Subject: **CHEMISTRY, MATHEMATICS & PHYSICS**

Paper Name: **JEE_ Main_ Sample Paper - IV**

Duration: 3 hours

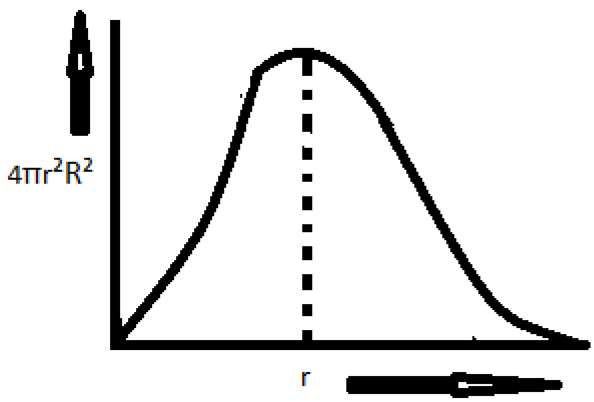
Maximum Marks: 360

Part – A – Chemistry

1

Ans: a

Exp:



2

Ans: d

Exp: Energy of nth Bohr's orbit of H – atom

$$= -\frac{2\pi^2 me^4k^2}{h^2} x \frac{1}{n^2}$$

Energy of first Bohr's orbit of H – atom

$$= -\frac{2\pi^2 me^4k^2}{h^2} = -13.6eV(\text{given})$$

Energy of fourth Bohr's orbit H – atom

$$= \frac{2\pi^2 me^4k^2}{h^2} x \frac{1}{4^2} = 13.6x\frac{1}{16} eV = -0.85eV$$

P.E. of electron in nth orbit = 2 x E_n

So P.E. of electron in 4^{th} orbit = 2 x (-0.85) = - 1.70eV

3)

Ans: b

Exp: The hybridization of the atomic orbital of nitrogen in NO_2^+, NO_3^-, NH_4^+ aresp, sp^2, sp^3 respectively.

4

Ans: a

Exp: Correct answer is (a), since HClO is weakest acid among HClO, $HClO_2$, $HClO_3$ and $HClO_4$.

5

Ans: c

Exp: Initial volume in both section = 22.4liter

The gas in sec B is compressed reversibility & adiabatically

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$
$$\Rightarrow T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow 27.3x(8)^{2/3} \Rightarrow 4T_1$$

Final pressure

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)x\left(\frac{T_2}{T_1}\right) \Rightarrow 1x8x4 \Rightarrow 32atm$$

∴ Final temperature in section A

$$\Rightarrow 0.1x\frac{3}{2}Rx(1638-27.3) = 483.21cal$$

6

Ans: b

Exp:

$$Dy(s) + \frac{3}{2}Cl_2(g) \rightleftharpoons DyCl_3(s)$$
$$\Delta H_f^o = -994.30 kJmol^{-1} = x$$
$$DyCl_3(s) \xrightarrow{aq.HCl} DyCl_3(aq.in 4.0M\ HCl)$$
$$\Delta H_f^o = -180.06 kJmol^{-1}$$
$$3HCl(aq.4M) \longrightarrow \frac{3}{2}H_2(g) + \frac{3}{2}Cl_2(g)$$
$$\Delta H_f^o = 3x158.31 KJmol^{-1}$$

$$Dy(s) + 3HCl \rightleftharpoons DyCl_3 + \frac{3}{2}H_2(g)$$
$$(aq., 4M) \qquad (aq., in\ MHCl)$$
$$\Delta H = -699.43 kJmol^{-1} = x$$

7

Ans: a

Exp: Potassium dichromate dissociates with evolution of O_2 on heating.

$$4K_2Cr_2O_7 \longrightarrow 4K_2CrO_4 + 2Cr_2O_3 + 3O_2$$
So, X is Cr_2O_3

8

Ans: a

Exp:

$$Cr_2O_7^{2-} + 6Fe^{2+} + 14H^+ \longrightarrow 2Cr^{3+} + 6Fe^{3+} + 7H_2O$$
Here $Cr_2O_7^{2-}$ is reduced to Cr^{3+}

9

Ans: c

Exp: $2No + Cl_2 \longrightarrow 2NOCl$

Let the order w.r.t. NO is x.

Let the order w.r.t. Cl_2 is y.

$(rate) = k[NO]^x [Cl_2]^y$

$= K\ 2^{x+y} [NO]^x [Cl_2]^y$

$2^{x+y} = 8$

$x+y = 3.$

10

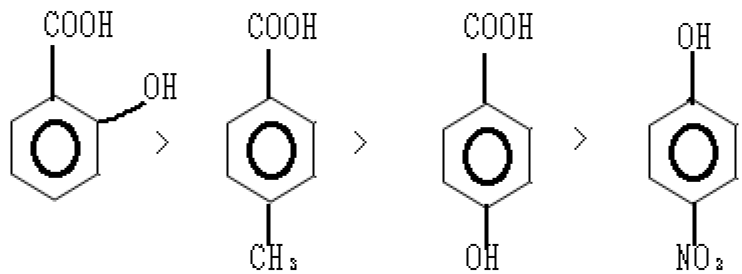
Ans: b

Exp: NA

11

Ans: c

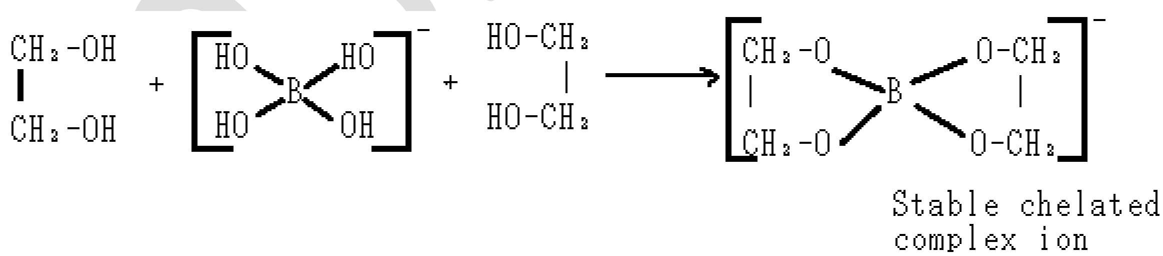
Exp: o-Hydroxybenzoic acid is strongest acid and the decreasing order of acidity is



12

Ans: b

Exp: cis-1,2-diol forms chelated complex ion with the product, $[B(OH)_4]^-$ causing the reaction to proceed in forward direction.



13

Ans: a

Exp: NA

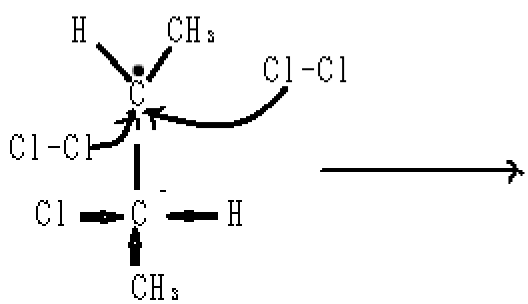
14

Ans: c

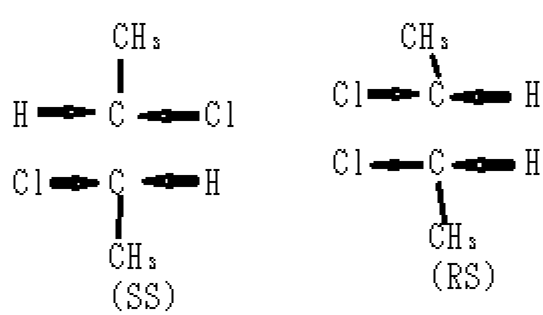
Exp: NA

15

Ans: d



Exp:



16

Ans: d

Exp: Packing efficiency

$$= \frac{\text{Area occupied by effective circles}}{\text{Area of square}}$$

$$= \frac{2\pi r^2}{L^2} x100 = \frac{2\pi r^2}{2\left(2\sqrt{2}r\right)^2} x100 = \frac{\pi}{4} x100 = 78.54\%$$

17

Ans: a

$$E_{Br^-/AgBr/Ag} = E^o_{Ag/Ag} + \frac{0.059}{1}\log K_{SP}$$

$$= E^o_{Ag^+/Ag} - 0.7257$$

$$\text{and } E^o_{Cl^-/AgCl/Ag} = E^o_{Ag^+/Ag} + \frac{0.059}{1}\log K_{SP}$$

$$= E^o_{Ag^+/Ag} - 0.59$$

Exp: Now cell reaction is

$$Ag + Br^- \longrightarrow AgBr + le^-$$

$$AgCl + le^- \longrightarrow Ag + Cl^-$$

$$Br^- + AgCl \xrightarrow{le^-} Cl^- + AgBr$$

$$0 = \left(0.7257 - 0.59\right) + \frac{0.059}{1}\log\frac{\left[Br^-\right]}{\left[Cl^-\right]} \Rightarrow \frac{\left[Br^-\right]}{\left[Cl^-\right]} = 0.005$$

18

Ans: d

Exp:
$$1.5x10^{-4}\,x\,\frac{1}{100} = e^{-E_a/RT} = 1.5x10^{-6} = e^{-E_a/Rx300}$$
$$\therefore E_a = 33.43\,kJ\,mole^{-1};$$
$$Using\ k = e^{-E_a/RT}, we\ get$$
$$\frac{dk}{dT} = \frac{E_a}{RT^2}k$$
$$k = 0.2x\frac{8.314x325x325}{33.43x1000} = 8.75x10^{-2}\,min^{-1}$$

19

Ans: c

Exp: As Sb_2S_3 is a negative sol, so $Al_2(SO_4)_3$ will be the most effective coagulant the to higher positive charge on Al (Al^{3+}) – **Hardy –Schulze rule.**

20

Ans: d

Exp: NA

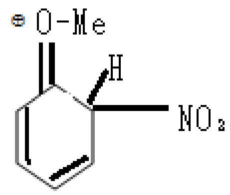
21

Ans: d

Exp: NA

22

Ans: b

Exp: 

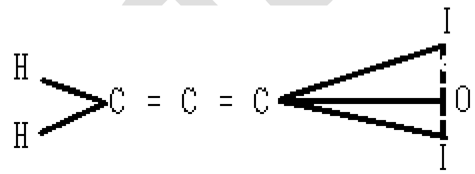
: In this structure, every atom (except, of course, H) has a stable octet of electrons. So it is the most stable.

23

Ans: c

Exp: $CH_2 = C = C = Cl_2$ has sp^2 – hybridized carbon and thus ICI bond angle is 120^o.

$\therefore$ IO/CI = sin 60^o



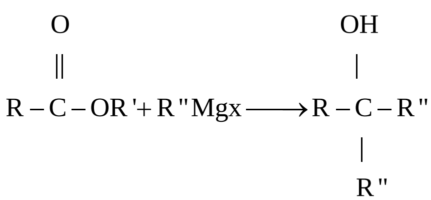
[In ΔICO, ∠ICO = 60^o and ∠IOC = 90^o or IO = CI sin 60^o = 2.10 x 0.866 = 1.8186 Å

$\therefore$ I – I distance = 2 x 1.8186 = 3.64 Å

24

Ans: a

Exp: Recall that, esters react with excess of Grignard reagents to form 3^{o} alcohols having at least two identical alkyl groups corresponding to Grignard reagent.

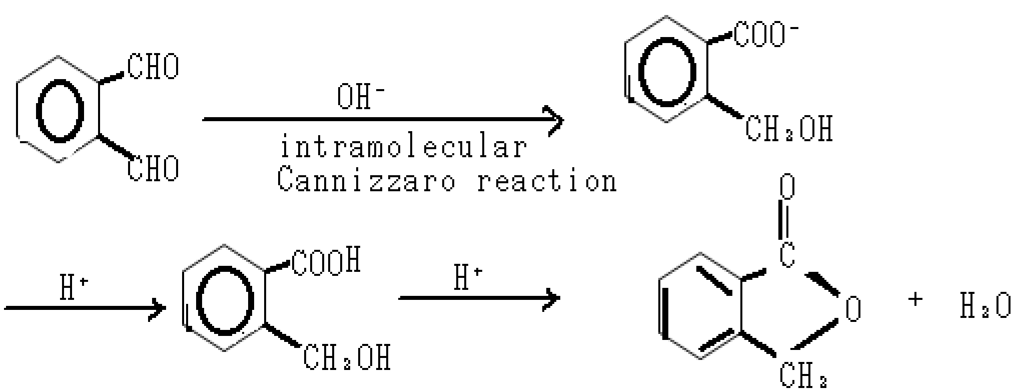


Since here Grignard reagent is CH_3MgBr, the 3^{o} alcohol should have at least two methyl groups Thus, the choice with at least two methyl groups at the carbon linked with – OH group will be the correct choice. Hence (a) is the correct choice.

25

Ans: d

Exp:



26

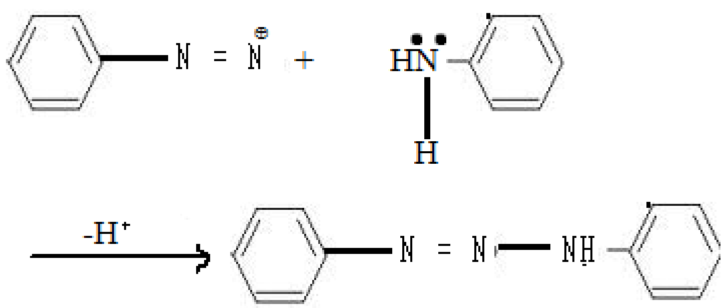
Ans: c

Exp: Only primary aromatic amines undergo diazotization followed by coupling.

27

Ans: c

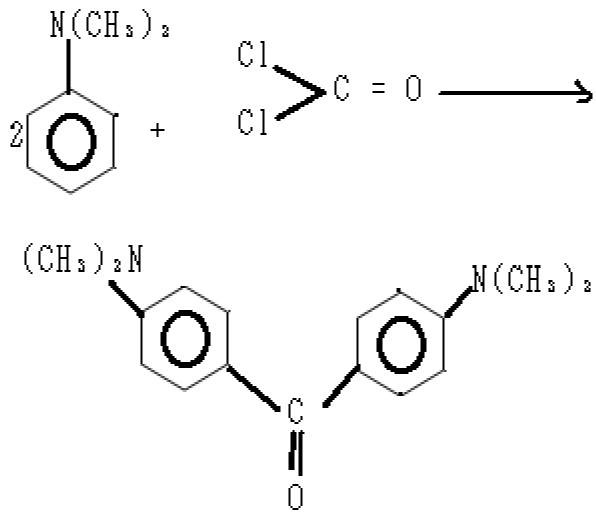
Exp: Diazoniumcation reacts with aniline in weakly acidic medium resulting in N, N-coupling rather than C – coupling



28

Ans: c

Exp: N, N – Dimethylaniline condenses with carbonyl chloride to form benzophenone derivatives



29

Ans: a

Exp: NA

30

Ans: d

Exp: NA

Part – B – Physics

31

Ans: b

Exp: RD = (weight in air)/(loss of weight in water)

$\rho = 5.00/1.00 = 5.00$

$d\rho/\rho = 0.05/5.00 + 0.1/1.00 = 0.11$ or 11%

$\therefore \quad \rho = 5.00 \pm 11\%$

32

Ans: b

Exp: Particle will strike the point B if velocity of particle with respect to platform is along AB or component of its relative velocity along AD is zero. i.e.,

$u\cos\theta = v$

or $\quad \theta = \cos^{-1}(v/u)$

33)

Ans: a

Exp: It needs to cross maximum height. So at maximum height it is just at rest relative to wedge.

$mu = 2mv, \quad \frac{1}{2} mu^2 = \frac{1}{2} \times 2mv^2 + mgH$

34

Ans: c

Exp:
$$\frac{a_1+a_2}{a_1-a_2}=5 \Rightarrow a_1+a_2=5(a_1-a_2)$$
$$\frac{a_1}{a_2}=\frac{3}{2};\frac{I_1}{I_2}=\left(\frac{a_1}{a_2}\right)^2=\frac{9}{4}$$

35

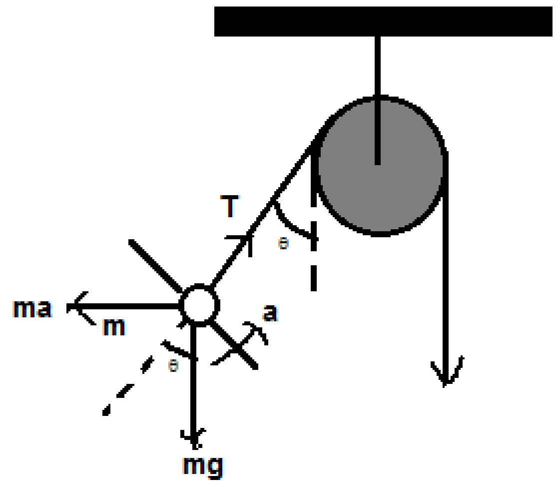
Ans: d

Exp: W=MB ($\cos\theta_1-\cos\theta_2$)

$$\therefore w_1=MB(\cos 0^\circ-\cos 60^\circ)=\frac{MB}{2}$$
$$W_2=MB(\cos 30^\circ-\cos 90^\circ)=\frac{\sqrt{3}MB}{2}$$
$$\therefore W_2=\sqrt{3}W_1$$

36

Ans: c



Exp:

(Force diagram in the frame of the car)

As frame is non-inertial we have to apply a pseudo force ma towards left on the mass

Applying Newton's law perpendicular to string

mg sin θ = ma cos θ , tan θ = a/g

applying Newton's second law on the string, then

$T - m\sqrt{(g^2 + a^2)} = ma$ or $T = m\sqrt{(g^2+a^2)} + ma$

37

Ans: a

Exp:

$$T\cos\theta + N = mg \qquad ....(1)$$

$$\text{and } T\sin\theta = m\omega^2 r \qquad ....(2)$$

$$\text{but} \qquad T = Kx$$

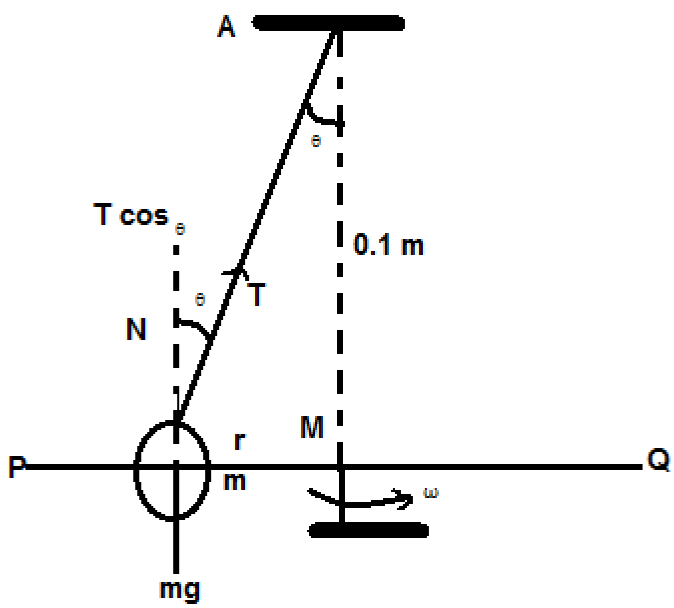
$$T = 1.47X10^2\,(0.1\sec\theta - 0.1)\left(k = 1.47x10^2\,N/m\right)$$

$$\text{Also, } r = 0.1\tan\theta$$

$$\text{put } \; T, r, m \text{ and } \omega \text{ and } \omega \text{ in eq.}(2)$$

$$\text{We have } \cos\theta = \frac{3}{5}$$

$$\text{and } T = 9.8N$$



38

Ans: C

Exp: Suppose r_1 be the distance of centre of mass of the remaining portion from centre of the bigger circle, then

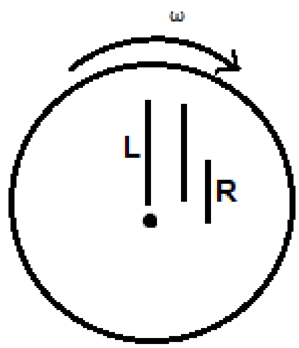
$$A_1 r_1 = A_2 r_2$$

$$r_1 = \left(\frac{A_2}{A_1}\right) r_2 = \frac{\pi(42)^2}{\pi\left[(56)^2 - (42)^2\right]} x7 = 9cm$$

39

Ans: a

Exp: Moment of inertia of the rod w.r.t. the axis through centre of the disc is: (by parallel axis theorem).



$I = mL^2/12 + mR^2$

& K.E. of rod w.r.t. disc = $\frac{1}{2} I\omega^2$

$= \frac{1}{2} m\omega^2[R^2+L^2/12]$

40

Ans: c

Exp: In horizontal direction

$$\text{Net force} = \frac{G\sqrt{3}mm}{12d^2}\cos 30° - \frac{Gm^2}{4d^2}\cos 60°$$

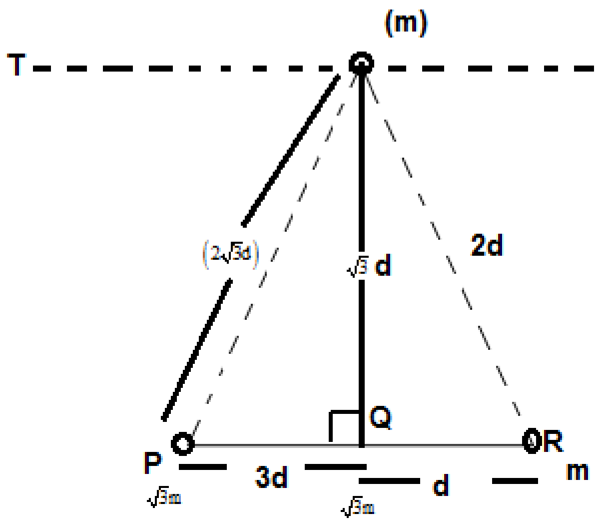
$$= \frac{Gm^2}{8d^2} - \frac{Gm^2}{8d^2} = 0$$

In vertical direction

$$\text{Net force} = \frac{G\sqrt{3}m^2}{12d^2}\cos 60° + \frac{G\sqrt{3}m^2}{3d^2} + \frac{Gm^2}{4d^2}\cos 30°$$

$$= \frac{\sqrt{3}Gm^2}{24d^2} + \frac{\sqrt{3}Gm^2}{3d^2} + \frac{\sqrt{3}Gm^2}{8d^2}$$

$$= \frac{\sqrt{3}Gm^2}{d^2}\left[\frac{1+8+3}{24}\right] = \frac{\sqrt{3}Gm^2}{2d^2} \text{ along SQ}$$



41

Ans: a

$$\omega = \sqrt{\frac{k}{m}}$$

General equation of motion,

$x = 2e\cos\omega t$

$e = 2e\cos\omega t$

Exp: $\omega t = \frac{\pi}{3}$

$\sqrt{\frac{k}{m}}t = \frac{\pi}{3}$

$\therefore \quad t = \frac{\pi}{3}\sqrt{\frac{m}{k}}$

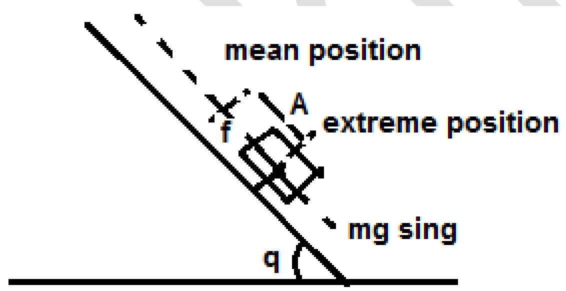
$\therefore \quad T = 2t = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$

42

Ans: c

Exp: The maximum static frictional force is

f = μmg cosθ = 2tanθ mg cos θ = 2mg sin θ



Applying Newton's second law to the block at lower extreme position

$f - mg\sin\theta = m\omega^2 A$, $f = m\omega^2 A + mg\sin\theta$

$\omega^2 A = g\sin\theta$ or $A = (3mg\sin\theta)/k$

43

Ans: b

Exp:
$f = \frac{c}{4l}$

$\therefore \quad \left|\frac{df}{dt}\right| = \frac{c}{4l^2}\left|\frac{dl}{dt}\right| = \frac{cv}{4l^2}$

44

Ans: A

Exp: Particle velocity v_p = -v(slope of y-x graph)

Here, v = + ve, as the wave is travelling in positive x-direction.

x-direction.

Slope at P is negative.

∴ Velocity of particle is in positive y (or $\hat{j}$) direction.

∴ Correct option is (a).

45

Ans: a

Exp:
$f \propto v \propto \sqrt{T}$

$f_{AB} = 2f_{CD}$

$\therefore \quad T_{AB} = 4T_{CD} \quad ....(i)$

Further $\sum \tau_p = 0$

$\therefore \quad T_{AB}(x) = T_{CD}(l - x)$

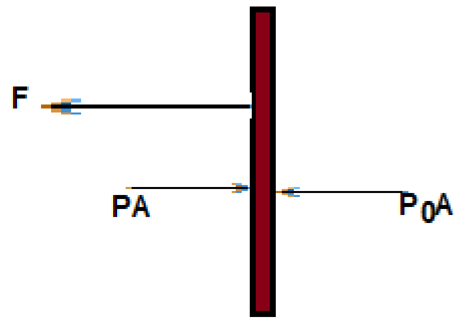
or $\quad 4x = l - x \quad (T_{AB} = 4T_{CD})$

or $\quad x = l/5$

46

Ans: b

Exp: Volume of the gas is constant V = constant



∴ $P \propto T$

i.e., pressure will be doubled if temperature is doubled

∴ $P = 2P_0$

Now let F be the tension in the wire. Then equilibrium of any one piston gives

$F = (P - P_0)A = (2P_0 - P_0)A = P_0A$

47

Ans: b

Exp: $y = \sin\left[\pi\left(\frac{t}{5} - \frac{x}{9}\right) + \frac{\pi}{6}\right]$ ......(1)

y= A sin $\left[2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)+\varnothing\right]$ ....(2)

From (1) and (2) $\frac{2\pi}{\lambda}=\frac{\pi}{9}\rightarrow\lambda=18cm$

48

Ans: d

Exp: Since tension is the two rods will be same, hence

$A_1Y_1\ \alpha_1\Delta\theta = A_2Y_2\alpha_2\Delta\theta$

$\Rightarrow A_1Y_1\alpha_1 = A_1Y_2\alpha_2$.

49

Ans: d

Exp: ω = λD/d

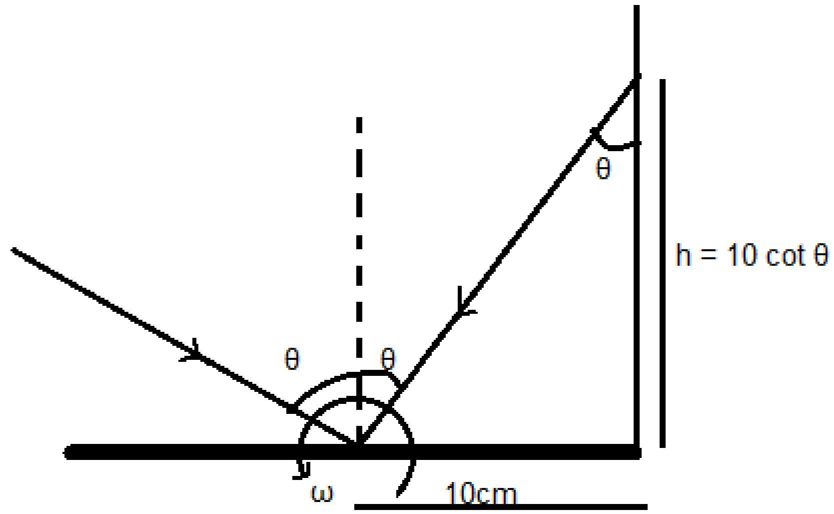
d is halved and D is doubled

∴ Fringe width ω will become four times.

50

Ans: b

Exp: When mirror is rotated with angular speed ω, the reflected ray rotates with angular speed 2ω (=36rad/s)



Speed of the spot = |dh/dt| = |d/dt (10cotθ)|

= |-10sec^{2}θ dθ/dt| = |- 10/(0.6)^2 x 36| = 1000m/s.

51

Ans: a

Exp: (n_A-1/1) 2/R_A = (n_B-1/1) 2/R_B or 0.63/R_A = n_B-1/R_B or n_B = 1.7

52

Ans: b

Exp: Z = $\sqrt{R^2+\left(X_L-X_C\right)^2}=\sqrt{25^2+12^2}=\sqrt{625+144}$

Z = 27.7 Ω

53)

Ans: a

Exp: $V_1 = V_2 = V_3$, $I_1R_1 = I_2R_2 = I_3R_3$, $R_1 = R_2 = R_3$

$\therefore I_1 = I_2 = I_3$

54)

Ans: d

Exp: In this question, the diode is forward biased but its forward bias resistance is not given, but as it is very small, we can get the approximate result. i = $\frac{V_s}{(R+r_f)}$.

where r_f is forward bias resistance.

$\therefore \quad i < \frac{9}{1000}$ = 9 mA.

i has to be slightly less than 9 mA.

55

Ans: a

Exp: Initial velocities of A and B are zero. So, initial relative velocity = 0, ie, u_{AB} = 0 ie, $a_A > a_B$ from graph. So, $a_{AB} = a_A - a_B = a$ = positive quantity

:. $v_{AB} = u_{AB} + a_{AB} \times t = at$,

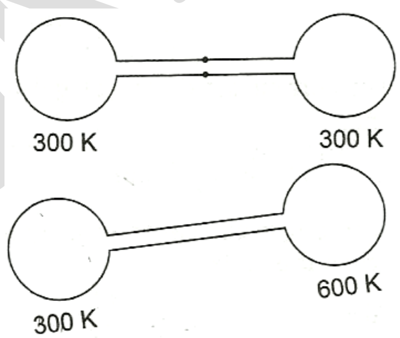
which is positive and continuously increasing with time.

56

Ans: c

Exp: Initial pressure, $P_1 = P_2 = P_0 = \frac{nRT_0}{V_0}$ when

P_0 = 1 atm, T_0 = 300 K.



Finally, let common pressure is p.

So, $\frac{pV_0}{R \times 2T_0} = n_1$

and $\frac{pV_0}{RT_0} = n_2$

and $n_1 + n_2 = 2n$

$$\frac{pV_0}{2RT_0} + \frac{pV_0}{RT_0} = 2\left(\frac{p_0V_0}{RT_0}\right)$$

$\Rightarrow$ $p = \frac{4}{3}\ p_0 = \frac{4}{3}$ atm

57)

Ans: b

Exp: The component of F_1 along the direction of the displacement is $F_1\cos 30° = (40\ N) \times (0.866)$

= 34.6 N

Hence, the work done by F_1 is (34.6 N) (0.80 m) = 28 J. F_2 does no work because, it has no component in the direction of the displacement.

The component of F_3 in the direction of the displacement is 30 N. Hence, the work done by F_3 is (30 N) (0.80 m) = 24 J.

58)

Ans: c

Exp: The centre will be at C as shown :

Coordinates of the centre are (r $\cos 60^\circ$, r sin 60°)

Where r = radius of circle

$$= \frac{mv}{Bq} = \frac{1X1}{1X1} = 1 i.e., \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

59)

Ans: c

Exp: $\frac{\vec{p}_m}{\vec{L}} = \frac{q}{2m}$

60)

Ans: c

Exp: By conservation of momentum, the momentum of the block bullet system just after the interaction is p = mv where m is the mass of bullet and v is its velocity before striking the block.

Hence, the kinetic energy of the system just after the lodging of bullet into the block is

$$K = \frac{p^2}{2(M+m)} = \frac{(mv)^2}{2(M+m)}$$

The frictional force does work $W_f = - f \times s = -\mu_k(m + M)\ gs$ in stopping the block where s is the distance traversed by block-bullet system on the table top.

From work-energy theorem,

$\Delta K = W_f$

$0 - K = -\mu_k (m+M)gs$

$\therefore \quad s = 0.37$ m

Part – C – Math

61)

Ans: d

Exp: $2^y = 2-2^x \Rightarrow y \log_2 (2-2^x)$ we must have $2-2^x > 0$

$\Rightarrow 2^x < 2 \Rightarrow x < 1$ or $x \in (-\infty, 1)$

62)is not equal to right hand limit

Ans: d

$$\lim_{x\to 0}\frac{\sqrt{1-\cos 2(x-1)}}{x-1}=\lim_{x\to 1}\frac{\sqrt{2\sin^2(x-1)}}{(x-1)}$$

$$\lim_{x\to 1}\sqrt{2}\frac{|\sin(x-1)|}{x-1} \qquad \left[\text{Note that } \sqrt{x^2}=|x|\right]$$

$$\text{L.H.L.}\to\lim_{x\to 0}\frac{\sqrt{2}\,|\sin\{(1-h)-1\}|}{\{(1-h)-1\}}$$

Exp: $$\lim_{h\to 0}\frac{\sqrt{2}\,|\sinh|}{-h}$$

$$=\lim_{h\to 0}\frac{\sqrt{2}(\sinh)}{(-h)}=-\sqrt{2}$$

$$\text{R.H.L.}\to\lim_{h\to 0}\frac{\sqrt{2}\,|\sin\{(1+h)-1\}|}{\{(1+h)-1\}}$$

$$=\lim_{h\to 0}\frac{\sqrt{2}\,|\sinh|}{h}=\sqrt{2}$$

$\therefore$ L.H.L. $\neq$ R.H.L. and hence option (d) is correct

63)

Ans: c

Exp: By trail, the two curves intersect at (π/2 , 1). The values of dy/dx to the two curves at (π/2, 1) are equal to 1/π and 0 respectively.

$$\Rightarrow \tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{\frac{1}{\pi}-0}{1+0}\right|=\frac{1}{\pi}$$

64)

Ans: d

Exp: Point of intersection of both curves can be O(0, 0) and P(4a, 4a).

Required area A is given by

$_0\int^{4a} (f(x) - g(x))\, dx$,

Where $f(x) = \sqrt{(4ax)}$ and $g(x) = x^2/4a$

$A = \int^{4a}_0 (\sqrt{(4ax)} - x^2/4a)dx = 16a^2/3\ (\text{unit})^2$

Hence choice (d) is correct.

65)

Ans: c

Exp: The equation can be written as

$2(x^2-4x) + 3(y^2-6y) = k - 35$

Or $2(x-2)^2 + 3(y-3)^2 - k$,

(a), (d) If $k > 0$, this represents and ellipse

$\Rightarrow$Choice (a) and (d) are wrong.

(b) If $k < 0$, then its represents no locus

$\Rightarrow$Choice (b) is wrong.

(c) If $k = 0$, then its represents a point (2, 3).

66)

Ans: c

Exp: Angle between two circles is defined as angle between the tangents drawn at the point of intersection which should be further same as angle between corresponding normals.

Since normals pass through centres we have

$\cos \theta = (r^2_1 + r^2_2 - d^2)/2r^2_1r^2_2$

67)

Ans: c

Exp: NA

68)

Ans: c

Exp: $R \times (P^C \cup Q^C)^C = R \times [(P^C)^C \cap (Q^C)^C]$

$= R \times (P \cap Q)$

$= (R \times P) \cap (R \times Q)$

69)

Ans: a

Exp: NA

70)

Ans: b

Exp: NA

71)

Ans: c

Exp: NA

72)

Ans: b

Exp: Suppose $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ represents a seven digit number. Then a_1 takes the value 1, 2, 3, …., 9. If we keep $a_1, a_2, a_3, \ldots, a_6$ fixed, then the sum $a_1 + a_2 + a_3 + \ldots + a_6$ is either even or odd. Since a_7 takes 10 values 0, 1, 2, ……, 9, five of the numbers so formed will be even and 5odd.

Hence the required number of numbers

$= 9 \times 10 \times 10 \times 10 \times 10 \times 10 \times 5$

$= 45 \times 10^5$

73)

Ans: b

Exp: NA

74)

Ans: b

Exp: NA

75)

Ans: d

Exp: $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \Rightarrow X^2 = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

Clearly for n = 2 the matrices in (a), (b), (c) do not match with $\begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

76)

Ans: b

Exp: NA

77)

Ans: a

$$\text{We have } \tan\frac{C}{2} = \tan\left(90^0 - \frac{A+B}{2}\right)$$

$$\cot\frac{A+B}{2} = \frac{\cot(A/2)\cot(B/2)-1}{\cot(A/2)+\cot(B/2)}$$

$$= \frac{\frac{6}{5}.\frac{37}{20}-1}{\frac{6}{5}+\frac{37}{20}} = \frac{222-100}{120+185} = \frac{122}{305} = \frac{2}{5}$$

Exp: $\text{Also } \tan\frac{A}{2}\tan\frac{C}{2}$

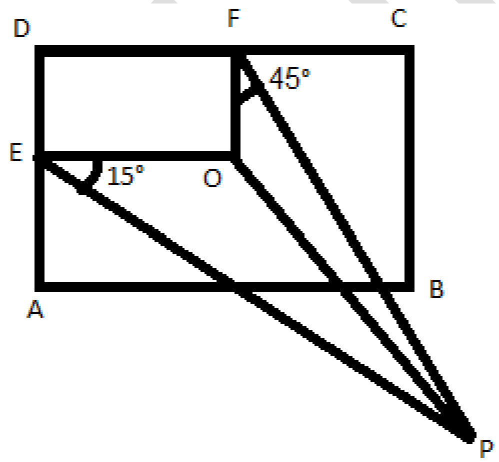
$$= \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$\Rightarrow \frac{5}{6}.\frac{2}{5} = \frac{s-b}{s} \Rightarrow 3(s-b) = s \Rightarrow 2s = 3b$$

$$\Rightarrow a+b+c = 3b \Rightarrow a+c = 2b$$

78)

Ans: c



Exp:

Let OP be the flagstaff of height 'h' standing at the centre O of the rectangular field ABCD subtending angles 15^o and 45^o at E the F, the midpoints of the sides AD and DC of the field (see fig.), then

OE = h cot 15^o = h $(2 + \sqrt{3})$

And OF = h cot45° = h

$$\Rightarrow EF = h\sqrt{1^2 + \left(2+\sqrt{3}\right)^2} = 2h\sqrt{2+\sqrt{3}}$$

$$\Rightarrow 1200 = AC = 2EF = 4h\sqrt{\left(2+\sqrt{3}\right)}$$

$$\Rightarrow h = \frac{300}{\sqrt{2+\sqrt{3}}} = 300\sqrt{2-\sqrt{3}}$$

79)

Ans: b

Exp:

$$|a\omega^2 + b + c\omega|^2 = \frac{1}{2}\left(\left(a-b\right)^2 + \left(b-c\right)^2 + \left(c-a\right)^2\right)$$

For minimum $b - c = -2, a - b = 2$ and $c - a = 4$

$$\Rightarrow |a\omega^2 + b + c\omega| = 2\sqrt{3}$$

80)

Ans: a

Exp: Given planes are $ax + by = 0$ ....(i)

and $z = 0$ ....(ii)

∴ Equation of any plane passing through the line of intersection of planes (i) and (ii) may be taken as

ax+ by + λz = 0 ....(iii)

The direction cosines of a normal to the plane (iii) are

$$\frac{a}{\sqrt{a^2+b^2+\lambda^2}}, \frac{b}{\sqrt{a^2+b^2+\lambda^2}}, \frac{\lambda}{\sqrt{a^2+b^2+\lambda^2}}$$

The direction cosines of a normal to the plane (i) are

$$\frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}}, 0$$

Since the angle between the plane (i) and (iii) is α,

$$\therefore \quad \cos\alpha = \frac{a.a+b.b+\lambda.0}{\sqrt{a^2+b^2+\lambda^2}\sqrt{a^2+b^2}} = \sqrt{\frac{a^2+b^2}{a^2+b^2+\lambda^2}}$$

$$\Rightarrow \lambda^2\cos^2\alpha^2 = a^2\left(1-\cos^2\alpha\right)+b^2\left(1-\cos^2\alpha\right)$$

$$\Rightarrow \lambda^2 = \frac{\left(a^2+b^2\right)\sin^2\alpha}{\cos^2\alpha}$$

$$\Rightarrow \lambda = \pm\sqrt{a^2+b^2}\tan\alpha = 0$$

Putting the value of λ in (iii we get, eq. of plane as

ax + by ± z √(a^2+b^2) tan α = 0

81)

Ans: a

Exp: We know that a number is divisible by 3 if the sum of its digits is divisibly by 3.

Now out of 0, 1, 2, 3, 4, 5 if we take 1, 2, 3, 4, 5 or 0, 1, 2, 4, 5 then the 5 digit numbers will be divisible by 3.

Case – I: Number of 5 digit numbers formed using the digits 1, 2, 3, 4, 5 = 5! = 120

Case – II: Taking 0, 1, 2, 4, 5 if we make 5 digit number then

I place can be filled in 4 ways (0 cannot come at I place)

II place can be filled in 3 ways

III place can be filled in 2 ways

IV place can be filled in 1 ways

V place can be filled in 1 ways

∴ Total numbers = 4 x 4! = 96

Thus total numbers divisible by 3 are = 120+96 = 216

82)

Ans: c

Exp:

Let

$$I=\int\frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}dx=\int\frac{\left(\cos^2 x+\cos^4 x\right)\cos x}{\sin^2 x\left(1+\sin^2 x\right)}$$

$$=\int\frac{\left[1-\sin^2 x+\left(1-\sin^2 x\right)^2\right]\cos x}{\sin^2 x\left(1+\sin^2 x\right)}dx$$

$$=\int\frac{\left(2-3\sin^2 x+\sin^4 x\right)\cos x}{\sin^2 x\left(1+\sin^2 x\right)}dx$$

Put $\sin x=t\Rightarrow \cos dx=dt$

$$I=\int\frac{2-3t^2+t^4}{t^4+t^2}dt$$

$$I=\int\left(1+\frac{2}{t^2}-\frac{6}{t^2+1}\right)dt=t-\frac{2}{t}-6\tan^{-1}(t)+C$$

$$=\sin x-2(\sin x)^{-1}-6\tan^{-1}(\sin x)+C$$

83)

Ans: a

$y^2 = t; 2y\frac{dy}{dx} = \frac{dt}{dx}$; hence the differential equation

becomes $\left(e^{x^2} + e^t\right)\frac{dt}{dx} + 2e^{x^2}\left(xt - x\right) = 0$

$e^{x^2} + e^t + 2e^{x^2}.x\left(t-1\right)\frac{dx}{dt} = 0$

put $e^{x^2} = z; e^{x^2}.2x\frac{dx}{dt} = \frac{dz}{dt}$

$z + e^t + \frac{dz}{dt}\left(t-1\right) = 0$

Exp: $\frac{dz}{dt} + \frac{z}{t-1} = -\frac{e^t}{t-1}$

$I.F. = e^{\int \frac{dt}{t-1}} = e^{\ln(t-1)} = t-1$

$\therefore z\left(t-1\right) = -\int\left(e^t\right)dt$

$z\left(t-1\right) = -e^t + C$

$e^{x^2}\left(y^2 - 1\right) = -e^{y^2} + C$

$e^{x^2}\left(y^2 - 1\right) + e^{y^2} = C$

84)

Ans: a

Exp: $\vec{A}.\left(\vec{B}+\vec{C}\right)x\left(\vec{A}+\vec{B}+\vec{C}\right)$ is scalar triple product of three vectors namely $\vec{A}, \vec{B}+\vec{C}$ and $\vec{A}+\vec{B}+\vec{C}$

Clearly first cross product will take place and then dot product

Hence the given product

$$= \vec{A}.\left[\vec{B}x\vec{A} + \vec{B}x\vec{B} + \vec{B}x\vec{C} + \vec{C}x\vec{A} + \vec{C}x\vec{B} + \vec{C}x\vec{C}\right]$$

$$= \vec{A}.\vec{B}x\vec{A} + \vec{A}.\vec{B}x\vec{C} + \vec{A}.\vec{C} + \vec{A}.\vec{C}x\vec{A} + \vec{A}.\vec{C}x\vec{B}$$

$$\left(\text{Using } \vec{a}x\vec{a} = 0\right)$$

$$= 0 + \left[\vec{A}\vec{B}\vec{C}\right] + 0 + \left[\vec{A}\vec{C}\vec{B}\right]$$

(as $\left[|\vec{a}\vec{b}\vec{c}|\right]=0$ if any two vector are equal out of $\vec{a},\vec{b},\vec{c}$)

$=\left[\vec{A}\vec{B}\vec{C}\right]-\left[\vec{A}\vec{B}\vec{C}\right]\ \left[\text{Using}\left[\vec{a}\vec{b}\vec{c}\right]=-\left[\vec{a}\vec{c}\vec{b}\right]\right]$

$=0$

85)

Ans: b

Exp:

$$P(w/E)=\frac{P(w\cap E)}{P(E)}$$

$$=\frac{\frac{1}{n}x\frac{2}{n+1}+\frac{1}{n}x\frac{4}{n+1}+\frac{1}{n}x\frac{6}{n+1}+...+\frac{1}{n}x\frac{n}{n+1}}{\frac{1}{n}+\frac{1}{n}+...+\frac{1}{n}\left(\frac{n}{2}\text{times}\right)}$$

$$=\frac{\frac{1}{n}x\frac{2}{n+1}\left[1+2+3....+\frac{n}{2}\right]}{\frac{1}{n}x\frac{n}{2}}(\text{n being even})$$

$$=\frac{2}{n(n+1)}\left[\frac{n}{2}\left(\frac{n}{2}+1\right)\right]$$

$$=\frac{n+2}{2(n+1)}$$

86)

Ans: A

Exp: The equation to the common chord of the two given circle sis

$2x^2 + 2y^2 + 8x + 4y - 7 - 2(x^2 + y^2 - 8x - 4y - 5) = 0$

i.e., $24x+12y+3=0$ i.e., $8x+4y+1=0$

any circle through the points of intersection of this line and the circle

$x^2 + y^2 - 8x - 4y - 5 = 0$, is of the form

$x^2 + y^2 - 8x - 4y - 5 + \lambda (8x + 4y + 1) = 0$

This meets x = 7, at points whose ordinates are given by

$49 + y^2 - 56 - 4y - 5 + \lambda (56 + 4y + 1) = 0$

i.e., $y^2 + 4(\lambda - 1) y 57\lambda - 12 = 0$

This quadratic has equal roots since x = 7 is to be a tangent.

$16(\lambda - 1)^2 - 4(57\lambda - 12) = 0$

i.e., $4\lambda^2 - 65\lambda + 16 = 0 \Rightarrow (4\lambda - 1)(\lambda - 16) = 0$

$\lambda = ¼; \lambda = 16$

for $\lambda = ¼$ one circle is $x^2 + y^2 - 8x - 4y - 5 + ¼ (8x + 4y + 1) = 0$

i.e., $\boxed{x^2 + y^2 + 18x - 2y + 32 = 0}$

for $\lambda = 16$, 2nd circle is $x^2 + y^2 - 8x - 4y - 5 + 16(8x + 4y + 1) = 0$

i.e., $\boxed{x^2 + y^2 + 120x - 60y + 11 = 0}$

87)

Ans: a

Exp: Let the required line by method $P+\lambda Q=0$ be

$\therefore$ Perpendicular from (0, 0) = $\sqrt{5}$ gives

$$\frac{1}{\sqrt{(1+2\lambda)^2+(5-3\lambda)^2}}=\sqrt{5}$$

Squaring and simplifying

$$(8\lambda-7)^2=0\Rightarrow\lambda=7/8$$

Hence the required line is

$$(x-3y+1)+7/8(2x+5y-9)=0$$
$$or\,22x+11y-55=0\Rightarrow 2x+y-5=0$$

88)

Ans: d

Exp: NA

89)

Ans: b

Exp: Let $(\alpha,3-\alpha)$ be any point on x+y=3

$\therefore$ equation of chord of contact is $\alpha x+(3-\alpha)y=9$
$i.e.,\alpha(x-y)+3y-9=0$

$\therefore$ the chord passes through the point (3,3) for all values of α.

90)

Ans: c

Exp: NA