

2006-VISVESVARAYA TECHNOLOGICAL UNIVERSITY

**B. E DEGREE EXAMINATION
ANALYSIS AND DESIGN OF ALGORITHMS
(COMPUTER SCIENCE AND ENGINEERING)**

**TIME-3HOUR
MARK-80**

ANSWER ANY FIVE QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS

1 a. Define O notation, Θ notation and Ω notation. If $f_1(n) \in O(g_1(n))$ and $f_2(n) \in O(g_2(n))$, prove that $f_1(n) + f_2(n) \in O(\max [g_1(n), g_2(n)])$.

b. Develop an algorithm to determine the minimum and maximum values in an array a_1, a_2, \dots, a_n of integers (Here $n \geq 1$ and the entries in the array need not be distinct). Determine the worst case complexity function for this algorithm.

c. What is wrong with the following argument? Since $n = O(n)$, $2n = O(n)$,..... we have $\sum_{k=1}^n k = O(n) = O(n^2)$.

2 a. Design a brute force algorithm for computing the value of a polynomial $p(x) = A_n x^n + A_{n-1} x^{n-1} + \dots + A_1 x + A_0$ are given point x_0 and determine its worst case complexity class.

b. If the algorithm designed in part (a) is in $\Theta(n^2)$. design a linear algorithm for this problem.

c. Write a quick sort algorithm. Derive worst-case and average-case complexities for this algorithm.

3 a. Write a decrease-by-one algorithm to generate all 2^n subsets of a set $\{a_1, a_2, \dots, a_n\}$ in quashed order i.e. subset involving a_j can be listed only after all subsets involving a_1, a_2, \dots, a_{j-1} ($j=1, 2, \dots, n-1$)

b. Design a decrease-by-one algorithm for generating a gray code of order n .

c. Solve the system of linear equations given below by gaussian elimination:

$$2x_1 - x_2 + x_3 = 1$$

$$4x_1 + x_2 - x_3 = 5$$

$$x_1 + x_2 + x_3 = 0$$

4 a. Define a heap. Prove that a n -element heap has height $\lceil \log n \rceil$. Show that there is a linear algorithm to construct a heap of size n .

b. What is the running time of heapsort on an array A of length n that is already sorted in the increasing order? what about decreasing order?

5 a. What is input enhancement? Apply this technique to design a linear sorting algorithm.

b. When does collision occur in hashing? What are different mechanisms used to resolve collisions?

c. Consider open hashing with linear probing policy. For the input : 1055, 1492, 1776, 1812, 1918, 1945 inserted in the order and hash function. $h(k) = 5k \pmod{8}$

i) Construct the open hash table

ii) Show the sequence of key comparisons needed to search for 1945 and 1543 in the table.

6 a. Write warshall's algorithm to find transitive closure of a diagraph. Prove that the time complexity of the algorithm is $\Theta(n^3)$.

b. Apply warshall's algorithm to find transitive closure of a diagraph defined by the following adjacency matrix.

[0 0 0 0
0 0 1 1
0 1 0 0
1 0 1 0]

7 a. What is a decision tree? Use decision trees to establish lower bound on worst-case and average case efficiency of comparison based sorting algorithm.

b. Define NP-complete problem. Prove that the hamiltonian circuit problem is polynomially reducible to the decision version of travelling salesman problem. (TSP).

8 a. What is a C-approximation algorithm ? Write a 2- approximation algorithm for a TSP with a Euclidian distances.

b. If $P \neq NP$, prove that there exists no C-approximation algorithm for TSP.

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