Practice Paper 6-Solutions

ANSWER KEY

PART-1 (PHYSICS)

SECTION I

1.(a) **2.**(b) **3.**(d) **4.**(b) **5.**(c) **6.**(a) **7.**(c) **8.**(a) **9.**(a)

SECTION II

10.(d) **11.**(c) **12.**(a) **13.**(d)

SECTION III

14.(c) **15.**(b) **16.**(b) **17.**(b) **18.**(b) **19.**(d)

SECTION IV

20. (A) →(Q), (B) →(P),(C) →(Q),(D) →(P) **21.** (A) →(Q), (B) →(P),(C) →(S),(D) →(R)

22. (A) →(S), (R) → (P),(C) →(P),(D) →(Q)

PART –II (CHEMISTRY)

Section I

23.(a) **24.**(b) **25.**(b) **26.**(b) **27.**(b) **28.**(b) **29.**(d) **30.**(b) **31.**(b)

Section II

32.(a) **33.**(a) **34.**(d)

Section III

36.(c) **37.**(c) **38.**(b) **39.**(a) **40.**(a) **41.**(c)

Section IV

42. (A) →(Q), (B) →(R),(C) →(S),(D) →(P) **43.** (A) →(Q), (B) →(R),(C) →(S),(D) →(P)

44. (A) →(S), (B) →(R),(C) →(Q),(D) →(P)

PART –III (MATHEMATICS)

Section I

45.(b) **46.**(b) **47.**(b) **48.**(a) **49.**(c) **50.**(c) **51.**(c) **52.**(a) **53.**(b)

Section II

54.(c) **55.**(a) **56.**(a) **57.**(a)

Section III

58.(a) **59.**(c) **60.**(c) **61.**(c) **62.**(b) **63.**(b)

Section IV

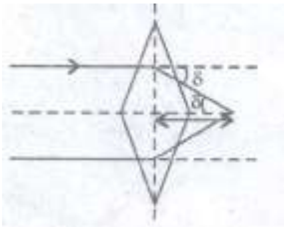
64. (A) →(S), (B) →(P),(C) →(Q),(D) →(R) **65.** (A) →(Q), (B) →(S),(C) →(Q),(D) →(R)

66. (A) →(R), (B) →(S),(C) →(P),(D) →(Q)

Solutions

Sol.1 (d)

Explanation:



$$tan\delta = \frac{h}{f}$$

δ=deviation suffered by small angled prism= (μ−1)A For small deviation $tan\delta = \delta$

Or $\delta = \frac{h}{f} = (\mu - 1)A$

$\Rightarrow f = \frac{h}{(\mu-1)A}$

Choices (a),(b) and (c)are wrong.

Sol.2 (b)

Explanation:

The $\mathbf{K_{\alpha\ line}}$ characteristics of an element is produced due to transition from L-shell (n_2=2) to the K-shell (n_1=1). Thus

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$\Rightarrow \frac{1}{\lambda} = \frac{3}{4}RZ^2$

Or $\frac{Z^2 4}{3R\lambda} = \frac{4}{3\times(1.0973\times10^7)\times(0.76\times10^{-10})}$

Or $Z^2 = 1599.25 \cong 1600 \Rightarrow$ or Z $= 40$

Choices (a),(c)and (d) are wrong.

Sol. 3 (d)

Explanation:

Change in electic field strength

$$\Delta V = \frac{\Delta V}{h}$$

But change in potential between thunder cloude and earth's surface is given by

$$\Delta V = \frac{\Delta Q}{C} = \frac{it}{C}$$

Hence, $\Delta E = \frac{\Delta V}{h} = \frac{it\backslash C}{h} = \frac{it}{Ch}$

This implies that electric field strength E has reduced by $\frac{it}{Ch}$.

Choices (a),(b) and (c) are wrong.

Sol.4 (b)

Explanation:

$$\vec{F}_{AOB} = \vec{F}_{AB} = i(\vec{\imath} \times \vec{B})$$

y- Co-ordinate of A is $y^2 = 2x = 2 \times 2 = 4$

$$y = 2 \ or \ AC = 2m.$$

Also BC=2m.

So AB=4m.

$$\vec{F}_{AB} = 2[(-4\hat{\jmath}) \times (-4\hat{k})] = 32\hat{\imath}$$

Sol.5 (c)

Explanation:

Let us consider dN number of turns of radius r and thickness dr. IF dE is the corresponding induced emf,then

$$|dE| = dN \frac{d}{dt} (\pi r^2 B) = dN\pi r^2 \frac{d}{dt}(B_0 \sin \omega t)$$

$$= \left(\frac{drN}{a}\right) \pi r^2 B_0 \omega \cos \omega t$$

$$|dE| = \frac{NrB_0\omega\cos\omega t}{a}r^2dr$$

Net induced emf

$$E = |dE| = \frac{NrB_0\omega\cos\omega t}{a}\int_0^a r^2dr$$

$$E = \frac{NrB_0\omega\cos\omega t}{a}.\frac{a^3}{3}$$

$$\Rightarrow E = \frac{1}{3}\pi Na^2B_0\omega\cos\omega t$$

So, amplitude of induced emf

$$Amp = \frac{1}{3}\pi Na^2B_0\omega$$

Choices (a),(b)and (d)are wrong.

Sol.6 (a)

Explanation:

For collision of bodies, the unit vector along relative Velocity is equal to unit vector along relative displacement

$$\hat{V}_{relative} = \frac{\vec{V}_1 - \vec{V}_2}{|\vec{V}_1 - \vec{V}_2|}$$

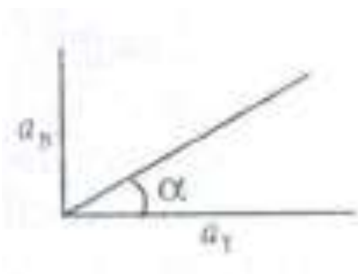
$$\hat{r} = \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}$$

For collision

$$\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \frac{\vec{V}_1 - \vec{V}_2}{|\vec{V}_1 - \vec{V}_2|}$$

<u>Sol.7</u> (c)

Explanation:



$V = a\sqrt{s}$

Tangential acceleration $a_r = \frac{a^s}{R}$

$\tan\alpha = \frac{a_N}{a_r} = \frac{a^2 s}{Ra^2} \times 2$

$\alpha = \tan^{-1}\left(\frac{2s}{R}\right)$

Choices (a),(b) and (d)are wrong.

<u>Sol.8</u> (a)

Explanation:

Thrust force depends on acceleration due to effect gravity. In freely condition effective i.e., net acceleration is zero.

<u>Sol.9</u> (a)

Explanation:

Change in momentum of bullet= mev-(-mv)

$F = \frac{dP}{dt} = nmv(1+e)$ for n bullets.

This force will balance the weight of plate.

nmv(1+e)=Mg

so $M = \frac{nmv(1+e)}{g}$

<u>Sol.10</u> (d)

Explanation:

Assertion and Reason are both wrong.

From conservation of linear momentum

$m_1u + m_2 \times 0 = m_1v_1 + m_2v_2$ … (1)

$e = \frac{v_2 - v_1}{u - 0}$ … (2)

$m_1v_1 + m_2v_2 = m_1u$

$-v_1m_1 + v_2m_1 = um1$

$v_2 = \frac{2m_1u}{m_1 + m_2}$

$\Rightarrow \quad v_2 = \frac{2u}{1 + \frac{m_2}{m_1}}$

v_2 is maximum, when $\frac{m_2}{m_1} \to 0$

i.e., $m_1 \gg m_2$

<u>Sol.11</u> (c)

Explanation:

Assertion is true but reason is false. From conservation of angular momentum

$I\omega = I\omega' + mvR$

$\frac{mR^2}{2}\omega = \frac{mR^2}{2}\omega' + mR^2\omega'$

$\Rightarrow \quad \omega' = \frac{\omega}{3}$, So Reason is wrong.

<u>Sol.12</u> (a)

Explanation:

According to low of vector addition $\int \overrightarrow{dt}$ is equal to the length vector $\overrightarrow{l'}$, joining initial to final points. So both assertion and Reason are true and Reason explains the assertion.

Sol.13 **(d)**

Explanation:

Ionisation energy of hydrogen atom.

$$E_H = -\frac{\delta e^4}{8\varepsilon_0^2 h^2} \cdot \frac{mM_H}{m + M_H}\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$$

Ionisation energy of deuterium atom

$$E_D = \frac{e^4}{8\varepsilon_0^2 h^2} \cdot \frac{mM_H}{m + M_H}\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$$

Now $E_D - E_H = 3.68 \times 10^{-3}$

Assertion is false but Reason is true.

Sol.14 **(c)**

Explanation:

Energy incident on the sphere

$$=4\pi R^2 . \frac{P}{4\pi r^2} - \frac{pR^2}{r^2}$$

Where P=Power, R=Radius of sphere , r=Distance of source

Choices (a) and (b) are dimensionally wrong.

Choice (c) is proved correct.

So, choice (d) is wrong.

Sol.15 **(b)**

Explanation:

Energy $=E = \frac{hc}{\lambda}$ $\quad\quad \therefore \lambda = \frac{hc}{E}$

It is not dependent on the number of photons or time. So , choice (b) is correct and others are wrong.

Sol.16 **(b)**

Explanation:

As the sphere release electrons, it will acquire net positive charge

$$Q = \frac{N}{t} e$$

Since potential $V = \frac{KQ}{R}$

Potential at any time t' is given by

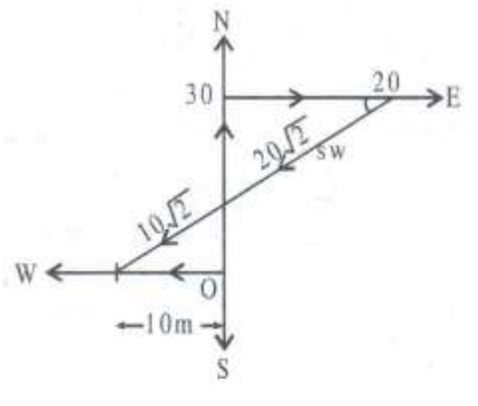
$$V = \frac{K}{R}\frac{N}{t} et'. V\alpha t'$$

So, choice (b) is correct and the rest of the choices are wrong.

Sol.17 **(b)**

Explanation:

As shown in diagram, displacement will be 10m. West. So choice (b) is correct. Common mistake is to draw SW intercepting at O, leading to choice (d).



Sol.18 **(b)**

Explanation:

For larger angular displacements, vector law of addition is not satisfied. So it will be scalar.

Torque of rotation is along the axis-Axial vector.

So, choice (b) is correct and rest are wrong.

<u>Sol.19</u> (d)

Explanation:

On reaching the horizontal position,

$$\frac{1}{2}mu^2 = mgl + \frac{1}{2}mv^2$$

$\Rightarrow \qquad v^2 = u^2 - 2gl$

$$v = \sqrt{u^2 - 2gl}$$

$v \perp$u. So change

$$= \sqrt{v^2 + u^2} = \sqrt{(u^2 - 2gl) + u^2} = \sqrt{2u^2 - 2gl}$$

Many students find (v-u). So choice (d) is correct and rest are wrong.

<u>Sol.20</u> (A)→(Q), (B)→(P),(C)→(Q), (D)→(P)

Explanation:

Least count of an ammeter is current required to produce unit deflection in needle of the meter while the range is the current required to produce full scale deflection in needle of the meter. Hence, range and least count are directly proportional to each other.

Hence, (A) → (Q)

Least count of an ammeter is current required to produce unit division deflection in the meter needle of the meter while sensitivity is number of divisions through which the needle is defected due to unit current. Hence, least count and sensitivity are inversely related to each other.

So,(B) → (P)

If ammeter has large resistance, then resistance of the circuit will be altered significantly. Hence, current will also be altered. To decrease this alteration, a very small resistance called shunt is connected in parallel with galvanometer. Hence, by using shunt, accuracy of the meter increases.

Range means the maximum current which can be measured by ammeter. On decreasing shunt resistance, more fraction of current passed through shunt, therefore, to produce full scale deflection in galvanometer, more current is required in the circuit. Hence, range increases.

This means range and accuracy carry linear relation.

So, (C) → (Q)

Let resistance of galvanometer be G and a resistance R be connected in its series to convert it into a voltmeter. If current I flows through the device then potential difference across galvanometer is equal to iG while potential difference across the voltmeter is equal to $i(G + R)$.

The Multiplication factor K $=\frac{i(G+R)}{iG} = \frac{G+R}{G}$

Let i_0= maximum current that can flow through galvanometer, then range is equal to

$i_0G + i_0R = i_0(R + G)$

Or range α K.

Hence, (D) → (P)

<u>Sol.21</u>

(A) →(Q), (B) →(P),(C) →(S),(D) →(R)

Explanation:

$V_A - i \times 2 + 10 - i \times 1 = V_B$

$V_A - V_B = 2i + i - 10 = -1$

$3i = 9$

Or $i = 3A$

So, (A) →(Q)

Let the current through 1Ω resistor be i_1 and that through cell is $i_2$then

$i = i_1 + i_2$

Also, $i_1 \times 1 = 6 + 2i_2 = 6 + 2(i - i_1)$

$i_1 = 6 + 2(3 - i_1 = 6 + 6 - 2i_1)$

$3i_1 = 12$

Or $i_1 = 4A$

Hence, (B)→ (P).

$V_c - 2 \times 3 - 3 - 3 \times 1 = V_D$

$V_C - V_D = 6 + 3 + 3 = 12V$

Hence,(D) →(R).

Or $V_A - 2 \times 3 + 10 - 3 \times 1 - 3 \times 3$

$$-4 \times 1 - 2 \times 3 - 3 - 3 \times 1 = V_D$$

$$V_A - 6 + 10 - 3 - 9 - 4 - 6 - 3 - 3 = V_D$$

$$V_A - 24 = V_D$$

Or $$V_A - V_D = 24V$$

Hence,(C) →(S).

Sol.22

(A) →(S), (B) →(R),(C) →(P),(D) →(Q)

Explanation: As a projectile is projected with a velocity V at an angle ‘θ’, the horizontal component of velocity V and θ will remain the same throughout and V Sin θ –the vertical component decreases to zero and comes back to the same value on reaching the target point. Minimum velocity will be at the highest point.

So,(C) →(P)

Change in magnitude of momentum is present in vertical direction only. Between the target and the point of throw, it is absent.

So, (A) →(S)

Magnitude of change in momentum between the initial and target point will be 2mVsin θ. However between any pair of points magnitude will vary in vertical direction only.

So, (D) →(Q)

Angular momentum about the point of throw as the projectile is at the highest point is

$$L = mVcos\,\theta \times h\_max$$

$$= \frac{mVcos\,\theta . V^2 \sin^2\theta}{2g} = \frac{mV^3 \sin^2\theta \cos\theta}{2g}$$

So, (B) →(R)

CHEMISTRY

Sol.23 (a)

Explanation:

(a) $pH = \frac{1}{2}(pK_w + pK_a - pK_b)$

$$= \frac{1}{2}(14 + 4 - 5) = \frac{13}{2} = 6.5$$

(b) is possible only if $K_a = K_b, i.e., pK_a = pK_b$

(c) and (d) are possible only if $K_b > K_a i.e., pK_b < pK_a$

Sol.24 (b)

Explanation:

$N_0 = 100N$

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{2.303}{t}\log\frac{N_0}{N}$$

$$\frac{0.693}{60} = \frac{2.303}{t}\log\frac{100N}{N}$$

$$\frac{0.693}{60} = \frac{2.303}{t}log100$$

$\Rightarrow \quad t = \frac{2.303\times60\times2}{0.693}$=398 days

Hence, choice (b) is correct. While (a), (c) and (d) are incorrect.

Sol.25 (b)

Explanation:

The structure of $C_3O_2 is\ O = \overset{+2}{\underset{1}{C}} = \underset{2}{C} = \overset{+2}{\underset{3}{C}}$

So each carbon has different oxidation states.

The C_1 and C_3 are in +2 oxidation state but as average oxidation number is asked so

$\Rightarrow 3x + (-2)2 = 0$

$x = +\frac{4}{3}$

For $M_{g_2}C_3$ $M_g = +2$

$\Rightarrow (+2)2 + 3x = 0$ $x = -\frac{4}{3}$

Hence choice (b) is correct, it implies (a),(c)and (d) are wrong.

Sol. 26 (b)

Explanation:

In (b) but -2-ene is CH_3−CH=CH-CH_3 the number of hypercojugation possible are 7. Hence it is the most stable.

In (b) but -1-ene is CH3CH_2−CH=CH_2 the numbers of hypercojugation are 3.

In (c) Pent-1 ene CH_3−CH_2− CH_2−CH=CH_2 the numbers of hypercojugation are 3.

In (d) Ethene there are no hyperconjugation.

Hence But -2-ene is the most stable so choice (b) is correct while (a),(c) and (d) are wrong.

Sol. 27 (b)

Explanation:

η= Efficiency of engine=$\frac{T_1 - T_2}{T_1}$

Where T_1= Temperature of source

T_2= Temperature of sink

$\Rightarrow \eta = \frac{400-300}{400} = \frac{1}{4} = 0.25.$

Hence choice (b) is correct while (a),(c) and (d) are false.

Sol. 28 **(b)**

Explanation:

(b) is correct answer as explained below, (a),(c) and (d) are ruled out.

$$t_{99}\% = \frac{2.303}{k}\log\frac{[A]_0}{\frac{1}{100}[A]_0} = \frac{4.606}{k}$$

$$t_{99}\% = \frac{2.303}{k}\log\frac{[A]_0}{\frac{1}{100}[A]_0} = \frac{2.303}{k}$$

$$t_{99}\% = 2t_{90}\% = 2 \times 100 = 200 min.$$

Sol. 29 **(d)**

Explanation:

(a) $Fe + H_2SO_4$ (dil) → $FeSO_4 + H_2$

(b) $Mg + 2HNO_3$ (5%) → $Mg(NO_3)_2 + H_2$

(c) $Sn + 2HCl$ (dil) → $SnCl_2 + H_2$

(d) $Cu + 2H_2SO_4$ (Conc) → $CuSO_4 + SO_2 + 2H_2O$

Because conc. H_2SO_4 is an oxidizing agent.

Sol. 30(b)

Explanation:

Gold number is defined as the number of milligrams of a lyophilic colloid that will just prevent the precipitation of 10ml of a gold sol an adding 1 ml of 10% NaCl Solution. Lower is the gold number, higher is the proactive power.

Hence, from the values given, the ascending order of gold number is Q<P<R<S.

Since Q has the lowest gold number and S has the highest gold number, thus protective power is Q<P<R<S.

Hence, choice (b) is correct while (a),(c) and (d) are incorrect.

Sol. 31(b)

Explanation: I^- is oxidized to IO_3^- in alkaline medium and I_2 in acidic medium,∴ (a) is not correct. (c) and (d) are not formed in alkaline medium.

Sol. 32 **(a)**

Explanation:

A is true, R is true and R is the correct explanation of A.

Sol. 33 **(a)**

Explanation:

A is true, R is true and R is the correct explanation of A.

Sol. 34 **(d)**

Explanation:

A is false but R is true.

Phenol is stronger acid than ethanol, therefore, phenoxide ion is weaker base than $C_2H_5O^{\ominus}$ Secondly. Phenyl group is electron withdrawing and also due to resonance effect. It will become weaker base.

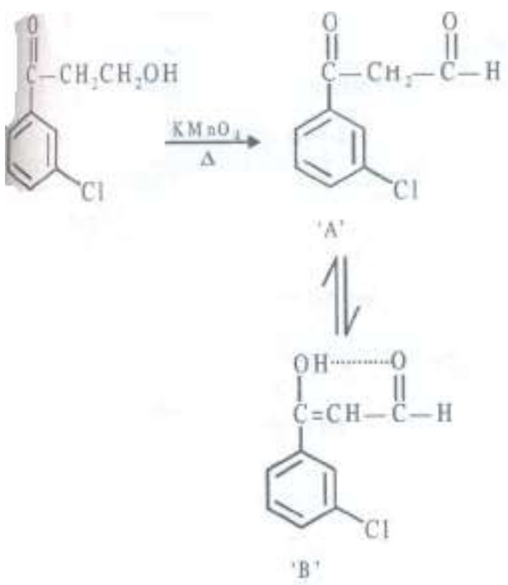
Sol. 35 **(a)**

Explanation:

A is true, R is true and R is the correct explanation of A.

Sol. 36 **(c)**

Explanation:



‘B’ is more stable because it is stabilized by intermolecular H-bonding.

(a),(b) and (d) are ruled out.

Sol. 37 (c)

Explanation:

(c) is correct answer as explained above.

Sol. 38 (b)

Explanation:

Will give violet colour with neutral $FeCl_3$ because enol form give violet complex with neutral $FeCl_3$.

(a),(b) and (d) are ruled out.

Sol. 39 (a)

Explanation:

The half-life of ${}_1^3H$ is 12.32 years, therefore, it is used for determining age of old samples of water and wine.

Sol. 40 (a)

Explanation:

Hydrogen is acting as oxidizing agent because sodium is better reducing agent. ‘H’ is gaining electron forming H^- ion.

Sol. 41 (c)

Explanation:

${}_1^3H$ is radioactive because $\frac{n}{p} = 2$, therefore, it is unstable.

Sol. 42 (A) →(Q), (B) →(R),(C) →(S),(D) →(P)

Explanation:

(A) → (Q): Fusion mixure is $K_2CO_3+Na_2CO_3$

(B) → (R): Diborane (B_2H_6) is a good reducing agent.

(C) → (S): The electronic configuration of NO is

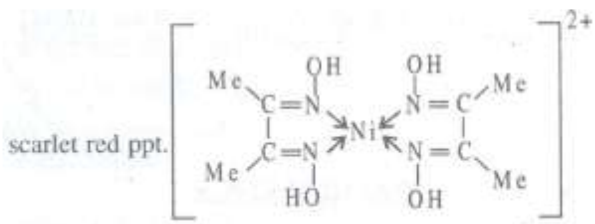
$(KK)\sigma 2s^2\sigma^*2s^2\sigma 2p_x^2\pi 2p_y^2\pi 2p_z^2\pi^*2p_y^1$. Hence it is paramagnetic and colourless.

(D) → (P): Producer gas is $CO+N_2$

<u>Sol. 43</u> (A) → (Q); (B) →(R);(C) →(S);(D) →(P)

Explanation:

(A) → (Q); $Ni^{2+} + DMG \rightarrow [Ni(DMG)_2]^{2+}$ which is



(B) → (R); Co^{2+} (NH_4) SCN $\xrightarrow{Acetone}$ $[Co(SCN)_4]^{2-}$

Which is blue coloured ppt.

(C) → (S); $Ba^{2+} + CrO_4^{2} \rightarrow BaCrO_4$

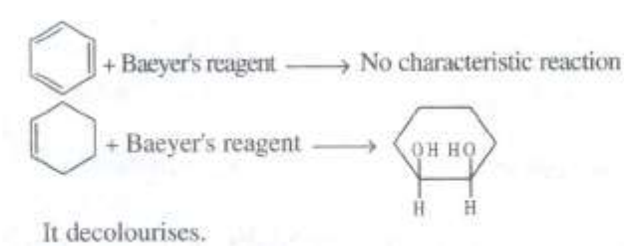
yellow ppt.

(D) → (P); $Ca^{2+} + C_2O_4^{2-} \rightarrow CaC2O_4$.

<u>Sol. 44</u> (A) → (S); (B) →(R);(C) →(Q);(D) →(P)

Explanation:

(A) → (S); Beayer's reagent is used to distinguish between benzene and cyclohexene.



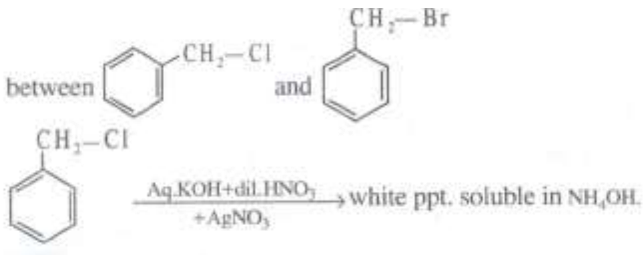
(B) → (R); Ammoniacal Cu_2Cl_2 is used to distinguish between but -1-yne and but -2-yne.

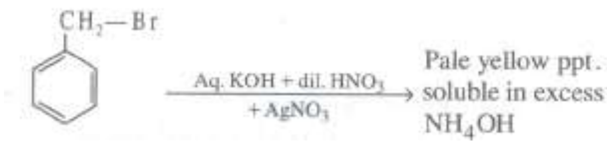
$CH_3CH_2{-}C{\equiv}CH \xrightarrow{Ammoniacal} CH_3CH_2{-}C{\equiv}CCU$.

$CH_3{-}C{\equiv}C{-}CH \xrightarrow[Cu_2Cl_2]{Ammoniacal}$ No reaction.

(C) → (Q);

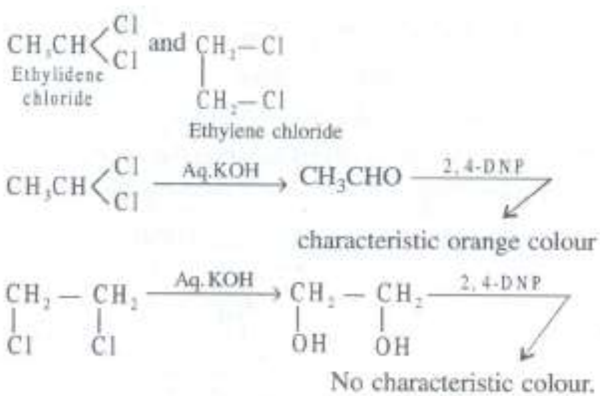
Aq.KOH +dil. HNO_3 + $AgNO_3$ is used to distinguish





(D) → (P);

Aq.KOH +2, 4-DNP is used to distinguish between



MATHEMATICS

Sol. 45 (b)

Explanation:

f (x) + f (y) = log $\frac{1+x}{1-x}$ + log $\frac{1+y}{1-y}$

= log $\frac{(1+x)(1+y)}{(1-x)(1-y)}$ = log $\frac{1+(x+y)+xy}{1-(x+y)+xy}$

= log $\left(\frac{1+\frac{x+y}{1+xy}}{1-\frac{x+y}{1+xy}}\right)$ = f $\left(\frac{x+y}{1+xy}\right)$

<u>Sol. 46</u> (b)

Explanation:

$81^{\sin^2 x} + (81)^{1-\sin^2 x} = 30$

Put $81^{\sin^2 x} = t \Rightarrow t + \frac{81}{t} = 30$

$\Rightarrow t^2 - 30t + 81 - 0 \Rightarrow (t - 27)(t - 3) = 0$

$\Rightarrow t = 27, t = 3 \Rightarrow 81^{\sin^2 x} = 27,$

$81^{\sin^2 x} = 3 \Rightarrow 3^{4\sin^2 x} = 3^3\ 3^{4\sin^2 x} = 3^1$

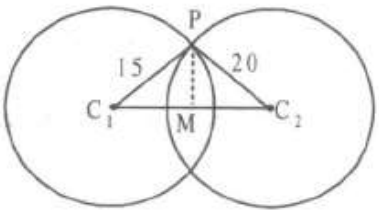
$\Rightarrow \sin^2 x = \frac{3}{4}, \sin^2 x = \frac{1}{4}$

$\Rightarrow \sin x = \frac{\sqrt{3}}{4}, \sin x = \frac{1}{2}$

$\Rightarrow$ smallest positive x is $\frac{\pi}{6}$.

<u>Sol. 47</u> (b)

Explanation:



Let PM = x

$C_1 M = \sqrt{225 - x^2}$, $C_2 M = \sqrt{400 - x^2}$

$\Rightarrow \sqrt{225 - x^2} + \sqrt{400 - x^2 = 25}$

Let us proceed by choices

Choice (a) is not correct because for x = 8

and L. H. S. = $\sqrt{225 - 64} + \sqrt{400 - 64} \neq 25$

Choice (b) is correct since for x = 12

L. H. S. = $\sqrt{225 - 144} + \sqrt{400 - 144}$

= 25 this x is satisfied

<u>Sol. 48</u> (a)

Explanation:

M is (t, 2), Slope AB = $\frac{4-0}{0-2t} = -\frac{2}{t}$

⇒ Slope MR = $\frac{t}{2}$

⇒ Equation of MR is y – 2 $\frac{t}{2}$ (x – t)

On putting x = 0, we get R as $\left(0, 2-\frac{t^2}{2}\right)$

If P is (x, y) then x = $\frac{t}{2}$, y = $\left(2+2-\frac{t^2}{2}\right)/2$

On eliminating t, we get

y = $\left(4-\frac{(2x)^2}{2}\right)/2$ Or y = 2 – x^2

which is the locus of R.

<u>Sol. 49</u> (c)

Explanation:

Since $(3)^2$ < 8 x 2, the point (2, 3) lies within the parab therefore no tangent can be drawn.

<u>Sol. 50</u> (c)

Explanation:

X = at_1 t_2, y = a (t_1 + t_2) (a = 1). But as slope if tange are $\frac{1}{t_1}, \frac{1}{t_2}$ and they are perpendicular ⇒ $\frac{1}{t_1}$ x $\frac{1}{t_2}$ = -1

⇒ t_1 t_2 = -1 ⇒ x = - 1

⇒ Locus is directrix.

<u>Sol. 51</u> (c)

Explanation:

Use $\sqrt{a^2+b^2+c^2}$

Sol. 52 **(a)**

Explanation:

Let OP = α, OQ = ß, OR = γ then the equation of plar

PQR is $\frac{x}{\alpha}+\frac{y}{ß}+\frac{z}{\gamma}$ = 1.

$\frac{a}{\alpha}+\frac{b}{ß}+\frac{c}{\gamma}$ = 1

Now sphere OPQR is

$x^2 + y^2 + z^2 - \alpha x - ßy - \gamma z = 0$

⇒ Centre of the sphere (x, y, z) is given by

$\left(\frac{\alpha}{2},\frac{ß}{2},\frac{\gamma}{2}\right)$.

Sol. 53 **(b)**

Explanation:

f (cos x) = $\frac{1-cosx}{1+\cos x}=\frac{2\sin^2 x/2}{2\cos^2 x/2}$ = $\tan^2\frac{x}{2}$

f (f (cos x) = $\frac{1-\tan^2 x/2}{1+\tan^2 x/2}$ = cos x

⇒ (b) is correct.

Sol. 54 **(c)**

Explanation:

$p_n = \frac{2.^nC_2}{.^{2n}C_2}=\frac{n-1}{2n-1}\longrightarrow\frac{1}{2}$ as n ⟶ ∞

⇒ A is true, R is false

⇒ Choice (c) is correct.

Sol. 55 **(a)**

Explanation:

The statement R is true since on integrating by parts

$I_n = \left|\log \cos x \left(\frac{sin2nx}{2n}\right)\right|_0^{\pi/2} - \int_0^{\pi/2} - \frac{\sin x}{\cos x}\left(\frac{\sin 2nx}{2n}\right) dx$

$= \frac{1}{2n}\int_0^{\pi/2} tan$ x sin 2nx dx

To prove assertion, let us prove that

n I_n + (n - 1) I_{n-1} = 0

L. H. S. = $\frac{1}{2}\int_0^{\pi/2} tan$ x sin 2nx + $\frac{n-1}{2(n-1)}\int_0^{\pi/2} tan$ x sin (2n – 2) x dx

= $\frac{1}{2}\int_0^{\pi/2} tan$ x {sin 2nx + sin (2n – 2) x} dx

= $\frac{1}{2}\int_0^{\pi/2} tan$ x. 2 sin (2n – 1) x. cos x dx

= $\int_0^{\pi/2} sin$ (2n - 1) x sin x dx

= $\frac{1}{2}\int_0^{\pi/2}${cos (2n - 2) n – cos 2nx} dx = 0

⇒ (a) is correct

Sol. 56 **(a)**

Explanation:

Both A and R are true and R is correct explanation for A since R = 8r.

⇒ R = 8 $\left(4R \sin\frac{A}{2}\sin\frac{B}{2}\ sin\frac{C}{2}\right)$

⇒ $\left(\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right)\sin\frac{C}{2} - \frac{1}{16}$

⇒ $\left(\frac{1}{2} - x\right)x = \frac{1}{16}$

⇒ x = $\frac{1}{4}$ whence

cos C = 1 – 2 $\sin^2\frac{C}{2} = \frac{7}{8}$

<u>Sol. 57</u> (a)

Explanation:

The LH rule is applicable for limit in (A) and after applying it, the limit boils down to the limit in R

⇒ (a) is correct.

<u>Sol. 58</u> (a)

Explanation:

Since chord is above the curve $f\left(\frac{x_1+x_2}{2}\right)$ must be smaller than $\frac{f(x_1)+f(x_2)}{2}$

⇒ (a) is correct

<u>Sol. 59</u> (c)

Explanation:

f' (x) is positive only in (c).

⇒ (c) is correct.

<u>Sol. 60</u> (c)

Explanation:

f' = 2 log x + 3

which is positive if x = 1 but is negative if $x = \frac{1}{e^2}$

⇒ (c) is correct.

<u>Sol. 61</u> (c)

Explanation:

(a) is false at x = 1

(b) is false at x = 1 since sin 1 < sin 60° ≅ .86 But $1 - \frac{1^3}{8} = \frac{7}{8} \cong .87$

(c) can be proved as follows,

$\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2} = 2 \tan \frac{x}{2} \cos^2 \frac{x}{2}$

$= 2\tan\frac{x}{2}\left(1 - \sin^2\frac{x^2}{2}\right) > 2.\frac{x}{2}\left(1 - \frac{x^2}{4}\right)\left(\because \sin x < x \tan\frac{x}{2} > \frac{x}{2}\right)$

$= x - \frac{x^3}{4}$

<u>Sol. 62</u> (b)

Explanation:

(a), (c) are false inequalities.

We will try to prove (b) by calculus.

Let f (x) = sin x - $\left(x - \frac{x^3}{6}\right)$, f (0) = 0

f' (x) = cos x - $\left(1 - \frac{x^2}{2}\right)$, f' (0) = 0

f'' (x) = x – sin x > 0 ∵ x > sin x

Since f'' (x) > 0, f' (x) is MI

⇒ f' (x) > f' (0) ⇒ f' (x) > 0

⇒ f (x) is MI ⇒ f (x) > f (0)

⇒ sin x – $\left(x - \frac{x^3}{6}\right)$ > 0

<u>Sol. 63</u> (b)

Explanation:

(a), (c) are false. The falsity can be established by counter examples.

The inequality in (b) is a better result since it does not require the condition that a, b, c are sides off a triangle.

<u>Sol. 64</u> (A) → (S), (B) → (P), (C) → (Q), (D) → (R)

Explanation:

(A) Required probability

$= \frac{4.\ {}^{n}C_4}{4\ {}^{n}C_4} = \frac{4n\,(n-1)(n-2)(n-3)}{4n\,(4n-1)(4n-2)(4n-3)}$

$= \frac{(n-1)(n-2)(n-3)}{(4n-1)(4n-2)(4n-3)}$

(B) Required probability

$= \frac{({}^{n}C_1)^4}{4\,{}^{n}C_4} = \frac{3n^3}{(2n-1)(4n-1)(4n-3)}$

(C) 2 colors can be chosen in ${}^{4}C_2$ ways. After choosing the colors, It can be (1, 3) (2, 2) and (3, 1).

⇒ Number of favorable ways

$= 4X_2\ [2.\ {}^{n}C_3.\ {}^{n}C_1 + {}^{n}C_2\ {}^{n}C_2]$

Required probability

$= \frac{\left[2n\frac{n(n-1)(n-2)}{6} + \frac{n^2(n-1)^2}{4}\right]}{{}^{4n}C_4}$

⇒ Required probability $= \frac{3(n-1)(7n+1)}{(4n-1)(2n-1)(4n-3)}$

(D) Three colors can be chosen in ${}^{4}C_3$ ways.

After which it can be 1, 1, 2; 1, 2, 1; 2, 1, 1,

⇒ Required Probability $= \frac{{}^{4}C_3\ [3.{}^{n}C_1.{}^{n}C_1.{}^{n}C_2]}{{}^{4n}C_4}$

$= \frac{3n\,(n-1)}{(4n-1)(2n-1)(4n-3)}$

<u>Sol. 65</u> (A) → (Q), (B) → (S), (C) → (P), (D) → (R)

Explanation:

(A) Sum of the series in ß

$= ß^{15}\ (1 + ß + ß^2 + \ldots\ldots\ldots\ldots\ldots ß^{36})$

$= ß^{15}.\frac{ß^{36}-1}{ß-1}$

$= ß^{15}.\ \frac{ß-1}{ß-1}$ ($\because ß^5 = 1 \Rightarrow ß^{35} = 1$)

= 1 ⇒ (A) ⇒ (Q)

(B) On replacing x by $\frac{1}{x}$, we get

$2f\left(\frac{1}{x_2}\right) + 3f\ (x^2) = \frac{1}{x^2} - 1$

On eliminating f $\left(\frac{1}{x_2}\right)$ between this and given reaction

We get, f $(x^2) = \frac{1}{5}\left[\frac{3}{x_2} - 2x^2 - 1\right]$ whence f (1) = 0

⇒ (B) ⟶ (S)

(C) We have $|x + 1|^2 = |x - 1|^2$

⇒ $(x + 1)^2 = (x - 1)^2$

⇒ 4x = 0 ⇒ x = 0

⇒ one solution ⇒ (C) ⟶ (Q)

(D) We have

$2^{2x} + 2^{2x-2} - 72 - 2^{2x-3} \geq 0$

⇒ $2^{2x}\left(1 + \frac{1}{4} - \frac{1}{8}\right) \geq 72$

⇒ $2^{2x} \geq 2^6 \Rightarrow \geq 3$

⇒ least positive integer is 3.

⇒ (D) ⟶ (R)

Sol. 66 (A) ⟶ (R), (B) ⟶(S), (C) ⟶ (P), (D) ⟶ (Q)

Explanation:

(A) The triangle is certainly equilateral ⇒ Circumcentre and in Centre coincide

⇒ (A) ⟶ (R)

(B) $2.\ {}^{2n}C_5 = {}^{2n}C_4 + {}^{2n}C_6$ ⇒ n = 7 ⇒ (B) ⟶ (S)

(C) By plotting the graph we easily get max value = 6 ⇒ (C) ⟶ (P)

(D) f (3) = [3] + $\sum_{r=1}^{2008} \frac{3+r-[3+r]}{2008}$

= 3 + $\sum_{r=1}^{2008} \frac{(3+r)-(3+r)}{2008}$ (∵ [3 + r] = 3 + r).

= 3 + 0 = 3

(D) ⟶ (Q).

Paper - II

Answer Sheet

Part – I (PHYSICS)

SECTION – I

1. (b)
2. (a)
3. (d)
4. (d)
5. (b)
6. (b)
7. (a)
8. (a)
9. (a)

SECTION – II

10. (a)
11. (a)
12. (c)
13. (c)

SECTION – III

14. (a)
15. (b)
16. (d)
17. (c)
18. (d)
19. (c)

SECTION IV

20. (A) → (R), (B) → (P) and (T) , (C) → (Q) and (S), (D) → (Q)
21. (A) → (R), (B) → (S), (C) → (Q) and (T), (D) → (P)
22. (A) → (R), (B) → (P), (C) → (P), (D) → (S) and (Q)

PART – II (CHEMISTRY)

SECTION I

23. (b)
24. (c)
25. (a)
26. (a)
27. (a)
28. (b)
29. (b)
30. (c)
31. (a)

SECTION II

32. (a)
33. (d)
34. (b)
35. (a)

SECTION III

36. (a)
37. (b)
38. (c)
39. (d)
40. (c)
41. (a)

SECTION IV

42. (A) → (P) and (Q), (B) → (P), (Q), (R) and (S), (C) → (S), (D) → (P) and (Q)

43. (A) → (R), (B) → (P), (C) → (S), (D) → (Q)

44. (A) → (R), (B) → (P), (C) → (S), (D) → (Q)

PART – III (MATHEMATICS)

SECTION I

45. (d)

46. (c)

47. (b)

48. (d)

49. (c)

50. (b)

51. (a)

52. (a)

53. (d)

SECTION II

54. (d)

55. (a)

56. (a)

57. (a)

SECTION III

58. (a)

59. (c)

60. (a)

61. (c)

62. (b)

63. (b)

SECTION IV

64. (A) → (Q), (B) → (R), (C) → (P)

65. (A) → (R), (B) → (Q), (C) → (P), (Q), and (R), (D) → (Q), (E) → (R)

66. (A) → (S), (B) → (R), (C) (P), (D) → (Q)

SOLUTIONS WITH CLEAR REASONING

PHYSICS

Sol 1. (b)

Explanation:

Rms velocity of molecules of gas v = $\sqrt{\frac{3KT}{m}}$

Where k = Boltzmann constant

T = Temperature in Kelvin scale,

m = mass of gas particle.

Or mv = $\sqrt{3mKT}$

De – Broglie wavelength

$\lambda = \frac{h}{mv}$ $\Rightarrow \lambda = \frac{h}{\sqrt{3mKT}}$

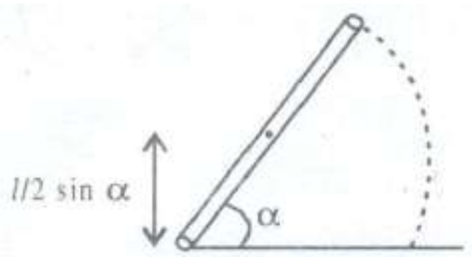
$$\frac{\lambda_H}{\lambda_{He}} = \frac{\sqrt{m_{He}T_{He}}}{\sqrt{m_H T_H}} = \frac{\sqrt{(4)(273+127)}}{\sqrt{(2)(273+27)}}$$

$$\frac{\lambda_H}{\lambda_{He}} = \sqrt{\frac{4\,(400)}{2\,(300)}} = \sqrt{\frac{8}{3}}$$

Choices (a), (c) and (d) are wrong.

Sol 2. (a)

Explanation:



Loss in P. E. = mg $\frac{1}{2}$ sin α

Gain in Rotational K. E. = $\frac{1}{2}$ m $\frac{l^2}{3}$ ω^2

Conserving energy, we get, ω $\sqrt{\frac{3g\sin\alpha}{l}}$

Choices (b), (c) and (d) are wrong

Sol 3. (d)

Explanation:

Breaking stress = $\frac{Force}{Area}$

$$\frac{S_A}{S_B} = \frac{T_A r_2^2}{T_B r_1^2} = \frac{\left(Mg+\frac{Mg}{3}\right).r_2^2}{\frac{Mg}{3}.r_1^2} = \frac{4r_2^2}{r_1^2}$$

$\Rightarrow \quad S_A = \frac{4S_B.r_2^2}{r_1^2}$

(a) is correct , since $S_A = 4S_B$ for $r_1 = r_2$ causing so A to break first.

(b) is incorrect since if $r_1 < 2r_2, S_A > S_B$.

For $r_1 = 2r_2, S_A = S_B$.So any of them may break. So,

(d) is correct.

Sol.4 (d)

Explanation:

Let I_1,I_2 and I_3 be the images formed by

(i) reflection from ABC

(ii) reflection from DEF

(iii) reflection from ABC

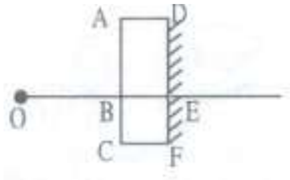
$$\frac{BI_1}{5} = \mu \; or \; BI_1 = 1.5 \times 5 = 7.5cm$$

Now $\quad EI_1 = 7.5 + 2.5 = 10cm$

So, $\quad EI_2 = 10\ cm$behind the mirror

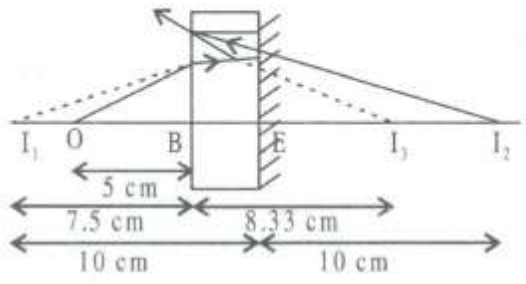
$BI_2 = 10 + 2.5 = 12.5cm$

So, $\quad BI_3 = \frac{12.5}{\mu} = \frac{12.5}{1.5} = 8.33cm$



The ray diagram has been shown.

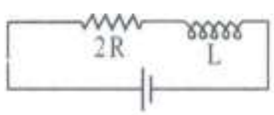
Choices (a),(b) and (c) are wrong.



<u>Sol. 5</u> (b)

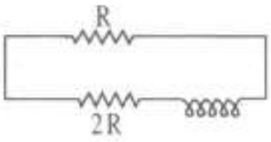
Explanation:

During charging the time constant $\tau_{1=\frac{L}{2R}}$



And during discharging the time constant $\tau_2 = \frac{L}{3R}$

So, $\frac{\tau_1}{\tau_2} = \frac{L/2R}{L/3R} = \frac{3}{2}$



Choices (a), (c) and (d) are wrong.

<u>Sol. 6</u> (b)

Explanation:

In absence of inductor the current in the circuit will

$I = \frac{E}{R} = 0.5A.$ Due to increase in resistance (Moving sliding contains) there will be decrease in main current, so an emf will induced which will oppose the decrease in current. Hence induced current will flow in the direction of main current. Thus current in the circuit will be more than 0.5A. Choices (a), (c) and (d) are wrong.

Sol. 7 (a)

Explanation:

Magnetic moment of loop

$$M = iA = 4 \times \pi(0.5)^2(-\hat{k}) = -\pi\hat{k}\ Am^2$$

Torque acting $\tau = \vec{M} \times \vec{B} = -\pi\hat{k} \times 10\hat{\imath} = -10\pi\hat{\jmath}$

Axis of rotation is along $\vec{\tau}$ i.e., the axis of rotation is the Y-axis.

Moment of inertia of ring about Y-axis is

$$I = \frac{mR^2}{2} = \frac{1}{2} \times 2.(0.5)^2 - \frac{1}{4} Kgm^2$$

So angular acceleration $= \alpha = \frac{|\vec{\tau}|}{I}$

$$\alpha = \frac{10\pi}{1/4} = 40\pi\ rad/s^2$$

Choices (b), (c) and (d) are wrong.

Sol.8 (a)

Explanation:

Sol. Ist $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$\Rightarrow$ $$\frac{d^2y/dx\text{^}2}{\frac{1}{R} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

Sol. IInd $a_c = \frac{v^2}{R}$

At highest point $v = u\cos\theta$

$\therefore$ $$R = \frac{u^2\cos^2\theta}{g}$$

Choice (b), (c) and (d) are wrong.

Sol.9 (a)

Explanation:

$$dQ = dU + dW$$

For ideal gas $U = f(T)$

Since T is constant.

$$\Delta T = 0, \Delta U = 0 \; and \; dQ = dW$$

<u>Sol. 10</u> (a)

Explanation:

Reason and Assertion, both are correct and reason explains Assertion.

<u>Sol. 11</u> (a)

Explanation:

Both are true by themselves and are not dependent on their path, but state only.

<u>Sol. 12</u> (c)

Explanation:

Rise of liquid in capillary tube is to compensate for the excess pressure in liquid and the pressure is measured in terms of vertical length of the liquid .

For the same liquid hr = constant

But for the different liquids hrp=constant

(i.e.) density should be considered to account for the height.

<u>Sol. 13</u> (c)

Explanation:

Assertion is correct , Reason is wrong, since

$v = \sqrt{2g'h}$ $\quad t = \sqrt{\frac{2h}{g'}}$

$g' = g\left(1 - \frac{\sigma}{p}\right)$ i.e. , g' depends on the density of material. Density of both balls are unequal, so their velocities and times are different.

Sol. 14 (a)

Explanation:

$$V_g = \frac{E_2 - E_1}{l_2 - l_1} = \frac{0.4}{0.6} = \frac{2}{3} V/m.$$

So, choice (a) is correct and the rest of the options are wrong.

Also, $E_1 = K_x$

$E_2 = K(x + 0.6)$

$E_2 - E_1 = 0.4 = K(0.6)$

Sol. 15 (b)

Explanation:

Potential= $V_g \times length = \frac{2}{3} \times 10 = \frac{20}{3} volt$

So (b) is correct and the rest are wrong.

Sol. 16 (d)

Explanation:

Balancing length=$\frac{V_{PQ}}{V_g} = \frac{3}{\left(\frac{20}{3}\right)} = \frac{9}{20}$

So (d) is correct and the rest are wrong.

Sol. 17 (c)

Explanation:

$$F = -\frac{dU(r)}{dr} = -\frac{d}{dr}(10r^3) = -30r^2$$

For a circular path $\frac{mv^2}{r} = F = 30r^2$

$$\Rightarrow \qquad v = \sqrt{\frac{30r^3}{m}} = 100\, m/s$$

So choice (c) is correct and rest are wrong.

<u>Sol. 18</u> (d)

Explanation:

Angular momentum = $L = mvr$

$$= 3 \times 100 \times 10 = 3000\ kg\,\frac{m^2}{sec}$$

So choice (d) is correct and rest are wrong.

<u>Sol. 19</u> (c)

Explanation:

$P.E = -10 \times 10^3 = -10{,}000$ joule

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m.\frac{30r^3}{m}$$

$$= \frac{30r^3}{2} = \frac{30\times10^3}{2} = 15{,}000 \text{ joule}$$

Total energy=5000 joule.

<u>Sol. 20</u> (A) → (R), (B) → (P) and (T), (C) → (Q) and (S), (D) → (Q)

Explanation:

For v= constant, the motion is uniform.

V=0, t=0 it refers to time axis of v−t graph. Constant v≠0 and t≠0 refers to a line parallel to time axis. The area has the dimensions of displacement. So (A)→ (R).

Slope of x –t graph represents velocity. For straight x−t graph, velocity has to be constant.

So (B) → (P)

As the two x−t graphs intersect, the position should be same at the that instant.

So (B) → (T)

Slope of v–t graph represents acceleration and will be constant for straight v–t graph.

So (C) → (Q) and (S)

Equations of motion are applicable for uniformly accelerated bodies only.

So (D) → (Q)

Sol. 21 **(A) → (R), (B) → (S),(C) → (Q) and (T), (D) → (P)**

Explanation:

(A) → (R)

Wires made of same material will have same young's modules.

(B) → (S)

Work done in stretching a wire

$$= \int \frac{Ayxdx}{L} = \frac{1}{2}\frac{Ayl^2}{L}$$

⇒ $\frac{1}{2} \times stress \times \text{strain} \times \text{Volume}$

(C) → (Q) and (T)

When volume is constant $\pi r^2 l$=constant

Differentiating $2\pi rl\ dr + \pi r^2\ dl=0$

Gives, $\frac{dr}{r} = \frac{1}{2}\frac{dl}{l}$

$$\sigma = \frac{\frac{dr}{r}}{\frac{dl}{l}} = \frac{1}{2} = 0.5$$

(D) → (P)

Material having better elasticity extend less for a given force since

$$Y = \frac{F}{A} \cdot \frac{l}{\Delta l}$$

Sol. 22 **(A) → (R), (B) → (P),(C) → (P), (D) → (S) and (Q)**

Explanation:

In rocket propulsion, the change in momentum with the rocket is due to the change in momentum of burnt fuel.

(i.e.) M dv=−u dM where M is instantaneous mass.

$$\therefore a = \frac{dv}{dt} = -\frac{u}{M}\frac{dM}{dt}$$

So (A) → (R)

By basic definition, $\int F\,dt$ or change in momentum or Impulse means the same. Since $\int F\,dt$refers to area under the force –time graph, (B) → (P) and (C) → (P).

When equal masses undergo one dimensional collision, they exchange their velocities. For inelastic collision, e=0 and $(v_2 - v_1) = 0$. So (D) → (S). Also it can be proved that the line of motion will be perpendicular for equal masses if one is at rest and the collision is elastically oblique.

So (D) → (Q)

CHEMISTRY

Sol. 23 (b)

Explanation:

$2KClO_3 \rightarrow 2KCl + 3O_2$

x 0 0

0 x $\frac{3}{2}x$

$4KClO_3 \rightarrow 3KClO_4 + KCl$

$1-x$ $\frac{3}{4}(1-x)$ $\frac{1}{4}(1-x)$

$\frac{3}{2}x = 1$ mole (given) $\Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$

Mole fraction of $KCLO_4$ = $\frac{1/4}{\frac{1}{4}+\frac{3}{4}} = \frac{1}{4} = 0.25$

No. of moles of $KClO_3$ =$\frac{3}{4}\left(1-\frac{2}{3}\right) = \frac{1}{4}$

No. of moles of KCl =$x + \frac{1}{4}(1-x)$

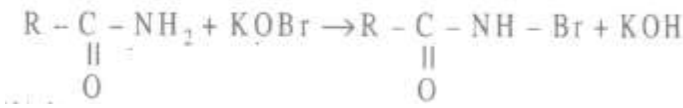
$$= \frac{2}{3} + \frac{1}{4}\left(1-\frac{2}{3}\right) = \frac{2}{3} + \frac{1}{12} = \frac{9}{12} = \frac{3}{4} = 0.75$$

$KClO_4$ obtained =1-0.75=0.25

Sol. 24 **(c)**

Explanation:

(a) is correct as the reaction taking place first is



(b) is correct

Choice (d) is incorrect as at no stage such compound is formed.

Sol. 25 **(a)**

Explanation:

Extraction of Pb from Pbs follows the following steps:

(i) Froth Floatation

(ii) Roasting

(iii) Self reduction

Extraction of Sn from SnO follows the following steps:

(i) Hydraulic washing to remove gangue

(ii) Calculation to remove water

(iii) Carbon reduction

Hence choice (a) is correct while (b), (c) and (d) are wrong.

Sol. 26 **(a)**

Explanation:

Superoxide KO_2 is basic hence it will react with acidic oxide to form K_2CO_3 and oxygen is libereated.

$4KO_2 + 2CO_2 \rightarrow 2K_2CO_3 + 3O_2$ Hence choice (a) is correct while (b),(c) and (d) are wrong.

Sol. 27 (a)

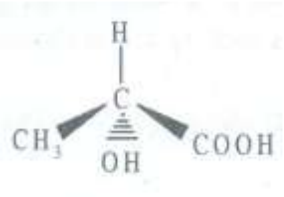
Explanation:

(a) The groups which are in horizontal position, i.e., CH_3–and–COOH are coming out of the plane whereas groups in vertical position are going into plane.

(b), (c) and (d) are ruled out.

Alternate Solution:

The fisher projection when converted wedge edge gives:



Hence CH_3–and –COOH are coming out of the plane.

Hence choice (a) is correct.

Sol. 28 (a)

Explanation:

$$\Delta E = \Delta m \times 931.48 MeV = 0.21 \times 931.48 = 195.6108$$

$$\Delta E \ per \ nucleon = \frac{195.6108}{16} = 12.2 MeV.$$

Sol. 29 (a)

Explanation:

(b) $S_n + 4HNO3 \rightarrow H_2SO_4 + 4NO_2 + H_2O$

$(a), (c) and \ (d) are \ ruled \ out.$

Sol. 30 (c)

Explanation:

(c) (i) and (iv) are isotonic because both of them are non-electrolytes.

(a) (i) and (ii) are not isotonic because $CaCl_2$ is an electrolyte whereas urea is not.

(b) (i),(ii),(iii),(iv) cannot be isotonic because $CaCl_2$ and MgSO4 are electrolytes whereas urea and glucose are non –electrolytes

(d) (ii) and (iii) cannot be isotonic because $MgSO_4$ is electrolyte , glucose is non-electrolyte.

<u>Sol. 31</u> (c)

Explanation:

1 Curie=3.7×10^{10} disintegration/sec

$-\frac{d[N]}{dt} = 3.7 \times 10^{10}$ disintegration/sec

$\lambda \times [N] = 3.7 \times 10^{10}$ disintegration/sec

$$\Rightarrow \lambda = 3.7 \times \frac{10^{10}}{[N]}$$

Since $[N] = \frac{1\times6.023\times10^{23}}{226} = 0.0266 \times 10^{23}$

$$\lambda = \frac{3.7 \times 10^{10}}{0.0266 \times 10^{23}} = 138.8 \times 10^{-13}$$

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{138.9\times10^{-13}\times365\times24\times60\times60}$$

$= 15.8 \times 10^2 = 1600\ years$

<u>Sol. 32</u> (a)

Explanation:

A is true, R is true and R is the correct explanation of A.

<u>Sol. 33</u> (d)

Explanation:

A is false because V.P of water i.e., aqueous tension is constant at 27^0C. It changes only with change in temperature. R is correct.

<u>Sol. 34</u> (b)

Explanation:

A is true, R is true but R is not the correct explanation of A.

Sol. 35 (a)

Explanation:

A is true, R is true but R is the correct explanation of A.

Sol. 36 (a)

Explanation:

At cathode, $2H^+ + 2e^- \rightarrow H_2$

At anode, $2OH^- \rightarrow O_2 + 2H^+ + 4e^-$

OH^- get discharged as compared to SO_4^{2-} due to over voltage. SO_2 will be formed if conc. H_2SO_4 is used.

Sol. 37 (b)

Explanation:

(b) Energy=current ×time× voltage.

$E = I \times \mathrm{t} \times \mathrm{V} = 1 \times 100 \times 115$

=11500J=11.5KJ

Sol. 38 (c)

Explanation:

1 F will deposit 8 g of oxygen $=\frac{16}{2} = 8 = Eq.wt.$

32 g of O_2=1mole=22.4L at STP

8 g of O_2=$\frac{1}{32} \times 8$ mole=5.6 L at STP

Sol. 39 (d)

Explanation:

The number of H-atoms which should be for 15 carbon atoms =32. But the compound $C_{15}H_{17}N$ contains 16 H-atoms hence degree of unsaturation $=\frac{16}{2} = 8.$

Hence number of phenyl groups should be =2.

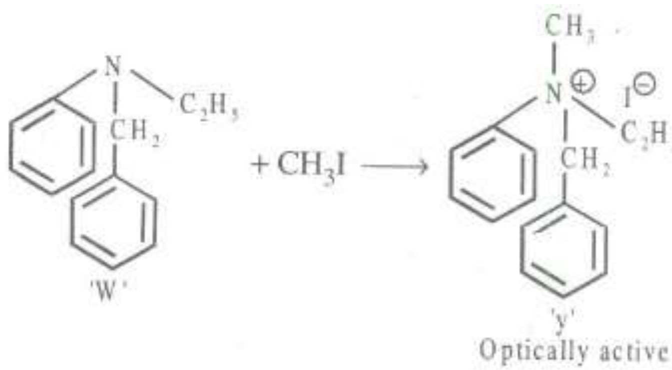
Hence choice (d) is correct while (a),(b) and (c) are wrong.

<u>Sol. 40</u> (c)

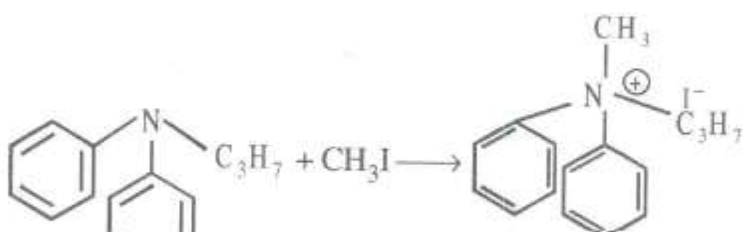
Explanation:

Since compound W does not react with benzene sulphonyl chloride so it is a tert-anime. Also when W is treated with CH_3I it gives an optically active quaternary ammonium salt Y.

Hence it should be (c)



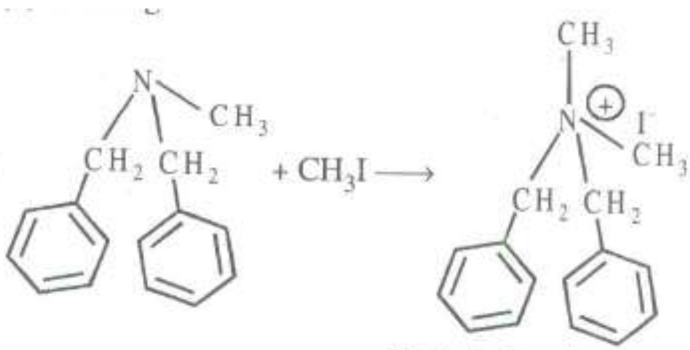
(a)



Optically inactive quaternary ammonium salt.

So (a) is wrong

(b)

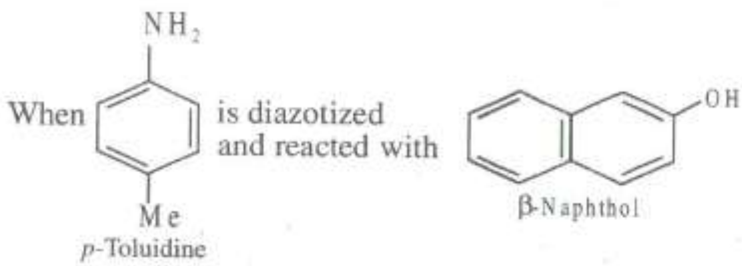


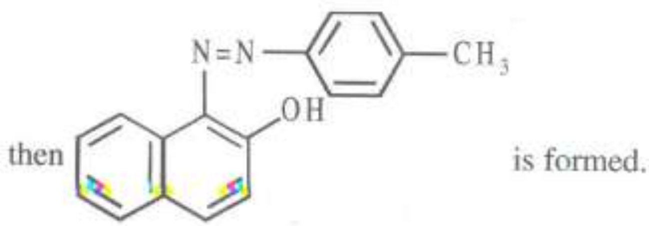
Optically inactive quaternary ammonium salt.

So (b) is wrong.

(d) It is a 2^0 amine hence it is wrong.

Sol. 41





Hence choice (a) is correct while (b),(c) and (d) are wrong.

Sol. 42 **(A) → (P) and (Q); (B) → (P), (Q), (R) and (S), (C) → (S), (D) → (P) and (Q)**

Explanation:

(A) Dilution changes discharge potential and discharging of ions.

Dilution also decrease reduction potential as concentration in mole/L decreases.

$$E_{M^{n+}/M} = E^0_{M^{n+}/M} - \frac{2.303RT}{nF}\log\frac{1}{[M^{n+}]}$$

(B) Increase in temperature increases vapour pressure.

Increase in temperature decrease reduction potential

Increase in temperature increases discharging of ion.

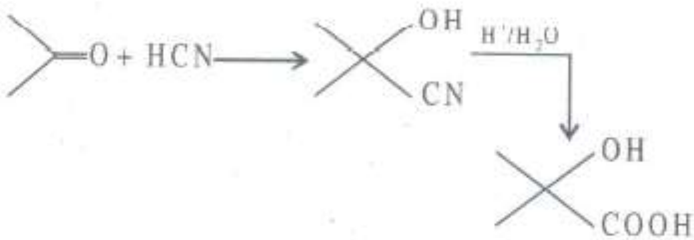
Increase in temperature may increase or decrease equilibrium constant .

(C) Atmospheric pressure changes vapour pressure.

(D) Nature of electrode changes both discharging of ions and reduction potential as $E_{M^{n+}/M}$ is a characteristic of electrode as well as it is a characteristic of discharging of ions.

<u>Sol. 43</u> (A) →(R);(B)→(P);(C)→(S);(D)→(Q)

Explanation:



α-Hydroxy carboxylic acid

$$R{-}COOH \xrightarrow[\text{synthesis}]{\text{Arndt Eistert}} R{-}CH_2COOH$$

$$R{-}CH=CH{-}CHO \xrightarrow{NaBH_4} R{-}CH=CH{-}CH_2(OH)$$

$NaBH_4$ is highly specific.

$$R{-}CH=CH_2 \xrightarrow{Hg(OAc)_2/AcOH} R{-}CH(OH){-}CH_2(HgOAc) \xrightarrow{NaBH_4} R{-}CH(OH){-}CH_3$$

<u>Sol. 44</u> (A) →(R);(B)→(P),(C)→(S),(D)→(Q)

Explanation:

2^{nd} most abundant element on earth crust=Si

Most abundant transaction element=Fe

Most abundant element on earth crust=O_2

Most abundant element in universe =H_2

MATHEMATICS

Sol. 45 (d)

Explanation:

Numerical expression

$= \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^{165}$

$= \cos 165\frac{\pi}{6} + i\sin 165\frac{\pi}{6}$

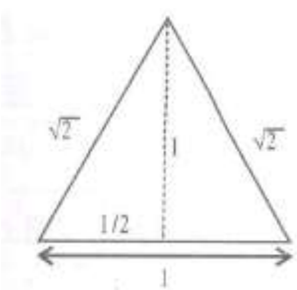
$= \cos 55\frac{\pi}{6} + i\sin 55\frac{\pi}{2} = -i$

$(Since \cos 55\frac{\pi}{2} = 0, \sin 55\frac{\pi}{2} = \sin\left(28\pi - \frac{\pi}{2}\right) = -\sin\frac{\pi}{2})$

Sol.46 (c)

Explanation:

Area can be easily found by determinant.



We can do it by calculating sides also.

$AB = \sqrt{2}, BC = \sqrt{2}, AC = 2$ (By distance formula)

$\Rightarrow$Area $\Delta ABC = 1$

<u>Sol. 47</u> (b)

Explanation:

$a = 2b \Rightarrow \sin A = 2\sin B$

Also $\sin A = \sin 3B$

On eliminating, $sin\ A$ we get

$2\ sin\ B = sin\ 3B = 3sin\ B - 4sin^3 B$

$\Rightarrow 4\sin^3 B - \sin B = 0$

$\Rightarrow \sin B(4\sin^2 B - 1) = 0$

$\Rightarrow \sin B = 0$ (Not possible)OR $\sin B = \frac{1}{2}$

$\Rightarrow \mathrm{B} = 60^0 \left(\sin B = -\frac{1}{2}\ not\ pssible\ either\right)$

$\Rightarrow \sin A = 2 \times \frac{1}{2} = 1 \Rightarrow \mathrm{A} = 90^0$ Whence $C = 30^0$

$\Rightarrow$ (b) is correct.

Q.48 (d)

Explanation:

If α is a solution then α + 2nπ is also a solution. It is sufficient to exhibit only one solution.

By trial $x = \pi/2$ is a solution.

$\Rightarrow \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$, ... all are solutions.

Q.49 (c)

Explanation:

Function	**Periods**
$sin\ x$ $tan\ x/2$ $cos\ 3x$	2π, 4π, 6π….. 2π, 4π, 6π…… $\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{6\pi}{3}, \ldots \ldots$

⇒Least common multiple is 2π ∵ $\frac{6\pi}{3} = 2\pi \Rightarrow$ (c) is correct.

Sol. 50 (b)

Explanation:

The given differential equation can be written as

$y^5xdx + ydx - xdy = 0.$

Multiplying by x^3/y^5, we have

$$x^4dx + \frac{x^3}{y^3}\left(\frac{ydx - xdy}{y^2}\right) = 0$$

Integrating, we get

$$\frac{x^5}{5} + \left(\frac{1}{4}\right)\left(\frac{x}{y}\right)^4 = C$$

Since $\frac{x^3}{y^3}\left(\frac{ydx-xdy}{y^2}\right) = u^3\ du\ where\ u = \frac{x}{y}$

Sol. 51 (a)

Explanation:

Apply $C_1 \to C_1 + C_2 + C_3$

Sol. 52 (a)

Explanation:

Since the given system of equations has a non-trivial solution.

$$\Delta = \begin{vmatrix} 1 & a & -1 \\ 2 & -1 & a \\ a & 1 & 2 = \end{vmatrix} = 0$$

Using $C_1 \to C_1 + C_3, C_2 \to C_2 + aC_3$, we get

$$\Delta = \begin{vmatrix} 0 & 0 & -1 \\ 2+a & -1+a^2 & -a \\ 2+a & 1+2a & 2 \end{vmatrix} = 0$$

$$\Rightarrow (-1)\begin{vmatrix} 2+a & -1+a^2 \\ 2+a & 1+2a \end{vmatrix} = 0 \Rightarrow (2+a)(1+2a+1-a^2) = 0$$

$$\Rightarrow a = -2, 1 \pm \sqrt{3}$$

Thus, there are three real values of a for which the system of equations has a non-trivial solution.

Sol. 53 **(d)**

Explanation:

Let $OA = x_1 i + y_1 j$ and $OB = x_2 i + y_2 j$.

Since $1 = OA.i = x_1$ and $-2 = OB.i = x_2$.

Moreover, $y_1 = x_1^2$ and $y^2 = x_2^2 = 4$.

So, $OA = i + j$ and $OB = -2i + 4j$

Hence $|2OA - 3OB| = |8i - 10j| = \sqrt{164} = 2\sqrt{41}$

Sol.54 **(d)**

Explanation:

Assertion is false. Let us prove it by a counter example. If $n = 3, r = 3,$ then answer must be $\frac{1}{6}$ (All letters in correct envelope)but

$$1 - \frac{{}^nC_r}{n!} = 1 - \frac{{}^3C_3}{3!} = \frac{5}{6}$$

The reason is obviously correct.

⇒ (d) is correct.

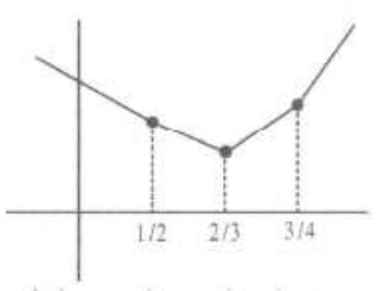
Note: The correct probability of the event described in assertion is

$$\frac{1}{r!}\left[\frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^{n-r}}{(n-r)!}\right]$$

Sol. 55(a)

Explanation:

The graph of the function will have a line segments of negative slopes, less negative slopes then at $\frac{2}{3}$ It will turn and yield positive slope and the more positive slope (See figure)



⇒ Absolute minimum is attained at

A=2/3 which is 2/3

⇒ Assertion A is true and Reason R is correct explanation.

Sol. 56(a)

Explanation:

The Reason R is true and can be proved by integrating by parts taking $\sin^{n-1} x$ as first function and $\sin x$ as second function. If n is odd then

$$I_n = \frac{n-1}{n} I_{n-2} = \frac{n-1}{n} . \frac{n-3}{n-2} I_{n-4}$$

$$= \frac{(n-1)(n-3)(n-5)}{n(n-2)(n-4)\dots 2} . I_1$$

Now $I_1 = \int_0^{\pi/2} \sin^n x \, dx = 1$

⇒ I_n is rational for all odd n.

⇒ A is true and R is correct explanation.

Note: Indeed I_n is irrational if n is even.

Sol. 57 (a)

Explanation:

The equality of expressions in (A) can be easily establishment. But all expressions are defined if

$3x \neq (2l+1)\frac{\pi}{2}$, $x \neq (2m+1)\frac{\pi}{2}$

$\frac{\pi}{3} - x \neq (2n+1)\frac{\pi}{2}, \frac{\pi}{3} + x \neq (2b+1)\frac{\pi}{2}$ The above four are equivalent to n≠$\frac{k\pi}{6}$

<u>Sol. 58</u> (a)

Explanation:

E_1 is $\frac{x^2}{1/a^2}+\frac{y^2}{1/b^2}$=2 since $\frac{1}{a^2}>\frac{1}{b^2}$

$e=\frac{\sqrt{b^2-a^2}}{b}$ and foci are $\left(\pm\frac{e}{a},0\right)$

E_2 is $\frac{x^2}{\frac{1}{b^2}}+\frac{y^2}{\frac{1}{a^2}}=1$

⇒ Foci are $\left(0,\pm\frac{e}{a}\right)$

⇒ Distance between some focus of E_1 and some focus of E_2 is $\sqrt{\frac{2e^2}{a^2}}=\frac{e\sqrt{2}}{a}$

$$=\frac{\sqrt{b^2-a^2}}{b}\frac{\sqrt{2}}{a}.=\frac{\sqrt{2}\sqrt{b^2-a^2}}{ab}$$

⇒ (a) is correct.

<u>Sol. 59</u> (c)

Explanation:

E_2 Intersects x-axis at $\left(\frac{1}{b},0\right)$

On solving, we easily get

$$x=y=\pm\frac{1}{\sqrt{a^2+b^2}}$$

By summary area =4 (area in first quadrant)

$$=4\int_0^{\frac{1}{\sqrt{a^2+b^2}}}\frac{\sqrt{1-a^2x^2}}{b}dx+4\int_{\frac{1}{\sqrt{a^2+b^2}}}^{\frac{1}{b}}\frac{\sqrt{1-b^2\alpha^2}}{a}dx$$

=α=$\frac{1}{\sqrt{a^2+b^2}},\beta=\frac{1}{b}$

Sol. 60 **(a)**

Explanation:

Apply $\int \sqrt{a^2 - x^\wedge 2dx} = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} sin^{-1}\frac{x}{a}$.

Sol. 61 **(a)**

Explanation:

$[f(n) - 1]^2 = 2^{2^n}.2^{2^n}$

$= 2^{2^n + 2^n} = 2^{n+1}$

$= f(n+1) - 1$

⇒ (c) is correct.

Sol. 62 **(b)**

Explanation:

Without loss of generality, we can assume that n>m.

$f(n) - 2 = 2^{2^n} - 1$

$= \left(2^{2^{n-1}} + 1\right)\left(2^{2^{n-1}} - 1\right)$

$= \left(2^{2^{n-1}} + 1\right)\left(2^{2^{n-2}} + 1\right)\left(2^{2^{n-2}} - 1\right)$

$= \left(2^{2^{n-1}} + 1\right)\left(2^{2^{n-2}} + 1\right)\left(2^{2^{n-2}} - 1\right)\left(2^{2^{n-3}} - 1\right) \ldots (2+1)$

⇒ $f(n) - 2$ is divisible by $f(m)$ since $f(m)$ will appear a factor on RHS.

⇒ Thus if there is a common factor of (m) and $f(n)$ then that common factor β must divide 2 also.

⇒ β=2 or β=1

But both $f(m)$ and $f(n)$ are odd therefore β cannot be 2

⇒ β=1

HCF of $f(m)$ and (n) is essentially 1.

Sol. 63 **(b)**

Explanation:

If the number of prime numbers were finite then the result in 18 is contradicted.

⇒ Number of prime numbers is infinite.

⇒ (b) is correct.

Sol. 64 **(A) → (Q), (B) →(R), (C) → (P)**

Explanation:

On integrating by parts

$$I_m = |\sin^m n(-e^{-x})|_0^\infty - \int_0^\infty m(\sin x)^{m-1}\cos x(-e^{-x})dx$$

$$= m\int_0^\infty (e^{-x})(\sin x)^{m-1}\cos x\, dx$$

$$= m[|\sin^{m-1} x\cos x\,(-e^{-x})|_0^\infty] - \int_0^\infty [(m-1)\sin^{m-2} x\cos^2 x - \sin^m x]]\ (-e^{-x})dx.$$

$$= m(m-1)\int_0^\infty \sin^{\mathrm{m}-2} x\,(1-\sin^2 x)e^{-x}\,dx - m^2\int_0^\infty \sin^m x.e^{-x}\,dx$$

$=m(m-1)I_{m-2} - m^2 I_{m-2}$

$\Rightarrow (1+m^2)I_m = m(m-1)I_{m-2}$

Now $I_0 = \int_0^\infty e^{-x}\,dx = 1$

$$I_2 = \frac{2\times 1}{5}I_0 = \frac{2}{5}$$

$$I_4 = \frac{12}{17}I_2 = \frac{24}{85} \Rightarrow 85\mathrm{I}_4 = 24$$

⇒(C) → (P)

On putting m=5 we get

$26I_5 = 5\times 4I_3 = 20I_3$

$\Rightarrow \mathrm{A} = 26, \mathrm{B} = 20 \Rightarrow (\mathrm{A}) \rightarrow (Q), (B) \rightarrow (R)$

<u>Sol. 65</u> (A) → (R), (B) →(P), (C) → (P),(Q),(R),(D)→(Q),(E)→(R)

Explanation:

Put $2x-3=\frac{1}{t}.\,dx=-\frac{1}{t^2}dt$

$$I=\int-\frac{\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{2\left(3+\frac{1}{t}\right)-\frac{1}{4}\left(3+\frac{1}{t}\right)^2}}=-\int\frac{2dt}{\sqrt{15t^2+12t-1}}$$

$$=C-\frac{1}{\sqrt{15}}\log\left[\frac{x+6+\sqrt{60x-15x^2}}{2x-3}\right]$$

<u>Sol. 66</u> (A) → (Q), (B) →(R), (C) → (P)

Explanation:

(A) Expression $=\sum_{r=0}^{n}\frac{n^2}{(n+r)^3}$

Limit of expression$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n^2}{(n+r)^3}$ (∵The first term → 0, we isolate it.)

$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{\left(1+\frac{r}{n}\right)^3}=\int_0^1\frac{dx}{(1+\alpha)^3}=\frac{3}{8}\Rightarrow$ (A) → (S)

(B) Expression$=\sum_{r-0}^{3n}\frac{1}{n+r}$

Limit $=\lim_{n\to\infty}\sum_{r=1}^{3n}n\left(1+\frac{r}{n}\right)=\int_0^3\frac{dx}{1+x}=\log 4$

⇒ (B → (R)

(C) Limit$=\int_0^3\frac{dx}{(3+x)^3}=\frac{1}{24}$

⇒ (C) → (P)

(D) Expression$=\sum_{r=1}^{n}\frac{1}{\sqrt{2rn-r^2}}$

$$=\sum_{r=1}^{n}\frac{1}{n\sqrt{\frac{2r}{n}-\frac{r^2}{n^2}}}=\int_0^1\frac{dx}{\sqrt{2x-x_2}}=\frac{\pi}{2}$$

⇒ (D) → (Q)