

2005-PUNJAB TECHNICAL UNIVERSITY

B.TECH IV SEMESTER DEGREE EXAMINATION

NUMERICAL ANALYSIS

(MECHANICAL ENGINEERING)

TIME-3HOUR

MARK-60

Note: Section A Is Compulsory. Attempt Any Four Questions From Section B And Any Two From Section C.

SECTION A MARKS 2 EACH

1. (a) With $a=0$, $b=1$, the following function changes sign in (a, b) , what point does the bisection method locate? Is this point a zero of $f(x)$?
- (b) Find the function whose first difference is $9x^2+11x+5$.
- (c) What are the major drawbacks of the Lagrange's form of interpolation?
- (d) Prove that (please see the attachment)
- (e) Show that the matrix (please see the attachment) is Invertible, but that A cannot be written as the product of a lower triangular matrix with an upper triangular matrix,
- (f) Calculate the number of additions and number of multiplications necessary to multiply an $n \times n$ matrix with an n -vector.
- (g) If third differences are constant, prove that:
- (h) Prove that:
- (i) Why higher order Newton-Cole's formula for numerical integration are not commonly used?
- (j) Convert the following second order initial value problem into a system of first order initial value problem. $y'' + 4t^3y = 0$. $Y(1) = 1$, $y'(1) = 2$.

SECTION B MARKS 5 EACH

2. Show that bisection method always converges and its order of convergence is one.

3. Solve the equations:

$$x_1 + x_2 + x_3 = 6$$

$$3x_1 + (3 + \epsilon)x_2 + 4x_3 = 20$$

$$2x_1 + x_2 + 3x_3 = 13$$

using Gauss elimination method, where ϵ is small such that $1 + \epsilon^2 \sim 1$.

4 (a) Define the operators d and m and prove that

$$d(f(x)g(x)) = mf(x)dg(x) + mg(x)df(x). \text{ where } d = \text{delta and } m \text{ is mu}$$

(b) Use Newton's formula for interpolation to find the number of deaths at 40-50 and 50-55 if the following are the number of deaths on for successive ten year age groups:

Age group Deaths

25-35 13229
35-45 18139
45-55 24225
55-65 31496

5. Use Stirling's formula to find the first derivative of the function $y = 2e^{x-1}$ tabulated below at the point $x = 0.6$

X y

0.4 1.5836494
0.5 1.7974426
0.6 2.0442376
0.7 2.3275054
0.8 2.6510818

Compare with the true value which is 2.044238

6. Derive Simpson's 1/3 formula for numerical integration and show that its local truncation error is of the order h^3 .

SECTION C MARKS 10 EACH

7. (a) Use Picard's method to approximate y where $x = 0.1, x = 0.2$, given that $y = 0$ when $x = 0$, $dy/dx = x + y$. Compare the results with exact value.

(b) Find the three term Taylor series solution for the third order initial value problem

$$W''' + WW'' = 0.$$

$$W(0) = 0, W'(0) = 0, W''(0) = 1.$$

Find the bound on the error for $t \in [0, 0.2]$

8. By considering the limit of the three-point Lagrange interpolating polynomial relative to (please see the attachment)

9 (a) Factorize the following matrix into LU decomposition using direct factorization (Please see the attachment) with $u_{ii} = 1$ for all i :

(b) The equation $e^x - 4x^2 = 0$ has a root between $x = 4$ and $x = 5$. Show that we cannot find this root using fixed point iteration with natural iteration function $x = \frac{1}{2} e^{x/2}$