3+}$ :  $1s^2 2s^2 2p^6 3s^2 3p^6 3d^2$ : two unpaired electron

$\text{Fe}^{2+}$ :  $1s^2 2s^2 2p^6 3s^2 3p^6 3d^6$ : four unpaired electrons

9) d

Exp: The minimum  $[\text{OH}^-] = 1.34 \times 10^{-5} \text{ M}$  will be no precipitation of  $\text{Mg}(\text{OH})_2$  can be obtained by

$$K_{sp} = [\text{Mg}^{2+}][\text{OH}^-]^2$$

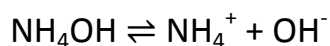
$$\Rightarrow 9.0 \times 10^{-12} = (0.05) \times [\text{OH}^-]^2$$

$$\therefore [\text{OH}^-] = 1.34 \times 10^{-5} \text{ M}$$

Thus, solution having  $[\text{OH}^-] = 1.34 \times 10^{-5} \text{ M}$  will not show precipitation of  $\text{Mg}(\text{OH})_2$  in  $0.05 \text{ M Mg}^{2+}$ . These hydroxyl ions are to be derived by basic buffer of  $\text{NH}_4\text{Cl}$  and  $\text{NH}_4\text{OH}$ .

$$\text{pOH} = \text{pK}_b + \log [\text{salt}]/[\text{base}]$$

$$\text{pOH} = \text{pK}_b + \log [\text{NH}_4^+]/[\text{NH}_4\text{OH}]$$



In presence of  $[\text{NH}_4\text{Cl}]$ , all the  $\text{NH}_4^+$  are provided by  $\text{NH}_4\text{Cl}$  as due to common ion effect, dissociation of  $\text{NH}_4\text{OH}$  is suppressed.

$$-\log [\text{OH}^-] = -\log 1.8 \times 10^{-5} \log [\text{NH}_4^+]/[0.05]$$

$$\therefore \text{NH}_4^+ = 0.67 \text{ M or } [\text{NH}_4\text{Cl}] = 0.67 \text{ M.}$$

10) a

Exp: One can calculate ionic product from given data and for precipitation ionic product  $> K_{sp}$ .

11) b

Exp: The rate increase with temperature but not directly.

12) d

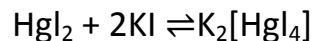
$$\text{Rate}(S_N 2) = 5.0 \times 10^5 \times 10^{-2} [R - X] = 5.0 \times 10^{-7} [R - X]$$

Exp:  $\text{Rate}(S_N 1) = 0.20 \times 10^{-5} [R - X]$

$$\% \text{ of } S_N 2 = \frac{5 \times 10^{-7} [R - X] \times 100}{5 \times 10^{-7} [R - X] + 0.20 \times 10^{-5} [R - X]} = 20$$

13) a

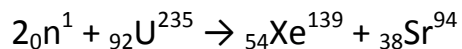
Exp: Addition of  $HgI_2$  to KI solution establishes the following equilibrium:



The above equilibrium decreases the number of ions (4 ions on left side of reactions becomes three ions on right side), hence rises the freezing point.

14) b

Exp: The balanced nuclear reaction is



15) b

Exp: b)



$$C(1+\alpha) \quad C\alpha \quad C\alpha$$

$$[\text{H}^+] = 10^{-2} \quad [\text{A}^-] = 10^{-2}$$

$$[\text{HA}] = 0.1 - 10^{-2} = 0.09$$

$$(1-\alpha) = \frac{0.09}{0.1} = 0.9$$

$$\alpha = 1 - 0.9 = 0.1$$

$$\therefore \pi = C(1+\alpha)RT = 0.1(1.1)RT = 0.11RT$$

16) b

Exp:  $\text{H}^+ \text{e}^- \Rightarrow (1/2)\text{H}_2(\text{g})$

$$E_1 = 0 - .0591 \log \frac{1}{(\text{H}^+)_1}$$

$$E_1 = 0 + .0591 \log [\text{H}^+]_1 = -.0591 \text{pH}_1$$

$$E_2 = -.0591 \text{pH}_2$$

$$pH_1 = pK_a + \log \frac{\text{salt}}{\text{acid}}; \quad pH_1 = pK_a + \log \frac{a}{b} \quad \dots\dots\dots(1)$$

$$pH_2 = pK_a + \log \frac{b}{a}; \quad pH_2 = pK_a - \log \frac{a}{b} \quad \dots\dots\dots(2)$$

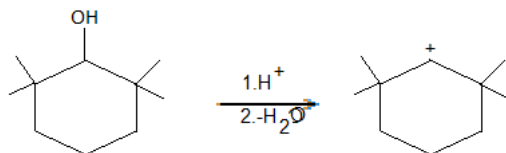
Add (1) & (2)  $pH_1 + pH_2 = 2pK_a$

$$2pK_a = - \frac{E_1}{.0591} - \frac{E_2}{.0591}; \quad pK_a = - \left[ \frac{E_1 + E_2}{0.118} \right]$$

17) b

Exp:  $\text{NaBH}_4$  reduces only carbonyl group, protecting the double bond and as well as acid.

18) a

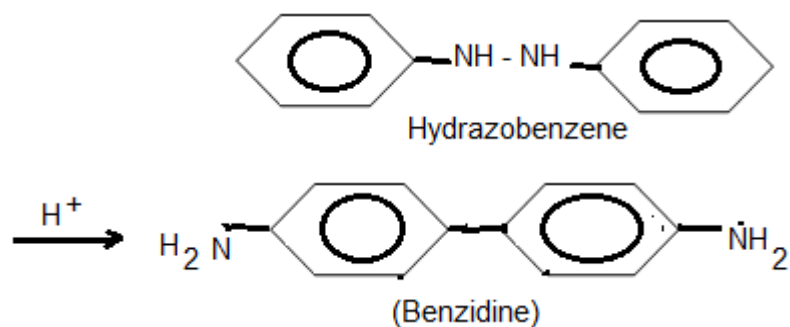


Exp:

2,2,6,6-tetramethyl cyclohexanol  $2^\circ$  carbocation





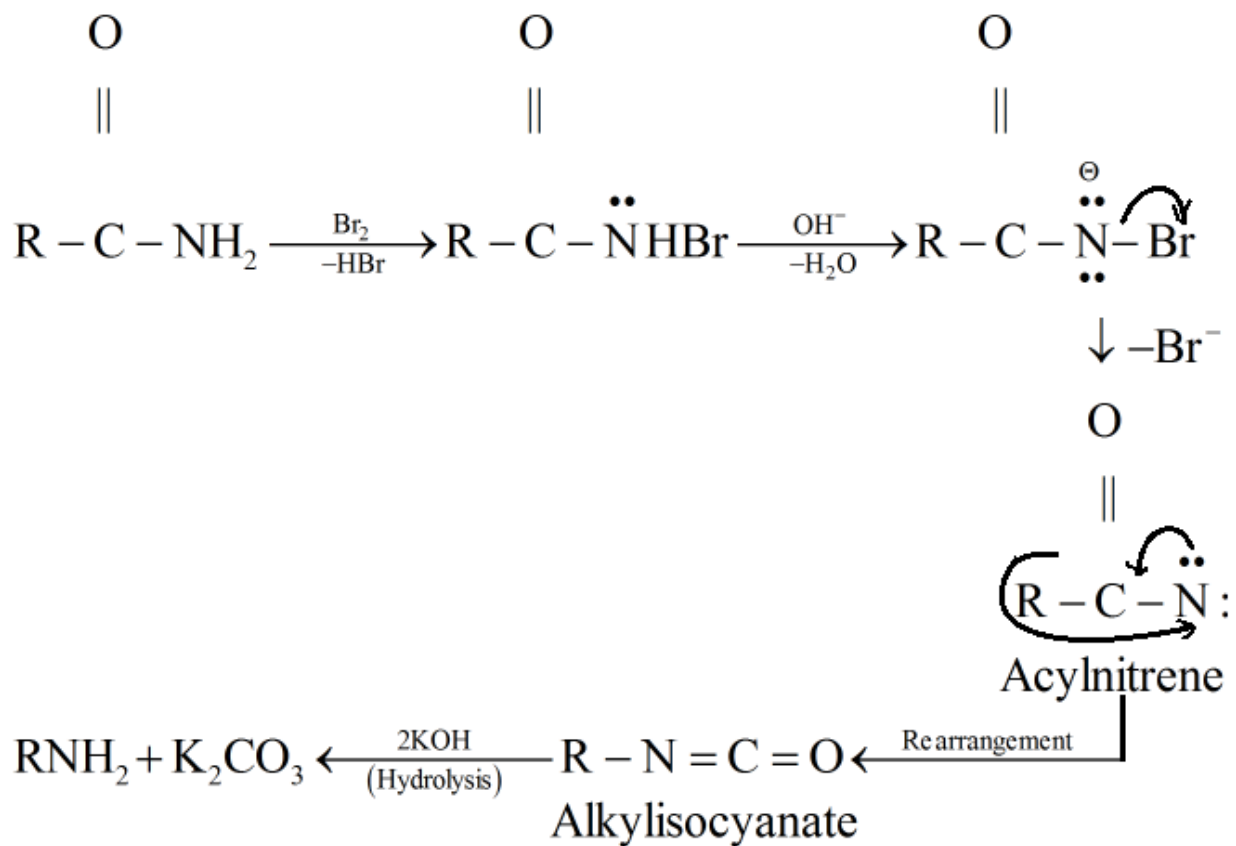


Exp:

Hydrazobenzene undergoes rearrangement in strongly acidic solution (i.e.,  $H^+$ ) to form 4,4'-diaminobiphenyl commonly called benzidine. This reaction is also known as benzidine rearrangement

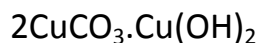
22) a

Exp:



23) c

Exp: Azurite is a basic carbonate ore of copper.



24) b

Exp: The colour exhibited by transition metal ions is due to the presence of unpaired electrons in d-orbitals which permits d – d excitation of electrons.

In  $\text{TiF}_6^{2-}$  - Ti is in + 4 O.S.;  $= 3d^0$  = colourless

In  $\text{CoF}_6^{3-}$  - Co is in +1 O.S.;  $= 3d^5$  = coloured

In  $\text{Cu}_2\text{Cl}_2$  – Cu is in + 1 O.S.;  $3d^{10}$  – colourless

In  $\text{NiCl}_4^{2-}$  - Ni is in +2 O.S.;  $3d^8$  – coloured

25) a

Exp:  $\text{Cr}_2\text{O}_7^{2-} + 2\text{OH}^- \rightarrow 2\text{CrO}_4^{2-} + \text{H}_2\text{O}$

Hence  $\text{CrO}_4^{2-}$  ion is obtained.

26) a

Exp: The green colour appears due to the formation of  $\text{Cr}^{+++}$  ion

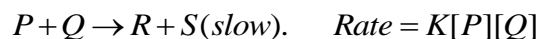
$\text{Cr}_2\text{O}_7^{2-} + 3\text{SO}_3^{2-} + 8\text{H}^+ \rightarrow 3\text{SO}_4^{2-} + 2\text{Cr}^{3+} + 4\text{H}_2\text{O}$

27) d

Exp: Amides react with solutions of chlorine or bromine in excess sodium hydroxide (NaOH) to give primary amines containing one carbon atom less than the original amide. This is Hoffman's bromamide reaction.

28) b

Exp: Rate of a reaction is always determined by the slowest step. Hence, rate determining step is:

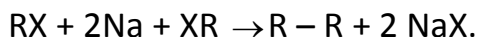


29) b

Exp: Enthalpy of neutralization of NaOH and  $CH_3COOH$  is less than  $57.1 \text{ kJ mol}^{-1}$  because  $CH_3COOH$  uses some heat energy to get partially dissociated.

30) a

Exp: Methane cannot be prepared by Wurtz reaction because in this reaction we use equimolecular amounts of two halides, alkyl groups of which join together.



### Part – B – Physics

31) d

Exp: Density  $\rho = m/\pi r^2 L$

$$\therefore \Delta\rho/\rho \times 100 = (\Delta m/m + 2 \Delta r/r + \Delta L/L) \times 100$$

After substituting the values, we get the maximum percentage error in density = 4%

32) b

$$t_{AC} = 6 / 2 = 3s$$

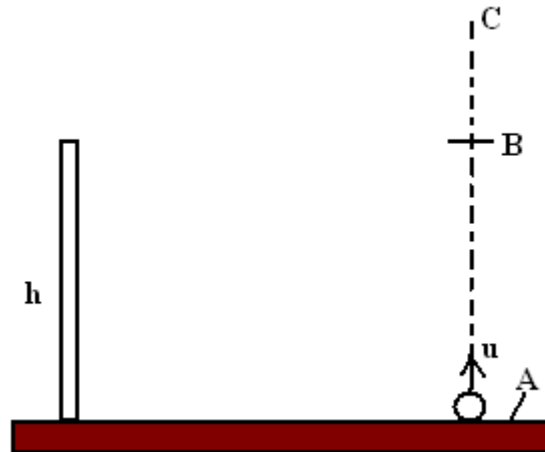
$$t_{AC} = 12 / 2 = 6s$$

$$\therefore t_{AB} = 3s$$

$$\therefore 0 = u - (10)6$$

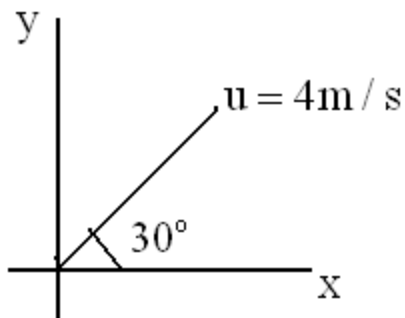
Exp: or  $u = 60m/s$

$$\begin{aligned} \text{Further } h &= ut_{AB} - \frac{1}{2}gt_{AB}^2 \\ &= (60)(3) - \frac{1}{2}(10)(3)^2 \\ &= 135m \end{aligned}$$



33) b

Exp: Components of velocity of ball relative to lift are:



$$U_x = 4 \cos 30^\circ = 2\sqrt{3} \text{ m/s}$$

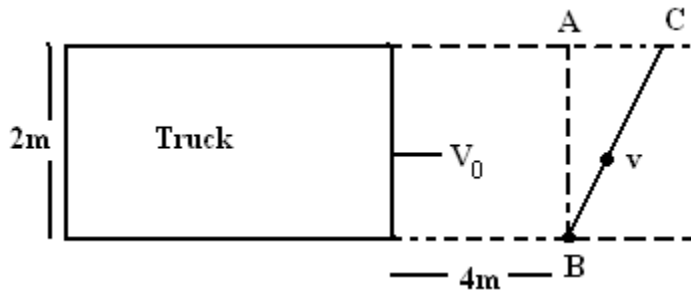
And  $u_y = 4 \sin 30^\circ = 2\text{m/s}$

And acceleration of ball relative to lift is  $12\text{m/s}^2$  in negative y-direction or vertically downwards. Hence time of flight

$$T = 2u_y/12 = u_y/6 = 2/6 = 1/3\text{s}$$

34) c

Exp: Let the man starts crossing the road at an angle  $\theta$  with horizontal as shown in figure. For safe crossing the condition is that the man must cross the road by the time the truck describe the distance  $4+AC$  or  $4+2\cot\theta$ .

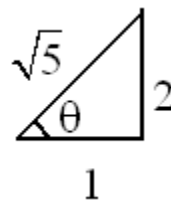


$$\therefore (4+2\cot\theta)/8 = 2/v\sin\theta$$

$$\text{Or } v=8/(2\sin\theta+\cos\theta) \dots\dots(1)$$

For minimum  $v$ ,  $dv/d\theta = 0$

$$\text{Or } -8(2\cos\theta-\sin\theta)/(2\sin\theta+\cos\theta)^2= 0$$



$$\text{Or } 2\cos\theta - \sin\theta = 0$$

Or  $\tan\theta = 2$

From Eq. (1)

$$V_{\min} = 8/(2(2/\sqrt{5}) + 1/\sqrt{5}) = 8/\sqrt{5} = 3.57\text{m/s}$$

35) d

Exp: Weight of lift = Mg

Maximum tension = n Mg

$$\therefore \text{Maximum acceleration} = (nMg - Mg)/M = (n-1)g$$

And maximum retardation = g

Corresponding velocity –time graph for shortest time will be as shown.

$$\text{Here } (n-1)g = v_m/t_1 \quad \dots\dots\dots(1)$$

$$\text{Or } t_1 = v_m/(n-1)g$$

$$\text{And } g = v_m/t_2 \quad \dots\dots\dots(2)$$

$$\text{Or } t_2 = v_m/g$$

Area under v-t graph is total displacement h.

$$\text{Hence } h = \frac{1}{2} (t_1 + t_2)v_m$$

From Eqs.(1), (2) and (3) we get,

$$v_m = \sqrt{2gh \left( \frac{n-1}{n} \right)}$$

36) c

Exp:  $N \sin \theta$  provides the force responsible for motion of the wedge,

$$N \sin \theta = Ma$$

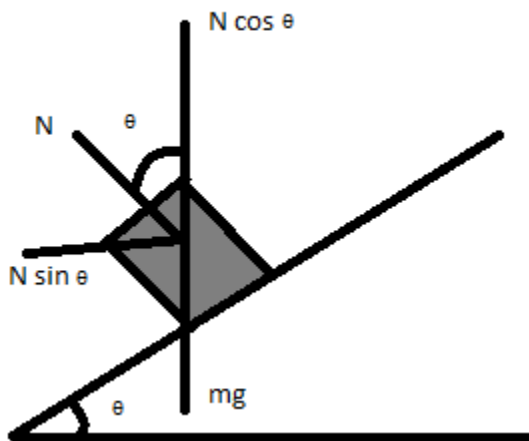
Let  $A$  be the acceleration of the block w.r.t the wedge. When wedge moves with acc.  $a$  then the acc. of the block is  $(A \cos \theta - a)$  in the opposite direction.

$N$  is the normal reaction b/w the two, for both of them,

$$N \sin \theta = Ma = m (A \cos \theta - a)$$

$$m A \cos \theta = Ma + ma$$

$$A = \frac{(M+m)2a}{m}$$



37) d



Exp: Extension in the spring is

$$x = AB - R = 2R \cos 30^\circ - R$$

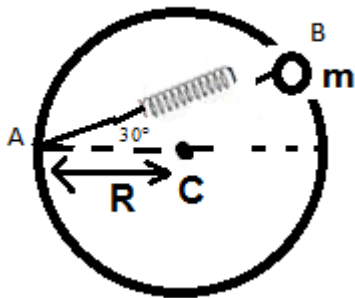
$$= (\sqrt{3} - 1) R$$

∴ Spring force

$$F = kx = (\sqrt{3} + 1)mg/R (\sqrt{3} - 1) R$$

$$= 2mg$$

Free body diagram of bead is shown in figure



$$N = (F + mg) \cos 30^\circ - mg \sin 30^\circ = (2mg + mg)\sqrt{3}/2$$

$$= 3\sqrt{3}/2$$

38) d

Exp: At the highest point, from conservation of energy,

$$\frac{1}{2} mv^2 = \frac{1}{2} mu^2 + 2mgl, \text{ also } mg = mu^2/l \text{ (at highest point)}$$

$$v^2 = u^2 + 4gl = 5gl$$

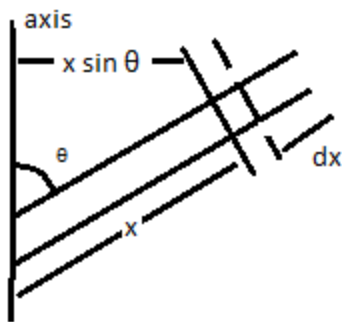
$$\text{Tension at lowest point, } T = mg + mv^2/l = 6mg$$

39) c

Exp: Mass of the element =  $(m/l) dx$ .

Moment of inertia of the element about the axis =  $(m/l dx) (x \sin\theta)^2$ .

$$I = m/l \sin^2\theta \cdot \int_0^l x^2 dx = ml^2/3 \sin^2\theta.$$



40) b

Exp:  $y = 4\cos^2(1/2 t) \sin(1000t) = 2(1+\cos t)\sin(1000t)$

$$= 2\sin(1000t) + 2\sin(1000t) \cos t$$

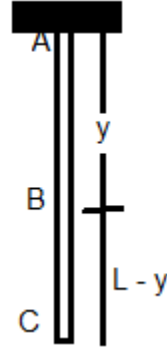
$$= 2\sin(1000t) + \sin(1001t) + \sin(999t).$$

41) b

Exp:  $V_p = -GM/V(r^2+a^2), V_c = -GM/a, \frac{1}{2} mv^2 = m[V_p - V_c]$

42) b

Exp: Mass of section BC =  $m/L (L-y)$ .



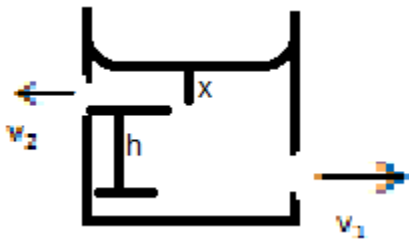
$\therefore$  tension at B =  $T = m/L (L-y)g$ .

$\therefore$  elongation of element  $dy$  at B

$$= dx = (dy) T / AY = m/L (L-y) g \, dy / AY$$

$$\text{Total elongation} = \int dx = mg/LAY \int_0^L (L-y) dy = MgL/2YA$$

43) b



Exp:

Let  $A$  = area of cross – section of the tube,  $\rho$  = density of the liquid.

Consider the section AB of the tube.

Mass of the liquid in AB =  $dAp$ .

Pressure at A and B =  $h_2\rho g$  and  $h_1\rho g$ .

Net force to the right on AB =  $(h_2\rho g - h_1\rho g)A$ .

$$\therefore (h_2 - h_1)\rho g A = (dA\rho)a$$

$$\text{Or } (h_2 - h_1)g = da \quad \text{or } h_2 - h_1 = ad/g.$$

44) a

$$\text{Exp: } \gamma(\text{mixture}) = (n_1 C_{p1} + n_2 C_{p2}) / (n_1 C_{v1} + n_2 C_{v2}) = 1.5$$

$$C_{p1} = 5R/2, \quad C_{v1} = 3R/2$$

$$C_{p2} = 7R/2, \quad C_{v2} = 5R/2, \quad \text{therefore, } n_1 = n_2$$

45) b

Exp: Problems like this are best solved by using their electrical analogues.

For a rod of lengths  $l$ , area of cross-section  $A$  and thermal conductivity  $k$ , we define the thermal resistance as

$$R = l/kA$$

The given situation is like two resistances in series. We define the heat current  $I = (\theta_1 - \theta_2)/R$

As the resistances are in series, they carry the same current. Let  $\theta$  be the temperature of their junction.

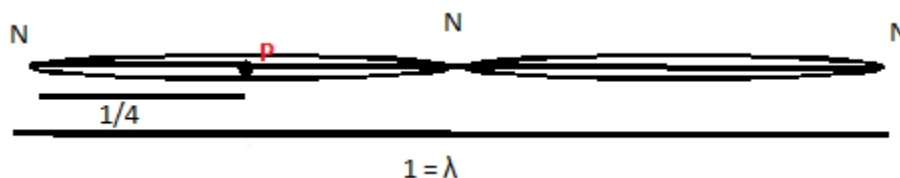
$$I = \theta_1 - \theta / R_1 = \theta - \theta_2 / R_2, \quad \text{where } R_1 = l/k_1 A \text{ and } R_2 = l/k_2 A$$

$$\text{Or } \theta_1 - \theta / \theta - \theta_2 = R_1 / R_2 = k_2 / k_1.$$

Solve for  $\theta$ .

46) d

Exp: In the first case



Point P is an anti node i.e., the string is vibrating in its second harmonic. Let  $f_0$  be the fundamental frequency.

Then

$$2f_0 = 100\text{Hz}$$

$$\therefore f_0 = 50\text{Hz}$$

Now P is an anti node (at length  $l/4$  from one end) so centre should be a node. So, next higher frequency will be sixth harmonic or  $6f_0$  which is equal to 300Hz as shown below:



47)c

Exp: Since current  $I$  is independent of  $R_6$ , it follows that the resistances  $R_1$ ,  $R_2$ ,  $R_3$  and  $R_4$  must form a balanced Wheatstone bridge.

48) d

Exp:  $2\Omega$  resistance is short circuited i.e., potential difference across it is zero. Or current passing through  $2\Omega$  resistance is zero. Further, potential difference across  $4\Omega$  and  $5\Omega$  resistance is 20 V. Therefore, current passing through them will be 5 A and 4A respectively.

49) d

Exp: Voltage across bulb  $B_2$  will be less than across  $B_3$ .  $\Rightarrow W_2 < W_3$ .

50) b

Exp:  $V = E - ir = E - \frac{Er}{R+r}$

$$= E \left[ \frac{R+r-r}{R+r} \right]$$

$$V = \frac{ER}{(R+r)}$$

$$\Rightarrow V = 0 \text{ at } R = 0$$

$$R = \infty, V = E$$

51) Ans: b

Exp:  $Q = Q_0 e^{-t/\tau} = Q_0 / \eta$

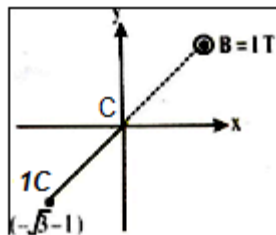
$$\text{Or } e^{-t/\tau} = \frac{1}{\eta} \quad \text{or} \quad t/\tau = \ln \eta$$

52) Ans: c

Exp:  $\frac{\vec{p}_m}{L} = \frac{q}{2m}$

53) Ans: c

Exp: The centre will be at C as shown :



Coordinates of the centre are  $(r \cos 60^\circ, r \sin 60^\circ)$

Where  $r$  = radius of circle

$$= \frac{mv}{Bq} = \frac{1 \times 1}{1 \times 1} = \text{i.e.,} \left( \frac{1}{2}, -\frac{\sqrt{3}}{2} \right)$$

54) Ans: d

Exp:  $F = q[v(-\hat{i})] \times B(\hat{i}) = 0$

Because  $B$  as well as  $v$  is along axis of circular  $CN$

55) Ans: d

Exp: The output D for the given combination

$$D = \overline{(A+B).C} = \overline{(A+B)} + \overline{C}$$

If A = B = C = 0 then

$$D = \overline{(0+0)} + \overline{0} = \overline{0} + \overline{0} = 1+1=1$$

If A = B = 1, C = 0 then

$$D = \overline{(1+1)} + \overline{0} = \overline{0} + \overline{0} = 1+1=1$$

56) Ans: b

Exp: We have,

$$6\pi \eta r v = \frac{4}{3} \pi r^3 p g - \frac{4}{3} \pi r^3 g \sigma$$

Where  $p \rightarrow p_{\text{water}}$  and  $\sigma \rightarrow p_{\text{air}}$

$$\begin{aligned} \Rightarrow \eta &= \frac{2gr^2(p-\sigma)}{9v} \\ &= \frac{2 \times 9.81 \times (0.2 \times 10^{-2})^2 \times 999}{9 \times 8.7} \\ &= 1 \times 10^{-3} \text{ poise} \end{aligned}$$

57)

Ans: c

Exp: Work done



$$\begin{aligned} &= \int_0^{x_0} p_0 A dx + \frac{kx_0^2}{2} \\ &= p_0 A x_0 + \frac{kx_0^2}{2} = 80 + 40 = 120 \text{ J.} \end{aligned}$$

where  $p$  = pressure of gas,

$p_0$  = atmospheric pressure

$k$  = spring constant

$x_0$  = compression in spring

58)

Ans: b

Exp:  $v \propto \frac{1}{n}$ , so the curve is rectangular hyperbola.

59) b

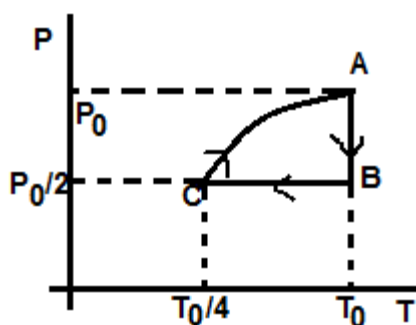
Exp: Clearly the coordinates of A are (2f, 2f)

$$\therefore f = \frac{40}{2}$$

$$= 20\text{cm}$$

60) c

Exp: Process AB is isothermal expansion, BC is isobaric compression and in process CA,  $P \propto V$



### Part – C – Math

61) b)

Exp:  $\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right)$

$$= \tan^{-1} \left( \frac{(x-1)/(x+1) + (2x-1)/(2x+1)}{1 - (x-1)/(x+1) \cdot (2x-1)/(2x+1)} \right)$$

$$= \tan^{-1} \left( \frac{2x^2 - 2x + x - 1 + 2x^2 + 2x - x - 1}{2x^2 + 2x + x + 1 - 2x^2 + 2x + x - 1} \right)$$

$$= \tan^{-1} \left( \frac{4x^2 - 2}{6x} \right) = \tan^{-1} \left( \frac{2x^2 - 1}{3x} \right)$$

But,

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left( \frac{x+y}{1-xy} \right), \text{ only where, } xy < 1$$

$$\text{i.e., } \left( \frac{(x-1)(2x-1)}{(x+1)(2x+1)} \right) < 1$$

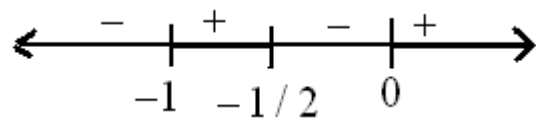
$$\frac{(x-1)(2x-1)}{(x+1)(2x+1)} - 1 < 0$$

$$\frac{(x-1)(2x-1) - (x+1)(2x+1)}{(x+1)(2x+1)} < 0$$

$$\frac{2x^2 - 2x - x + 1 - 2x^2 - 2x - x - 1}{(x+1)(2x+1)} < 0$$

$$\frac{-6x}{(x+1)(2x+1)} < 0$$

$$x((x+1)(2x+1)) > 0$$



Plotting on the number line,

$$\therefore X \in (-1, -1/2) \cup (0, \infty)$$

$$\tan^{-1} 2x^2 - 1/3x = 23/36$$

$$2x^2 - 1/3x = \tan^{-1} 23/36$$

$$12(2x^2 - 1) = 23x$$

$$24x^2 - 12 = 23x$$

$$24x^2 - 23x - 12 = 0$$

$$24x^2 - 32x + 9x - 12 = 0$$

$$8x(3x-4) + 3(3x-4) = 0$$

$$(8x+3)(3x-4) = 0$$

$$x = 4/3, -3/8.$$

But  $x = -3/8$  does not lie in the specified interval.

$$\therefore x = 4/3.$$

62) b

Exp: The determinant is

$$p(qr-bc) + b(ca - ar) + c(ab - aq) = 0$$

$$\therefore pqr + 2abc - pbc - qca - rab = 0$$

Add the following to both sides

$$2pqr - 2abc + 2 pbc + 2 qca + 2qca + 2rab - 2aqr - 2bqr - 2cpq$$

Then

$$\text{LHS} = 3pqr + pbc + qca + rab - 2aqr - 2bqr - 2cpq$$

$$= p(q-b)(r-c) + q(p-q)(r-c) + r(p-a)(q-b)$$

$$\text{RHS} = 2(p-a)(q-b)(r-c)$$

$$\therefore p(q-b)(r-c) + q(p-a)(r-c) + r(p-a)(q-b) = 2(p-a)(q-b)(r-c)$$

Divide by  $(p-a)(q-b)(r-c)$

$$p/p-a + q/q-b + r/r-c = 2.$$

$\therefore$  the value of the required expression is 2.

63) b

Exp:

$$\alpha = e^{2\pi i/7} \Rightarrow \alpha^7 = e^{2\pi i} = 1, \alpha \neq 1$$

$$\text{or } \alpha^7 - 1 = 0$$

$$\text{or } (\alpha - 1)(\alpha^6 + \alpha^5 + \alpha^4 + \dots + 1) = 0$$

$$\Rightarrow 1 + \alpha + \alpha^2 + \dots + \alpha^6 = 0$$

$$= \frac{1 - (\alpha^k)^7}{1 - \alpha^k} = \frac{1 - \alpha^{7k}}{1 - \alpha^k} = \frac{1 - 1}{1 - \alpha^k} = 0$$

where  $k \neq 7m$

$$f(x) = A_0 + \sum_{k=1}^{20} A_k x^k \text{ (given)}$$

Replace  $x$  by  $x, \alpha x, \alpha^2 x, \dots, \alpha^6 x$  in the above and add the seven results thus obtained.

$$\text{L.H.S.} = (A_0 + A_0 + \dots) + \sum_{k=1}^{20} A_k x^k (1 + \alpha^k + \alpha^{2k} + \dots + \alpha^{6k})$$

$$= 7A_0 + 0 \text{ as shown above}$$

$\therefore$  Given expression =  $7A_0$  which is independent of  $\alpha$

64) B

$$\text{Exp: } (1+x+2x^3)(3x^2/2 - 1/3x)^9 = (1+x+2x^3)$$

$$\sum_{r=0}^9 {}^9C_r \left(\frac{3x^2}{2}\right)^{9-r} \left(\frac{-1}{3x}\right)^r$$

$$= \sum_{r=0}^9 (-1)^r {}^9C_r \frac{3^{9-2r}}{2^{9-2r}} x^{18-3r} + \sum_{r=0}^9 (-1)^r {}^9C_2 \frac{3^{9-2r}}{2^{9-r}} x^{19-3r} \\ + 2 \sum_{r=0}^9 (-1)^r {}^9C_r \frac{3^{9-2r}}{2^{9-r}} x^{21-3r}$$

Making the power of the variable x zero one be one, we get  $18 - 3r = 0$  (or)  $r = 6$  in the first series and the corresponding term

$$= {}^9C_6 (3^{-3}/2^3) = {}^9C_6 (1/6^3) = 7/18$$

$$19-3r = 0 \Rightarrow r = 19/3$$

Not an integer in the second series and hence there is no term independent of x in the second series.

$21-3r=0 \Rightarrow r = 7$  in the third series and the corresponding term

$$= 2(-1)^7 {}^9C_7 (3/2)^{9-7} (1/3)^7 = -2, 9.8/1.2 = 1/2^2 1/3^5 = - 2/27$$

Hence the term independent of x =  $7/18 - 2/27 = 21-4/54 = 17/54$

65) D

Exp: If (-10, 2) is an interior point, the circle is more likely to touch  $x - y = 0$  at P(-4, -4) in the 3<sup>rd</sup> quadrant since  $OP = 4\sqrt{2}$ .

The line  $x + y = 0$  meets the circle at A and B such that  $AB = 6\sqrt{2}$ ;  $CM = OP = 4\sqrt{2}$  and  $AM = \frac{1}{2} AB = 3\sqrt{2}$ .

$$CA = \sqrt{CM^2 + AM^2} = 5\sqrt{2} = \text{radius of the circle} = CP.$$

∴ C lies on PC whose equation is  $x+y+8=0$  and on CM whose equation is  $x - y + 10 = 0$ .

This is for the reason that if M be  $(-x, x)$ , then  $\sqrt{(-x)^2 + x^2} = 5\sqrt{2}$  giving  $x = 5$  i.e., M is  $(-5, 5)$ .

$x + y + 8 = 0$  and  $x - y + 10 = 0$  when solved simultaneously fixes the centre at  $(-9, 1)$ .

$$\therefore \text{the equation to the circle is } (x+9)^2 + (y-1)^2 = (5\sqrt{2})^2$$

$$\text{i.e., } \boxed{x^2 + y^2 + 18x - 2y + 32 = 0} \quad \dots\dots(i)$$

Substituting  $x = -10, y = 2$  on the L.H.S of the above equations, we get

$$100 + 4 - 180 - 4 + 32 = -48 < 0.$$

∴  $(-10, 2)$  lies inside the circle. Hence the circle (i) is the required circle since it satisfies all the conditions stated in the problem.

66) C

Exp: Any tangent to the ellipse  $x^2/a^2 + y^2/b^2 = 1$  is  $y = mx \pm \sqrt{a^2m^2 + b^2}$

Let P be  $(h, k)$

$$k = mh \pm \sqrt{a^2m^2 + b^2}$$

$$(k - mh)^2 = a^2m^2 + b^2$$

$$m^2(h^2 - a^2) - 2hkm + (k^2 - b^2) = 0 \quad \dots\dots(i)$$

$$m_1 + m_2 = 2hk/(h^2 - a^2) \text{ and } m_1 m_2 = (k^2 - b^2)/(h^2 - a^2)$$

.....(ii)

If  $\theta_1$  and  $\theta_2$  are the angles of inclination of tangents to the x-axis, then

$$\tan\theta_1 + \tan\theta_2 = 2hk/h^2 - a^2 \text{ and } \tan\theta_1 \tan\theta_2 = (k^2 - b^2)/(h^2 - a^2)$$

.....(iii)

given that  $\tan^2\theta_1 + \tan^2\theta_2 = \lambda$

$$(\tan\theta_1 + \tan\theta_2)^2 - 2 \tan\theta_1 \tan\theta_2 = \lambda$$

$$(2hk/h^2 - a^2) - 2(k^2 - b^2)/(h^2 - a^2) = \lambda \text{ (from (ii))}$$

$$4h^2k^2 - 2(k^2 - b^2)(h^2 - a^2) = \lambda(h^2 - a^2)^2$$

$$2(h^2k^2 + k^2a^2 + h^2b^2 - a^2b^2) = \lambda(h^2 - a^2)^2$$

required locus is  $2(x^2y^2 + a^2y^2 + x^2b^2 - a^2b^2) = \lambda(x^2 - a^2)^2$

67) a

Exp: Let the required line by method  $P + \lambda Q = 0$  be

$\therefore$  Perpendicular from  $(0, 0) = \sqrt{5}$  gives

$$\frac{1}{\sqrt{(1+2\lambda)^2 + (5-3\lambda)^2}} = \sqrt{5}$$

Squaring and simplifying

$$(8\lambda - 7)^2 = 0 \Rightarrow \lambda = 7/8$$

Hence the required line is



$$(x-3y+1)+7/8(2x+5y-9)=0$$

$$\text{or } 22x+11y-55=0 \Rightarrow 2x+y-5=0$$

68) c

Exp:  $f(x) = \left[ \frac{2 \sin x [1 + \cos x] 1 - \cos x}{2 \cos x (1 + \sin x) 1 - \sin x} \right]^{3/2}$

Hence  $x \neq \frac{\pi}{2}$  and  $x \neq \frac{3\pi}{2}$

69) b

Exp: Let  $(\alpha, 3-\alpha)$  be any point on  $x+y=3$

$\therefore$  Equation of chord of contact is  $\alpha x + (3-\alpha)y = 9$   
i.e.,  $\alpha(x-y) + 3y - 9 = 0$

$\therefore$  The chord passes through the point  $(3,3)$  for all values of  $\alpha$ .

70) a

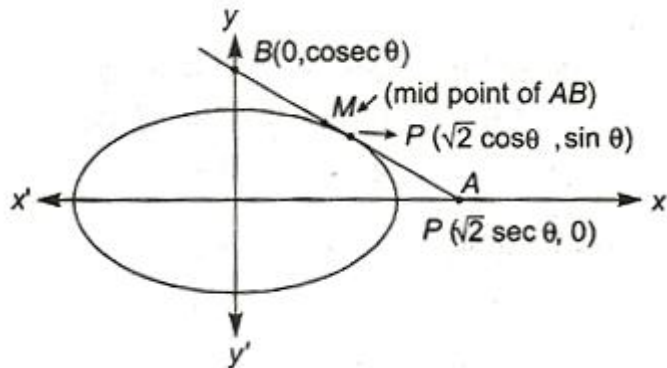
Exp: Let any point be  $P(\sqrt{2} \cos \theta, \sin \theta)$  on

$$\frac{x^2}{2} + \frac{y^2}{1} = 1.,$$

Equation of tangent is,

$$\frac{x\sqrt{2}}{2} \cos \theta + \frac{y}{1} \sin \theta = 1$$

Whose intercept on coordinate axes are  $A(\sqrt{2} \sec \theta, 0)$  and  $B(0, \operatorname{cosec} \theta)$



∴ Mid-point of its intercept between axes is

$$\left( \frac{\sqrt{2}}{2} \sec \theta, \frac{1}{2} \operatorname{cosec} \theta \right) = (h, k) \quad (\text{say})$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}h}$$

And  $\sin \theta = \frac{1}{2k}$

Now,  $\cos^2 \theta + \sin^2 \theta = \frac{1}{2h^2} + \frac{1}{4k^2}$

$$\Rightarrow \frac{1}{2h^2} + \frac{1}{4k^2} = 1$$

Thus, locus of mid-point M is

$$\frac{1}{2x^2} + \frac{1}{4y^2} = 1$$

71) b

Exp:  $g(x) = \int_0^x f(t) dt \Rightarrow g(2) = \int_0^2 f(t) dt$

$$\Rightarrow g(2) = \int_0^1 f(t) dt + \int_1^2 f(t) dt \quad \dots\dots\dots (i)$$

Now,  $\frac{1}{2} \leq f(t) \leq 1$  for  $t \in [0,1]$ , we get

$$\int_0^1 \frac{1}{2} dt \leq \int_0^1 f(t) dt \leq \int_0^1 1 dt$$

(apply line integral inequality)

$$\frac{1}{2} \leq \int_0^1 f(t) dt \leq 1 \quad \dots\dots\dots (ii)$$

Again,  $0 \leq f(t) \leq \frac{1}{2}$  for  $t \in [1,2]$

$$\int_1^2 0 dt \leq \int_1^2 f(t) dt \leq \int_1^2 \frac{1}{2} dt$$

(apply line integral inequality)

$$\Rightarrow 0 \leq \int_1^2 f(t) dt \leq \frac{1}{2} \quad \dots\dots\dots (iii)$$

From Eqs. (ii) and (iii), we get

$$\frac{1}{2} \leq \int_0^1 f(t) dt + \int_1^2 f(t) dt \leq \frac{3}{2}$$

$$\Rightarrow \frac{1}{2} \leq g(2) \leq \frac{3}{2} \quad \text{[From Eq. (i)]}$$

$$\Rightarrow 0 \leq g(2) \leq \frac{3}{2}$$

72) b

Exp: Given,

$$\begin{vmatrix} 1 + \sin^2 \theta & \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & 1 + \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & \cos^2 \theta & 1 + 4 \sin 4\theta \end{vmatrix} = 0$$

Applying  $R_1 \rightarrow R_1 - R_3$ ,  $R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2 \theta & \cos^2 \theta & 1 + 4 \sin 4\theta \end{vmatrix} = 0$$

Applying  $C_3 \rightarrow C_3 + C_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ \sin^2 \theta & \cos^2 \theta & 1 + \sin^2 \theta + 4 \sin 4\theta \end{vmatrix} = 0$$

$$\Rightarrow 1 + \sin^2 \theta + 4 \sin 4\theta + \cos^2 \theta = 0$$

$$\Rightarrow 2 + 4 \sin 4\theta = 0 \Rightarrow \sin 4\theta = -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right)$$

$$\Rightarrow 4\theta = n\pi + (-1)^n \left(-\frac{\pi}{6}\right)$$

$$\Rightarrow \theta = \frac{n\pi}{4} - (-1)^n \frac{\pi}{24}$$

For  $0 \leq \theta \leq \frac{\pi}{2}$ , we have  $\theta = \frac{7\pi}{24}, \frac{11\pi}{24}$ .

73) a

Exp: Here,  $\sin C = \frac{1 - \cos A \cos B}{\sin A \sin B} \leq 1$  ..... (i)

$$\Rightarrow \frac{1 - \cos A \cos B}{\sin A \sin B} - 1 \leq 0$$

$$\Rightarrow 1 - \cos A \cos B - \sin A \sin B \leq 0$$

$$\Rightarrow 1 - \cos (A - B) \leq 0$$

$$\Rightarrow \cos (A - B) \leq 1$$

But  $\cos (A - B)$  cannot be greater than 1

$$\Rightarrow \cos (A - B) = 1$$

$$\Rightarrow A - B = 0$$

$$\Rightarrow A = B$$

$$\Rightarrow \sin C = \frac{1 - \cos^2 A}{\sin^2 A} [\text{from Eq.(i)}]$$

$$= \frac{\sin^2 A}{\sin^2 A} = 1$$

$$\Rightarrow C = 90^\circ$$

$$\Rightarrow A = B = 45^\circ$$

Using sine rule,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\Rightarrow \frac{1}{\sqrt{2a}} = \frac{1}{\sqrt{2b}} = \frac{1}{c}$$

$$\Rightarrow a : b : c = 1 : 1 : \sqrt{2} .$$

74) C

Exp:  $\Rightarrow S_1 S_2 = 2ae$

$$\therefore \text{ordinate of } P \equiv \frac{a}{e} (1 - e^2)$$

$$\Delta PS_1S_2 = \frac{1}{2} \times 2ae \times \frac{a}{e} (1 - e^2)$$

$$\Rightarrow a^2(1 - e^2) \leq \frac{a^2}{2}$$

75) b

Exp: The lines given by  $ax^2 + 5xy + 2y^2 = 0$  are mutually perpendicular if  $a + 2 = 0$

$$\therefore a = -2.$$

76) c

Exp: Let  $(h, k)$  be the coordinates of the midpoint of a chord which subtends a right angle at the origin. Then equation of the chord is

$$kx + ky - 4 = h^2 + k^2 - 4(u \sin g T = S')$$

$$\text{or } hx + ky = h^2 + k^2$$

The combined equation of the pair of lines joining the origin to the points of intersection of  $x^2 + y^2 = 4$  and  $hx + ky = h^2 + k^2$  is

$$x^2 + y^2 - 4 \left( \frac{hx + ky}{h^2 + k^2} \right)^2 = 0$$

Lines given by the above equations of the pair of lines joining the origin to therefore coeff. of  $x^2 + \text{coeff. of } y^2 = 0$

$$\Rightarrow 2(h^2 + k^2) - (4h^2 + 4k^2) = 0 \Rightarrow h^2 + k^2 = 2$$

$$\therefore \text{Locus of } (h, k) \text{ is } x^2 + y^2 = 2$$

77) a

Exp: since the line passing through the focus and perpendicular to the directrix is s x-axis, therefore axis of the required parabola is x-axis. Let the coordinates of the focus be(a, 0). Since the vertex is the midpoint of the line joining the focus and the point (-5, 0) where the directrix  $x + 5 = 0$  meets the axis. Therefore

$$-3 = \frac{a-5}{2} \quad \Rightarrow a = -1$$

Thus, the coordinates of the focus are (-1, 0).

Let P (x, y) be a point on the parabola. Then by definition

$$\sqrt{(x+1)^2 + y^2} = \left(\frac{x+5}{\sqrt{1}}\right)^2 \quad \Rightarrow y^2 = 8(x+3)$$

78) b

$$\text{Let } D = \begin{bmatrix} d_1 & 0 & 0 & \dots & 0 \\ 0 & d_2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & d_n \end{bmatrix}$$

Then  $|D| = d_1 d_2 \dots d_n$

and cofactor of  $D_{11} = d_2 d_3 \dots d_n$

cofactor of  $D_{22} = d_1 d_3 \dots d_n$  etc

co factor of  $D_{ij} = 0$  when  $i \neq j$ :  $\therefore D^{-1} = \frac{1}{|D|} \text{adj}D$

$$= \frac{1}{d_1 d_2 \dots d_n} \begin{bmatrix} d_2 d_3 \dots d_n & 0 & 0 & \dots & 0 \\ 0 & d_2 d_3 \dots d_n & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & d_1 d_2 \dots d_{n-1} \end{bmatrix}$$

Exp:

$$= \begin{bmatrix} \frac{1}{d_1} & 0 & 0 & \dots & 0 \\ 0 & \frac{1}{d_2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \frac{1}{d_n} \end{bmatrix} = \text{diag}(d_1^{-1} d_2^{-1} \dots d_n^{-1})$$

79) c



For the non-trivial solution, we must have

$$\begin{vmatrix} 1 & a & a \\ b & 1 & b \\ c & c & 1 \end{vmatrix} = 0 \text{ or } \begin{vmatrix} 1-a & 0 & a \\ b-1 & 1-b & b \\ 0 & c-1 & 1 \end{vmatrix} = 0$$

Applying  $C_1 \rightarrow C_1 - C_2$ ;  $C_2 \rightarrow C_2 - C_3$

or  $(1-a)[(1-b)-b(c-1)] + a(b-1)(c-1)$ , we get

$$\frac{1}{c-1} + \frac{b}{b-1} + \frac{a}{a-1} = 0$$

Adding 1 on both sides, we get

$$\left(\frac{1}{c-1} + 1\right) + \frac{b}{b-1} + \frac{a}{a-1} = 1$$

$$\text{or } \frac{c}{c-1} + \frac{b}{b-1} + \frac{a}{a-1} = 1$$

Exp:  $\Rightarrow \frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = -1.$

80) b

Exp: Clearly  $AB = \sqrt{2}$ . The given set of lines represent square of side =  $\sqrt{2}$ . So its area =  $(\sqrt{2})^2 = 2$ .

81) a

Exp:  $f(x) = \tan^{-1}x$

$$\therefore f(n) = \frac{1}{n} f\left(\frac{n}{n}\right)$$

$$= \frac{1}{n} f(1) = \frac{\pi}{4n}.$$

82) b

$$\text{Exp: } S = \lim_{n \rightarrow \infty} \sum_{r=0}^{2n-1} \frac{1}{n} \sec^2 \left( \frac{r}{n} \right)$$

$$= \int_0^2 \sec^2 x dx = [\tan x]_0^2$$

$$= \tan 2$$

83) b

Exp: If the given the plane contains the given line then the normal to the plane, must be perpendicular to the line and the condition for the same is

$$al + bm + cn = 0$$

84) a

Exp:  $n(S) = 36$

Let E be the event of getting the sum of digit on the dice equal to 7, then  $n(E) = 6$ .

$$P(E) = \frac{6}{36} = \frac{1}{6} = p, \text{ then } P(E') = q = \frac{5}{6}$$

Probability of not throwing the sum 7 in first m trails =  $q^m$

$$\therefore P(\text{at least one 7 in m throws}) = 1 - q^m = 1 - \left( \frac{5}{6} \right)^m$$

$$1 - \left(\frac{5}{6}\right)^m > 0.95$$

According to the question  $\Rightarrow \left(\frac{5}{6}\right)^m < 0.05$

$$\Rightarrow m \{ \log_{10} 5 - \log_{10} 6 \} < \log_{10} 1 - \log_{10} 20$$

$$\therefore m > 16.44$$

Hence, the least number of trials = 17.

85) b

Exp: The equation of given curve is

$$x + y = e^{xy} \quad \dots\dots\dots (i)$$

On differentiating w.r.t. x, we get

$$1 + \frac{dy}{dx} = e^{xy} \left( y + x \frac{dy}{dx} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{ye^{xy} - 1}{1 - xe^{xy}}$$

Since, tangent is parallel to the y-axis, then

$$\frac{dy}{dx} = \frac{1}{0}$$

$$\Rightarrow 1 - xe^{xy} = 0$$

$$\Rightarrow 1 - x(x + y) = 0 \quad \text{[from Eq. (i)]}$$

This holds for  $x = 1, y = 0$ .

86) c

Exp: Since, sum of the coefficients in the expansion of  $(1 + 2x)^n$  is 6561.

$$\therefore 3^n = 6561$$

$$\Rightarrow 3^n = 3^8$$

$$\Rightarrow n = 8$$

$$\begin{aligned}\therefore \frac{T_{r+1}}{T_r} &= \frac{8+1-r}{r} \times 2x \\ &= \frac{9-r}{r} \times 2 \quad (\because x = 1)\end{aligned}$$

$$\text{Now, } \frac{T_{r+1}}{T_r} > 1$$

$$\Rightarrow 18 - 2r > r \Rightarrow r < 6$$

Thus, 6th and 7th terms are greatest and equal in magnitude.

$$\begin{aligned}\text{Now, } T_6 &= {}^8C_5 (1)^{8-5} (2)^5 \\ &= 1792\end{aligned}$$

87) b

$$P(E_2 / E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

$$\frac{1}{2} = \frac{P(E_1 \cap E_2)}{1/4}$$

Exp:  $\Rightarrow P(E_1 \cap E_2) = \frac{1}{8} = P(E_2) \cdot P(E_1 / E_2)$

$$= P(E_2) \cdot \frac{1}{4} = \frac{1}{8} \Rightarrow P(E_2) = \frac{1}{2}$$

Since  $P(E_1 \cap E_2) = \frac{1}{8} = P(E_1) \cdot P(E_2)$

$\Rightarrow$  Events are independent

Also  $P(E_1 \cup E_2) = \frac{1}{2} + \frac{1}{4} - \frac{1}{8} = \frac{5}{8}$

$\Rightarrow E_1$  &  $E_2$  are non exhaustive

88) b

Exp: Let an equation of the required plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

The plane meets the coordinate axes in

$$A(a, 0, 0), B(0, b, 0) \text{ and } C(0, 0, c).$$

So that the coordinates of the centroid of the triangle ABC are  $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) =$

$$(1, r, r^2) \text{ (given)} \Rightarrow a = 3, b = 3r, c = 3r^2$$

Hence the required equation of the plane is

$$\frac{x}{3} + \frac{y}{3r} + \frac{z}{3r^2} = 1 \text{ or } r^2x + ry + z = 3r^2$$

89) c

$$\int \frac{e^x (1 + nx^{n-1} - x^{2n})}{(1-x^n)\sqrt{1-x^{2n}}} dx$$

Exp:  $= \int e^x \left( \sqrt{\frac{1+x^n}{1-x^n}} + \frac{nx^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}} \right) dx$

$$= \frac{e^x \sqrt{1-x^{2n}}}{1-x^n} + C$$

90) d

Exp: Number of ways = Arrangement of (m - 1) things of one kind and (n-1) things of the other kind =  $\frac{(m+n-2)!}{(m-1)!(n-1)}$