HINTS AND EXPLANATIONS

CHEMISTRY

Sol.1

Since the solution becomes basic on addition of solute X , so the solute is a salt of weak acid and strong base.

Sol.2

On reaction with dilute HCI, the salt containing sulphide (S^{2-}) or sulphite (SO_3^{-}) ions liberate H_2S and SO_2 gases respectively; sulphate ions do not give any reaction. Thiosulphate ($S_2O_3^{2-}$) gives SO_2 gas which has a pungent odour and yellow ppt of colloidal sulphur.

$S_2O_3^{2-} + 2H^+ \rightarrow SO_2\uparrow + S\downarrow + H_2O$

Sol.3

Hydrogen peroxide acts as both oxidising and reducing agent.

$H_2O_2 \rightarrow H_2O + O$: $H_2O_2 + O \rightarrow H_2O + O_2$

Sol.4

Fe^{2+} requires 2F for conversion to Fe; Fe^{3+} requires 3F so if equal quantities of electricity are passed the amount of iron deposited from $Fe(NO_3)_3$ is 2/3 of that deposited from $FeSO_4$.

Sol.5

In the reaction

$2HI + H_2SO_4 \rightarrow I_2 + SO_2 + 2H_2O$; iodine changes its oxidation state from – 1 (in HI) to 0 (oxidation), i.e., H_2SO_4 oxidizes HI to I_2.

Sol.6

F^- is not oxidized by MnO_2.

Sol.7

If the reverse reaction is endothermic, heat liberated is less but if it is exothermic more heat is liberated as compared to the forward reaction. Thus the activation energy for the reverse reaction can be can be less than or more than E_a.

Sol.8

In the reaction $I_2 \rightarrow 2I^-$, the change is of 2 electrons; $I_2 + 2e \rightarrow 2I^-$, therefore, the equivalent mass of I_2 is $^1/_2$ of its molecular mass.

Sol.9

On addition of dilute HCI, cations Hg_2^{2+} and Pb^{2+} are precipitated as chlorides Hg_2CI_2. The cations Hg^{2+} and Cd^{2+} are not precipitated as chlorides in 1^{st} group of salt analysis.

Sol.10

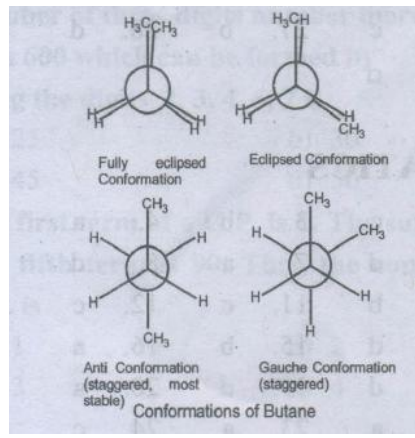
Metal oxides in the lower oxidation states are predominantly ionic, in higher oxidation state they are predominantly covalent.

Sol.11

Orbitals used for sp^3d hybridization are s, p_x, p_y, p_z and d_{z^2}.

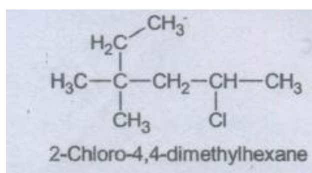
Sol.12

The most stable conformation of n-butane is the one in which bulky groups are maximum distance apart as in staggered anti conformation.



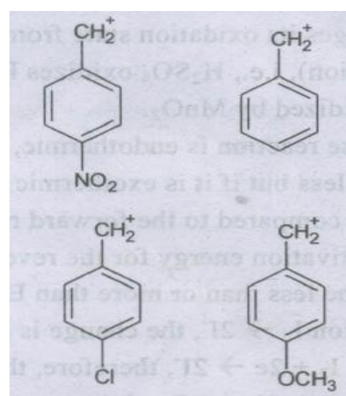
Sol.13

IUPAC name of the given organic compound is



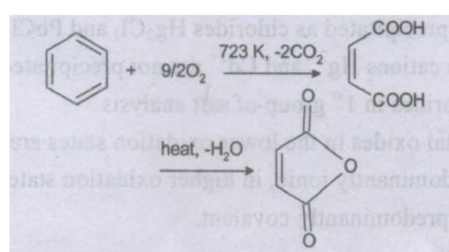
Sol.14

Electron releasing groups (OCH_3) stabilize carbocation, electron withdrawing groups (CI, NO_2) destabilize it.



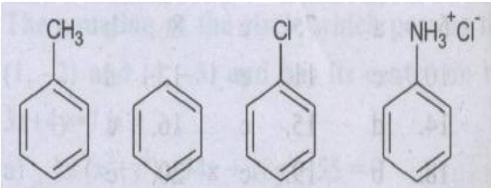
Sol.15

When benzene vapours are allowed to react with V_2 O_5 at 723K, , ,maleic anhydride is formed as shown below.



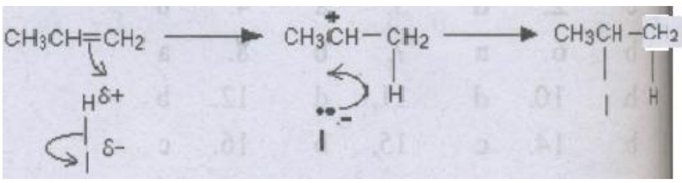
Sol.16

Reactivity of aromatic compounds towards electrophilic substitution reactions decreases when electron withdrawing groups are attached and increase when electron releasing groups are attached.



Sol.17

Addition of HI to propene involves the formation of more stable secondary carbocation to yield isopropyl iodide. The formation of n-propyl iodide involve the formation of a less stable primary carbocation.



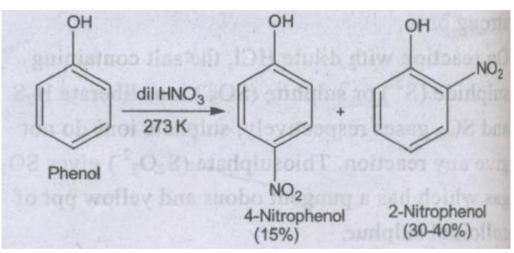
Sol.18

When an organic compound containing both nitrogen and sulphur is fused with sodium, NaCNS is formed

Na + C + N + S →NaCNS

Sol.19

Treatment of phenol with dilute nitric acid gives a mixture of ortho and para nitro phenols.



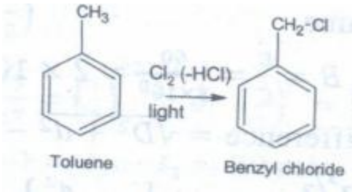
Sol.20

CI > F > Br > I. Electron affinity in halogen family decreases down the group with increase in the atomic size. However, fluorine has a lower electron affinity than chlorine. This can be explained by the small size of fluorine, compared to chlorine.Electron Affinity of halogens (kJ/mol)

-328.0 (F), -349.0 (CI), -324.6 (Br), -295.2 (I).

Sol.21

Toluene reacts with chlorine in the presence of light to give benzyl chloride



Sol.22

Reducing behavior of alkali metals increases down the group due to decrease in the ionization energies. Lithium, however, is the strongest reducing agent among all alkali metals because of high hydration energy of Li^+ ion.

Sol.23

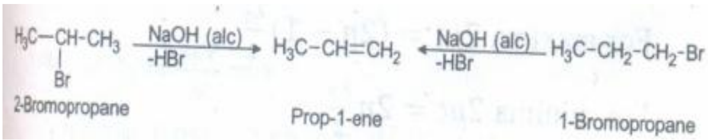
The given reaction takes place in ammonia recovery tower.

Sol.24

Sodium benzoate is a preservative; aspirin is analgesic, Aluminium hydroxide is antacid and sulphadiazine is not disinfectant.

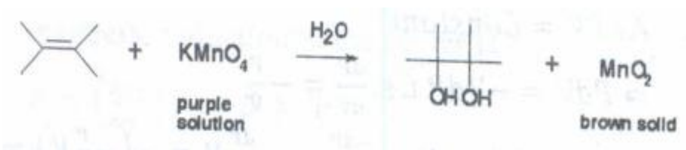
Sol.25

A mixture of 1-bromo propene and 2-bromo propane on heating with alcoholic solution of sodium hydroxide gives propene.



Sol.26

In Baeyer's test of unsaturation, olefinic hydrocarbons are treated with an alkaline solution if $KMnO_4$ gets discharged, unsaturation is present.



Sol.27

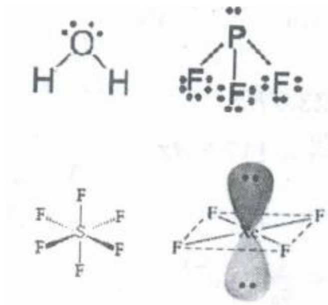
Reducing behaviour of hydrides of group 15 elements increases down the group due to decrease in their stabilities. PH_3 is a stronger reducing agent than NH_3.

Sol.28

Alkali metals are strong reducing agents.

Sol.29

SF_6 has regular geometry.



Sol.30

$Ag_2CrO_4 \leftrightarrows 2Ag^+ + CrO_4^{2-}$

$K_{sp} = [Ag^+]^2[CrO_4^2] = (2C)^2C = 4C^3$.

PHYSICS

Sol.1

[RC] = [T]

Sol.2

Number of revolutions to cover 1.5 cm $n = \frac{1.5}{1/12} = 18$. Angular speed = ω $2\pi\nu = 2\pi x \frac{216}{60} = 7.2\pi$ rad / s

As $\omega = \frac{\theta}{t} \Longrightarrow t = \frac{\theta}{\omega} = \frac{2\pi n}{\omega} = \frac{2\pi x 18}{7.2\pi} = 5$ seconds

Sol.3

Mass of lift T = m (g - a) = 5(9.8 – 5) =24N

Sol.4

It is given that collision is inelastic. After the collision, the spheres of same mass move. The angle between the two directions will be different from 90°

Sol.5

It has been given that external force is zero. Therefore the velocity of centre of mass does not change by mutual force of attraction whatever may be the relative velocity of approach.

Sol.6

Mass of element of rod,

D_m = D x dl = D x $\frac{\text{sda}}{\cos\ a}$ = D x $\frac{s}{\cos^2 a}$ da

Gravitational force

dF = $\frac{\text{GMdm}}{\left(\frac{s}{\cos\ a}\right)^2}$ cos = $\frac{\text{MG}\,D^a}{s}$ cos a da

Total force F = $\int_{-\pi/2}^{\pi/2} \frac{\text{MGD}}{s}$ cos a da = $\frac{2\text{MGD}}{s}$

Sol.7

If y is the weight of the cube outside

Then 3(l – y) l² = l^3 = $\frac{2}{3}$ L

Sol.8

Volume is decreased during the melting of ice i.e. a positive work is done by ice water system on the atmosphere. As the heat is being absorbed by ice to melt so internal energy of ice water system increases.

Sol.9

As PV = Constant⇒ PdV =–VdP i.e. $\frac{dP}{dV} = -\frac{P}{V}$Bulk modulus, K = $-\frac{-dP}{dV/V} = -\frac{dP}{dV}V = -\left(-\frac{P}{V}V\right)$= P

Sol.10

Let n be the Frequency of standard fork, $n_1 = \frac{103}{100}$ nFrequency of second fork, $n_2 = \frac{98}{100}$ n

Number of beats $n_1 - n_2$ = 6⇒ $\frac{103}{100}n - \frac{98}{100}n = \frac{5n}{100} = 6 \Rightarrow n = \frac{600}{5} = 120$ Hz⇒ $n_1 = 123.6$ Hz

Sol.11

$n_2 = \frac{98\text{x}120}{100} = 117.6$ Hz

Sol.12

As $Q = \frac{q}{\epsilon_0}$We get $Q = \frac{1}{\epsilon_0}\ {\epsilon_0}^{-1}$

Sol.13

Three resistors of resistance R each has total resistance in series = 3Rand in parallel = $\frac{3}{R}$Two resistors in series and one in parallel give total resistance

$= \frac{2R x R}{R+2R} = \frac{2}{3} R$

Two resistors in parallel and one is series give total resistance

$= \frac{R}{2} + R = \frac{3}{2} R$

Obviously $\frac{R}{2}$ i.e. $\frac{2}{4}$ R is not possible

Sol.14

Only statement 1 is correct

Sol.15

Magnetic field at the middle of the solenoid

$B = \mu_0 nI = \mu_0 \frac{N}{L} I = 4\pi \times 10^{-7} \times \frac{500}{0.4} \times 3 = 4.713 \times 10^{-3} T$

Magnetic dipole moment of the coil

$M = NIA = NI\pi r^2$ = 10 x 0.4 x 3.142 x $(0.01)^2$

= 1.26 x $10^{-3} Am^2$

Torque acting on the coil

$\tau = MB \sin\theta$ As $\theta = 90^0$, $C = MB$

$\tau = 1.26 \times 10^{-3} \times 4.713 \times 10^{-3}$

= 5.94×10^{-6} Nm

Sol.16

As per Lenz's law it shows no polarity

Sol.17

Input and output powers for 100% efficiency, should be same.

Sol.18

As $C = \frac{E}{B} \Rightarrow B = \frac{E}{C} = \frac{60}{3\times10^8} = 2 \times 10^{-7} T$

Sol.19

Here path difference, = $\sqrt{D^2 + D^2}$ – D = D $\left[1 + \frac{d^2}{D^2}\right]^{1/2} - D = D\left[1 + \frac{d^2}{2D^2}\right] - D = \frac{a^2}{2D} = \frac{\lambda}{2}$

Or $\lambda = \frac{d^2}{D}$

Sol.20

Given distance between lenses $= f_0 + f_e$

and magnification, $M = \frac{f_0}{f_e}$

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{u}{v} + 1 = \frac{u}{f}$

i.e. $M = \frac{f}{u-f}$ But u becomes $(f_0 + f_e)$ when objective is replaced by slit

Then $\frac{l}{L} = \frac{f_e}{(f_0+f_e)-f_e} = \frac{f_e}{f_0} = \frac{1}{M}$ or $M = \frac{L}{l}$

Sol.21

As we know

For maxima $2\mu t = (2n-1)\frac{\lambda_1}{2}$

For minima$2\mu t = 2n\frac{\lambda_2}{2}$

Given $(2n-1)\lambda_1 = 2n\lambda_2$ Or $(2n-1)6000 = 2n(4500)$

$(2n-1)4 = 2n.3 \Rightarrow 8n - 4 = 6n \Rightarrow n = 2$

$\therefore 2 \times 1.33 \times t = 2 \times 2 \times \frac{4500}{2} \Rightarrow t = 3.38 \times 10^{-5}$cm

Sol.22

The de-Broglie wave formed between inter atomic spacing is given by

$d_1 = \frac{n\lambda}{2}$ and $d_2 = (n+1)\frac{\lambda}{2}$

$\therefore \frac{\lambda}{2} = (d_2 - d_1)$or $\lambda = 2(d_2 - d_1) \Rightarrow \lambda = 2(2.5 - 2) = 1A^0$

$\therefore d_{min} = \frac{1}{2}A^0 = 0.5A^0$

Sol.23

Both the statement are self explanatory

Sol.24

As we know $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$ For 33% decay ,$\frac{N}{N_0} = \frac{67}{100}$

$\Rightarrow \left(\frac{67}{100}\right) = \left(\frac{1}{2}\right)^{t_1/20}$ For 67% decay ,$\frac{N}{N_0} = \frac{33}{100} \Rightarrow \frac{33}{100} = \left(\frac{1}{2}\right)^{\frac{t_2 - t_1}{20}}$ Or $\left(\frac{1}{2}\right)^1 = \left(\frac{1}{2}\right)^{(t_2 - t_1)/20}$ Or $\frac{t_2 - t_1}{20} = 1 \Rightarrow$ $t_2 - t_1 = 20$ min

Sol.25

Electrons of emitter reach the collector through the base

Sol.26

Modulation index, $\mu = \frac{A_m}{A_c} = \frac{1}{2} = 0.5$

Sol.27

The net charge shared between the two capacitors is $Q' = Q_2 - Q_1 = 4CV - CV = 3CV$

The two capacitors will have the same potential, V'

The net capacitance of the parallel combination of two capacitors will be $C' = C_1 + C_2 = C + 2C = 3C$

The potential difference across the capacitor will be V' = $\frac{Q'}{C'} = \frac{3CV}{3C} = V$

The electrostatic energy of the capacitors will be $U' = \frac{1}{2}C'V'^2 = \frac{1}{2}(3CV)^2 = \frac{3}{2}CV^2$

Sol.28

For a current flowing into a circular arc, the magnetic induction at the centre is

$B = \left(\frac{\mu_0 I}{4\pi r}\right)\theta$ or $B \propto I\theta$

In the given questions, the total current is divided into two area

$I \propto \frac{1}{\text{Resistance of arc}} \propto \frac{1}{\text{Length of arc}} \propto \frac{1}{\text{angle subtended at the centre } (\theta)}$ or $I\theta = \text{Constant}$

$\Rightarrow$ Magnetic field at centre due to arc is equal and opposite to the magnetic field at centre due to arc or the net magnetic field at centre is zero.

Sol.29

$\Delta l = \frac{Fl}{AY} = \frac{Fl}{\left(\frac{\pi d^2}{4}\right)Y}$ or $(\Delta l) \propto \frac{1}{d^2}$

Sol.30

Internal energy of n moles of an ideal gas at temperature T is given by U = n $\left(\frac{f}{2}RT\right)$

Here

f = degree of freedom which is 5 for O_2 and 3 for Ar

$\therefore U = U_{O_2} + U_{A_r} = 2\left(\frac{5}{2}RT\right) + 4\left(\frac{3}{2}RT\right) = 11RT$

MATHEMATICS

Sol.1

Let A and B be two sets having m and n elements respectively. Then, Number of subsets of A = 2^m

Number of subsets of B = 2^n

Given that $2^m - 2^n = 56$

$\Rightarrow 2^n(2^{m-n} - 1) = 2^3\ (2^3 - 1)$

$\Rightarrow n = 3\ and\ m - n = 3 \Rightarrow n = 3\ and\ m = 6$

Sol.2

Given that $f(x) = \frac{4-x}{x-4}$

At $x = 4, f(x)$ takes indeterminate form $\frac{0}{0}$

$\therefore f(x)$ is defined for all x except x = 4

$\therefore$ domain (f) = R – {4}

For any x ∈ domain (f), we have

$f(x) = \frac{4-x}{x-4} = -\frac{(x-4)}{x-4} = -1$

$\therefore$ Range (f) = $\{-1\}$

Sol.3

We know

$(cos\theta + sin\theta)^2 + \ (cos\theta - sin\theta)^2 = 2$

$\Rightarrow \left(\sqrt{2}cos\theta\right)^2 + \ (cos\theta - sin\theta)^2 = 2$

$\Rightarrow (cos\theta - sin\theta)^2 = 2 - 2\,cos^2\,\theta$

$\Rightarrow (cos\theta - sin\theta)^2 = 2\,sin^2\,\theta$

$\Rightarrow cos\theta - sin\theta = \ \pm\sqrt{2}\,sin\theta$

Sol.4

We have cotα=$\frac{1}{2}$ $\Rightarrow tan\alpha$ =Now tanβ= $\pm\sqrt{sec^2\,\beta - 1} = \pm\sqrt{\frac{25}{9} - 1} = \pm\frac{4}{3}$

$\Rightarrow$ tan β = $\frac{-4}{3}$ [$\because$ β lies n second quadrant]

$\Rightarrow$ tan$(\alpha + \beta) = \ \frac{tan\ \alpha + tan\ \beta}{1 - tan\ \alpha\ tan\ \beta} = \frac{2 - \frac{4}{3}}{1 - 2 \times \frac{-4}{3}} = \frac{2}{11}$

Sol.5

$sin^4\,\frac{7\pi}{8} = sin^4\left(\pi - \frac{\pi}{8}\right) = sin^4\,\frac{\pi}{8}$

and$sin^4\,\frac{5\pi}{8} = sin^4\left(\pi - \frac{3\pi}{8}\right) = sin^4\,\frac{3\pi}{8}$

L.H.S. = $sin^4\,\frac{\pi}{8} + sin^4\,\frac{3\pi}{8} + sin^4\,\frac{5\pi}{8} + sin^4\,\frac{7\pi}{8}$

$= 2\ \left[sin^4\,\frac{\pi}{8} + sin^4\,\frac{3\pi}{8}\right]$

$= 2\ \left[\left(sin^2\,\frac{\pi}{8}\right)^2 + \ \left(sin^2\,\frac{3\pi}{8}\right)^2\right]$

$=2\left[\left\{\frac{1 - cos\frac{\pi}{4}}{2}\right\}^2 + \ \left\{\frac{1 - cos\frac{3\pi}{4}}{2}\right\}^2\right]$

$\frac{=2}{4}\left[\left(1 - cos\,\frac{\pi}{4}\right)^2 + \ \left(1 - cos\,\frac{3\pi}{4}\right)^2\right]$

$\frac{=1}{2}\left[\left(1 - \frac{1}{\sqrt{2}}\right)^2 + \ \left(1 + \frac{1}{\sqrt{2}}\right)^2\right]$

$\frac{=\ 1}{2}\left[\left(1 + \frac{1}{2} - \sqrt{2}\right) + \left(1 + \frac{1}{2} + \sqrt{2}\right)\right] = \frac{3\ =}{2}$R.H.S.

Sol.6

Let $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = K$

Then a = K sin A, b = K sin B, c = K sin C

$\Rightarrow \frac{c - a\cos B}{b - a\cos C} = \frac{(a\cos B + b\cos A) - a\cos B}{(a\cos C + c\cos A) - a\cos C} = \frac{b\cos A}{c\cos A}$

$= \frac{b}{c} = \frac{K\sin B}{K\sin C} = \frac{\sin B}{\sin C}$

Sol.7

$\left(\frac{3+2i}{2-3i}\right) + \left(\frac{3-2i}{2+3i}\right)$

$= \frac{3+2i}{2-3i} \times \frac{2+3i}{2+3i} + \frac{3-2i}{2+3i} \times \frac{2-3i}{2-3i}$

$= \frac{(3+2i)(2+3i)}{4-9i^2} + \frac{(3-2i)(2-3i)}{4-9i^2}$

$= \frac{13i}{13} - \frac{13i}{13} = 0$

Sol.8

We have $5x - 3 < 3x + 1 \Longrightarrow 5x - 3x < 3 + 1 \Longrightarrow 2x < 4 \Longrightarrow x < 2$

$\therefore x \in (-\infty, 2)$

Sol.9

The committee can be formed in the following ways:

(i) By selecting 2 men and 1 woman.
(ii) By selecting 1 man and 2 woman.

Now 2 men out of 5 men and 1 woman out of 2 woman can be chosen in ${}^5C_2 \times {}^2C_1$ways ∴ Total number of ways of forming the committee = ${}^5C_2 \times {}^2C_1 + {}^5C_1 \times {}^2C_2 = 20 + 5 = 25$

Sol.10

The old integers between 2 and 100 which are divisible by 3 are 3, 9, 15, 21 ……99. Clearly it is an A.P. with the first term a = 3 and common difference d = 6. Let there are n term is the sequence. Then $T_n = a + (n - 1)d$

⇒ 99 = 3 + (n − 1) d⇒ n = 17

∴ Required sum = $\frac{n}{2}[a + l] = \frac{17}{2}[3 + 99] = 867$

Sol.11

Let the coordinates of the third vertex be (x, y),

then$\frac{x+3-7}{3} = 2 \; and \; \frac{y-5+4}{3} = -1$

⇒ x – 4 = 6 and y – 1 = −3

⇒ x = 10 and y = −2

∴ third vertex is (10, −2)

Sol.12

The equation of given circle is

$x^2 + y^2 + 8x - 16y + 64 = 0$

⇒ $(x^2 + 8x + 16) + (y^2 - 16y + 64) = 16$

⇒ $(x+4)^2 + (y-8)^2 = 4^2$

Therefore centre of circle is (−4, 8) and radius = 4.

The image of the circle in the line mirror has centre (4, 8) and radius = 4

∴ equation of required circle is

$(x-4)^2 + (y-8)^2 = (4)^2$

⇒ $x^2 + y^2 - 8 - 16y + 64 = 0$

Sol.13

Let the equation of requires ellipse be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (i)

The coordinate of its vertices and foci are (± a, 0)

and (± ae, o) respectively.

∴ a = 5 and be = 4 ⇒ e = $\frac{4}{5}$Now $b^2 = a^2(1 - e^2)$

⇒ $b^2 = 25\left(1 - \frac{16}{25}\right) = 9$

Substituting the values of $a^2 \; and \; b^2 \; in \; (i)$,

we get $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Sol.14

L.H.L. = $Lt_{x\to 0_-} f(x) = Lt_{h\to 0} f\ (0-h)$

$= Lt_{h\to 0} \frac{-h-|-h|}{(-h)} = Lt_{h\to 0} \frac{-h-h}{-h}$

$= Lt_{h\to 0} \frac{-2h}{-h} = Lt_{h\to 0}\ 2 = 2$

R.H.L.= $Lt_{x\to 0_+} f(x) = Lt_{h\to 0} f\ (0+h)$

$= Lt_{h\to 0} \frac{-h-|h|}{h} = Lt_{h\to 0} \frac{h-h}{h}$

$= Lt_{h\to 0} \frac{0}{h} = Lt_{h\to 0}\ 0 = 0$

Clearly L.H.L. ≠ R.H.L.

$\therefore Lt_{h\to 0} f(x)\ does\ not\ exists.$

Sol.15

There are 9 persons. Out of these 9 persons 4 persons can be selected in $^9C_2$ = 126 ways. Exactly 2 children can be selected from 4 children in $^4C_2$ number of ways. 2 persons can be selected from 5 persons in $^5C_2$ number of ways.Therefore favourable number of events = $^4C_2 \times {}^5C_2$ = 60 So required probability = $\frac{60}{126} = \frac{10}{21}$

Sol.16

$\sin^{-1}\left(sin\frac{2\pi}{3}\right) \neq \frac{2\pi}{3}\ as\ \frac{2\pi}{3}$ does not lie between $\frac{-\pi}{2}\ and\ \frac{\pi}{2}$

now $\sin^{-1}\left(sin\frac{2\pi}{3}\right) = sin^{-1}\left\{sin\left(\pi-\frac{\pi}{3}\right)\right\} = sin^{-1}\left(sin\frac{\pi}{3}\right) = \frac{\pi}{3}$

Sol.17

Let $\Delta = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ Multiplying R_1, R_2, R_3, by a,b,c respectively

$\Delta = \frac{1}{abc}\begin{vmatrix} ab^2c^2 & abc & ab+ac \\ bc^2a^2 & abc & bc+ba \\ ca^2b^2 & abc & ac+bc \end{vmatrix} \Delta = \frac{(abc)^2}{abc}\begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ba \\ ab & 1 & ac+bc \end{vmatrix}$ Applying $c_3 \to c_3 + c_1$

$\Delta = abc\ \begin{vmatrix} bc & 1 & ab+bc+ca \\ ca & 1 & ab+bc+ca \\ ab & 1 & ab+bc+ca \end{vmatrix}$ = abc(ab + bc + ca) $\begin{vmatrix} bc & 1 & 1 \\ ca & 1 & 1 \\ ab & 1 & 1 \end{vmatrix}$ = abc (ab + bc + ca) . 0 = 0

Sol.18

L.H.L. = $Lt_{x\to 1-}\ f(x)$

$= Lt_{x\to 1-}(5ax - 2b) = \ 5a - 2b$

R.H.L.$= Lt_{x\to 1+} f(x)$

$= Lt_{x\to 1+}(3ax + b) = \ 3a + b$

and f(1) = 11

Given that f(x) is continuous at x = 1

$\therefore Lt_{x\to 1-} f(x) = \ Lt_{x\to 1+} f(x) = \ f(1)$

Therefore $5a - 2b = 3a + b = 11 \Rightarrow$ a = 3 and b = 2

Sol.19

L.H.D = $Lt_{x\to 0_-} \frac{f(x) - f(0)}{x-0}$

$= Lt_{h\to 0} \frac{f(0-h)-(f0)}{0-h-0}$

$= Lt_{h\to 0} \frac{f(-h) - f(0)}{-h}$

$= Lt_{h\to 0} \frac{|-h| - |0|}{-h}$

L.H.D.$= Lt_{h\to 0} \frac{h}{-h} = Lt_{h\to 0} - 1 = -1$

R.H.D.$= Lt_{x\to 0+} \frac{f(x) - f(0)}{x\ -\ 0}$

$= Lt_{h\to 0} \frac{f(0+h) - f(0)}{0+h-0}$

$= Lt_{h\to 0} \frac{f(h) - f(0)}{h}$

$= Lt_{h\to 0} \frac{|h| - |0|}{h}$

$= Lt_{h\to 0} \frac{h}{h} = Lt_{h\to 0}\ 1 = 1$

L.H.D. ≠ R.H.D. at x = 0

∴ f(x) is not differentiable at x = 0

Sol.20

Given that y = $\sqrt{\frac{1+e^x}{1-e^x}}$

Differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1+e^x}{1-e^x}\right)^{\frac{1}{2}-1} \frac{d}{dx}\left(\frac{1+e^x}{1-e^x}\right)$

$= \frac{1}{2}\sqrt{\frac{1-e^x}{1+e^x}} \times \frac{2e^x}{(1-e^x)^2}$

$= \frac{e^x}{\sqrt{1+e^x}(1-e^x)^{3/2}} = \frac{e^x}{\sqrt{1+e^x}\sqrt{1-e^x}\,(1-e^x)}$

$\Rightarrow \frac{dy}{dx} = \frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}$

Sol.21

The equation of the curve is y = x^2 – 5x + 6 (i)

$\frac{dy}{dx} = 2x - 5$

$m_1 = slope\ tangent\ at\ (2, 0)$

$= \left(\frac{dy}{dx}\right)_{(2,0)} = 2 \times 2 - 5 = -1$

$m_2 = slope\ of\ tangent\ at\ (3, 0)$

$= \left(\frac{dy}{dx}\right)_{(3,0)} = 2 \times 3 - 5 = 1$

Clearly $m_1 m_2 = (-1) \times 1 = -1$

product of slopes = − 1

Sol.22

We have $4x^3 – 24x^2 + 44x – 24 > 0$

$\Rightarrow x^3 – 6x^2 + 11x – 6 > 0$

$\Rightarrow (x-1)(x^2 - 5x + 6) > 0$

$\Rightarrow (x-1)(x-2)(x-3) > 0$

$\Rightarrow 1 < x < 2\ or\ x > 3 \Rightarrow x \in (1, 2) \cup (3, \infty)$

Sol.23

Let y = f(x) = $x^3 - 6x^2 + 9x - 8$

Differentiating both sides, we get

$\frac{dy}{dx} = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3)$

For local maxima or local minima,we have $\frac{dy}{dx} = 0$

$\Rightarrow 3\ (x^2 -\ 4x\ + 3) =\ 0\ \Rightarrow x = 1, 3$

$\frac{dy}{dx} changes\ sign\ from + ve\ to - ve\ as\ x\ increases\ through\ 1$

∴ x = 1 is a point of local maxima

$\frac{dy}{dx} changes\ sign\ from - ve\ to + ve\ as\ x\ increases\ through\ 3$ ∴ x = 3 is a point of local minima

Sol.24

Let I = $\int \frac{1+cos\,4x}{cot\ x - tan\ x} dx$

$= \int \frac{2\,cos^2 2x\ cos\ x\ sin\ x}{cos^2 x - sin^2 x}\ dx$

$= \int\ cos\ 2x\ sin\ 2x\ dx$

$= \frac{1}{2} \int\ sin\ 4x\ dx =\ -\frac{1}{8} cos\ 4x + c$

Sol.25

I $=\ \int \frac{a^x}{1-a^{2x}} dx = \int\ \frac{a^x}{1-(a^x)^2} dx$

Let $a^x\ = t \quad Then\ d\ (a^x) =\ dt$

$\Rightarrow a^x log\text{a}\ dx = dt$

$\Rightarrow dx = \frac{dt}{a^x\ loga}$

I = $\int \frac{a^x}{\sqrt{1-t^2}} \frac{dt}{a^x\ log\ a}\ = \frac{1}{log\ a} \int \frac{dt}{\sqrt{1-t^2}}$

I = $\frac{1}{log\ a} \times sin^{-1}(t) +\ c = \frac{1}{loga} sin^{-1}(a^x) +\ c$

Sol.26

Clearly repetition of digits is allowed. Since a three digit number greater than 600 will have 6 or 7 at hundred's place. So, hundred,s place can be filled in 2 ways. Each of ten's and one's place can be filled in 5 ways.∴ Total number of required numbers = 2 × 5 × 5 = 50

Sol.27

Let r be the common ratio of the G.P. It is given that the first term a = 1

Now $a_3 + a_5 = 90 \Rightarrow ar^2 + ar^4 = 90 \Rightarrow r^2 + r^4 = 90 \Rightarrow r^4 + r^2 - 90 = 0 \Rightarrow r^4 + 10r^2 - 9r^2 - 90 = 0$

$\Rightarrow (r^2 + 10)(r^2 - 9) = 0 \Rightarrow r^2 - 9 = 0 \Rightarrow r = \pm 3$

Sol.28

Here a = 4, b = −3, So equation of the line is $\frac{x}{a} + \frac{y}{b} = 1$ or $\frac{x}{4} + \frac{y}{-3} = 1$Or 3x – 4y = 12

Sol.29

Let the equation of the required circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ (i) It press through (1, −2) and (4, −3)

∴ 5 + 2g – 4f + c = 0 (ii)

25 + 8g – 6f + c = 0 (ii) The centre (−g, −f) lies on 3x + 4y = 7

Therefore – 3g – 4f = 7 (iv) Subtracting (ii) from (iii) we get

20 + 6g – 2f = 0⇒10 + 3g – f = 0 (v)

Solving (iv) and (v) simultaneous equations We get $g = -\frac{47}{15}, \ f = \frac{3}{5}$

Substituting the values of g and f in (ii), we get $5 - \frac{94}{15} - \frac{12}{5} + c = 0 \Rightarrow c = \frac{55}{15} = \frac{11}{5}$

Putting the values of g, f and c in (i), we get $x^2 + y^2 - \frac{94}{15}x + \frac{6}{5}y + \frac{11}{5} = 0$

$\Rightarrow 15(x^2 + y^2) - 94x + 18y + 55 = 0$

Sol.30

$$Lt_{x\to\frac{\pi}{6}} \frac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$$

$$Lt_{x\to\frac{\pi}{6}} \frac{(2\sin x - 1)(\sin x + 1)}{(2\sin x - 1)(\sin x - 1)}$$

$$Lt_{x\to\frac{\pi}{6}} \frac{\sin x + 1}{\sin x - 1} = \frac{\frac{1}{2}+1}{\frac{1}{2}-1} = -3$$