MATHEMATICS

Standard

X

Question Pool

Government of Kerala
Department of Education

Prepared by

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Preface

This book contains a collection of model questions of the public examination. For each question, corresponding learning outcomes, score, time and scoring indicators are given.

Students themselves can assess their achievements by practicing these questions. We hope that this will be a great help for the students to face the public examination with confidence.

Regards,

Director

Contents

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ARITHMETIC SEQUENCES

1. Learning Outcomes

- Formation of sequences from practical situations.
- Formulating the relation between terms, position of terms and expressing it in algebraic form.

Write the sequence obtained by adding two adjacent consecutive terms in counting numbers starting from 1. Write the algebraic expression of this sequence.

(Score: 3, Time: 4 minutes)

Scoring indicators

- Counting numbers : 1, 2, 3, 4, 5, ... (1)
- Sequence obtained by adding two adjacent consecutive terms : 1 + 2, 2 + 3, 3+ 4, 4 + 5, ... (1)
  3, 5, 7, 9, ...
- Algebraic expression of above sequence : $n + (n + 1) = 2n + 1$ (1)

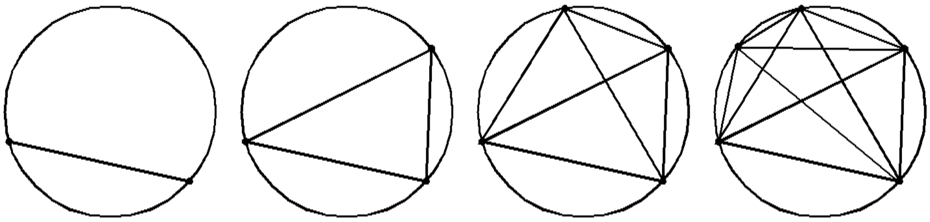
2. Learning Outcomes

- Formation of sequence from practical situations.
- Formulating the relation between terms, position of terms and expressing it in algebraic form.

Consider circles, points on its circumference and chords as shown in the figure. Mark two points on the circle and draw a chord. Mark one more point and draw three chords. Continue this process by adding one more point each time.

a) Write the number of chords in each figure as a sequence.
b) Write the algebraic expression of this sequence
c) Find the number of chords in the 10th figure.

(Score: 4, Time: 7 minutes)



■ Scoring indicators

(a) No. of chords in figure.1 = 1
No. of chords in figure.2 = $1 + 2 = 3$
No. of chords in figure.3 = $1 + 2 + 3 = 6$ (1)
Sequence of number of chords = 1, 3, 6, 10, ... (1)
(b)No. of chords in figure. n = $1 + 2 + 3 + ... + n$
$= \frac{n(n+1)}{2}$ (1)
(c)No. of chords in the 10th figure = $\frac{10 \times 11}{2} = 55$ (1)

3. Learning Outcomes

- Formulating sequence from practical situations.
- Formulating the relation between terms, position of terms and expressing it in algebraic form.
- Finding a particular term from the algebraic expression.

A pattern is formed using sticks of equal length as shown below:

a) Write the sequence of number of sticks used in each figure.

b) Write the sequence of number of squares and rectangles in each figure.

c) Write the algebraic expression in the above two sequences

d) Find the number of sticks and squares in the 10th figure

(Score: 4, Time: 9 minutes)

■ Scoring indicators

(a) No of sticks in the figure.1 = $1 + 3 = 4$
No of sticks in the figure.2 = $1 + 3 + 3 = 1 + 2 \times 3 = 7$
No of sticks in the figure.3 = $1 + 3 \times 3 = 10$
No of sticks in the figure.4 = $1 + 4 \times 3 = 13$
Sequence of number of sticks = 4, 7, 10, 13, ... (1)
(b) No of squares and rectangles in the figure.1 = 1
No of squares and rectangles in the figure.2 = $2 + 1 = 3$
No of squares and rectangles in the figure.3 = $3 + 2 + 1 = 6$
No of squares and rectangles in the figure.4 = $4 + 3 + 2 + 1 = 10$
Sequence of squares and rectangles = 1, 3, 6, 10, ... (1)
(c) No of sticks in the nth figure = $1 + n \times 3 = 3n+1$
No of squares and rectangles in the nth figure = $1 + 2 + 3 ... + n$
$= \frac{n(n+1)}{2}$ (1)

(d) No of sticks in the 10th figure $= 3 \times 10 + 1 = 31$

No of squares and rectangles in the 10th figure $= \frac{10 \times 11}{2} = 55$ (1)

4. Learning Outcomes

- Formulating an arithmetic sequence when a term, position of the term, common difference are known.
- The difference between any two terms of an arithmetic sequence will be a multiple of common difference.

Consider an arithmetic sequence with common difference 6 and 7th term 52. Find the 15th term of the arithmetic sequence. Is it possible, to get a difference of 100 between any two terms of this sequence?

(Score: 3, Time: 5 minutes)

Scoring indicators

- 15th term can be obtained by adding 8 times the common difference to the 7th term (1)

  ie., $x_{15} = x_7 + 8d$

  $= 52 + 8 ' 6 = 100$ (1)

  The difference between any two terms of an Arithmetic sequence will be a multiple of common difference

  100 can't be the difference between any two terms of this sequence, since it is not a multiple of 6. (1)

5. Learning Outcomes

- In an arithmetic sequence the difference between any two terms will be the product of common difference and difference of term positions of each term.
- Finding any term of an arithmetic sequence where a term, its position and, common difference are given

Consider an arithmetic sequence whose 7th term is 34 and 15th term is 66.

(a) Find the common difference.

(b) Find the 20th term.

(Score: 3, Time: 5 minutes)

Scoring indicators

(a) 15th term can be obtained by adding 7th term and 8 times the common difference.

$x_{15} = x_7 + 8d$ (1)

$66 = 34 + 8d$

$8d = 66 - 34 = 32$

$d = \frac{32}{8} = 4$ (1)

(b) 20th term can be obtained by adding 15th term and 5 times the common difference.

$$x_{20} = x_{15} + 5d$$
$$= 66 + 5 \times 4 = 86 \quad (1)$$

6. Learning Outcomes

- Formulating an algebraic expression connecting terms and its position in arithmetic sequence.
- Finding a particular term using algebraic expression in an arithmetic sequence.

❓ Consider an arithmetic sequence $\frac{17}{7}, \frac{20}{7}, \frac{23}{7}$, ...

(a) Write the algebraic expression of the sequence.

(b) Write the sequence of counting numbers in the above given sequence. Is the newly obtained sequence an arithmetic sequence.

(Score: 4, Time: 6 minutes)

■ **Scoring indicators**

(a) Common Difference= $\frac{20}{7} - \frac{17}{7} = \frac{3}{7}$ (1)

Algebraic expression of the sequence = $\frac{17}{7} + (n-1)\frac{3}{7}$

$$x_n = \frac{3}{7}n + \frac{17}{7} - \frac{3}{7}$$

$$= \frac{3}{7}n + 2 \quad (1)$$

(b)
$$x_n = \frac{3}{7}n + 2$$
$$x_7 = \frac{3}{7} \times 7 + 2 = 5$$
$$x_{14} = \frac{3}{7} \times 14 + 2 = 8$$
$$x_{21} = \frac{3}{7} \times 21 + 2 = 11 \quad (1)$$
$$x_n = \frac{3}{7} \times n + 2$$

If we give multiples of 7 to n in the expression $\frac{3}{7}n + 2$, we get counting numbers.

ie., $x_n = \frac{3}{7} \times 7m + 2$

$= 3m + 2$

Thus 5, 8, 11..... is an arithmetic sequence with common difference 3 (1)

7. Learning Outcomes

- Formulating an algebraic expression connecting terms and its position in arithmetic sequence.
- Finding a particular term using algebraic expression in an arithmetic sequence.

Considering an arithmetic sequence $\frac{17}{7}, \frac{31}{7}, \frac{45}{7}, \ldots$

(a) Write the algebraic expression of the sequence.

(b) Is there any counting number in this sequence? Justify your answer?

(Score: 3, Time: 6 minutes)

Scoring indicators

a. Common difference $= \frac{31}{7} - \frac{17}{7} = \frac{14}{7} = 2$ (1)

Algebraic expression of the sequence $= \frac{17}{7} + (n-1)\, 2$

$= 2n + \frac{17}{7} - 2$

$= 2n + \frac{3}{7}$ (1)

For any counting number 'n', $2n$ will be a counting number. But when $\frac{3}{7}$ is added to $2n$ it becomes a fraction. So this sequence will not have a counting number. (1)

8. Learning Outcomes

- In an arithmetic sequence the differnce between any two terms will be the product of common difference and difference of term position of each term.

x_n is the n^{th} term of an arithmetic sequence. If $x_a, x_b, x_c, \ldots$ are in arithmetic sequence, then prove that a,b,c are in arithmetic sequence.

(Score: 4, Time: 8 minutes)

Scoring indicators

- If 'd' is the common difference of an arithmetic sequence, then

$x_n - x_m = (n-m)d$

$\Rightarrow x_b - x_a = (b-a)d$ (1)

$x_c - x_b = (c-b)d$ (1)

Given ; $x_a, x_b, x_c, \ldots$ are in arithmetic sequence.

$\Rightarrow x_b - x_a = x_c - x_b$

$\Rightarrow (b-a)\, d = (c-b)d$ (1)

$b - a = c - b$

$\therefore$ $a, b, c, \ldots$ are in arithmetic sequence. (1)

9. Learning Outcomes

- In an arithmetic sequence the differnce between any two terms will be equal to the product of common difference and difference between the position of the terms.
- Finding other terms in an arithmetic sequence when common difference, a term and its position are given.

Find the 20th term of an arithmetic sequence if its 6th term is 14 and 14th term is 6.

(Score: 2, Time: 3 minutes)

■ Scoring indicators

$$x_6 = 14; \; x_{14} = 6$$

$$x_{14} - x_6 = (14 - 6)\,d \qquad (1)$$

$$6 - 14 = 8d$$

$$d = -1 \qquad (1)$$

$$x_{20} = x_{14} + 6d$$

$$= 6 + 6 \times -1 = 0 \qquad (1)$$

10. Learning Outcomes

- In an arithmetic sequence the differnce between any two terms will be equal to the product of common difference and difference between the position of the terms.
- Finding other terms in an arithmetic sequence when common difference, a term and its position are given.

Consider an arithmetic sequence whose m^{th} term is 'n' and n^{th} term is 'm'.

a) Find the common difference of the sequence
b) Prive that $(m + n + p)^{th}$ term of the sequence is '$-p$'

(Score: 3, Time: 7 minutes)

■ Scoring indicators

(a) $x_m = n \,; \; x_n = m$

$$x_m - x_n = (m - n)\,d \qquad (1)$$

$$n - m = (m - n)\,d$$

$$d = \frac{n - m}{m - n} = \frac{-(m - n)}{m - n} = -1 \qquad (1)$$

(b) $x_{m+n+p} = x_m + (\,m + n + p - m)\,d$

$$= n + (n + \mathrm{p}\,) \times -\,1$$

$$= n - n - p = -\,p \qquad (1)$$

11. Learning Outcomes

- Finding other terms of an arithmetic sequence whose common difference, a term and its position are given

Find the 13th term of an arithmetic sequence if 5 times the 5th term is equal to 8 times the 8th term

(Score: 4, Time: 6 minutes)

Scoring indicators

- 5th term = x_5
- 8th term = $x_5 + 3d$ (1)

$$5x_5 = 8(x_5 + 3d) \quad (1)$$

$$5x_5 = 8x_5 + 24d$$

$$-3x_5 = 24d \quad (1)$$

$$x_5 = -8d$$

$$x_{13} = x_5 + 8d$$

$$= -8d + 8d = 0 \quad (1)$$

12. Learning Outcomes

Understanding properties of an arithmetic sequence from its algebraic expression

Prove that the arithmetic sequence 7, 11, 15, ... does not contain perfect square.

(Score: 4, Time: 6 minutes)

Scoring indicators

- The algebraic expression of the arithmetic sequence is $x_n = 4n + 7 - 4 = 4n + 3$ (1)
- Each term in this sequence will give a remainder 3 when divided by 4. All counting numbers greater than 7, when divided by 4 will give a remainder 3 will be a term in this sequence. (1)

  Any counting number can be written in the form $4n$, $4n \pm 1$, $4n \pm 2$

$$\Rightarrow (4n)^2 = 16n^2$$

$$(4n \pm 1)^2 = 16n^2 \pm 8n + 1$$

$$(4n \pm 2)^2 = 16n^2 \pm 16n + 4 \quad (1)$$

- This shows that $(4n)^2$, $(4n \pm 2)^2$ are exactly divisible by 4. ie., remainder will be zero. But when $(4n \pm 1)^2$ is divided by 4 the remainder is 1. This shows that when a perfect square is divided by 4 there remainder is 0 or 1. But in the sequence 7,11,15......when terms are divided by 4 remainder is 3. So a perfect square will not be a term in this sequence. (1)

13. Learning Outcomes

Understanding the properties of an arithmetic sequence from its algebraic expression.

Let the algebraic expression of an arithmetic sequence be $5n + b$. If there is no perfect square in this sequence, find the counting number less than 5 that can be the value of 'b'.

(Score: 4, Time: 5 minutes)

Scoring indicators

- Any counting number can be written in the form $5n$, $5n \pm 1$, $5n \pm 2$
  $(5n)^2 = 25\ n^2$; when divided by 5, remainder = 0 (1)
  $(5n \pm 1)^2 = 25n^2 \pm\ 10n + 1$;when divided by 5, remainder = 1 (1)
  $(5n \pm 2)^2 = 25n^2 \pm\ 20n + 4$; when divided by 5, remainder = 4 (1)

⇒ Any perfect square when divided by 5 leaves remainder 0,1,4. So if remainder is 2 and 3 then it will not be a perfect square.

⇒ If there is not perfect square in the sequence, the only possibility for value of 'b' less than 5 is 2 and 3 (1)

14. Learning Outcomes

- Understanding the properties of an arithmetic sequence from its algebraic expression.

Consider the arithmetic sequence 10, 16, 22, ...Can you findout any terms as the sum or difference of any two terms of this sequence.

(Score: 5, Time: 5 minutes)

Scoring indicators

- Algebraic expressions of the sequence $x_n = 6n + 4$

  Each term when divided by 6, remainder is 4. (1)

$$n^{th}\text{ term} = 6n + 4$$

$$m^{th}\text{ term} = x_m = 6m + 4$$

$$x_n + x_m = 6n + 4 + 6m + 4 \quad (1)$$

$$= 6\ (n + m) + 8 = 6\ (n + m) + 6 + 2$$

⇒ $x_n + x_m$ when divided by 6 will give remainder '2'. Therefore $x_n + x_m$ can't be a term in the given sequence (1)

Similarly; $x_n - x_m = (6n + 4) - (6m + 4)$

$$= 6n - 6m = 6\ (n - m) \quad (1)$$

$x_n - x_m$ when divided by 6 will give remainder '0'. Therefore $x_n - x_m$ can't be a term in the given sequence. (1)

15. Learning Outcome

- Understanding the properties of an arithmetic sequence from its algebraic expression.

Prove that the square of any term of the arithmetic sequence 7,11,10..... will not be a term of the sequence.

(Score: 3, Time: 6 minutes)

Scoring indicators

- Algebric expression of the sequence $x_n = 4n + 3$. (1)

$\Rightarrow$ Each term when divided by 4 will leave the remainder 3.

$$x_n^2 = (4n + 3)^2 = 16n^2 + 24n + 9$$

$$= 16n^2 + 24n + 8 + 1 \quad (1)$$

x_n^2 when divided by '4' will leave remainder '1'.

$\Rightarrow$ x_n^2 will not be a term of the given sequence. (1)

16. Learning Outcomes

- Formulating the algebraic expression of an arithmetic sequence when a term and common difference is given
- Finding the position of term using the algebraic expression.

Find the 110th term in the arithmetic sequence 5, 12, 19, ...

(Score: 2, Time: 4 minutes)

Scoring indicators

- Common difference = 7

110 will be equal to sum of 'n' times the common difference and first term.

$$5 + 7n = 110 \quad (1)$$

$$7n = 110 - 5 = 105$$

$$n = \frac{105}{7} = 15$$

$$\Rightarrow 110 = x_1 + 15\,d$$

$$= x_{16} \quad (1)$$

$\therefore$ 110 be the 16th term in the given sequence.

17. Learning Outcomes

- Comparing sequences using their algebraic expressions.

Consider two arithmetic sequences given below: 11, 19, 27, ...and 50, 55, 60, ...

Is there a common number to these sequences at same term position? If yes, find the term positions. Find the term?

(Score: 4, Time: 8 minutes)

Scoring indicators

For the sequence 11, 19, 27, ... ;$x_n = 8n + 11 - 8 = 8n + 3$ (1)

For the sequence 50, 55, 60, ... ;$x_n = 5n + 50 - 5 = 5n + 45$ (1)

If n^{th} terms are equal for both sequences

$$8n + 3 = 5n + 45 \qquad (1)$$

$$8n - 5n = 45 - 3 = 42$$

$$3n = 42$$

$$n = \frac{42}{3} = 14 \qquad (1)$$

The 14^{th} term of both sequences are equal.

14^{th} term = $8 \times 14 + 3 = 112 + 3 = 115$

18. Learning Outcomes

- Understanding the properties of an arithmetic sequence using their algebraic expression.

Consider the arithmetic sequence -74,-68,-62,......... How many negative numbers are there in this sequence? Find the first positive number in this sequence?

(Score: 5, Time: 9 minutes)

Scoring indicators

- Common difference = 6
- Algebraic expression of the sequence $= 6n + -74 - 6$

$$\Rightarrow x_n = 6n - 80 \qquad (1)$$

If zero is a term in the sequence, then

$$6n - 80 = 0$$

$$6n = 80 \qquad (1)$$

$$n = \frac{80}{6} = 13 + \frac{2}{6} \qquad (1)$$

Here n is not a counting number. This shows that zero is not a term in the sequence. Hence upto zero there are 13 terms.

Therefore there are 13 negative numbers in this sequence. (1)

The first positive number will be 14^{th} term $= x_{14}$

$= 6 \times 14 - 80$

$= 84 - 80 = 4$ (1)

19. Learning Outcome

- Understanding the properties of an arithmetic sequence using their algebraic expression.

Is it possible that the product of any two terms in the arithmetic sequence 9,16,23,.... a term in it? Justify your answer.

(Score: 4, Time: 8 minutes)

Scoring indicators

- 9, 16, 23, ... ; the algebraic expression = $7n + 2$ (1)

  Assume that $7n + 2$ and $7m + 2$ are two terms in this sequence (1)

$(7n + 2)(7m + 2) = 49mn + 14n + 14m + 4$ (1)

This shows that when $(7n + 2)(7m + 2)$ divided by '7' will leave the remainder '4'. But the terms of the sequence when divided by '7' should leave a remainder '2'. Hence the product of any two terms is not a term in the sequence. (1)

20. Learning Outcome

- The sum of first and third term is equal to twice the second term.

❓ $2x + 1, 4x - 1, 5x + 1, \ldots$ are in an arithmetic sequence.

(a) Find x?
(b) Write the algebraic expression of the sequence.
(c) Find the position of 195 in this sequence

(Score: 4, Time: 7 minutes)

■ Scoring indicators

- If a, b, c are three consecutive terms of an arithmetic sequence, then $2b = a + c$

$$\begin{aligned} 8x - 2 &= 7x + 2 \\ 2(4x - 1) &= 2x + 1 + 5x + 1 \quad (1) \\ 8x - 2 &= 7x + 2 \\ 8x - 7x &= 2 + 2 \\ x &= 4 \quad (1) \end{aligned}$$

∴ the sequence is 9, 15, 21, ...

The algebraic expression of this sequence ; $x_n = 6n + 3$

$$\begin{aligned} 6n + 3 &= 195 \quad (1) \\ 6n &= 195 - 3 \\ &= 192 \\ n &= \frac{192}{6} \\ &= 32 \quad (1) \end{aligned}$$

∴ 32^{nd} term of the sequence is 195

21. Learning Outcome

- In an arithmetic sequence when the sum of odd number of terms divided by the number of terms we get the middle term.

❓ The angles in a 9 sided polygon are in arithmetic sequence. Is 100° the smallest angle of the polygon? Justify your answer.

(Score: 5, Time: 10 minutes)

■ Scoring indicators

The sum of angles in a 9 sided polygon $= 7 \times 180$ (1)

The 5th term in the arithmetic sequence = $\frac{7 \times 180}{9}$ = 140 (1)

If 100° is the first angle, then

$x_5 = x_1 + 4d$

$140 = 100 + 4d$

$d = 10$ (1)

9th angle ; $x_9 = x_5 + 4d$

$140 + 4 \times 10 = 180$ (1)

Therefore 100° is not the smallest angle in the polygon since 180° is not an angle of the polygon (1)

22. Learning Outcome

- The sum of first n counting numbers is $\frac{n(n+1)}{2}$

(a) Find the sum of first 25 counting numbers

(b) Find the sum of first 25 even numbers

(c) Find the sum of first 25 odd numbers

(Score: 4, Time: 9 minutes)

■ Scoring indicators

(a) $1 + 2 + 3 + ...+ 25 = \frac{25 \times 26}{2} = 25 \times 13 = 325$ (1)

(b) $2 + 4 + 6 + ...+ 50 = 2\,(1 + 2 + 3 + ... + 25) = 2 \times \frac{25 \times 26}{2} = 650$ (1)

(c) $1 + 3 + 5 \,.\,.\,.\, + 49 = (2 - 1) + (4 - 1) + (6 - 1) + \,.\,.\, ... + (50 - 1)$

$2 + 4 + 6 + ...+ 50 - (1 + 1 + \,.\,.\,.\, + 1)$ (1)

$= 2\,(1 + 2 + 3 + \,.\,.\,.\, + 25) - (1 \times 25)$

$= 650 - 25 = 625$ (1)

23. Learning Outcome

- The sum of first n counting numbers is $\frac{n(n+1)}{2}$

Prove that the sum of first n odd numbers is n^2.

(Score: 4, Time: 7 minutes)

■ Scoring indicators

- $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$ (1)
- $1 + 3 + 5 + ... + 2n - 1 = (2 - 1) + (4 - 1) + (6 - 1) + ... (2n - 1)$ (1)

$= (2 + 4 + 6 ... + 2n) - \underbrace{(1+1+1+...+1)}_{n\ times}$

$= 2\,(1 + 2 + 3 ... + n) - (1 \times n)$ (1)

$$= 2\frac{n(n+1)}{2} - n = n^2 + n - n = n^2 \qquad (1)$$

24. Learning Outcome

- The sum of first n terms of an arithmetic sequence whose nth term 'an + b' is

  'a$\frac{n(n+1)}{2}$ + bn'.

Find the sum of first 25 terms of the arithmetic sequence 5,8,11,.....

(Score: 4, Time: 6 minutes)

■ Scoring indicators

The nth term of the sequence 5,8,11,..... is

$$x_n = 3n + 2 \qquad (1)$$

$$x_1 = 3 \times 1 + 2$$

$$x_2 = 3 \times 2 + 2$$

$$x_3 = 3 \times 3 + 2$$

$$x_{25} = 3 \times 25 + 2 \qquad (1)$$

$$x_1 + x_2 + \dots x_{25} = 3\ (1 + 3 + \dots + 25) + 2 \times 25 \qquad (1)$$

$$= 3 \times \frac{25 \times 26}{2} + 50 \qquad (1)$$

$$= 1025$$

25. Learning Outcomes

- In an arithmetic sequence the difference between two terms is equal to the product of common difference and difference between the term position.
- The sum of first n terms of an arithmetic sequence whose first term is x_1 and nth term x_n is $\frac{n}{2}(x_1 + x_n)$

Consider an arithmetic sequence whose 6th term is 40 and 9th term is 58

(a) Find the 25th term of the sequence

(b) Find the sum of first 25 terms of the sequence

(c) Find the sum of first n terms of the sequence

(Score: 5, Time: 7 minutes)

■ Scoring indicators

(a)

$$x_9 = x_6 + 3d \qquad (1)$$

$$58 = 40 + 3d$$

$$3d = 58 - 40 = 18$$

$$d = 6$$

$$x_1 = x_6 - 5d = 40 - 5 \times 6 = 10 \qquad (1)$$

$x_{25} = x_6 + 19d = 40 + 19 \times 6 = 154$ (1)

(b) Sum of first 25 terms $= \frac{25}{2} (x_1 + x_{25})$

$= \frac{25}{2} (10 + 154)$

$= \frac{25}{2} \times 164 = 2050$ (1)

(c) n^{th} term; $x_n = 6n + 4$

Sum of first 'n' terms $= \frac{n}{2} (x_1 + x_n)$

$= \frac{n}{2} (10 + 6n + 4)$

$= n (3n + 7)$

$= 3n^2 + 7n$ (1)

26. Learning Outcome

- The sum of first n terms of an arithmetic sequence whose n^{th} term $an + b$ is '$\frac{an(n+1)}{2}$ + bn

Let the algebraic expression of an arithmetic sequence be $6n + 3$.

(a) Find the sum of first 20 terms of the sequence
(b) Write the algebraic expression of the sum.

(Score: 5, Time: 9 minutes)

Scoring indicators

Gvien ; $x_n = 6n + 3$ (1)

(a) Sum of 20 terms $= \frac{6 \times 20 \times 21}{2} + 3 \times 20$ (1)

$= 1260 + 60 = 1320$ (1)

(b) Sum of n terms $= \frac{6n(n+1)}{2} + 3 + n$ (1)

$= 3n^2 + 3n + 3n$

$= 3n^2 + 6n$ (1)

27. Learning Outcome

- The sum of first n terms of an arithmetic sequence whose n^{th} term is '$an + b$' is '$\frac{an(n+1)}{2}$ + bn'.

(a) Find the sum of first 20 terms of counting numbers.

(b) Consider an arithmetic sequence whose common difference is '7' and sum of first 20 terms is '1530'. Write the algebraic expression of the sequence.

(Score: 4, Time: 6 minutes)

Scoring indicators

(a) $1 + 2 + 3 + ... + 20 = \frac{20 \times 21}{2} = 210$ (1)

(b) The nth term of an arithmetic sequence with common difference 7 is $x_n = 7n + b$

Sum of first '20' terms $= 7 \times 210 + b \times 20 = 1530$ (1)

$$\Rightarrow 1470 + b \times 20 = 1530$$

$$20b = 60 \quad (1)$$

$$b = 3$$

Therefore the algebraic expression of the sequence is $7n + 3$ (1)

28. Learning Outcome

- The sum of first n terms of an arithmetic sequence whose n^{th} term an+b is $\frac{an(n+1)}{2} + bn$

Consider an arithmetic sequence whose sum of first 10 terms is 250 and sum of first 16 terms is 592

(a) Write the algebraic expression of the sequence

(b) Write the algebraic expression of the sum of the sequence

(Score: 5, Time: 10 minutes)

Scoring indicators

(a) Assume that the algebraic expression of the sequence $x_n = an + b$

Sum of first 10 terms $= a \times \frac{10 \times 11}{2} + b \times 10 = 250$

$$\Rightarrow \frac{11}{2}a + b = 25 \text{\_\_\_\_\_\_\_\_} (1) \quad (1)$$

Sum of first 10 terms $= a \times \frac{16 \times 17}{2} + b \times 16 = 592$

$$\Rightarrow \frac{17}{2}a + b = 37 \text{\_\_\_\_\_\_\_\_} (2) \quad (1)$$

$$(2) - (1) \Rightarrow 3a = 12$$

$$a = 4$$

(1) $$\Rightarrow \frac{11}{2} \times 4 + b = 25$$

$$b = 3 \quad (1)$$

Therefore the algebraic expression,

$$x_n = an + b$$

$$= 4n + 3 \quad (1)$$

(b) Assume that the algebraic expression for the sum of first n terms,

$$S_n = \frac{an(n+1)}{2} + bn$$

$$= \frac{4n(n+1)}{2} + 3n$$

$$= 2n\ (n + 1) + 3n$$

$$= 2n^2 + 2n + 3n$$

$$= 2n^2 + 5n \qquad (1)$$

29.Learning Outcome

- The sum of first n terms of an arithmetic sequence whose n^{th} term is $an + b$ *is* $\frac{an(n+1)}{2} + bn$

If the sum of first (n+1) terms of an arithmetic sequence is $pn^2 + qn + r$; then show that p+ r = q. Which of the following is the sum of first (n + 1) term of an arithmetic sequence

a) $2n^2 + 3n + 4$ b) $2n^2 + 3n + 1$

(Score: 5, Time: 10 minutes)

Scoring indicators

- nth term of an arithmetic sequence $x_n = an + b$ (1)
- The sum of first 'n' terms of the sequence = $\frac{a(n+1)(n+2)}{2} + b\ (n + 1)$ (1)

$$= \frac{a}{2}\ (n^2 + 3n + 2) + bn + b$$

$$= \frac{a}{2}\ n^2 + \left(\frac{3}{2}\ a + b\right) n + a + b$$

$$= pn^2 + q\ n + r$$

$$p = \frac{a}{2},\ q = \frac{3}{2}\ a + b,\ r = a + b$$

$$p + r = \frac{a}{2} + a + b = \frac{3}{2}\ a + b = q \qquad (1)$$

$$p + r = q$$

(a) Since in $2n^2 + 3n + 4$; $2+4 \neq 3$, so $2n^2 + 3n + 4$ can't be the sum of first (n + 1) terms of an arithmetic sequence (1)

(b) Since in $2n^2 + 3n + 1$; $2 + 1 = 3$. So $2n^2 + 3n + 1$ is a sum of first n terms of an arithmetic sequence (1)

30. Learning Outcome

- Explaining the method of finding the sum of a particular number of terms in arithmetic sequence whose two terms and their term positions are given.

Consider an arithmetic sequence whose sum of first 9 terms is 261 and sum of next 6 terms is 444.

a) Find the first term and common difference

b) Write the algebraic expression of the sequence

c) Write the algebraic expression of the sum of the sequence

(Score: 5, Time: 6 minutes)

- **Scoring indicators**

a) Sum of first 9 terms = 261

Middle term = 5th term = $x_5 = \frac{261}{9} = 29$ (1)

Sum of first 15 terms = 261 + 444 = 705 (1)

Middle term = 8th term = $x_8 = \frac{705}{15} = 47$

$x_8 = x_5 + 3d$

$3d = x_8 - x_5 = 47 - 29 = 18$

$d = \frac{18}{3} = 6$ (1)

$x_1 = x_5 - 4d = 29 - 4 \times 6 = 5$ (1)

b) $x_n = 6n - 1$

c) $s_n = 3n^2 + 2n$ (1)

31. Learning Outcome

- The method of finding an arithmetic sequence when the sum of a particular number of terms is given.

Consider an arithmetic sequence whose sum of first 10 terms is 230 and sum of first 16 terms is 560

a) Find the first term and common difference

b) Write the algebraic expression of the sequence

c) Write the algebraic expression of the sum of the sequence

(Score: 5, Time: 7 minutes)

- **Scoring indicators**

a) The sum to first 10 terms = 230

$S_{10} = 230$

$\frac{10}{2}(x_1 + x_{10}) = 230$

$x_1 + x_{10} = 46$ .... (1)

The sum to first 16 terms = 560

$S_{10} = 560$

$\frac{16}{2}(x_1 + x_{16}) = 560$

$x_1 + x_{16} = 70$ .... (2) (1)

(2) – (1) ⇒ $x_{16} - x_{10} = 24$

$6d = 24$

$d = 4$ (1)

(1) ⇒ $x_1 + x_{10} = 46$

$x_1 + x_1 + 9d = 46$

$2x_1 = 46 - 9d = 10$ (1)

$x_1 = 5$ (1)

(b) $x_n = 4n + 1$

(c) $s_n = 3n^2 + 3n$ (1)

32. Learning Outcome

- Comparing the sum of terms of two arithmetic sequences.

Consider two arithmetic sequences with common difference 4 and difference between their 5th terms is 8

a) Find the difference between the first terms

b) Find the difference between the sum of first 20 terms of the arithmetic sequences.

(Score: 5, Time: 7 minutes)

Scoring indicators

a) Sequence 1: $a, a+4, a+8, a+12, a+16, \ldots$

Sequence 2: $b, b+4, b+8, b+12, b+16, \ldots$

Difference between the 5th terms $= (a+16) - (b+16) = 8$

$a - b = 8$ (1)

(b)

$$\begin{array}{ccccccccc} a & + & (a+4) & + & (a+8) & + & (a+12) & + \ldots + & a+76 \\ b & + & (b+4) & + & (b+8) & + & ((b+12) & + \ldots + & b+76 \\ \hline 8 & + & 8 & + & 8 & + & 8 & + \ldots + & 8 \end{array}$$ (1)

Difference between the sum of first 20 terms $= 8 \times 20$

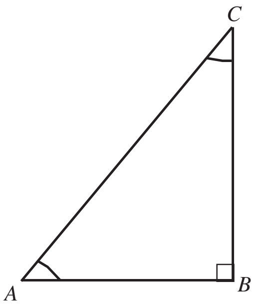
$= 160$ (1)

2

CIRCLES

1. Learning Outcome

- The lines drawn from the end points of a diameter of a circle to any point on the circle make a right angle.



In the figure ΔABC is a right triangle

(a) If a circle is drawn with AC as diameter find the position of B?

(b) If a circle is drawn with BC as diameter, find the position of A ?

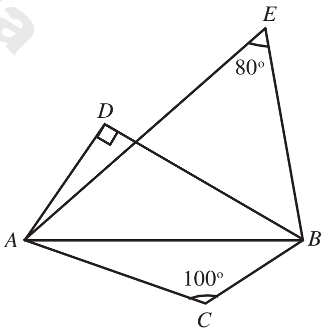
(Score: 2, Time: 3 minute)

Scoring indicators

On the circle	(1)
Outside the circle	(1)

2. Learning Outcome

- Angle in a semi circle is right.



A circle is drawn with AB as diameter. Find the positions of the points C, D, E related to the circle.

(Score: 3, Time: 5 minutes)

Scoring indicators

C inside the circle.	(1)
D on the circle.	(1)
E outside the circle.	(1)

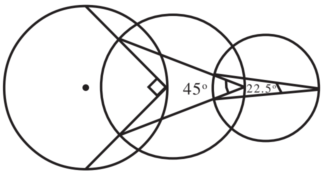
3. Learning Outcome

- The half of the central angle of an arc is equal to the angle in the alternate arc.

Construct an angle $22\frac{1}{2}^{o}$ without using a protractor.

(Score: 3, Time: 5 minutes)

- **Scoring indicators**



Constructing 90^{o} (1)
Constructing 45^{o} (1)
Constructing $22\frac{1}{2}^{o}$ (1)

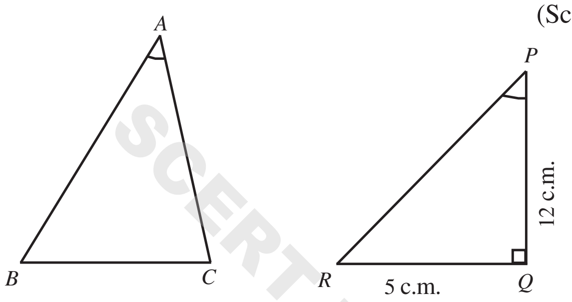
4. Learning Outcome

- Angles in the same arc are equal.
  Angles in a semi circle is 90^{o}

In ΔABC and $\Delta PQR, BC = QR$, $\angle A = \angle P$, $\angle Q = 90^{o}$, $QR = 5$ cm, $PQ = 12$ cm.

Find the diameter of the circumcircle of ΔABC

(Score : 4, Time 6 : minutes)



- **Scoring indicators**

$QR = BC$ (1)

$\angle A = \angle P$ (1)

$PR =$ Diameter of the circumcircle of ΔABC (1)

$PR = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$ cm (1)

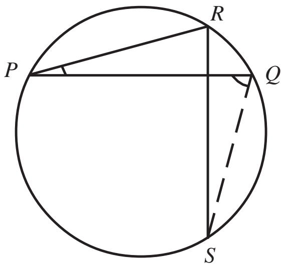
5. Learning Outcome

- Angle in a segment are equal.

PQ & RS are two mutually perpendicular chords of a circle. $\angle QPR = 50^{o}$ find $\angle PQS$?

(Score : 3, Time : 4 minutes)

- **Scoring indicators**



Draw the picture and mark the angles. (1)

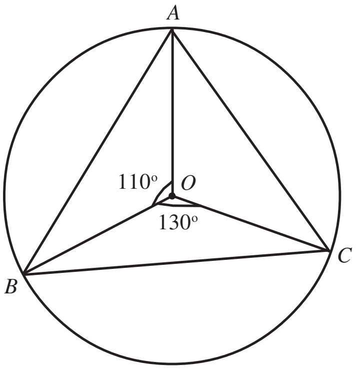
$\angle PRS = 90 - 50 = 40^{o}$ (1)

$\angle PQS = 40^{o}$ (1)

6. Learning Outcome

- Relation between the central angle of an arc and an angle in the point on a circle.

O is the centre of the circle . If $\angle BOC = 130^\circ$ and $\angle AOB = 110^\circ$. What is $\angle AOC$? Find all angles of ΔABC.



(Score 3, Time 3 minutes)

Scoring indicators

$\angle A = \dfrac{130}{2} = 65^\circ$ (1)

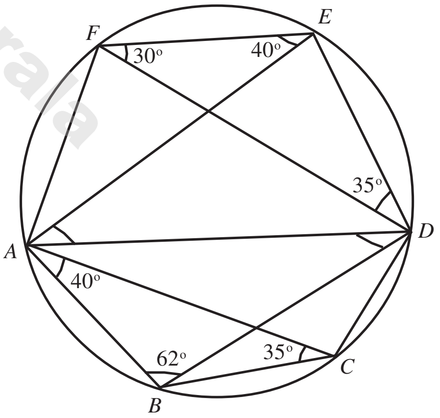
$\angle C = \dfrac{110}{2} = 55^\circ$ (1)

$\angle B = 180 - (65 + 55) = 60^\circ$ (1)

7. Learning Outcome

- Relation between the central angle of an arc and the angles made by that arc points on a circle.

Find all angles of the hexagon *ABCDEF*.



(Score: 4, Time: 3 minutes)

Scoring indicators

$\angle EFD = \angle EAD = 30^\circ$

$\angle FEA = \angle FDA = 40^\circ$

$\angle FDE = \angle FAE = 35^\circ$ (1)

$\angle BAC = \angle BDC = 45^\circ$

$\angle ABD = \angle ACD = 62^\circ$

$\angle ACB = \angle ADB = 35^\circ$ (1)

$\angle A = 148^\circ$

$\angle B = 100^\circ$

$\angle C = 97^\circ$ (1)

$\angle D = 155^\circ$

$\angle E = 115^\circ$

$\angle F = 105^\circ$ (1)

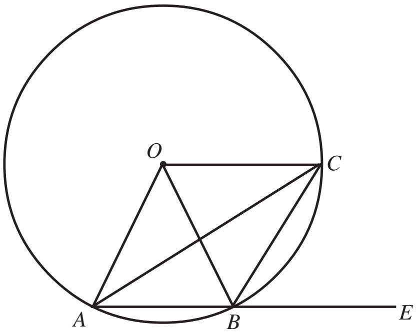
8. Learning Outcome

- If the vertues of a quadrilateral are on a circle, the angles at the opposite vertices are supplementary.

O is the centre of the circle and AB is a chord. AC is the bisector of $\angle OAB,\ \angle OAB = 56^{o.}$

(a) Prove that OC and AB are parallel

(b) Find $\angle ABC$ and $\angle OBE$.



(Score : 5, Time: 8 minutes)

■ **Scoring indicators**

(a) $\angle OAC = \angle BAC$ (bisector) (1)

$\angle OAC = \angle OCA$ (Isosceles triangle)

$\therefore\ \angle BAC = \angle OCA$

$\therefore\ AB$ and OC are parallel (1)

(b) $\angle AOC = 180 - 56 = 124^o$

$\angle ABC = \dfrac{360-124}{2}$

$= \dfrac{236}{2} = 118^o$ (1)

$\angle CBE = 180 - 118 = 62^o$

$\angle COB = 2 \times 28 = 56^o$

$\angle AOB = 124 - 56 = 68^o$ (1)

$\angle OBA = 180 - (56 + 68)$

$= 180 - 124 = 56^o$

$\therefore \angle OBC = 180 - (56 + 62)$ (1)

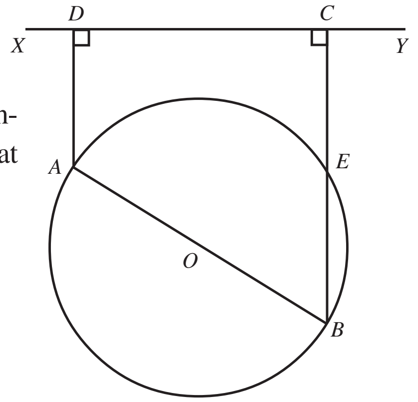
$= 180 - 118 = 62^o$

$\therefore\ \angle OBE = 62 + 62 = 124^o$

9. Learning Outcome

- Angle in a semi circle is right angle.

O is the centre of the circle. AD and BC are perpendicular to XY. CB cuts the circle at E. Prove that $CE = AD$.



(Score : 3, Time: 4 minutes)

- **Scoring indicators**

$\angle AEB = 90^\circ$ (Angle in a semi circle) (1)

$\angle AEC = 90^\circ$

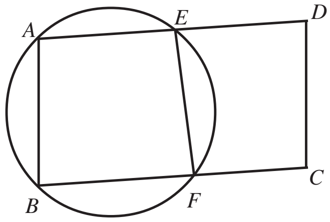
$\therefore AECD$ is a rectangle (1)

$\therefore AD = CE$ (1)

10. Learning Outcome

- The opposite angles of a cyclic quadrilateral are supplementary.

ABCD is a parallelogram. *A, B, E, F* are the points on a circle. $\angle DEF = 80^\circ$ Find out the angles of the quadrilateral *AEFB*.



(Score : 4, Time: 4 minutes)

- **Scoring indicators**

$\angle AEF = 180 - 80 = 100^\circ$ (1)

$\angle ABF = 180 - 100 = 80^\circ$ (1)

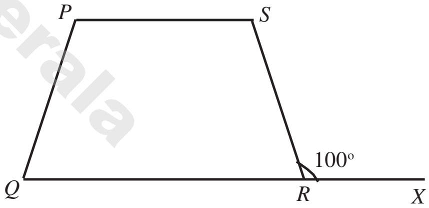
$\angle A = 180 - 80 = 100^\circ$ (1)

$\angle EFB = 180 - 100 = 80^\circ$ (1)

11. Learning Outcome

- Opposite angles of a cyclic quadrilateral are supplementary.

PQRS is a cyclic quadrilateral. QR is extended upto *X*. $\angle SRX = 100^\circ$, $\angle RPS = 50^\circ$. Find $\angle RPQ$.



(Score: 3, Time : 4 minutes)

- **Scoring indicators**

$\angle SRQ = 180 - 100 = 80^\circ$ (1)

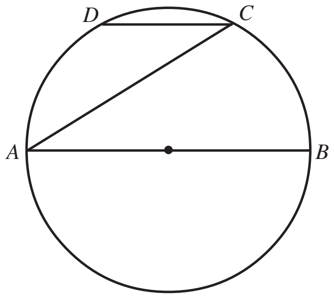
$\angle SPQ = 180 - 80 = 100^\circ$ (1)

$\angle RPQ = 100 - 50 = 50^\circ$ (1)

12. Learning Outcome

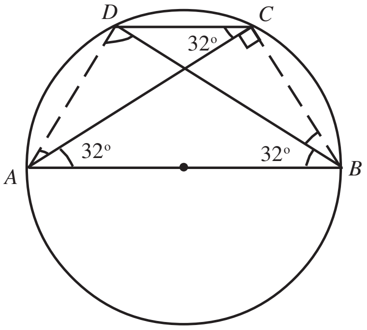
- Angle in a semi circle is right.

AB is the diameter of the circle. *CD* is parallel to *AB*, $\angle CAB = 32^\circ$. Find $\angle ADC$ and $\angle DAC$.



(Score 4, Time: 8 minutes)

Scoring indicators



$\angle ACB = \angle ADB = 90^\circ$ (1)

$\angle CAB = \angle ACD = 32^\circ$

$\angle DCA = \angle DBA = 32^\circ$ (1)

$\angle DAB = 90 - 32 = 58^\circ$ (1)

$\angle CDB = 32^\circ$

$\angle ADC = 90 + 32 = 122^\circ$

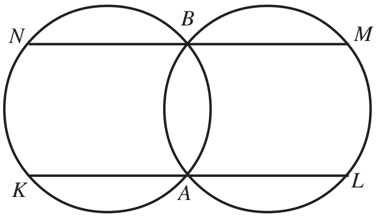
$\angle DAC = 58^\circ - 32^\circ$ (1)

$= 26^\circ$

13. Learning Outcome

- Recognising if the vertices of a quadrilateral are on a circle, opposite angles are supplementary.

Two circles intersect at A & B. The lines KAL, NBM are parallel. Prove that $KLMN$ is a parellelogram.



(Score: 4, Time: 4 minutes)

Scoring indicators

$\angle N = x$ (1)

$\angle BAL = x$ (1)

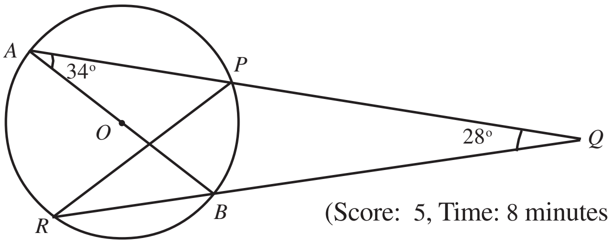
$\angle M = 180 - x$ (1)

$\therefore$ $KLMN$ is a parellelogram (1)

14. Learning Outcome

- Understanding the relationship between the central angle of an arc and angles formed at various points on a circle.

AB is a diameter of the circle. A, P, B, R are four points on the circle. The lines AP, RB intersect at Q. Find $\angle PRB$, $\angle PBR$, $\angle BPR$.



(Score: 5, Time: 8 minutes)

Scoring indicators

$\angle PAB = \angle PRB = 34^\circ$ (1)

$\angle APB = 90^\circ$

$\angle ARB = 90^\circ$ (1)

$\therefore ARP = 90 - 34 = 56^\circ$ (1)

$\angle ABP = 56^o$

$\angle RPQ = 180 - (28 + 34) = 118^o$

$\angle APR = 62 = \angle ABR$

$\angle PBR = 62 + 56 = 118^o$ (1)

$\angle RAP = 180 - 118 = 62^o$

$\angle RAB = 62 - 34 = 28^o$

$\angle BPR = 28^o$ (1)

15. Learning Outcome

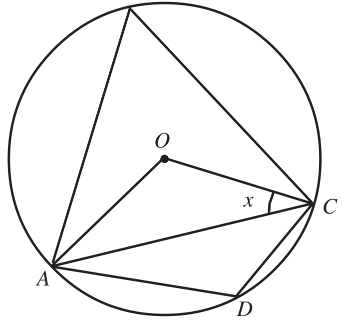
- Understanding the relationship between the central angles of an arc and the angles by that arc at various points on a circle.

O is the centre of the circle. $\angle OCA = x^o$. Find

a) Find $\angle OAC$

b) Prove that $\angle OCA + \angle ABC = 90^o$.

c) Prove that $\angle ADC - \angle OCA = 90^o$.



(Score: 5, Time: 8 minutes)

Scoring indicators

(a) $\angle OAC = x$ (1)

$\angle AOC = 180 - 2x$ (1)

$$\angle B = \frac{180-2x}{2} = 90 - x$$

(b) $\angle OCA + \angle ABC = x + 90 - x = 90^o$ (1)

(c) $\angle ADC = 180 - (90 - x)$

$= 90 + x$ (1)

$\angle ADC - \angle OCA = 90 + x - x = 90^o$ (1)

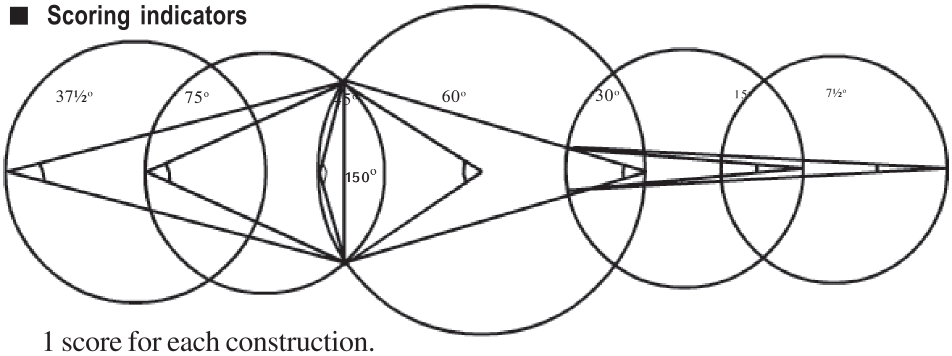
16. Learning Outcome

- Understanding half the central angle of an arc is equal to the angle made by that are on the alternate arc.

Draw an equilateral triangle. Draw a circle with centre as the vertex of the equilateral triangle and passes through other two vertices of it. Construct angles with measurements 30^o, 15^o, $7\frac{1}{2}^o$, 150^o, 75^o using scale and compasses with the help of the drawn figure.

(Score 5, Time: 5 minutes)

Scoring indicators



1 score for each construction.

17. Learning Outcome

- Understanding the half the central angle of an arc is equal to the angle made by that arc on the alternate arc.

Draw a circle with radius 5cm. Draw a triangle with its vertices on the circle and having angles 35°, 72°, 73°.

(Score: 4, Time: 5 minutes)

Scoring indicators

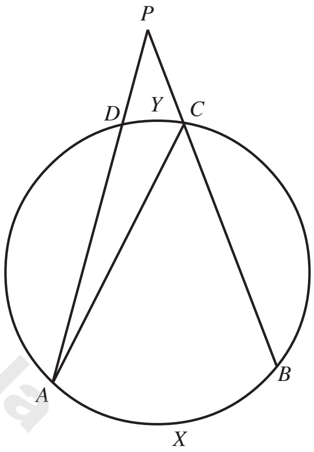
Draw the circle.	(1)
Draw 70°, 144° angles at the centre.	(1)
Completing the triangle.	(1)
Measuring the sides.	(1)

18. Learning Outcome

- Understanding the half the central angle of an arc is equal to the angle made by that arc on the alternate arc.

In the figure central angle of arc *AXB* is 100° central angle of arc *CYD* is 30°.

Find $\angle CAD$ and $\angle ACB$. Find the angles of ΔAPC.



(Score: 4, Time: 5 minutes)

Scoring indicators

$\angle CAD = \frac{30}{2} = 15^{\circ}$ (1)

$\angle ACB = \frac{100}{2} = 50^{\circ}$ (1)

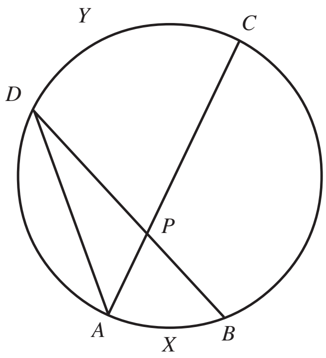
$\angle PCA = 180 - 50 = 130^{\circ}$ (1)

$\angle P = 35^{\circ}$ (1)

19. Learning Outcome

- Understanding the half the central angle of an arc is equal to the angle made by that arc on the alternate arc.

In the figure, the central angle of arc AXB is 40°, central angle of arc CYD is 70°. Find the angles of ΔAPD



(Score : 3, Time: 4 minutes)

Scoring indicators

$\angle D = \frac{40}{2} = 20^{o}$ (1)

$\angle A = \frac{70}{2} = 35^{o}$ (1)

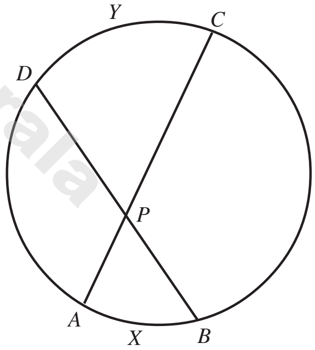
$\angle APD = 180 - (35 + 20)$

$= 125^{o}$ (1)

20. Learning Outcome

- Understanding the half the central angle of an arc is equal to the angle made by that arc on the alternate arc.

In the figure, prove that $\angle APB$ is equal to half the sum of the central angles of arc AXB and are DYC.



(Score : 3, Time: 4 minutes)

Scoring indicators

$\angle ADP = \frac{\text{Central angle of arc } AXB}{2}$ (1)

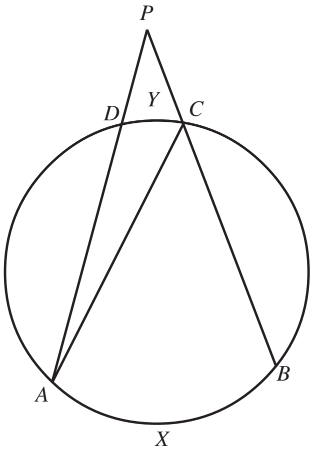
$\angle DAC = \frac{\text{Central angle of arc } DYC}{2}$ (1)

$\angle APB = \angle ADP + \angle DAC$ (1)

21. Learning Outcome

- Understanding the half the central angle of an arc is equal to the angle made by that arc on the alternate arc.

Prove that $\angle APB$ is equal to half the difference of the central angles of arc AXB and are DYC.



(Score: 3, Time: 4 minutes)

- **Scoring indicators**

$$\angle ACB = \frac{\text{Central angle of arc } AXB}{2} \quad (1)$$

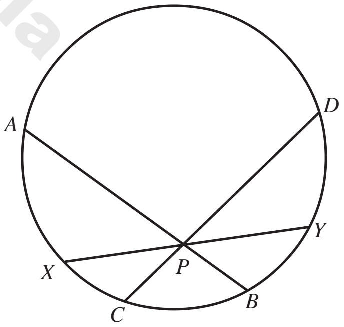
$$\angle DAC = \frac{\text{Central angle of arc } CYD}{2} \quad (1)$$

$$\angle P = \angle ACB - \angle DAC \quad (1)$$

22. Learning Outcome

- Understanding the relation between the parts into which chords of a circle cut each other.

In the figure, the chords AB, CD, XY intersect at P. $AP = 9$ cm, $AB = 13$ cm, $PD = 12$ cm. Find CD. If $PX = PY$, find XY.



(Score :3, Time: 4 minutes)

- **Scoring indicators**

$PA \times PB = PC \times PD$

$9 \times 4 = PC \times 12$

$PC = 3$

$CD = 12 + 3 = 15$ (1)

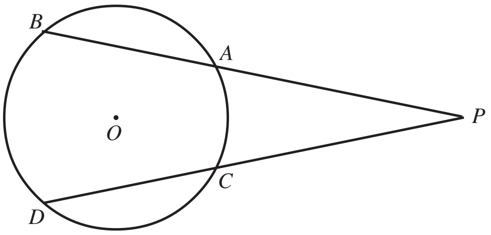
$PX \times PY = 36$ (1)

$PX = PY = \sqrt{36} = 6$ cm (1)

23. Learning Outcome

- Understanding the relation between the parts into which chords of a circle cut each other.

In the figure $PA = 3$ cm, $AB = 9$ cm, $PC = 4$ cm Find CD .



(Score : 3, Time: 4 minutes)

Scoring indicators

$PA \times PB = PC \times PD$

$3 \times 12 = 4\,(4 + CD)$ (1)

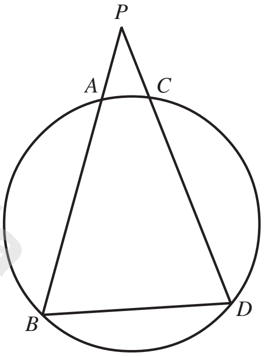
$36 = 16 + 4\,CD$ (1)

$CD = 5$ cm (1)

24. Learning Outcome

- Understanding the relation between the parts into which chords of a circle cut each other.

In the figure $PA = PC$. Prove that the sides PB and PD of the triangle are equidistant from the centre of the circle.



(Score: 5, Time :5 minutes)

Scoring indicators

$PA \times PB = PC \times PD$ (1)

$PA = PC$ (1)

$\therefore\ PB = PD$ (1)

$\therefore\ AB = CD$ (1)

Equal chords of a circle are equidistant from the centre. (1)

25. Learning Outcome

- Converting a rectangle into another rectangle or square without changing the area.

The sides of a rectangle are 5 cm and 4 cm. Construct a rectangle of the same area with width 7cm.

(Score: 4, Time: 6 minutes)

Scoring indicators

Drawing the rectangle (1)
Drawing a circle with a chord of length equal to the sum of the sides of the rectangle (1)
Drawing a chord $PC = 7$cm in the circle. (1)
Construct the rectangle with sides PC and PD (1)

26. Learning Outcome

- Realising that the product of the parts into which a diameter of a circle is cut by a perpendicular chord, is equal to the square of half the chord.

Draw a line of length $\sqrt{7}$ cm.

(Score 3, Time: 4 minutes)

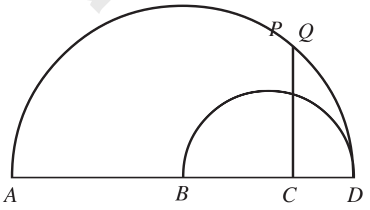
Scoring indicators

- Draw a semicircle having diameter 7 + 1 = 8 cm. (2)
- Draw a perpendicular line. (1)

27. Learning Outcome

- Realising that the product of the parts into which a diameter of a circle is cut by a perpendicular chord, is equal to the square of half the chord.

In the figure $AD = 10$ cm, $BD = 6$ cm, $CD =$ 2 cm. Find PQ.



(Score : 4, Time: 4 minutes)

Scoring indicators

$AC = 8$ cm
$AC \times CD = CQ^2$ (1)
$8 \times 2 = 16$
$CQ = \sqrt{16} = 4$ (1)
$BC \times CD = CP^2$
$4 \times 2 = CP^2$
$CP = \sqrt{8}$ (1)
$PQ = 4 - \sqrt{8}$ (1)

28. Learning Outcome

- Converting a rectangle into another rectangle or square without changing the area.

Draw a rectangle of sides 5 cm and 3 cm. Construct a square of the same area.

(Score: 4, Time: 4 minutes)

■ Scoring indicators

Drawing a rectangle of sides 5 cm and 3 cm.	(1)
Drawing a semicircle having diameteras 5 + 3 = 8 cm	(1)
Drawing a perpendicular	(1)
Constructing the square	(1)

29. Learning Outcome

- Converting a rectangle into another rectangle or square without changing the area.

The sides of a triangle are 4 cm, 7 cm and 8 cm. Draw it and construct a square of the same area.

(Score 5, Time: 8 minutes)

■ Scoring indicators

Drawing the triangle	(1)
Drawing rectangle with equal area	(1)
Drawing semi circle with diameter equal to the sum of the sides of the rectangle	(1)
Drawing the perpendicular	(1)
Construct the square	(1)

30. Learning Outcome

- Converting a rectangle into another rectangle or square without changing the area.

Draw an isosceles triangle of hypotenuse 7 cm

(Score: 3, Time: 4 minutes)

■ Scoring indicators

Drawing a semicircle of diameter 7 cm.	(1)
Drawing the perpendicular to the diameter from the centre.	(1)
Construct the isosceles right triangle.	(1)

3

MATHEMATICS OF CHANCE

1. Learning Outcomes

- Explaining probability as a number.
- Explaining the method to calculate probability in different contexts.

One is asked to say a two digit number. What is the probability of being the number not a perfect square?

(Score : 3, Time : 3 Minutes)

Scoring Indicators

Total number of two digit number = 90 (1)

Total number of two digit perfect squares = 6 (1)

Number of two digit numbers which are not perfect squares = 90 - 6 = 84

Probability $= \frac{84}{90} = \frac{42}{45} = \frac{14}{15}$ (1)

2. Learning Outcomes

- Explaining probability as a number
- Explaining the method to calculate probability in different contexts.

A bag contains 10 blue balls and 12 yellow balls. Another contains 15 blue balls and 7 yellow balls.

a) What is the probability of getting a yellow ball from the first bag?

b) What is the probability of getting a yellow ball from the second bag?

c) If all the balls are put in a single bag, what is the probability of getting a yellow ball from it?

(Score : 4 , Time: 4 Minutes)

Scoring Indicators

a) Total number of balls in the first bag = $10 + 12 = 22$

Number of yellow balls = 12

Probability of getting a yellow ball = $\frac{12}{22} = \frac{6}{11}$ (1)

b) Total number of balls in the second bag = $15 + 7 = 22$

Number of a yellow ball = 7

Probability of getting a yellow ball = $\frac{7}{22}$ (1)

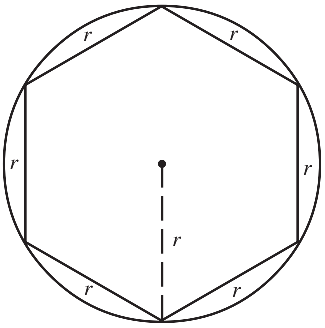
c) Total number of balls = $22+22=44$

Number of yellow balls = $12+7=19$ (1)

Probability of getting a yellow ball = $\frac{19}{44}$ (1)

3. Learning Outcomes

- Explaining probability as a number
- Explaining the method to calculate probability in different contexts.



A regular hexagon, is drawn with its vertices on a circle. Without looking into the picture, if one put dot in that picture, what is the probability of being the dot not in the regular hexagon?

(Score : 4 Time: 3 Minutes)

■ Scoring Indicators

Area of the circle = πr^2 (1)

Area of regular hexagon = $\frac{\sqrt{3}r^2}{4} \times 6 = \frac{3\sqrt{3}\,r^2}{2}$ (1)

Required probability = $\frac{\frac{3\sqrt{3}\,r^2}{2}}{\pi r^2} = \frac{3\sqrt{3}\,r^2}{2} \times \frac{1}{\pi r^2}$ (1)

= $\frac{3\sqrt{3}}{2\pi}$ (1)

4. Learning Outcomes

- Explaining probability as a number
- Explaining the method to calculate probability in different contexts

A box contains slips numbered 1, 2, 3, 4. Another box contains slips numbered 1, 2, 3. If one slip is taken from each, what is the probability of getting a sum which is a multiple of three? Also find the probability of getting a sum , which is multiple of 2

(Score: 5, Time: 6 Minutes)

■ Scoring Indicators

Pair of numbers obtained by taking one slip from each box at a time:

(1, 1) (2, 1) (3, 1) (4, 1)

(1, 2) (2, 2) (3, 2) (4, 2) 12 Numbers (1)

(1, 3) (2, 3) (3, 3) (4, 3)

pairs having a sum, which is a multiple of 3 (1)

(2, 1) (1, 2) (4, 2) (3, 3) - 4 Numbers (1)

Probability = $\frac{4}{12} = \frac{1}{3}$

pairs having a sum, which is a multiple of 2

(1, 1) (3, 1) (1, 3) (2, 2) (3, 3) (4, 2) - 6 Numbers (1)

Probability = $\frac{6}{12} = \frac{1}{2}$ (1)

5. Learning Outcome

- Justifying the need to analyse probability using numbers in practical situations.

A bag contains 6 red beads and 4 green beads. Another contains 2 red and 2 green seads more. The probability of getting red bead from which bag is more?

(Score: 4 , Time: 3 Minutes)

■ Scoring Indicators

Number of red beads in first bag = 6

Probability of getting red beads from the first bag = $\frac{6}{10}$ (1)

Number of red beads in the second bag = 6 + 2 = 8 (1)

Probability of getting red beads from the second bag = $\frac{8}{14}$ (1)

$\frac{6}{10} > \frac{8}{14}$

$\therefore$ Probability of getting red bead from the first bag is more (1)

6. Learning Outcome

- Justifying the need to analyse probability using numbers in practical situations.

One is asked to say a two digit number: what is the probability of

(1) Both digits being different?

(2) The first digit being larger?

(3) The first digit being smaller?

(Score: 6, Time: 8 Minutes)

■ Scoring Indicators

(1) Total two digit numbers : 90

Numbers having equal digits

11, 22, 33, 44, 55, 66, 77, 88, 99 - (9 Numbers) (1)

Therefore, numbers having different digits - 81 Numbers (1)

Probability $= \frac{81}{90} = \frac{9}{10}$ (1)

(2) The probability of the first digit being larger and second being smaller

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = \frac{9\times10}{2}$$

$$= 45$$

Probability $= \frac{45}{90} = \frac{1}{2}$ (1)

10 − 1

20 21 − 2

30 31 32 − 3

40 41 42 43 − 4

....

90 91 92 93 ....99 − 9

$\frac{9\times10}{2} = 45$

(3) The probability of the fist digit being smaller and second digit being larger

$= 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = \frac{8\times9}{2}$ (1)

$= 36$

Probability $= \frac{36}{90} = \frac{4}{10} = \frac{2}{5}$ (1)

7. Learning Outcome

- Justifying the need to analyse probability using numbers in practical situations.

There are 60 students in a class among which 30 are boys. In another class there are 50 students among which 25 of them are boys. If one from each class is selected,

1) What is the probability of both being girls?

2) What is the probability of having atleast one girl?

(Score: 4, Time: 5 Minutes)

■ Scoring Indicators

Total number of students in the first class = 60

No. of boys = 30

No. of girls = 30

Total number of students in the second class = 50

No. of boys = 25

No. of girls = 25

1) Probability of both being girls $= \frac{30\times25}{60\times50}$ (1)

$= \frac{750}{3000}$

$= \frac{1}{4}$ (1)

2) Probability of at least one girl $= \dfrac{30\times 25+30\times 25+30\times 25}{3000}$ (1)

$= \dfrac{2250}{3000}$

$= \dfrac{3}{4}$ or $\dfrac{3000-750}{3000}$

$= \dfrac{3}{4}$ (1)

8. Learning Outcome

- Justifying the need to analyse probability using numbers in practical situations.

A vessel contains 4 black beads, 6 white beads and 10 red beads. In another vessel contains 7 black beads, 5 white beads and 8 red beads. If we take one bed from each vessel, without looking into it.

(a) What is the probability of both being same colour?

(b) What is the probability of both being different colours?

(c) What is the probability of getting atleast one black bead?

(Score: 5, Time: 8 Minutes)

■ Scoring Indicators

4 B **6 W** **10 R**	**7 B** **5 W** **8 R**

Number of beads in each vessel = 20

Total number of ways of selecting two beads = 20 × 20 = 400 (1)

(a) No. of beads both being same colour $= 4\times 7+6\times 5+10\times 8$

$= 138$

$= \dfrac{138}{400}$ (1)

(b) Probability of both being different colour $= \dfrac{400-1388}{400}$

$= \dfrac{262}{400}$ (1)

(c) Number of beads having atleast one black bead

$= 4\times 7+4\times 5+4\times 8+6\times 7+10\times 7$

$= 28+20+32+42+70$ (1)

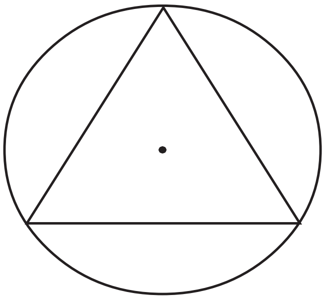
$= 192$

Its probability $= \dfrac{192}{400} = \dfrac{12}{25}$ (1)

9. Learning Outcomes

- Justifying the need to analyse probability using numbers in practical situations.

An equilateral triangle is drawn with its vertices if we put a dot init without looking into the picture, what is the probability of being the dot inside the triangle? Also find the probability of being the dot outside the triangle?



(Score: 4, Time: 7 Minutes)

Scoring Indicators

If the radius of the circle is r, then the side of the triangle will be $\sqrt{3}\, r$

Area of the circle $= \pi r^2$ (1)

Area of the equilateral triangle $= \frac{\sqrt{3}}{4}(\sqrt{3}r)^2 = \frac{\sqrt{3}}{4}\times 3r^2$ (1)

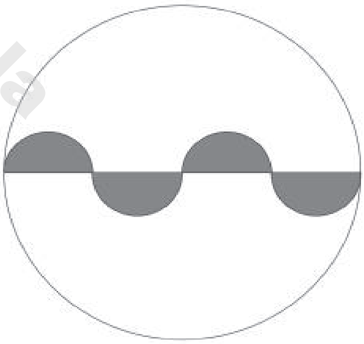
Proabability of being the dot inside the triangle $= \frac{\frac{\sqrt{3}}{4}\times 3r^2}{\pi r^2} = \frac{4\pi - 3\sqrt{3}}{4\pi}$ (1)

Probability of being the dot out side the triangle $= \frac{4\pi - 3\sqrt{3}}{4\pi}$ (1)

10. Learning Outcome

- Justifying the need to analyze probability using numbers on practical situations

In the figure all the four shaded semicircles have same area. If we put a dot in the figure without looking into it, what is the probability of being the dot in the shaded semicircles?



(Score : 3 , Time: 5 Minutes)

Scoring Indicators

If the radius of the shaded semicircle is r,

Area of all 4 semicircles $= 4 \times \frac{\pi r^2}{2} = 2\pi r^2$ (1)

Area of the bigger circle $= \pi(4r)^2 = 16\pi r^2$ (1)

Probability $= \frac{2\pi r^2}{16\pi r^2} = \frac{1}{8}$ (1)

11. Learning Outcome

- Justifying the need to analyse probability using numbers in practical situations.

What is the probability of getting 5 sundays in December in a calendar year?

(Score: 3, Time: 5 Minutes)

Scoring Indicators

There are 31 days in December.
That means 4 full weeks and 3 days.
The probable three days are as shown below.

Sunday	Monday	Tuesday
Monday	Tuesday	Wednesday
Tuesday	Wednesday	Thursday
Wednesday	Thursday	Friday
Thursday	Friday	Saturday
Friday	Saturday	Sunday
Saturday	Sunday	Monday

Total possibilities = 7 (1)

Number of possible ways having sunday

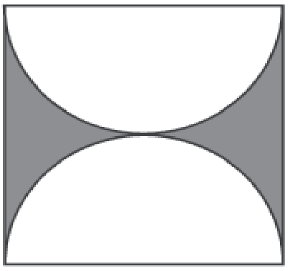
= 3 (1)

Probability = $\frac{3}{7}$ (1)

12. Learning Outcome

- Justifying the need to analyse probability using numbers in practical situations.

Two semicircles are drawn in a square as shown. If we put a dot in the figure, without looking into it, what is the probability of being the dot in the shaded region?



(Score: 3, Time: 5 Minutes)

Scoring Indicators

If a is the side of a square,

Area of the two circular part = $\pi \, \frac{a}{2}^{2} = \frac{\pi a^2}{4}$ (1)

Area of the shaded portion = $a^2 - \frac{\pi a^2}{4} = \frac{4-\pi}{4} a^2$ (1)

Probability = $\frac{\frac{4-\pi}{4} a^2}{a^2} = \frac{4-\pi}{4}$ (1)

4

SECOND DEGREE EQUATIONS

1. Learning outcome

- Framing and solving second degree equations

When the sides of a square are increased by 8 cm each, its area becomes 1225 sq. cm. Frame an equation using the above data by taking the side of the smaller square as x cm. Find the sides of both the squares.

(Score: 3, Time: 4 Minutes)

Scoring Indicators

- One side of smaller square $= x$
- One side of bigger square $= x + 8$
- Area $= (x + 8)^2 = 1225$ (1)

$x + 8 = 35$

$x = 35 - 8 = 27$ (1)

- Side of smaller square = 27 centimetre
- Side of bigger square = 35 centimetre (1)

2. Learning outcome

- Framing and solving second degree equations

The difference of two positive numbers is 6. Their product is 216. Find the numbers.

(Score: 3, Time: 4 Minutes)

Scoring Indicators

Let the numbers be x, $x + 6$

Products: $x (x + 6) = 216$ (1)

$x^2 + 6x = 216$

$x^2 + 6x + 9 = 216 + 9 = 225$

$(x + 3)^2 = 225$ (1)

$x + 3 = 15$

$x = 15 - 3 = 12$

Numbers: 12, 18 (1)

3. Learning outcome

- Framing and solving second degree equations

In a right triangle one of the perpendicular sides is one less than 2 times the smaller side. Hypotenuse is one more than 2 times the same smaller side. By taking the smaller side as x cm, write the algebraic expression for the other two sides. Compute all the 3 sides of the right triangle.

(Score: 4, Time: 5 Minutes)

Scoring Indicators

Smaller side $= x$

Perpendicular side $= 2x - 1$

Hypotenuse $= 2x + 1$ (1)

$x^2 + (2x - 1)^2 = (2x + 1)^2$ (1)

$x^2 + 4x^2 - 4x + 1 = 4x^2 + 4x + 1$

$x^2 - 8x = 0$ (1)

$x(x - 8) = 0$

$x = 0$ or $x = 8$

Sides : 8 cm, 15 cm, 17 cm (1)

4. Learning outcome

- Framing and solving second degree equations

Find the sides of a rectangle whose perimeter is 100 metres and area 600 sq. metres.

(Score: 4, Time: 5 Minutes)

Scoring Indicators

Perimeter = 100 metre

Length + breadth = 50 metre

Length $= 25 + x$

Breadth $= 25 - x$ (1)

Area, $(25 + x)(25 - x) = 600$

$25^2 - x^2 = 600$ (1)

$x^2 = 625 - 600 = 25$

$x = 5$ (1)

Sides, $25 + 5 = 30$ metre

$25 - 5 = 20$ metre (1)

5. Learning outcome

- Solving practical problems by farming second degree equations

The one's place of a two digit number is 4. The product of the number and digit sum is 238.

a) If ten's place digit is taken as x, write the number.

b) Frame a second degree equation and find the number.

(Score: 4, Time: 6 Minutes)

Scoring Indicators

$$\text{Digit in the ten's place} = x$$

$$\text{Number} = 10x + 4 \qquad (1)$$

$$\text{The product of the number and digit sum} = (x + 4)\,(10x + 4) = 238 \qquad (1)$$

$$10\,x^2 + 44x + 16 = 238$$

$$10x^2 + 44x - 222 = 0$$

$$5x^2 + 22x - 111 = 0$$

$$x = \frac{-22 \pm \sqrt{22^2 - 4\times 5\times -111}}{2\times 5} \qquad (1)$$

$$= \frac{-22 \pm \sqrt{2704}}{10}$$

$$= \frac{-22 \pm 52}{10}$$

$$\text{Number} = 34 \qquad (1)$$

6. Learning outcome

- Framing and solving second degree equations

How many consecutive natural numbers from 1 should be added to get 465?

(Score: 4, Time: 5 Minutes)

Scoring Indicators

$$\text{Sum of first } n \text{ natural numbers: } \frac{n(n+1)}{2} = 465 \qquad (1)$$

$$n^2 + n = 930$$

$$n^2 + n + \frac{1}{4} = 930 + \frac{1}{4} \qquad (1)$$

$$\left(n+\frac{1}{2}\right)^2 = \frac{3721}{4}$$

$$n+\frac{1}{2} = \frac{61}{2}$$ (1)

$$n = 30$$

The sum of first 30 natural numbers =465 (1)

7. Learning outcome

- Solving practical problems by framing second degree equations.

The product of the digits of a two digit number is 12. When 36 is added to this number, got a new number with digits reversed. Find the number.

(Score: 5, Time: 7 Minutes)

Scoring Indicators

Digits : $x, \frac{12}{x}$ (1)

When 36 is added to the number, the digits became reversed

Then, $$10x+\frac{12}{x}+36 = 10\times\frac{12}{x}+x$$ (1)

$$10x^2+12+36x = 120+x^2$$

$$9x^2+36x = 108$$

$$x = 2$$ (1)

$$x^2+4x = 12$$

$$(x+2)^2 = 16$$

$$x+2 = 4$$ (1)

$$\text{Number} = 26$$ (1)

8. Learning outcome

- Solving practical problems by framing second degree equations.

For a two digit number, one's place is 3 more than its ten's place. The product of this number and its digitsum is square of double the digitsum. What is the number.

(Score: 5, Time: 6 Minutes)

■ Scoring Indicators

digits: $x, x+3$

Number: $10x + x + 3 = 11x + 3$ (1)

Product of the number and digit sum: $(2x+3)(11x+3) = (4x+6)^2$ (1)

$$22x^2+39x+9 = 16x^2+48x+36$$
$$6x^2-9x-27 = 0$$
$$2x^2-3x-9 = 0 \quad (1)$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4\times 2\times -9}}{2\times 2}$$

$$= \frac{3\pm\sqrt{9+72}}{4}$$

$$= \frac{3\pm\sqrt{81}}{4}$$

$$= \frac{3+9}{4} \text{ or } \frac{3-9}{4} \text{ (being a fraction not acceptable)}$$

$$x = \frac{3+9}{4} = \frac{12}{4} = 3 \quad (1)$$

Number $= 36$ (1)

9. Learning outcome

- Solving geometrical problems by framing second degree equations.

A pavement of width 2 metres is built around a square shaped garden. The area of the pavement alone is 116 square metres. Find one side of the garden.

(Score: 3, Time: 4 Minutes)

■ Scoring Indicators

Side of garden = x .

Side of garden including pavement $= x+4$. (1)

Area of the pavement $= (x+4)^2 - x^2 = 116$

$$x^2 + 8x + 16 - x^2 = 116 \quad (1)$$
$$8x + 16 = 116$$
$$x = \frac{116-16}{8} = 12.5 \text{ metre}$$

Side of garden = 12.5 metre (1)

10. Learning outcomes

- Solving geometrical problems by framing second degree equations.

A rectangle of width 8 centimetres is cut off from a square sheet along its side. The remaining rectangular portion has an area 84 sq. metres. Calculate the side of the square.

(Score: 4, Time: 6 Minutes)

Scoring Indicators

$$\begin{aligned}
\text{Let the side of square} &= x \\
\text{Sides of rectangle} &= x\ ,\ x-8 && (1)\\
\text{Area} &= x\,(x-8) = 84 \\
x^2 - 8x &= 84 && (1)\\
x^2 - 8x+16 &= 84+16 \\
(x-4\ )^2 &= 100 && (1)\\
x-4 &= 10 \\
x &= 14
\end{aligned}$$

Side of square = 14 centimetre (1)

11. Learning outcome

- Solving geometrical problems by framing second degree equations.

The length of a rectangle is 3 metre more than 3 times its breadth. Its diagonal is 1 metre more than the length. Find the length and breadth of the rectangle.

(Score: 4, Time: 7 Minutes)

Scoring Indicators

$$\begin{aligned}
\text{Side of rectangle} &= x\ . \\
\text{Length} &= 3x\ +3 \\
\text{Diagonal} &= 3x\ +4 && (1)\\
(3x+4)^2 &= x^2 + (3x+3)^2 && (1)\\
9x^2 + 24x + 16 &= x^2 + 9x^2 + 18x + 9 \\
x^2 - 6x &= 7 \\
x^2 - 6x + 9 &= 16 && (1)\\
(x-3)^2 &= 16 \\
x-3 &= 4 \\
x = 4+3 &= 7 && (1)\\
\text{length of rectangle} &= 3 \times 7 + 3 = 24 \text{ metre} \\
\text{breadth of rectangle} &= 7 \text{ metre} && (1)
\end{aligned}$$

12. Learning outcome

- Applying the method to frame and solve a second degree equation in practical contexts.

In the figure AB is the diameter of the circle. CD = 10 cm. BC is 15 cm less than AC. Find AB?

(Score: 4, Time: 5 Minutes)

Scoring Indicators

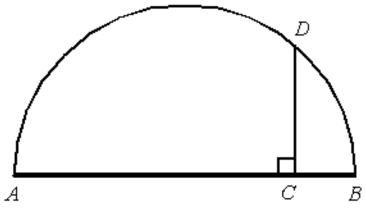
$AC \times CB = CD^2$ (1)

$AC = x$

$x(x - 15) = 10^2$

$x^2 - 15x = 100$ (1)

$x^2 - 15x + \frac{225}{4} = 100 + \frac{225}{4}$



$(x - \frac{15}{2})^2 = \frac{625}{4}$ (1)

$x - \frac{15}{2} = \frac{25}{2}$

$x = \frac{25}{2} + \frac{15}{2} = 20$

$\therefore AB = 20 + 5 = 25$ centimetre (1)

13. Learning outcome

- Applying the method to frame and solve a second degree equation in practical contexts.

The chords AB and CD of a circle intersect at M. If MA = 6 cm, MB = 8 cm and CD = 16 cm, Find MC and MD.

(Score: 4, Time: 5 Minutes)

Scoring Indicators

$CD = 16$ centimetre

$MC = 8 - x$, $MD = 8 + x$ (1)

then, $MA \times MB = MC \times MD$

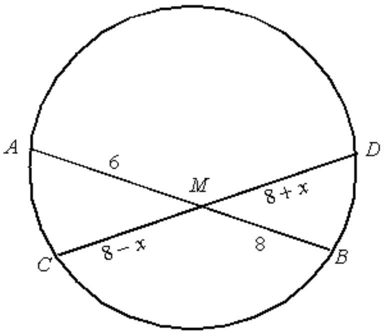
$6 \times 8 = (8 - x)(8 + x)$

$48 = 64 - x^2$ (1)

$x^2 = 16$, $x = 4$ (1)

MC = 8 – 4 = 4 centimetre (1)

MD = 8 + 4 = 12 centimetre



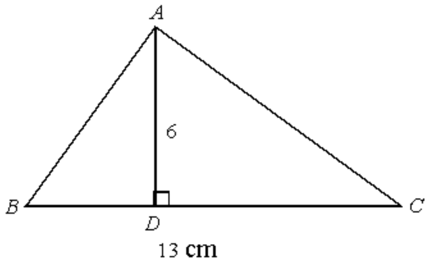
14. Learning outcome

- Applying the method to frame and solve a second degree equation in practical contexts.

In the figure AD is drawn perpendicular to the side opposite to the right angled vertex A. BC = 13 cm and AD = 6 cm.

(a) Take BD = x and express DC interms of x.

(b) Frame a second degree equation and find the lengths BD and DC.



(Score: 4, Time: 5 Minutes)

Scoring Indicators

$$\text{let BD} = x$$

$$\text{DC} = 13 - x$$

$$\text{BC} \times \text{DC} = \text{AD}^2 \quad (1)$$

$$x(13 - x) = 6^2$$

$$13x - x^2 = 36$$

$$x^2 - 13x + 36 = 0$$

$$x = \frac{13 \pm \sqrt{(-13)^2 - 4 \times 1 \times 36}}{2 \times 1} \quad (1)$$

$$\therefore \frac{13 \pm \sqrt{169 - 144}}{2} \quad (1)$$

$$\frac{13 \pm \sqrt{25}}{2} \quad \frac{13 \pm 5}{2}$$

BD = 9 centimetre, DC = 4 centimetre

15. Learning outcome

- Identifying the nature of solution in problems involving second degree equations.

Can the sum of a number and its reciprocal be $\frac{2}{3}$? Why?

(Score: 4, Time: 5 Minutes)

Scoring Indicators

let number = x, then its reciprocal $= \frac{1}{x}$

$$\text{then, } x + \frac{1}{x} = \frac{2}{3} \quad (1)$$

$$(x^2 + 1) \times 3 = 2x$$

$$3x^2 - 2x + 3 = 0 \quad (1)$$

$$x = \frac{+2 \pm \sqrt{(-2)^2 - 4\times 3\times 3}}{4\times 3}$$

$$= \frac{2 \pm \sqrt{4-36}}{12}$$

$$= \frac{2 \pm \sqrt{-32}}{12} \qquad (1)$$

Since the discriminant is negative, we do not get a solution

$\therefore$ The sum of a number and its reciprocal never gives $\frac{2}{3}$. (1)

16. Learning outcome

- Framing and solving second degree equations.

The sum of a number and its reciprocal is $\frac{13}{6}$. What is the number?

(Score: 4, Time: 5 Minutes)

Scoring Indicators

$$\text{Let number} = x$$

$$\text{then its reciprocal} = \frac{1}{x}$$

$$x + \frac{1}{x} = \frac{13}{6} \qquad (1)$$

$$(x^2 + 1) \times 6 = 13\,x$$

$$6x^2 - 13x + 6 = 0 \qquad (1)$$

$$x = \frac{13 \pm \sqrt{(-13)^2 - 4\times 6\times 6}}{2\times 6}$$

$$= \frac{13 \pm \sqrt{169-144}}{12}$$

$$= \frac{13 \pm \sqrt{25}}{12}$$

$$= \frac{13 \pm 5}{12} \qquad (1)$$

$$\text{Number} = \frac{3}{2} \text{ or } \frac{2}{3} \qquad (1)$$

17. Learning outcome

- Framing and solving second degree equations.

The sum of two numbers is 12 and sum of its reciprocal is $\frac{3}{8}$. Find the numbers

(Score: 4, Time: 5 Minutes)

Scoring Indicators

If we take numbers as $6+x$, $6-x$ (1)

$$\frac{1}{6+x}+\frac{1}{6-x} = \frac{3}{8} \quad (1)$$

$$12 \times 8 = 3\,(36 - x^2)$$

$$96 = 108 - 3x^2 \quad (1)$$

$$3x^2 = 12\; x^2 = 4$$

$x = 2$ or -2

Number $= 6+2 = 8$, $6-2=4$ (1)

18. Learning outcome

- Solving mathematical problems involving second degree equations.

Consider an arithmetic sequence with common difference 20. If the sum of reciprocals of two consecutive terms of this sequence is $\frac{1}{24}$, find the first term of the arithmetic sequence

(Score: 4, Time: 6 Minutes)

Scoring Indicators

Terms $x - 10$, $x + 10$ (1)

$$\frac{1}{x-10}+\frac{1}{x+10} = \frac{1}{24}$$

$$\frac{2x}{x^2-100} = \frac{1}{24}$$

$$2x \times 24 = x^2 - 100 \quad (1)$$

$$x^2 - 48x = 100$$

$$x^2 - 48x + 24^2 = 100 + 576$$

$$(x-24)^2 = 676 \quad (1)$$

$$x - 24 = \pm 26$$

$x = 26 + 24 =$ or $x = -26 + 24$ (1)

First term $= 40$ or -12

19. Learning outcomes

- Solving practical problems with the help of second degree equations.

Prove that the difference of a number and its reciprocal will be always positive.

(Score: 3, Time: 5 Minutes)

■ **Scoring Indicators**

$$\text{Number} = x$$

$$\text{Its reciprocal} = \frac{1}{x} \qquad (1)$$

$$\text{then; } x - \frac{1}{x} = k \qquad (1)$$

$$x^2 - kx - 1 = 0$$

$$x = \frac{+k \pm \sqrt{k^2+4}}{2}$$

Since k being a positive number, $k^2 + 4$ also negative (1)

20. Learning outcome

- Solving geometrical problems with the help of second degree equations.

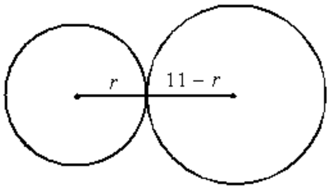
Two circles touch each other as shown below. Their centres are at a distance of 11 cenitmetres. The sum of the areas of the circles is 65π sq. centimetres. Find their radii.

(Score: 4, Time: 6 Minutes)

■ **Scoring Indicators**

Radius of first circle = r

Radius of second circle = $11 - r$ (1)



Sum of their areas:

$$\pi r^2 + \pi\,(11 - r)^2 = 65\pi$$

$$r^2 + 121 - 22\,r + r^2 = 65$$

$$2r^2 - 22r = -56$$

$$r^2 - 11\text{r} = -28 \qquad (1)$$

$$r^2 - 11r + \left(\frac{11}{2}\right)^2 = -28 + \frac{121}{4}$$

$$\left(r - \frac{11}{2}\right)^2 = \frac{9}{4} \qquad (1)$$

$$r - \frac{11}{2} = \frac{3}{2}$$

$$r = \frac{3+11}{2} = 7\text{ cm}$$

Radius of first circle = 7 cm

Radius of second circle = 11 − 7= 4 cm (1)

21. Learning outcome

- Applying the concept of second degree equations in different mathematical contexts.

The algebraic expression for the sum of 'n' consecutive terms of an arithmetic sequence is $2n^2 + 5n$. How many consecutive terms are needed to get a sum 1375?

(Score: 4, Time: 5 Minutes)

Scoring Indicators

$$2n^2 + 5n = 1375$$

$$2n^2 + 5n - 1375 = 0 \qquad (1)$$

$$n = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times -1375}}{2 \times 2} \qquad (1)$$

$$= \frac{-5 \pm \sqrt{25 + 11000}}{4}$$

$$= \frac{-5 \pm 105}{4} \qquad (1)$$

$$n = \frac{100}{4} = 25$$

Sum of first 25 terms is 1375 (1)

22. Learning outcome

- Solving problems using the nature of solution of second degree equations.

If $x^2 + ax + b = 0$ has only one solution, show that $a^2 = 4b$.

(Score: 3, Time: 4 Minutes)

Scoring Indicators

$x^2 + ax + b = 0$, has only one solution

It means the left side of the equation is a perfect square (1)

$$\therefore \left(\frac{a}{2}\right)^2 = b \qquad (1)$$

$$\frac{a^2}{4} = b$$

$$a^2 = 4b \qquad (1)$$

23. Learning outcome

- Applying second degree equations in different mathematical contexts.

Observe the given number pattern formed using natural numbers. Which line has the last number 210?

(Score: 4, Time: 5 Minutes)

1

2 3

4 5 6

7 8 9 10

....

....

- **Scoring Indicators**

$$\frac{n(n+1)}{2} = 210 \qquad (1)$$

$$n^2 + n = 420$$

$$n^2 + n + \frac{1}{4} = 420 + \frac{1}{4} \qquad (1)$$

$$\left(n+\frac{1}{2}\right)^2 = \frac{1681}{4} \qquad (1)$$

$$n + \frac{1}{2} = \frac{41}{2}$$

$$n = \frac{41-1}{2} = 20$$

20th line has the last number 210 (1)

24. Learning outcome

- Solving practical problems using the idea of second degree equations.

Can 240 be the sum of consecutive terms of an arithmetic sequence 7, 11, 15,... Justify.

(Score: 4, Time: 5 Minutes)

- **Scoring Indicators**

let $S_n = 2n^2 + 5n = 240$

$$2n^2 + 5n - 24 = 0 \qquad (1)$$

$$n = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times -24}}{2 \times 2}$$

$$= \frac{-5 \pm \sqrt{25 + 1920}}{4} \qquad (1)$$

$$= \frac{-5 \pm \sqrt{1945}}{4} \qquad (1)$$

sin a $\sqrt{1945}$ is not a natural number, 'n' cannot be a natural number

'n' should be a natural number. Hence sum cannot be 240 (1)

25. Learning outcome

- Relation between second degree equations and polynomials.

Can the polynomial $2x$-x^2 get 2 for any x ? What about getting $\frac{1}{2}$?

(Score: 5, Time: 7 Minutes)

- **Scoring Indicators**

$$2x - x^2 = 2 \qquad (1)$$

$$x^2 - 2x = -2$$

$$x^2 - 2x + 1 = -2 + 1$$

$$(x-1)^2 = -1 \qquad (1)$$

The square of any number cannot be negatives

So, for any x, we never get $2x - x^2 \quad = 2$ (1)

$$\text{Now, } 2x - x^2 = \frac{1}{2}$$

$$x^2 - 2x = \frac{1}{2}$$

$$x^2 - 2x + 1 = -\frac{1}{2} + 1$$

$$(x - 1)^2 = \frac{1}{2} \text{, a positive number} \quad (1)$$

$$\text{then } \therefore 2x - x = \frac{1}{2} \quad (1)$$

26. Learning outcome

- Applying the method of finding the solution of a second degree equation in different mathematical contexts.

In triangle ABC, AB = AC and ∠A = 36°. The bisector of ∠B intersect AC at D. if $\frac{BC}{CD} = x$, prove that $x = 1 + \frac{1}{x}$. Find the value of x also.

(Score: 5, Time: 7 Minutes)

- **Scoring Indicators**

The angles of triangle *ABC* and triangle *BCD* are equal

let BC = 1 and AC = AB = x (1)

$$\text{then, we have } \frac{x-1}{1} = \frac{1}{x} \quad (1)$$

$$x = 1 + \frac{1}{x} \quad (1)$$

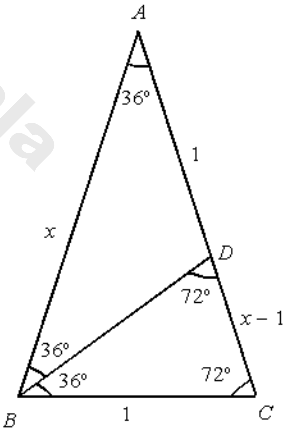
$$x = \frac{x+1}{x}$$

$$x^2 = x + 1$$

$$x^2 - x - 1 = 0 \quad (1)$$

$$x = \frac{1 \pm \sqrt{1-(-4)}}{2} = \frac{1 \pm \sqrt{5}}{2} \text{, not a negative number}$$

$$\text{Hence, } x = \frac{1+\sqrt{5}}{2} \quad (1)$$



27. Learning outcome

- Applying the method of solving a second degree equation in practical problems.

The distance from town A to town B is 180 kilometres. A cyclist rides half the distance from A to B in a specific speed, and the next half distance, at a speed of 15 kilometre/hr more than the first half. He travelled for 5 hours. Find the speed of the journey in each half?

(Score: 5, Time: 7 Minutes)

■ Scoring Indicators

$$\frac{90}{x} + \frac{90}{x+15} = 5 \qquad (1)$$

$$90x + 1350 + 90x = 5x^2 + 75x$$

$$5x^2 - 105x - 1350 = 0 \qquad (1)$$

$$x^2 - 21x - 270 = 0$$

$$= \frac{-21 \pm \sqrt{(-21)^2 - 4 \times 1 \times -270}}{2 \times 1} \qquad (1)$$

$$= \frac{21 \pm \sqrt{1521}}{2}$$

$$= \frac{21 \pm 39}{2}, \text{ not a negative number} \qquad (1)$$

$$\text{Hence, } x = \frac{60}{2} = 30$$

Speed in the first half : 30 km/hr

Speed in the second half : 45 km/hr (1)

28. Learning outcome

- Applying the method of finding the solution of a second degree equation in solving practical problems.

A traveller travels half of his journey in a specific speed and the next half, a speed 15 kilometre/hour more than the first half. The average speed of the total journey is 36 kilometre/hour. Find his speed in each half?

(Score: 5, Time: 7 Minutes)

■ Scoring Indicators

$$\frac{1}{x} + \frac{1}{x+15} = \frac{2}{36} = \frac{1}{18} \qquad (1)$$

$$(2x + 15) \times 18 = x^2 + 15x$$

$$x^2 + 15x - 36x - 270 = 0 \qquad (1)$$

$$x = \frac{-21 \pm \sqrt{441 - 4 \times -270}}{2 \times 1} \qquad (1)$$

$$= \frac{21 \pm \sqrt{1521}}{2}$$

$$= \frac{21 \pm 39}{2}$$

$$x = \frac{60}{2} = 30 \qquad (1)$$

Speed in the first half = 30 km/hr

Speed in the second half = 45 km/hr (1)

29. Learning outcome

- Applying the method of finding the solution of a second degree equation in solving practical problems.

If the price of a toy is lessened by 2 rupees, one could have bought 2 more toys for Rs.360. Find the actual price of the toy.

(Score: 5, Time: 7 Minutes)

■ Scoring Indicators

let the price of a toy = x

Number of toys for 360 rupees = $\frac{360}{x}$ (1)

If the price is lessened by 2 rupees per toy

Number of toys for 360 rupees = $\frac{360}{x-2}$ (1)

$$\text{Difference}: \frac{360}{x-2} - \frac{360}{x} = 2$$

$$360x - 360x + 720 = 2x^2 - 4x$$

$$2x^2 - 4x = 720$$

$$x^2 - 2x = 360 \qquad (1)$$

$$x^2 - 2x + 1 = 361$$

$$(x-1)^2 = 361$$

$$x - 1 = 19 \qquad (1)$$

$$x = 20$$

Price of a toy = 20 rupees (1)

30. Learning outcome

- Applying the method of finding the solution of a second degree equation in solving practical problems.

Ravi needs 16 more days to complete a work than Raju to complete the same work. If both of them work together, only 15 days are sufficient. How many days Raju alone needed to complete that work.

(Score: 5, Time: 7 Minutes)

■ Scoring Indicators

Number of days Raju needed for the job = x (1)

Number of days Ravi needed for the same job = $x + 16$

$$\therefore \frac{1}{x} + \frac{1}{x+16} = \frac{1}{15} \qquad (1)$$

$$(2x + 16) \times 15 = x^2 + 16x$$

$$x^2 + 16x - 30x = 240 \qquad (1)$$

$$x^2 - 14x = 240$$

$$x^2 - 14x + 49 = 269 \qquad (1)$$

$$(x-7)^2 = 269$$

$$x - 7 = 17$$

$$x = 17 + 7 = 24$$

Number days Raju needed to complete the job = 24

31. Learning outcome

- Applying the method of finding the solution of a second degree equation in solving practical problems.

Using 1 litre of petrol , car *B* travels 5 kilometres more than another car *A* for travelling 400 kilometres, car *B* uses 4 litres petrol less than that of car *A*. Find the distance travelled by car *A* using 1 litre of petrol.

(Score: 5, Time: 7 Minutes)

Scoring Indicators

Distance travelled by car *A* using 1 litre of petrol = x

Distance travelled by car *B* usign 1 litre of petrol = x +5

$$\frac{400}{x} - \frac{400}{x+5} = 4 \qquad (1)$$

$$400x + 2000 - 400x = 4x^2 + 20x$$

$$4x^2 + 20x = 2000$$

$$x^2 + 5x = 500 \qquad (1)$$

$$x^2 + 5x + \left(\frac{5}{2}\right)^2 = 500 + \frac{25}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{2025}{4} \qquad (1)$$

$$x + \frac{5}{2} = \frac{45}{2}$$

$$x = \frac{45-5}{2} = 20 \qquad (1)$$

Car A travels 20 kilometres using 1 litre of petrol (1)

32. Learning outcome

- Applying the method of finding the solution of a second degree equation in solving practical problems.

While computing the sides of a rectangle having specific perimeter and area, perimeter was taken as 52 centimetres instead of 50 centimetres. Then the side was calculated as 18 centimetres. Find the actual length of the rectangle.

(Score: 5, Time: 7 Minutes)

- **Scoring Indicators**

$$\text{Perimeter} = 52$$

$$\text{length} + \text{breadth} = 26$$

$$\text{Sides: } 18,\ 26 - 18 = 8 \qquad (1)$$

$$\text{Area: } 18 \times 8 = 144 \text{ Sq. centimetres}$$

$$\text{Perimeter} = 50$$

$$\text{length} + \text{breadth} = 25$$

$$\text{length} = x,\ \text{breadth} = 25 - x \qquad (1)$$

$$\text{Perimeter: } x(25 - x) = 144$$

$$x^2 - 25x + 144 = 0$$

$$x = \frac{25 \pm \sqrt{25^2 - 4 \times 144}}{2 \times 1} \qquad (1)$$

$$= \frac{25 \pm \sqrt{625 - 576}}{2}$$

$$= \frac{25 \pm \sqrt{49}}{2} \qquad (1)$$

$$= \frac{25 \pm 7}{2}$$

$$x = \frac{25 \pm 7}{2} = 16 \text{ or } \frac{25 - 7}{2} = \frac{18}{2} = 9$$

sides : 16 cm, 9 cm (1)

33. Learning outcome

- Finding the relation between polynomials and second degree equations.

Prove that the second degree polynomial x^2 - 2x + 6 will never be less than 5 for any number x. Also find for which number, gets the value 5.

(Score: 4, Time: 5 Minutes)

- **Scoring Indicators**

$$p(x) = x^2 - 2x + 6$$

$$= x^2 - 2x + 1 + 5$$

$$= (x-1)^2 + 5 \qquad (1)$$

Since $(x-1)^2$ a perfect square, the smallest value for $(x-1)^2$ is zero (1)

Thus $x^2 - 2x + 6$ never gets the value smaller than 5 for any x (1)

When $x = 1, (x-1)^2 = 0, (x-1)^2 + 5 = 5$ (1)

34. Learning outcome

- Applying the method of solving a second degree equation in other practical problems.

A water tank is connected to two pipes, the bigger one is to fill and a smaller one to drain. Time taken to fill the tank is 1 minute less than that of the time taken to drain out. When one uses both pipes simultaneously, the tank woll fill in 56 minutes. Find the time taken to fill up the tank, if one uses the bigger one only.

(Score: 5, Time: 8 Minutes)

■ **Scoring Indicators**

Time taken to drain out the tank using smaller pipe $= x$ only

Time taken to fill the tank using the bigger piper $= x-1$ only (1)

When both pipes are used simultaneously, we have

$$\frac{1}{x-1}-\frac{1}{x}=\frac{1}{56} \quad (1)$$

$$(x-x+1)\times 56 = x^2 - x$$

$$x^2 - x - 56 = 0$$

$$x = \frac{1 \pm \sqrt{1-4\times -56}}{2} \quad (1)$$

$$= \frac{1 \pm \sqrt{1+224}}{2}$$

$= \dfrac{1 \pm 15}{2}$, which never gets a negative number

Hence, $x = \dfrac{1+15}{2} = 8$ minute (1)

Time taken to fill the tank using the bigger pipe $= 7$ minutes only (1)

35. Learning outcome

- Applying the method of finding the solution of a second degree equation in solving practical problems.

In copying a second degree equation, the number without x was written as –30 instead of 30. The answers found were 15 and –2. What are the answers of the correct problem.

(Score: 5, Time: 8 Minutes)

■ **Scoring Indicators**

$$ax^2 + bx - 30 = 0$$

$x = 15$, $225a + 15b - 30 = 0$ $15a + b = 2$ (1) (1)

$x = -2$ $4a - 2b - 30 = 0$ $2a - b = 15$ (2) (1)

$(1) + (2) \rightarrow 17a = 17$, $a = 1$

$$2\times1-b=15 \quad b=-13 \qquad (1)$$

Then the equation is $x^2-13x-30=0$

for that equation, $x=\frac{13\pm\sqrt{169-120}}{2}$ (1)

$$x=\frac{13\pm7}{2}$$

$$x=\frac{13+7}{2}=10 \text{ or } \frac{13-7}{2}=3 \qquad (1)$$

Solutions of the correct problem 10 or 3

5

TRIGONOMETRY

1. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

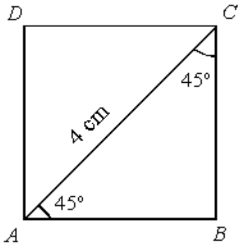
The diagonal of a square is 4cm long. Find its perimeter and area.

(Score : 2, Time : 3 minute)

Scoring indicators

Angles of ΔABC are 45°, 45°, 90°.

$$\begin{array}{ccccc} 45^0 & & 45^0 & & 90^0 \\ 1 & : & 1 & : & \sqrt{2} \\ \downarrow & & \downarrow & & \downarrow \\ 2\sqrt{2} & & 2\sqrt{2} & & 4 \end{array}$$



(1)

One side $= 2\sqrt{2}$ cm

Perimeter $= 4 \times 2\sqrt{2} = 8\sqrt{2}$ cm. (1)

Area $= 2\sqrt{2} \times 2\sqrt{2}$ sq.cm. (1)

2. Learning Outcome

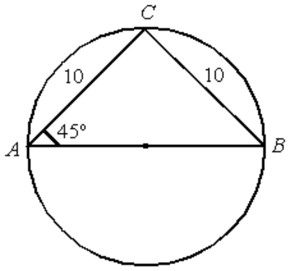
- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

AC and BC are two equal chords of a circle with diameter AB. If the equal chords have lengths 10cm find the area of the circle.

(Score : 3, Time : 3 minute)

Scoring indicators

$$\begin{array}{ccccc} 45^0 & & 45^0 & & 90^0 \\ 1 & : & 1 & : & \sqrt{2} \\ \downarrow & & \downarrow & & \downarrow \\ 10 & & 10 & & 10\sqrt{2} \end{array}$$



Diameter AB $= 10\sqrt{2}$ cm (1)

Radius $= 5\sqrt{2}$ cm (1)

Area $= \pi r^2 = \pi \times \left(5\sqrt{2}\right)^2 = 50\pi$ sq.cm (1)

3. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

In ΔABC, $AB = 10cm$. $AC = 8$ cm, $\angle A = 45°$

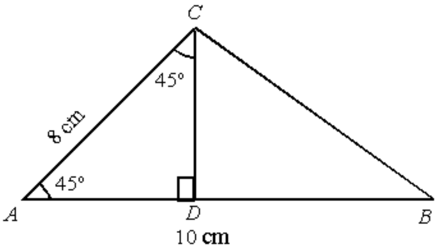
(a) Find the perpendicular distance from C to AB.

(b) Find the area of the triangle.

(Score : 2, Time : 3 minute)

Scoring indicators

(a) Draw CD perpendicular to AB.



Angles of ΔADC are 45°, 45°, 90°

45°		45°		90°
1	:	1	:	$\sqrt{2}$
↓		↓		↓
$4\sqrt{2}$		$4\sqrt{2}$		8

$CD = AD = 4\sqrt{2}$ cm (1)

(b) Area $= \frac{1}{2} \times 10 \times 4\sqrt{2}$

$= 20\sqrt{2}$ sq.cm (1)

4. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

One side of a rhombus is 12cm and one angle is 135°

(a) Find the distance between the parallel sides?

(b) Find the area of the rhombus?

(Score : 3, Time : 3 minute)

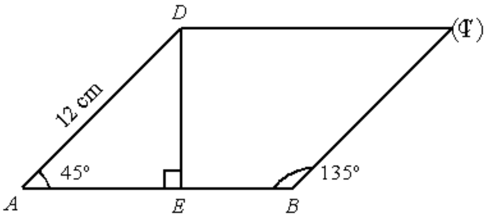
Scoring indicators

(a) ABCD is a rhombus

$AB = AD = 12$ cm; $\angle B = 135°$

$\angle A = 180 - 135 = 45°$

Angles of ΔADE are 45°, 45°, 90°



45°		45°		90°
1	:	1	:	$\sqrt{2}$
↓		↓		↓
$6\sqrt{2}$		$6\sqrt{2}$		12

$DE = 6\sqrt{2}$ cm (1)

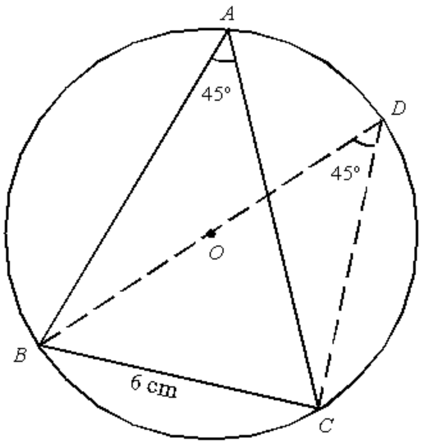
(b) Area of the rhombus $= 12 \times 6\sqrt{2}$

$= 72\sqrt{2}$ sq.cm (1)

5. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

In ΔABC, $\angle A = 45^\circ$, $BC = 6$ cm. Find the diameter of the circumcircle.



(Score : 4, Time : 4 minute)

Scoring indicators

(a) Draw diameter BD and Join CD Angles of (1)

Angles of ΔBCD is $45^\circ, 45^\circ, 90^\circ$

$$\begin{array}{ccccc} 45^\circ & & 45^\circ & & 90^\circ \\ 1 & : & 1 & : & \sqrt{2} \\ \downarrow & & \downarrow & & \downarrow \\ 6 & & 6 & & 6\sqrt{2} \end{array}$$ (1)

$BD = 6\sqrt{2}$ cm (1)

6. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

12 centimetre long diagonal of a rectangle makes an angle 30° with its one side. Find its perimeter and area.

(Score : 3, Time : 4 minute)

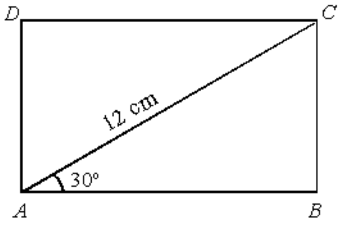
Scoring indicators

ABCD is a square

$AC = 12$ cm; $\angle BAC = 30^\circ$

Angles of ΔABC are $30^\circ, 60^\circ, 90^\circ$

$$\begin{array}{ccccc} 30^\circ & & 60^\circ & & 90^\circ \\ 1 & : & \sqrt{3} & : & 2 \\ \downarrow & & \downarrow & & \downarrow \\ 6 & & 6\sqrt{3} & & 12 \end{array}$$



$$AB = 6\sqrt{3}\ \text{cm} \qquad (1)$$
$$BC = 6\ \text{cm}$$
$$\text{perimeter} = 2\ (6 + 6\sqrt{3}) = (12 + 12\sqrt{3})\ \text{cm} \qquad (1)$$
$$\text{Area} = 6 \times 6\sqrt{3} = 36\sqrt{3}\ \text{sq.cm} \qquad (1)$$

7. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

In triangle ABC, AB = 20cm, $\angle A = 30°$, AC = 12cm

(a) Find the length of the perpendicular from C to AB.

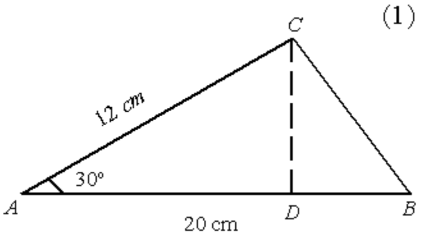
(b) Find the area of the triangle.

(Score : 3, Time : 4 minute)

Scoring indicators

(a) Draw CD perpendicular to AB.
Angles of ΔADC are 30°, 60°, 90° (1)

$$\begin{array}{ccccc} 30° & & 60° & & 90° \\ 1 & : & \sqrt{3} & : & 2 \\ \downarrow & & \downarrow & & \downarrow \\ 6 & & 6\sqrt{3} & & 12 \end{array}$$



CD = 6 cm (1)

(b) Area of the triangle $= \frac{1}{2} \times 20 \times 6 = 60$ sq.cm (1)

8. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

In ΔABC, AB = 8 cm, BC = 10cm and $\angle B = 60°$

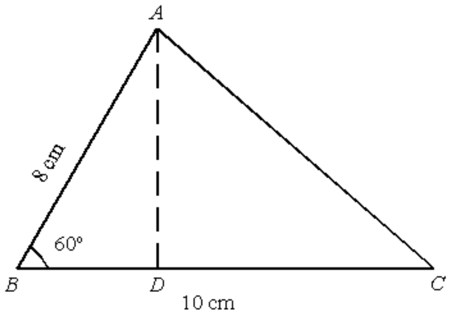
(a) Find the area of the triangle ΔABC?

(b) Find AC? (Score : 4, Time : 6 minute)

Scoring indicators

(a) Draw AD perpendicular to BC
Angles of ΔADB are 30°, 60°, 90°

$$\begin{array}{ccccc} 30° & & 60° & & 90° \\ 1 & : & \sqrt{3} & : & 2 \\ \downarrow & & \downarrow & & \downarrow \\ 4 & & 4\sqrt{3} & & 8 \end{array}$$



(1)

BD = 4 cm; AD $= 4\sqrt{3}$ cm

Area of $\Delta ABC = \frac{1}{2} \times 10 \times 4\sqrt{3} = 20\sqrt{3}$ sq.cm (1)

(b) $BC = 10$ cm

$CD = 10 - 4 = 6$ cm (1)

In ΔADC $AC^2 = AD^2 + CD^2$

$= \left(4\sqrt{3}\right)^2 + 6^2$

$= 48 + 36 = 84$

$AC = \sqrt{84} = 2\sqrt{21} =$ cm (1)

9. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

One angle of a triangle is 150° and its opposite side 3 centimetre. Find the diameter of its circum circle.

(Score: 3, Time: 5 minute)

Scoring indicators

In ΔABC

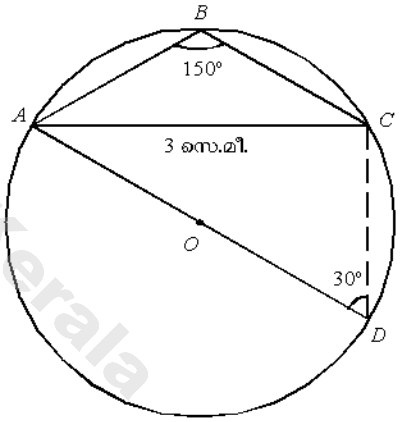
$\angle B = 150^\circ$, $AC = 3$ cm

Draw diameter AD and join CD

$\angle ADC = 180 - 150 = 30^\circ$ (1)

$\angle ACD = 90^\circ$

Angles of ΔADC are 30°, 60°, 90°



30°		60°		90°
1	:	$\sqrt{3}$	:	2
↓		↓		↓
3		$3\sqrt{3}$		6

(1)

Diameter, AD = 6cm (1)

10. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

In ΔABC, $AB = 12$ cm, $\angle A = 45^\circ$ and $\angle B = 30^\circ$

(a) Find the area of the triangle ABC.

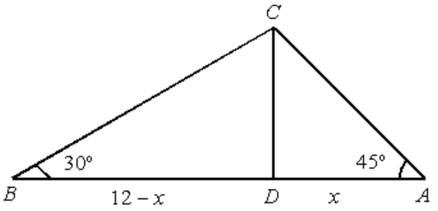
(b) Find the ratio of the sides of the triangle having angles 30°, 45°, 105°.

(Score: 5, Time: 8 minute)

Scoring indicators

Angles of Δ BDC are 30°, 60°, 90°

$$\begin{array}{ccccc} 30^0 & & 60^0 & & 90^0 \\ 1 & : & \sqrt{3} & : & 2 \\ \downarrow & & \downarrow & & \downarrow \\ x & & x\sqrt{3} & & 2x \end{array}$$



(1)

$$x\sqrt{3} = 12 - x$$

$$x\left(\sqrt{3}+1\right) = 12$$

$$x = \frac{12}{\sqrt{3}+1} = \frac{12\left(\sqrt{3}-1\right)}{3-1} = 6(\sqrt{3}-1)$$ (1)

$$\text{Area} = \frac{1}{2} \times 12 \times 6\left(\sqrt{3}-1\right)$$

$$= 36\left(\sqrt{3}-1\right)\text{sq.cm}$$ (1)

(b) AD = x, BD = $\sqrt{3}\,x$, AB = $\left(\sqrt{3}+1\right)x$ cm (1)

BC = $2x$, AC = $\sqrt{2}\,x$

$$\text{AC : BC : AB} = x : 2x : \left(\sqrt{3}+1\right)x$$

$$= \sqrt{2} : 2 : \sqrt{3} + 1$$ (1)

11. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

❓ One side of a rhombus is 10 cm and one of its angles measures 120°.

(a) Find the area of the rhombus

(b) Find the length of diagonals of the rhombus

(Time : 5, Score : 7 minute)

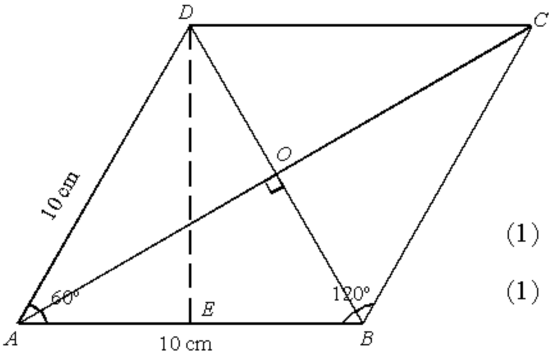
Scoring indicators

In Δ AED, $\angle A = 60^\circ$, $\angle AED = 90^\circ$, $\angle ADE = 30^\circ$

AD = 10 cm.

$$\begin{array}{ccccc} 30^\circ & & 60^\circ & & 90^\circ \\ 1 & : & \sqrt{3} & : & 2 \\ \downarrow & & \downarrow & & \downarrow \\ 5 & & 5\sqrt{3} & & 10 \end{array}$$ (1)

AE = 5 cm; DE = $5\sqrt{3}$ cm (1)



(a) Area $= 10 \times 5\sqrt{3} = 50\sqrt{3}$ sq.cm (1)

Angles of ΔAOB are $30^\circ, 60^\circ, 90^\circ$

$$\begin{array}{ccccc} 30^\circ & & 60^\circ & & 90^\circ \\ 1 & : & \sqrt{3} & : & 2 \\ \downarrow & & \downarrow & & \downarrow \\ 5 & & 5\sqrt{3} & & 10 \end{array}$$ (1)

OA $= 5\sqrt{3}$ cm; OB = 5 cm

AC $= 10\sqrt{3}$ cm; BD = 10 cm (1)

12. Learning Outcome

- Identifying that sides of a triangle are proportional to the sine values of the corresponding opposite angles.

In ΔABC, $\angle A = 40^\circ$, BC = 8 cm
Find the circumdiameter of the triangle.

[Sin 40 = 0.64] (Score: 3, Time: 5 minute)

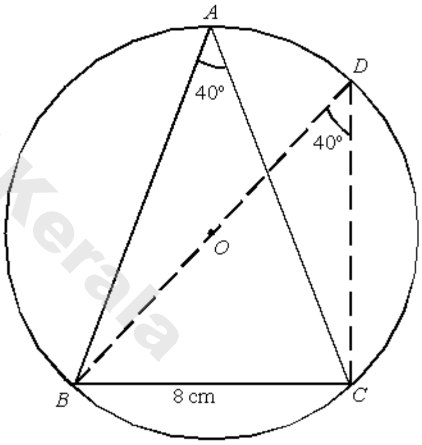
■ Scoring indicators

Draw Diameter BD and join CD.

In ΔBCD, $\angle D = 40^\circ$ (1)

$\sin 40 = \frac{BC}{BD} = \frac{8}{BD}$ (1)

BD $= \frac{8}{\sin 40} = \frac{8}{0.64} = 12.5$ cm (1)



13. Learning Outcome

- Identifying that sides of a triangle are proportional to the sine values of the corresponding opposite angles.

In Δ ABC, $\angle A = 50^\circ$, $\angle B = 70^\circ$, AB = 6 cm.

(a) Find the diameter of its circum circle

(b) Find the lengths of other two sides AC and BC.

[$\sin 60^\circ = 0.87$; $\sin 50^\circ = 0.77$; $\sin 70^\circ = 0.94$]

(Score: 5, Time: 8 minute)

■ Scoring indicators

(a) Draw diameter AD and join BD. (1)

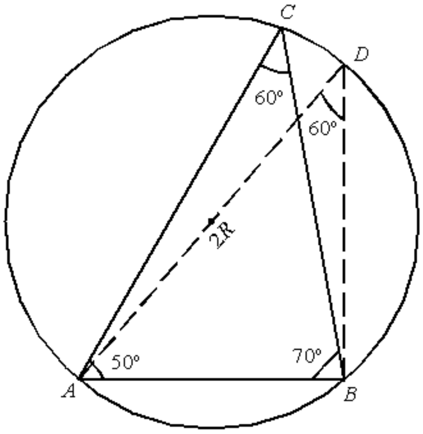
$\sin 60 = \frac{AB}{AD} = \frac{6}{AD}$ (1)

$2R = AD = \frac{6}{\sin 60} = \frac{6}{0.87} = 6.9$ (1)

$BC = 2R \sin 50 = 6.9 \times 0.77$

$= 5.31$ cm (1)

$AC = 2R \sin 70 = 6.9 \times 0.94 = 6.49$ cm (1)



14. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

In Δ ABC, AB = 10 cm; AC = 6cm; ∠A = 70°

(a) Find the area of the triangle

(b) Find BC

[cos 70° = 0.34; sin 70° = 0.94]

(Score: 4, Time: 6 minute)

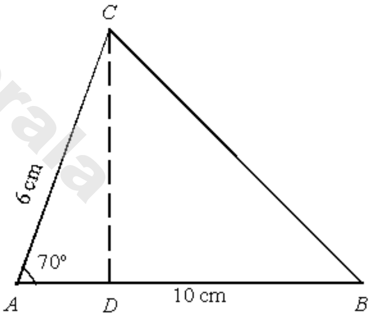
■ Scoring indicators

$\sin 70° = \frac{CD}{6}$

$CD = 6 \times \sin 70$

$= 6 \times 0.94 = 5.64$ cm (1)

$\cos 70° = \frac{AD}{6}$



$AD = 6 \times \text{Cos } 70° = 6 \times 0.34 = 2.04$ cm (1)

$BD = 10 - 2.04 = 7.96$ cm (1)

$BC = \sqrt{CD^2+BC^2} = \sqrt{(5.64)^2+(7.96)^2}$

$= \sqrt{31.81+63.36} = \sqrt{95.17}$ cm (1)

15. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

In ΔABC, ∠A = 60°, ∠B = 45° and AB = 10 cm

a) Find the perpendicular distance from C to AB?

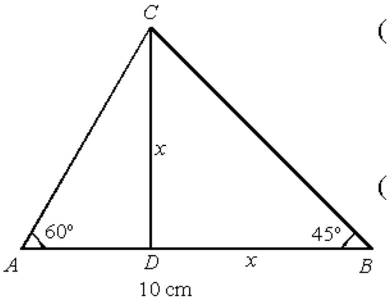
(b) Find the area of the triangle?

(Score: 4, Time: 6 minute)

■ Scoring indicators

(a) Draw CD perpendicular to AB

If CD $= x$ then BD $= x$; AD $= \frac{\sqrt{3}x}{3}$ (1)



$$\frac{\sqrt{3}x}{3} + x = 10$$

$$\sqrt{3}x + 3x = 30 \qquad (1)$$

$$x = \frac{30}{3+\sqrt{3}} = \frac{30(3-\sqrt{3})}{6}$$

$$= 5(3-\sqrt{3}) \text{ cm} \qquad (1)$$

(b) $\text{Area} = \frac{1}{2} \times 10 \times 5(3-\sqrt{3})$

$$= 25(3-\sqrt{3}) \text{ sq.cm} \qquad (1)$$

16. Learning Outcome

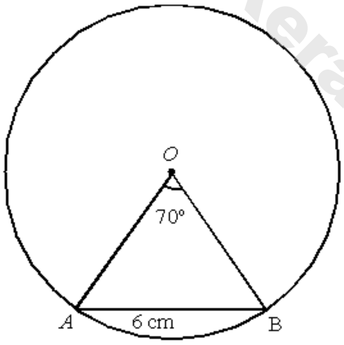
- The relation between the chord of a circle and its central angle can be represented using the trigonometric measurement sine.

AB is a chord of a circle with centre 'O'.

$\angle AOB = 70°$ and, AB = 16 cm. Find the diameter of the circle

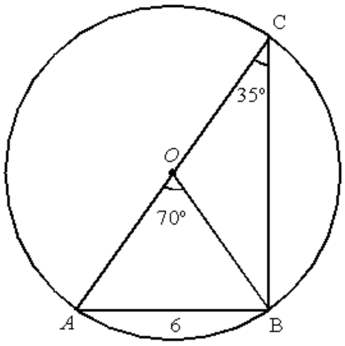
[sin 35° = 0.57]

(Score : 3, Time : 5minute)



■ Scoring indicators

Draw diameter AC and join BC.



$$\sin 35° = \frac{AB}{AC} \qquad (1)$$

$$AC = \frac{AB}{\sin 35} = \frac{6}{0.57} \qquad (1)$$

$$= 10.52 \text{ cm} \qquad (1)$$

17. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

In ΔABC, $\angle A = \angle B = 30^\circ$, $AB = 12$ cm.

(a) Find the perimeter of the triangle

(b) Find the area of the triangle

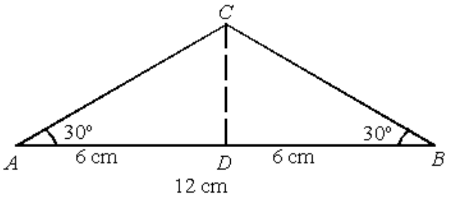
(c) Find the ratio of the sides of a triangle having angles 30°, 30°, 120°

(Score: 5, Time: 8 minute)

Scoring indicators

Angles of ΔACD 30°, 60°, 90°

$$\begin{array}{ccccc} 30^\circ & & 60^\circ & & 90^\circ \\ 1 & : & \sqrt{3} & : & 2 \\ \downarrow & & \downarrow & & \downarrow \\ 2\sqrt{3} & & 6 & & 4\sqrt{3} \end{array}$$ (1)



$CD = 2\sqrt{3}$ cm

$AC = 4\sqrt{3}$ cm

$BC = 4\sqrt{3}$ cm (1)

(a) Perimeter of $\Delta ABC = 4\sqrt{3} + 4\sqrt{3} + 12 = (12 + 8\sqrt{3})$ cm (1)

(b) Area of $\Delta ABC = \frac{1}{2} \times 12 \times 2\sqrt{3} = 12\sqrt{3}$ sq.cm (1)

(c) $AC = BC = 4\sqrt{3}$, $AB = 12$ cm

Ratio $= 4\sqrt{3} : 4\sqrt{3} : 12 = 1 : 1 : \sqrt{3}$ (1)

18. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

In ΔABC, $AB = 8$ cm; $\angle A = 45^\circ$ and $\angle B = 60^\circ$

(a) Find the perpendicular distance from C to AB.

(b) Find the area of the triangle

(c) Find the ratio of the sides of a triangle having angles 45°, 60°, 75°

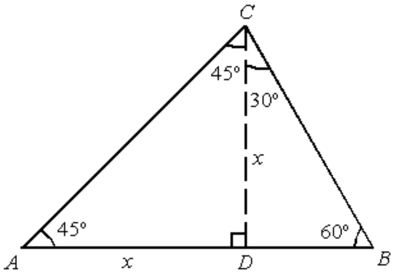
(Score: 5, Time: 8 minute)

Scoring indicators

Draw CD perpendicular to AB

In Δ ADC,

$$\begin{array}{ccccc} 45^\circ & & 45^\circ & & 90^\circ \\ 1 & : & 1 & : & \sqrt{2} \\ \downarrow & & \downarrow & & \downarrow \\ x & & x & & \sqrt{2}\,x \end{array}$$



(1)

In Δ BDC,

$$\begin{array}{ccccc} 30^\circ & & 60^\circ & & 90^\circ \\ 1 & : & \sqrt{3} & : & 2 \\ \downarrow & & \downarrow & & \downarrow \\ \frac{x}{3}\sqrt{3} & & x & & \frac{2x}{3}\sqrt{3} \end{array}$$ (1)

$$AB = x + \frac{x}{3}\sqrt{3} = 8$$

$$x(3 + \sqrt{3}) = 24$$

$$x = \frac{24}{3+\sqrt{3}} = 4(3-\sqrt{3})$$ (1)

Area of Δ ABC $= \frac{1}{2} \times 8 \times 4(3-\sqrt{3}) = 16(3 - \sqrt{3})$ sq.cm (1)

$$AC = \sqrt{2}\,x,\ BC = \frac{2\sqrt{3}}{3}x : AB = x + \frac{x\sqrt{3}}{3}$$

$$BC : AC : AB = \frac{2\sqrt{3}}{3}x : \sqrt{2}\,x : \left(x + \frac{\sqrt{3}}{3}\right) = \frac{2\sqrt{3}}{3} : \sqrt{2} : 1 + \frac{\sqrt{3}}{3}$$

$$= 2\sqrt{3} : 3\sqrt{2} : 3 + \sqrt{3}$$

$$= 2 : \sqrt{6} : \sqrt{3} + 1$$ (1)

19. Learning Outcome

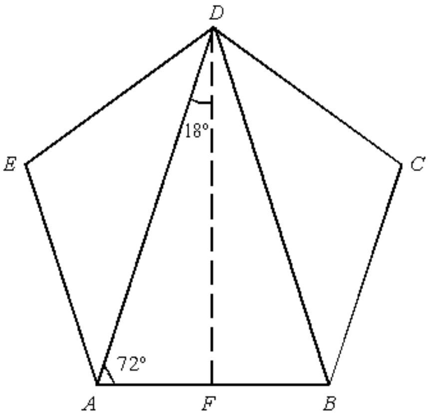
- Interpreting sine, cosine and tangent as measures of an angle using straight lines.

If one side of a regular pentagon is 6cm, find the length of its diagonal?

[sin 18° = 0.31]

(Score: 3, Time: 5 minute)

Scoring indicators



ABCDE is a regular pentagon

Angles of ΔADF

are 18°, 72°, 90° (1)

$$\sin 18^\circ = \frac{3}{AD}$$ (1)

$$AD = \frac{3}{\sin 18} = \frac{3}{0.31} = 9.68 \text{ cm}$$ (1)

20. Learning Outcome

- Interpreting sine, cosine and tangent as measures of an angle using straight lines.

Two sides of a triangle are 9cm and 10cm and the angle between those sides is 105°. find the area of the triangle.

$[\sin 75^\circ = 0.97]$

(Score: 3, Time: 6 minute)

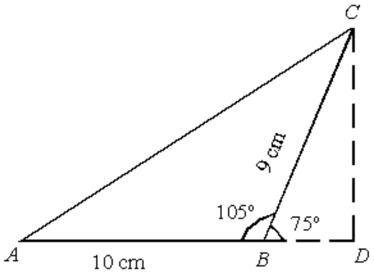
Scoring indicators

In $\Delta BDC\ \angle CBD = 180^\circ - 105^\circ = 75^\circ$

$\sin 75^\circ = \frac{CD}{9}$ (1)

$0.97 = \frac{CD}{9}$

$CD = 0.97 \times 9 = 8.73$ cm (1)



$\text{Area} = \frac{1}{2} \times 10 \times 8.77$

$= 43.65$ sq.cm (1)

21. Learning Outcome

- Interpreting sine, cosine and tangent as measures of an angle using straight lines.

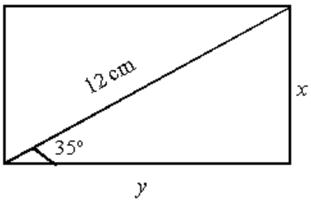
The diagonal of a rectangle is 12cm and it makes an angle 35° with one side. Find the perimeter of the rectangle.

$[\sin 35^\circ = 0.57, \cos 35^\circ = 0.82]$

(Score: 3, Time: 5 minute)

Scoring indicators

Let the breadth of the rectangle = x and length = y



$\sin 35^\circ = \frac{x}{12}$

$x = 12$ ´ $\sin 35 = 12 \times 0.57 = 6.84$ cm (1)

$\cos 35^\circ = \frac{y}{12}$

$y = 12$ ´ $\cos 35 = 12$ ´ $0.82 = 9.84$ cm (1)

$\text{Perimeter} = 2(6.84 + 9.84) = 2$ ´ $16.68 = 33.36$ cm (1)

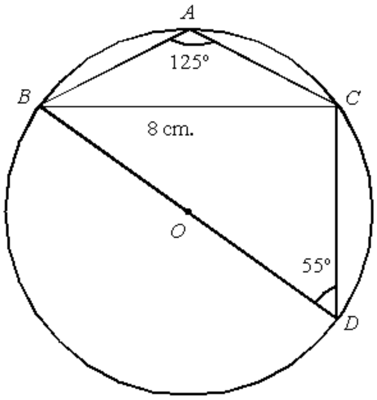
22. Learning Outcome

- Identifying that sides of a triangle are proportional to the sine values of the corresponding opposite angles.

In ΔABC, $\angle A = 125^\circ$, BC = 8 cm.
Find the diameter of the circumcircle. [sin 55 = .82]

(Score: 2, Time: 6 minute)

■ Scoring indicators



Draw Diameter BD and jon CD. (1)

In ΔBCD, $\sin 55^\circ = \frac{BC}{BD}$ (1)

$BD = \frac{BC}{\sin 55^\circ} = \frac{8}{0.82} = 9.76$ cm (1)

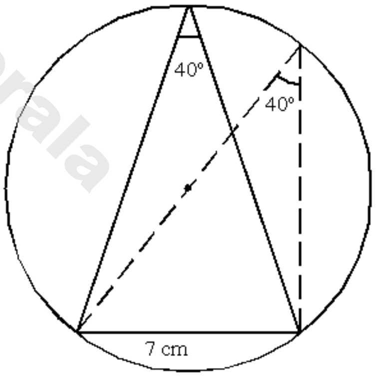
23. Learning Outcome

- Identifying that sidesof a triangle are proportional to the sine values of the corrresponding opposite angles.

Can one cut out a triangle of one side 7cm and its opposite angle 40° from a circular sheet of diameter 10 cm. Justify your answer. [sin 40° = 0.64]

(Score: 4, Time: 7 minute)

■ Scoring indicators



The diameter of the circum circle of a triangle with one angle 40° and its opposite side 7cm = $\frac{7}{\sin 40}$ (1)

$= \frac{7}{0.64} = 10.93$ cm (1)

(1)

Diameter of the paper is 10cm, which is less than 10.93 cm. Hence triangle cannot be cut out. (1)

24. Learning Outcome

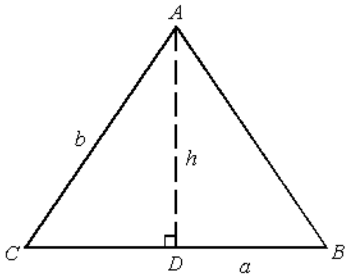
- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

Find the area of a triangle whose sides are *a* and *b* and the angle between those sides is *C*.

(Score: 2, Time: 3 minute)

■ Scoring indicators

In ΔABC, Draw AD perpendicular to BC.



$$\sin C = \frac{h}{b}$$ (1)

$$h = b \sin C$$

$$\text{Area} = \frac{1}{2}ah$$

$$= \frac{1}{2}ab \sin C$$ (1)

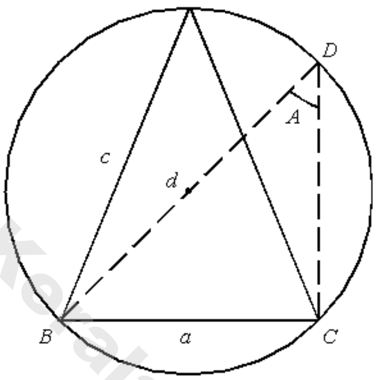
25. Learning outcome

- Identifying that sides of a triangle are proportional to the sine values of the corrresponding opposite angles.

Find the sides of a triangle whose angles are A, B and C and its circumdiameter d.

(Score: 3, Time: 5 minute)

■ Scoring indicators



Draw a diameter BD and join CD

$\angle BDC = A$; $BC = a$

$$\sin A = \frac{a}{BD} = \frac{a}{d}$$

$a = d \sin A$ (1)

Also $b = d \sin B$ and (1)

$c = d \sin C$

26. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

The parallel sides of an isosceles trapezium are 10cm and 6 cm and the non parallel sides 8 cm. If one angle is 110°.

(a) Find the distance between the parallel sides

(b) Find the area of the isosceles trapezium

[sin 70 = 0.94] (Score: 3, Time: 6 minutes)

■ Scoring indicators

In trapezium ABCD

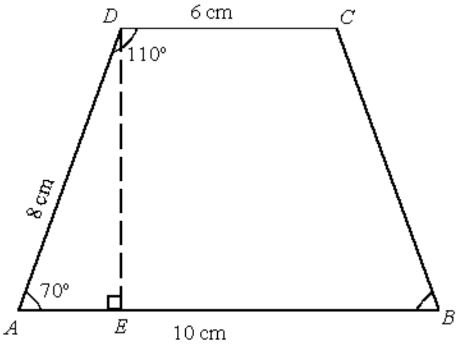
AB = 10 cm

CD = 6 cm

$AD = 8$ cm

$\angle ADC = 110°, \angle A = 180 - 110 = 70°$

Draw DE perpendicular to AB



$\sin 70° = \frac{DE}{8}$ (1)

$0.94 = \frac{DE}{8}$

DE = $0.94 \times 8 = 7.52$ cm (1)

Area of the trapezium $= \frac{1}{2} \times DE \times (AB + CD)$

$= \frac{1}{2} \times 7.52\ (10 + 6)$

$= 60.16$ sq.cm (1)

27. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

❓ In ΔABC, $AB = 8$ cm; $\angle A = 50°$, $\angle B = 70°$

(a) Find the perpendicular distance from C to AB.

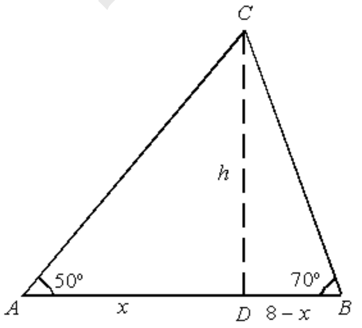
(b) Find the area of the triangle

[$\tan 50° = 1.2$, $\tan 70° = 2.75$]

(Score: 5, Time: 8 minute)

■ Scoring indicators

Draw CD perpendicular to AB.



$$\tan 50 = \frac{h}{x}$$ (1)

$$h = x \tan 50$$

$$\tan 70 = \frac{h}{8-x}$$

$$h = (8-x) \tan 70$$

$$x \tan 50 = (8-x) \tan 70$$ (1)

$$x \tan 50 = 8 \times \tan 70 - x \tan 70$$

$$x \tan 50 + x \tan 70 = 8 \times \tan 70$$

$$x = \frac{8 \times \tan 70}{\tan 50 + \tan 70}$$ (1)

$$h = x \tan 50$$

$$= \frac{8\times\tan 70\times\tan 50}{\tan 50+\tan 70}$$

$$= \frac{8\times 2.75\times 1.2}{2.75+1.2}$$

$$= 6.68 \text{ cm} \qquad (1)$$

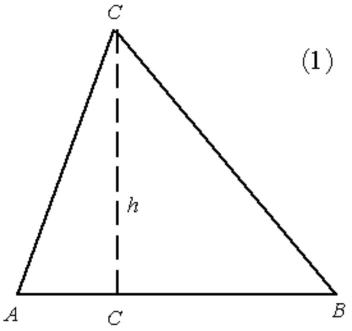
$$\text{Area} = \frac{1}{2}\times 8\times 6.68$$

$$= 26.72 \quad \text{sq.cm} \qquad (1)$$

let h be the height from C to AB,

$$h = \frac{c\ \tan A\times\tan B}{\tan A+\tan B}$$

$$\text{Area} = \frac{1}{2}ch = \frac{1}{2}c^2\ \frac{\tan A\,.\,\tan B}{\tan A+\tan B}$$



28. Learning Outcome

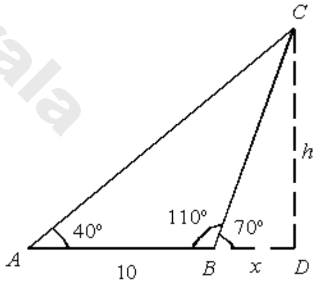
- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

In ΔABC, AB = 10cm , $\angle A = 40°$, $\angle B = 110°$. Find the area of the triangle.
$[\tan 40 = 0.84,\ \tan 70 = 2.75]$

(Score: 5, Time: 8 minute)

■ Scoring indicators

In ΔBDC, $\angle CBD = 70$



$$\tan 70 = \frac{h}{x} \qquad (1)$$

$$h = x\tan 70$$

$$\tan 40 = \frac{h}{10+x}$$

$$h = (10+x)\tan 40$$

$$x\tan 70 = (10+x)\tan 40$$

$$x(\tan 70-\tan 40) = 10\tan 40 \qquad (1)$$

$$x = \frac{10\tan 40}{\tan 70-\tan 40} \qquad (1)$$

$$h = x\tan 70$$

$$= \frac{10\tan 40\tan 70}{\tan 70-\tan 40} \qquad (1)$$

$$\text{Area} = \frac{1}{2}\times 10\times h$$

$$= \frac{1}{2}\times 10\times\frac{10\tan 40\tan 70}{\tan 70-\tan 40}$$

$$= \frac{1}{2}\times\frac{100\times 0.84\times 2.75}{2.75-0.84} = 60.47 \text{ sq.cm} \qquad (1)$$

29. Learning Outcome

- Explaining how distances and heights which cannot be directly measured can be computed using trigonometry.

When the sun is at an elevation of 40°; the shadow of a flagpost is 15 metres.

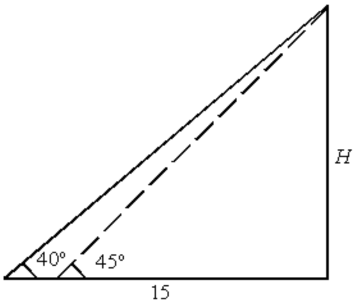
(a) Find the height of the flagpost?

(b) What would be the length of the shadow, when the sun is at an elevation of 45°

[$\tan 40° = 0.84$; $\sin 40° = 0.64$]

(Score: 4, Time: 6 minute)

■ Scoring indicators



Let H be the height of the flagpost (1)

$$\tan 40° = \frac{H}{15}$$ (1)

$$H = 15 \times \tan 40°$$

$$= 15 \times 0.84 = 12.60 \text{ metre}$$ (1)

length of shadow when the sum is at an elevation of 45° $= H = 12.60$ metre (1)

30. Learning Outcome

- Explaining how distances and heights which cannot be directly measured can be computed using trigonometry.

Two buildings in a plane ground are 20 metres apart. From the top of the smaller building, one sees the base of the building at a depression of 50° and its top at an elevation of 25°

(a) Draw a rough figure and mark the measurements

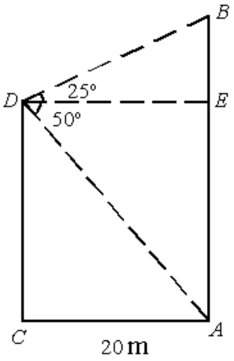
(b) Find the height of the smaller building

(c) Find the heights of the bigger building

[$\tan 50 = 1.2$; $\tan 25 = 0.4$]

(Score: 5, Time: 8 minute)

■ Scoring indicators



In ΔADE,

$$\tan 50° = \frac{AE}{DE}$$ (1)

$$AE = 20 \times \tan 50°$$

$$= 20 \times 1.2 = 24 \text{ m}$$ (1)

Height of the smaller building = 24 metre (1)

In $\Delta BED,$

$$\tan 25^\circ = \frac{BE}{20}$$

$$BE = 20^\circ \times \tan 25^\circ$$

$$= 20 \times 0.47 \quad = \quad 9.4 \text{ metre} \qquad (1)$$

$$\text{Height of the bigger building} = 24 + 9.4$$

$$= 33.4 \text{ metre} \qquad (1)$$

31. Learning Outcome

- Explaining how distances and heights which cannot be directly measured can be computed using trigonometry.

One sees the top of a tree on the bank of a river at an elevation of 70° from the other bank. Stepping 20 metres back, he sees the top of the tree at an elevation of 55°. Height of the person is 1.4 metres.

(a) Draw a rough figure and mark the measurements

(b) Find the height of the tree

(c) Find the width of the river.

[tan 70° = 2.75; tan 55° = 1.43]

(Score: 5, Time: 8 minute)

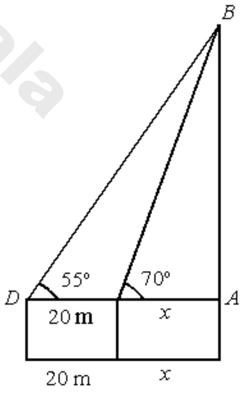
■ Scoring indicators

In $\Delta ABC,$

$$\tan 70^\circ = \frac{AB}{x}$$

$$AB = x \tan 70^\circ \qquad (1)$$

In $\Delta ABD,$



$$\tan 55^\circ = \frac{AB}{x+20}$$

$$AB = (x + 20) \tan 55^\circ$$

$$x \tan 70^\circ = (x + 20) \tan 55^\circ \qquad (1)$$

$$x \tan 70^\circ = x \tan 55^\circ + 20 \tan 55^\circ$$

$$x (\tan 70^\circ - \tan 55^\circ) = 20 \tan 55^\circ$$

$$x = \frac{20 \times \tan 55^\circ}{\tan 75^\circ - \tan 55^\circ}$$

$$= \frac{20 \times 1.43}{2.75 - 1.43} = \frac{28.6}{1.32} = 21.67 \qquad (1)$$

$$AB = x \tan 70 = 21.67 \times 2.75 = 59.59 \text{ m}$$

$$\text{Height of the tree} = 59.59 + 1.4 = 60.99 \text{ m}$$

$$\text{Width of the river} = 21.67 \text{ m} \qquad (1)$$

32. Learning Outcome

- Explaining how distances and heights which cannot be directly measured can be computed using trigonometry.

A tower is built in a river of width 80 metres. One sees the top of the tower at an elevation of 55° and 65° from either banks of the river.

(a) Draw a figure using the given measurements

(b) Find the distance from the water level of river to the top of the tower.

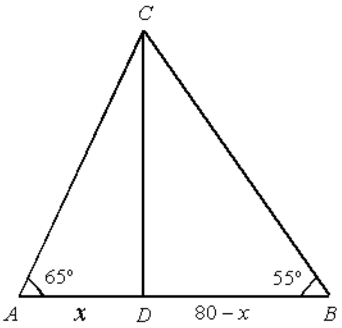
(c) Find the distances to either banks from the foot of the tower.

[$\tan 55^\circ = 1.43$; $\tan 65^\circ = 2.14$]

(Score: 5, Time: 9 minute)

Scoring indicators

From the figure,



$$\begin{aligned} CD &= x \tan 65^\circ \\ CD &= (80 - x) \tan 55^\circ \\ x \tan 65^\circ &= (80 - x)\tan 55^\circ \\ x \tan 65^\circ + x \tan 55^\circ &= 80 \tan 55^\circ \quad (1) \\ x &= \frac{80 \tan 55^\circ}{\tan 65^\circ + \tan 55^\circ} \\ &= \frac{80 \times 1.43}{2.14 + 1.43} = \frac{114.4}{3.57} = 32 \text{ metre} \quad (1) \end{aligned}$$

$$\begin{aligned} CD &= x \tan 65^\circ \\ &= 32 \times 2.14 \\ &= 68.48 \text{ metre} \quad (1) \end{aligned}$$

Height of the tower = 68.48 metre (1)

Distance from the foot of the tower to one bank of the river = 32 metre

Distance from the foot of the tower to other bank of the river = 80 - 32 = 48 metre (1)

33. Learning Outcome

- Explaining how distances and heights which cannot be directly measured can be computed using trigonometry.

A man sees the bottom and top of a building at a depression of 55° and 35° respectively from the top of a 40 metres high tower.

(a) Draw a rough figure using the given data and mark the measurements

(b) Find the distance from tower to the building

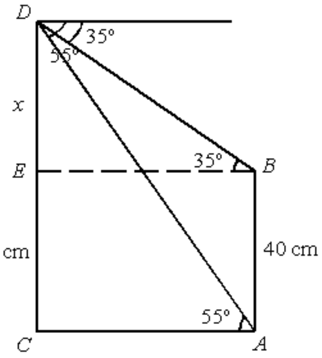
(c) Find the height of the tower.

[$\tan 55^\circ = 1.43$; $\tan 35^\circ = 0.7$] (Time : 5, Score : 8 minute)

Scoring indicators

Tower ⇒ $CD = 40 + x$

building ⇒ AB



In ΔBED,

$$\tan 35^\circ = \frac{x}{BE}$$

$$BE = \frac{x}{\tan 35^\circ} \qquad (1)$$

In ΔACD,

$$\tan 55^\circ = \frac{40+x}{\text{AC}}$$

$$AC = \frac{40+x}{\tan 55^\circ}$$

$$BE = AC \Rightarrow \frac{x}{\tan 35^\circ} = \frac{40+x}{\tan 55^\circ} \qquad (1)$$

$$x \tan 55^\circ = (40 + x) \tan 35^\circ$$

$$x (\tan 55^\circ - \tan 35^\circ) = 40 \tan 35^\circ$$

$$x = \frac{40 \times \tan 35^\circ}{\tan 55^\circ - \tan 35^\circ}$$

$$= \frac{40 \times 0.7}{1.43 - 0.7}$$

$$= \frac{28}{0.73} = 3835 \qquad (1)$$

Height of the tower $= CD = 40 + x$

$= 40 + 38.35 = 78.35$ metre (1)

Distance from the tower to the building $= BE = \dfrac{x}{\tan 35^\circ} = \dfrac{38.35}{0.7}$

$= 54.8$ metre (1)

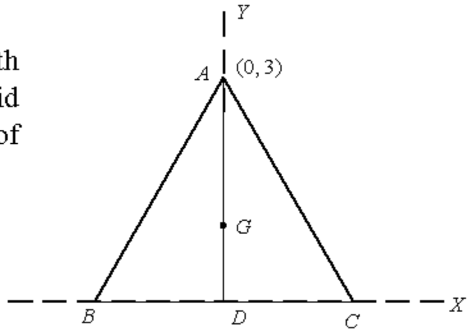
6

COORDINATES

1. Learning Outcome

- Finding the coordinates of various points of a figure by drawing axes of coordinates.

Consider an equilateral triangle ABC with A(0, 3)AD is the height. If G is the centroid and D is the origin, find the coordinates of B,C,D and G



■ Scoring Indicators

(Score: 4, Time: 9 minutes)

- D is the origin $\therefore$ coordinates is $(0, 0)$ (1)

  G divides AD in the ratio $2:1$

  $\therefore$ coordinates of G is $(0, 1)$ (1)

Triangle ADB is a right triangle with 30^0, 60^0, 90^0

$\therefore BD = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\therefore$ coordinates of B is $(-\sqrt{3}\ , 0)$ (1)

$\therefore$ coordinates of C is $(\sqrt{3}\ , 0)$ (1)

2. Learning Outcome

- Finding the distance between two points using their coordinates.

Find the coordinates of a point on the x-axis which is at a distance of 5 units from (4, -5) . Find the coordinates on the y-axis

(Score: 3, Time: 6 minutes)

■ Scoring Indicators

- The point on x-axis which is at a distance of 5 units from (4,–5) is (4,0). (1)
- The point on y-axis which is at a distance of 5 units from (4,–5) are (0, -2), (0,-8) (1)

3. Learning Outcome

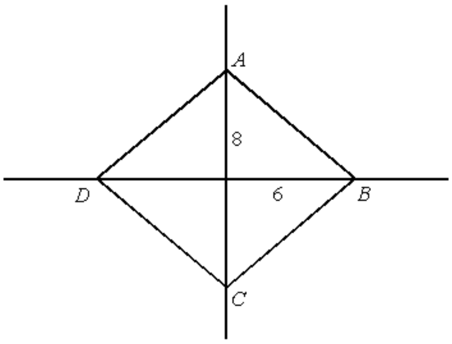
- Finding the co ordinates of various points of a figure by drawing axis of coordinates.

Consider a rhombus ABCD whose diagonals meet at origin. The length of diagonals are 8 units and 6 units. Find the coordinates of the vertices.

(Score: 4, Time: 4 minutes)

■ Scoring Indicators

- Finding the coordinates of the vertices by drawing the figure.
  (3, 0), (–3, 0), (0, 4), (0, –4)



(4)

4. Learning Outcome

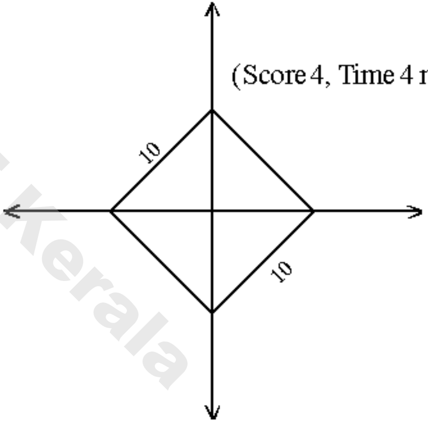
- Finding the coordinates of various points of a figure by drawing axis of coordinates.

Find the coordinates of the vertices of a square of side 10 units whose diagonals meet at origin.

(Score 4, Time 4 minutes)

■ Scoring Indicators

- $(0, 5\sqrt{2})$ $(0, -5\sqrt{2})$, $(5\sqrt{2}, 0)$, $(-5\sqrt{2}, 0)$



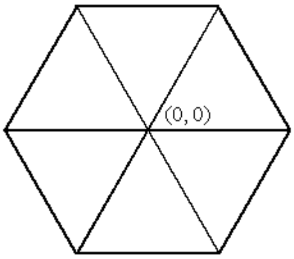
(4)

5. Learning Outcome

Finding the coordinates of various points of a figure by drawing axis of coordinates.

Consider a regular hexagon of side 6 units whose diagonals meet at the origin. Find the coordinates of the vertices.

(Score 5, Time 8 minutes)



(1)

■ Scoring Indicators

(6, 0) (0, 6)

Finding the coordinates of the vertices using the concept of right angle triangles 30°, 60°, 90°

$(3, 3\sqrt{3})$, $(-3, 3\sqrt{3})$, $(3, -3\sqrt{3})$ $(-3, -3\sqrt{3})$

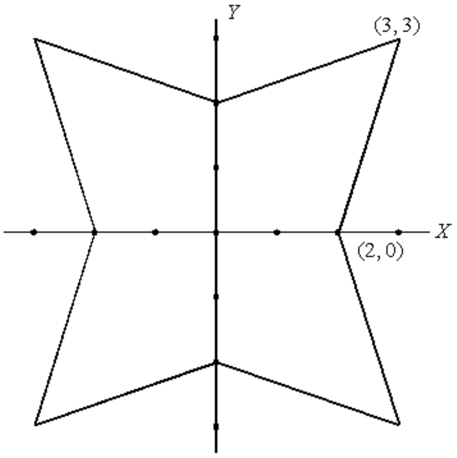
(4)

6. Learning Outcome

- Finding the coordinates of various points of a figure by drawing axis of coordinates.

Find the coordinates of other points.

(Score 5, Time 8 minutes)



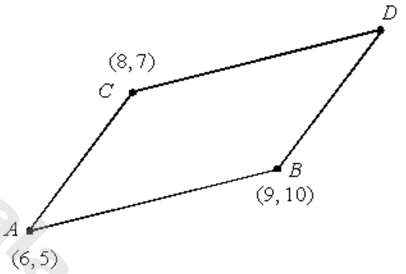
Scoring Indicators

- Finding $(-2, 0)$, $(0, 2)$, $(0, -2)$ (2)
- Finding $(3, -3)$, $(-3, -3)$, $(-3, 3)$ (2)

7. Learning Outcome

- Finding the coordinates of various points of a figure by drawing axis of coordinates.

Find the coordinates of the fourth vertices of the parallelogram. Find the length of the sides of the parellelogram. Write the length of diagonals



Scoring Indicators

The difference of x-coordinate of the points A,C = 2

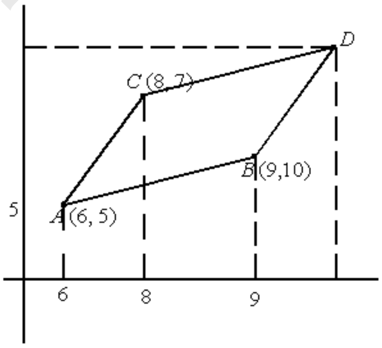
The difference of x-coordinate of the points B,D = 2

Hence the x-coordinate of the point

$x = 11$

The difference of y-coordinate of the points A,C = 7-5 = 2

The difference of y-coordinate of the points B,D = 2



Hence the y-coordinate of the point $x = 10 + 2 = 12$

Therefore the coordinate of D is (11, 12) (1)

$AB = \sqrt{3^2 + 5^2} = \sqrt{24} = CD$ (1)

$AC = \sqrt{2^2 + 2^2} = \sqrt{8} = BD$ (1)

$AD = \sqrt{5^2 + 7^2} = \sqrt{74}$ (1)

$BC = \sqrt{1^2 + 3^2} = \sqrt{10}$ (1)

8. Learning Outcome

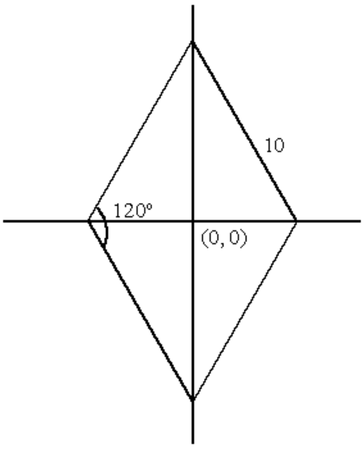
- Finding the coordinates of various points of a figure by drawing axis of coordinates.

Consider a rhombus of side 10units whose diagonals are coordinate axis. If an angle is 120°, find the coordinates of the other vertices.

(Score 4, Time 6 minutes)

Scoring Indicators

Understanding that a rhombus is formed by joining two equilateral triangle. Using the right triangle of 30°, 60°, 90° find the diagonals as $10, 10\sqrt{3}$ units. Hence the coordinates of the vertices of the rhombus are (5, 0) (-5, 0), $(0, 5\sqrt{3})$, $(0, -5\sqrt{3})$



(4)

9. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

Find the coordinates of the points on x-axis which are equidistant from the points (-5, 8) and (6,-4)

(Score 4, Time 8 minutes)

Scoring Indicators

The y-coordinate of points on x-axis will be zero. Hence assume that a point on x-axis is $(x,0)$

Distance between $(x, 0)$ and $(-5, 8) = \sqrt{(x+5)^2+8^2}$ (1)

Distance between $(x, 0)$ and $(6,-4) = \sqrt{(x-6)^2+(-4)^2}$ (1)

$$(x + 5)^2 + 8^2 = (x - 6) + (-4)^2$$

$$x^2 + 10x + 25 + 64 = x^2 - 6x + 36 + 16 \quad (1)$$

$$16x = 36 + 16 - 25 - 64 = 52 - 89 = -37$$

$$x = \frac{-37}{16}$$

$\therefore$ coordinate $\left(\frac{-37}{16}, 0\right)$ (1)

10. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

Prove that $A(4, 5)$, $B(4, 2)$, $C(8, 2)$ represents the vertices of a right angled triangle. find the coordinate of the circumcentre. What is the radius of the circumcircle.

(Score 4, Time 8 minutes)

■ **Scoring Indicators**

Distance between (4, 5) and (4, 2) $= 5 - 2 = 3$

Distance between (4, 2) and (8, 2) $= 8 - 4 = 4$

Distance between (4, 5) and (8, 2) $= \sqrt{(4-8)^2 + (5-2)^2} = \sqrt{4^2 + 3^2} = 5$ (1)

3, 4, 5 are sides of a right angled triangle, since $5^2 = 3^2 + 4^2$ (1)

The coordinates of circumcentre is (6,3.5) (1)

Radius of circumcircle = 2.5 units (1)

11. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

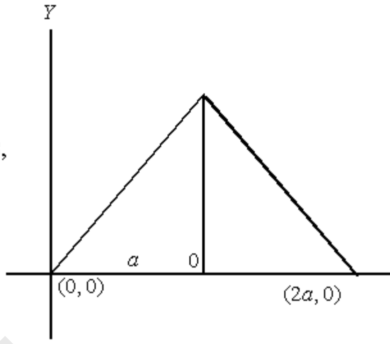
Show that the coordinates of all vertices of an equilateral triangle can never be rational numbers at the same time.

(Score 3, Time 5 minutes)

■ **Scoring Indicators**

Let one of the side of an equilateral triangle be x-axis. (1)

If one side of an equilateral triangle is '2a' units, then the coordinate of the vertices are (0, 0), $(2a, 0)$, $(0, \sqrt{3}\,a)$. Since the height is always $\sqrt{3}$ times the half of a side, the coordinates of vertices will always have a irrational number (2)



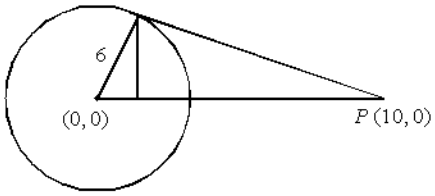
12. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

Consider a circle with centre at origin and radius 6 units. Let PA be a tangent to the circle from P(10,0). Find the length of the tangent. Find the coordinates of A

(Score 5, Time 8 minutes)

■ **Scoring Indicators**



Distance between (0, 0) and (10, 0) $= 10$

Given radius of circle $= 6$

Distance of the third side $= \sqrt{10^2 + 6^2}$

$= \sqrt{100 + 36} = \sqrt{64} = 8$ (1)

$OC \times OP = r^2$

$OC = \frac{r^2}{OP} = \frac{36}{10} = 3.6$ (2)

$AC = \sqrt{6^2 + (3.6)^2} = \sqrt{36 - 12.96} = \sqrt{23.04} = 4.8$ (1)

Coordinate of A is (3.6, 4.8)

13. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

Consider a circle centered at origin with (12,5) a point on it. Find the coordinates of other four points on the circle. What is the radius of the circle.

(Score 5, Time 8 minutes)

Scoring Indicators

Given (12, 5) a point on the circle

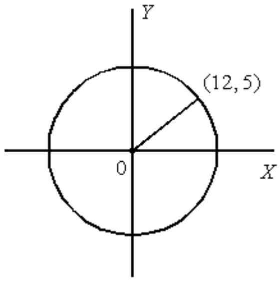
radius of the circle $= \sqrt{12^2 + 5^2} = 13$

Points of the circle

$(13, 0), (0, 13), (-13, 0), (0, -13)$

$(12, 5), (12, -5), (-12, -5), (-12, 5)$

$(5, 12), (5, -12), (-5, 12), (-5, 12)$



Writing any four of the points (4)

Finding the radius (1)

14. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

Consider a circle centered at (2,3) and (8,11) be a point on it.

a) What is the radius of the circle?

b) Find the coordinates of four points on the circle.

(Score 5, Time 8 minutes)

Scoring Indicators

Radius = Distance between (2, 3) and (8, 11)

$= \sqrt{(8-2)^2 + (11-3)^2}$ (1)

$= \sqrt{6^2 + 8^2} = 10$

Points 10 units away from (2, 3) are (12, 3), (-8, 3), (2, 13), (2, -7).

Again point obtained by shifting x coordinate by 6 units and y coordiante by 8 units are

$(2 + 6, 3 + 8); (2 + 6, 3 - 8); (2 - 6, 3 + 8); (2 - 6, 3 - 8)$

$\Rightarrow (8, 11)\ (8, -5), (-4, 11), (-4, -5)$

Again; $(2 + 8, 3 + 6)\ ; (2 + 8, 3 - 6); (2 - 8, 3 + 6)\ ; (2 - 8, 3 - 6)$

$\Rightarrow (10, 9)\ (10, -3), (-6, 9), (-6, -3)$

Writing any four points

(4)

15. Learning Outcome

- Formation of different geometric figures using number pairs

Draw *x, y* axis and mark the points *A* (2, 3), *B* (-2, 3), *C* (-2, -3), *D* (2, -3). Draw the polygon connecting these points taken in order and name the polygon.

(Score 4 Time 8 minutes)

■ **Scoring Indicators**

Drawing the coordinate axis	(1)
Marking the coordinates	(2)
Connecting the points and identifying the figure	(1)

16. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

Show that (4, 2), (7, 5), (9, 7) lie in a line

(Score 4 Time 8 minutes)

■ **Scoring Indicators**

Finding distance between (4, 2) and (7, 5)	(1)
Finding distance between (7, 5) and (9, 7)	(1)
Finding distance between (4, 2), (9, 7)	(1)
Showing the sum of smaller distance is equal to long distance	(1)

17. Learning Outcome

- Using the coordinates of vertices of geometric figures to determine various lengths

P is a point on the perpendicular bisector of the line joining (2 , 5) and (6,5). If the x coordinate and y coordinate of P are equal write the coordinate of P

(Score 5 Time 8 minutes)

■ **Scoring Indicators**

Understanding the concept that the distance of a point on the perpendicular bisector is equidistant from the end points.

Assume that the coordinate of P is (a,a).

Then ; Distance between (a,a) and (2,3) = $\sqrt{(a-2)^2+(a-3)^2}$ (1)

Distance between (a,a) and (6,5) = $\sqrt{(a-6)^2+(a-5)^2}$ (1)

They are equal

$$\sqrt{(a-2)^2+(a-3)^2} = \sqrt{(a-6)^2+(a-5)^2}$$

$(a-2)^2 + (a-3)^2 = (a-6)^2 + (a-5)^2$ (1)

$a^2 - 4a + 4a^2 - 6a + 9 = a^2 - 12a + 36 + a^2 - 10a + 25 - 4a - 6a + 12a + 10a$

$= 36 + 25 - 9 - 4$

$12a = 48$

$$a = \frac{48}{12} = 4$$

Coordinates of P (4, 4) (1)

18. Learning Outcome

- Using the coordinates of vertices of geometric figures to determine various lengths

Write the coordinate of the centre of a circle passing through the points (9, 3) , (7, -1) (1, -1). Find the radius of the circle.

(Score 5 Time 8 minutes)

Scoring Indicators

Assume that the (x, y) be centre of the circle

Distance between(9, 3) and (x, y) $= \sqrt{(x-9)^2+(y-3)^2}$

Distance between $(7, -1)$ and (x, y) $= \sqrt{(x-7)^2+(y+1)^2}$ (1)

They are equal

$\therefore \sqrt{(x-7)^2+(y+1)^2} = \sqrt{(x-1)^2+(y+1)^2}$ (1)

$(x-7)^2 = (x-1)^2$

$x^2 - 14x + 49 = x^2 - 2x + 1$

$12x + 48$

$\therefore x = \frac{48}{12} = 4$ (1)

$\sqrt{(x-9)^2+(y-3)^2} = \sqrt{(x-1)^2+(y+1)^2}$

$(x-9)^2 + (y-3)^2 = (x-1)^2 + (y-1)^2$

when $x = 4$

$(4-9)^2 + y^2 - 6y + 9 = (4-7)^2 + y^2 - 2y + 1$ (1)

$25 + y^2 - 6y + 9 = 9 + y^2 - 2y + 1$

$8y^2 = 24$

$y = \frac{24}{8} = 3$

The coordinate of the centre of the circle is (4,3)

$\therefore$ Radius = Distance between (4, 3) and (1, –1)

$= \sqrt{(4-1)^2+(3+1)^2} = \sqrt{3^2+4^2} = 5$ (1)

19. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

If (x, y) be a point equi distant from the points (7, 5), (4, 3) , then show that $6x + 4y = 49$

(Score 4 Time 7 minutes)

Scoring Indicators

Distance between (x, y) and (7, 5) $= \sqrt{(x-7)^2+(y-5)^2}$ (1)

Distance between (x, y) and (4, 3) $= \sqrt{(x-4)^2+(y-3)^2}$ (1)

They are equal

$$(x-7)^2 + (y-5)^2 = (x-4)^2 + (y-3)^2$$

Solving and concluding

$$6x + 4y = 49 \qquad (2)$$

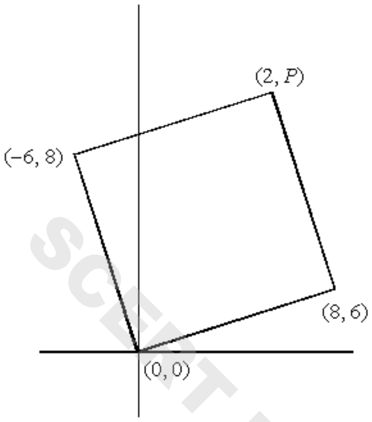
20. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

Three vertices of a square are given. If the fourth vertex is (2, P) find the ratio of P. Find the area of the square.

(Score 4 , Time 7 minutes)

Scoring Indicators



Distance between (−6, 8) and (2, P) = $\sqrt{(-6-1)^2+(8-P)^2}$ (1)

Distance between (8, 6) and (2, P) = $\sqrt{(8-2)^2+(6-P)^2}$ (1)

They are equal

$$8^2 + (8-P)^2 = 6^2 + (6-P)^2$$

$$64 + 64 - 16P + P^2 = 36 + 36 - 12P + P^2 \qquad (1)$$

$$4P = 56$$

$$P = \frac{56}{4} = 14$$

Distance between (0, 0) and (8, 6) = $\sqrt{8^2+6^2}$ = 10 (1)

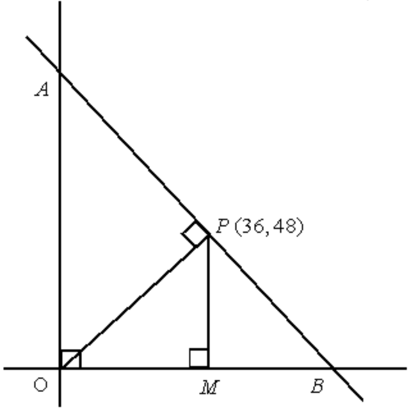
Area = $10^2 = 100$

21. Learning Outcome

- Using the coordinates of vertices of geometric figures to determine various lengths

In the figure if P is (36, 48), then find the coordinates of A,B, M

(Score 4 Time 7 minutes)



Scoring Indicators

Δ OMP is a right triangle

$\therefore$ OP = $\sqrt{48^2+36^2}$ = 60

ΔPAO, ΔOMP are similar (1)

$= \frac{OA}{OP} = \frac{OP}{PM} \therefore \quad OA = \frac{OP^2}{PM} = \frac{60^2}{48} = 75$

Δ OMP, Δ OPB are similar (1)

$= \frac{OB}{OP} = \frac{OP}{OM} \therefore OB = \frac{OP^2}{OM} = \frac{60^2}{36} = 100$ (1)

Coordinates of M (36, 0) (1)

Coordinates of A (0, 75) (1)

Coordinates of B (100, 0) (1)

7

TANGENTS

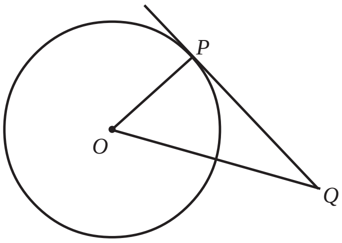
1. Learning outcome

- Realising that the tangent at a point on a circle is perpendicular to the diameter through that point.

PQ is a tangent to the circle with centre *O*.

(a) Find $\angle P$?

(b) If $\angle O = 42^\circ$, what is $\angle Q$?



(Score: 2, Time: 3 minutes)

■ Scoring indicators

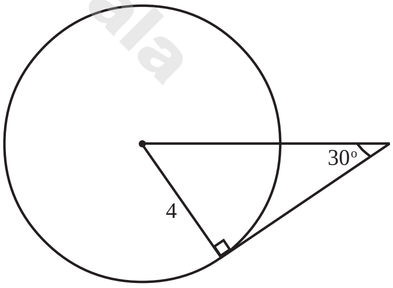
$\angle P = 90^\circ$ (1)

$\angle Q = 90 - 42 = 48^\circ$ (1)

2. Learning outcome

- Realising that the tangent at a point on a circle is perpendicular to the diameter through that point.

Draw this figure using the given measurements.



(Score: 3, Time: 4 minutes)

■ Scoring indicators

For drawing a circle of radius 4 centimetres. (1)

For drawing a radius and a perpendicular to it. (1)

For completing the triangle after marking angle 60° at the centre. (1)

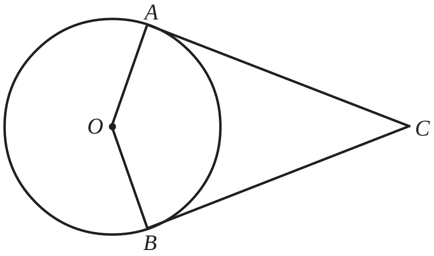
3. Learning outcome

- Understanding the centre of a circle, any two points on the circle and the point of intersection of the tangents through those points constitute a cyclic quadrilateral.

In the figure, AC and BC are tangents to the circle from C. Centre of the circle O

(i) Find $\angle A$?

(ii) If $\angle C$ is is 2 times $\angle O$, then what is $\angle C$?



(Score: 3, Time: 4 minutes)

- **Scoring indicators**

$\angle A = 90^{o}$	(1)
Writing $\angle C + \angle O = 180^{o}$	(1)
Finding $\angle C = 60^{o}$	(1)

4. Learning Outcome

- Realising that the angle between the radii through any two points of a circle and the angle between the tangents at these points are supplementary.

The radius of a circle touching all sides of an equilateral triangle is 3 centimetres. Draw this triangle.

(Score: 3, Time: 4 minutes)

- **Scoring indicators**

Drawing a circle of radius 3 cm.	(1)
For marking 120^{o} at the centre of circle.	(1)
For completing the equilateral triangle	(1)

5. Learning Outcome

- Realising that the angle between the radii through any two points of a circle and the angle between the tangents at these points are supplementary.

Radius of an incircle to a triangle is 3 centimetres. Two angles of this triangle are 55^{o} and 63^{o}. Draw this triangle.

(Score: 5, Time: 8 minutes)

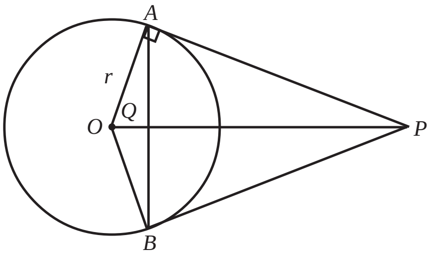
- **Scoring indicators**

For drawing a circle of radius 3 cm.	(1)
For marking the angles $180 - 55 = 125^{o}$, $180 - 63 = 117^{o}$, at the centre.	(1)
For drawing the tangents.	(1 + 1)
For completing the triangle.	(1)

6. Learning Outcome

- Realising that the angle between the radii through any two points of a circle and the angle between the tangents at these points are supplementary.

In the figure PA, PB are tangents through A and B of a circle with centre O. If the radius of the circle is r, then prove that $OP \times OQ = r^2$.



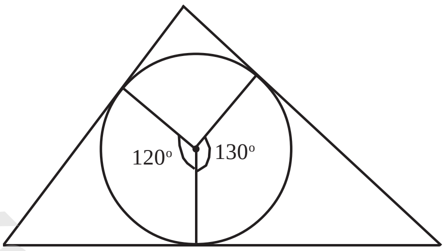
(Score: 3, Time: 5)

- **Scoring indicators**
  - For identifying ΔOQA, ΔOPA as triangles with equal angles. (1)
  - For writing the ratio of sides opposite to the equal angles. (1)
  - For finding $OP \times OQ = r^2$. (1)

7. Learning Outcome

- Realising that the angle between the radii through any two points of a circle and the angle between the tangents at these points are supplementary.

In the figure, angles formed by the radius segement of the meeting points of the tangent to incircle are given.Find all angles of the triangle.



(Score: 3, Time: 5 minutes)

- **Scoring indicators**
  - Angles $180 - 120 = 60^{o}$ (1)

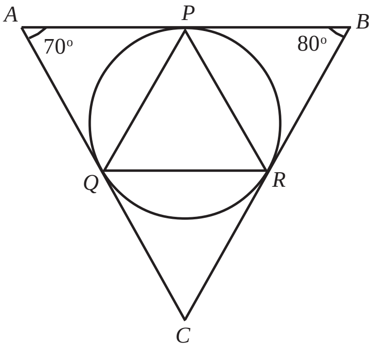
    $180 - 130 = 50^{o}$ (1)

    Third angle $180 - (60 + 50) = 70^{o}$ (1)

8. Learning Outcome

- Realising that the angle between the radii through any two points of a circle and the angle between the tangents at these points are supplementary..

The incircle triangle ABC touches the triangle sides at P,Q & R as shown in the figure.Find all angles of ΔPQR, ΔABC.



(Score: 4, Time: 6 minutes)

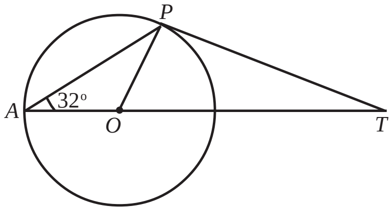
Scoring indicators

- For finding the angles at the centre of the circles are $180 - 70 = 110^o$, $180 - 80 = 100$ and $360+(110 + 100) = 150^o$ (2)
- For finding the angles of triangle *PQR* as $\frac{110}{2} = 55^o$, $\frac{100}{2} = 50^o$, $\frac{150}{2} = 75^o$ (2)

9. Learning Outcome

- Understanding that in a circle, the angle between a chord and tangent at either end is half the central angle of the chord.

Find all angles of triangles *AOP* and *OPT*.



(Score: 4, Time: 7 minutes)

Scoring indicators

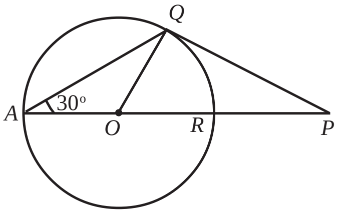
For finding the angles of ΔAOP as 32°, 32°, 116° (2)

For finding the angles of ΔOPT as 64°, 26°, 90° (2)

10. Learning Outcome

- Understanding that in a circle, the angle between a chord and tangent at either end is half the central angle of the chord.

QP is a tangent of the circle with centre *O*. *AR* is a diameter. Find all angles of triangle *PQR*.



(Score: 4, Time: 6 minutes)

Scoring indicators

$\angle PQR = \angle QAR = 30^o$ (1)

$\angle PRQ = 180 - 60 = 120^o$ (1)

$\angle P = 180 - (120 + 30) = 30^o$ (2)

11. Learning Outcome

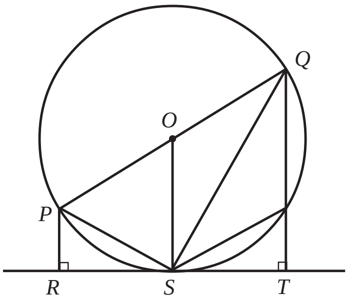
- Realising that the tangent at a point on a circle is perpendicular to the diameter through that point.

In the figure, *PQ* is a diameter and *O* is the centre of the circle

$\angle R = \angle T = 90^o$

(1) Prove that $\angle PSR = \angle OSQ$

(2) Prove that ΔPSR and ΔSQT are similar



(Score: 5, Time: 8 minutes)

- **Scoring indicators**

$\angle PSR = \angle PQS$ (1)

PQ is a diameter

$\therefore \angle PSQ = 90^o$ (1)

$\therefore \angle PSR + \angle QST = 90^o$ (1)

$\angle PSR = 90 - \angle QST = \angle OSQ$

$\angle PSR = \angle SQT$ (1)

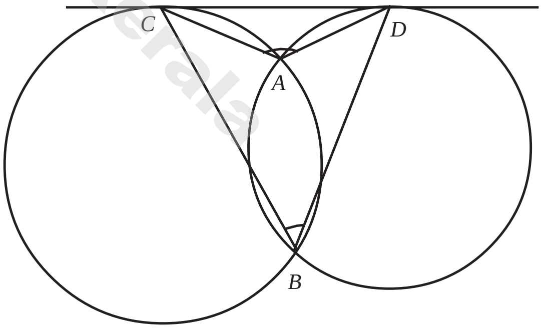
$\therefore$ Triangles are similar (1)

12. Learning Outcome

- Realising that in a circle, the angle between a chord and tangent at either end is half the central angle of the chord.

(Score: 5, Time: 8 minutes)

- **Scoring indicators**



$\angle CDA = \angle ABD$ (1)

$\angle DCA = \angle ABC$ (1)

$\angle CAD + \angle ACD + \angle ADC = 180$ (1)

$\angle CAD + \angle ABD + \angle ABC = 180$ (1)

$\angle CAD + \angle CBD = 180$ (1)

13. Learning Outcome

- Realising that the tangent at a point on a circle is perpendicular to the diameter through that point.

Draw a circle of radius 3 cm. Draw a chord *AB* = 4 cm of this circle. Draw tangents through *A* and *B*.

(Score: 3, Time: 5 minutes)

- **Scoring indicators**

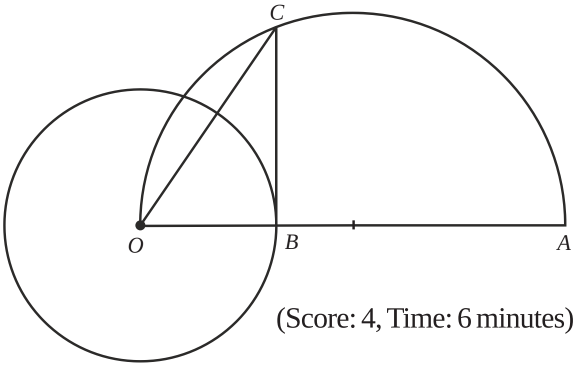
For drawing a circle. (1)
For drawing the chord. (1)
For drawing tangents. (1)

14. Learning Outcome

- Understanding that any tangent of a circle is perpendicular to the radius at the point of tangency.

In the figure, O is the centre, C is a point on the semicircle with diameter OA. BC is a tangent through B.

If $OB = 1$ cm, $AB = 3$ cm, then what is BC ? Find all angles of triangle OBC ?



(Score: 4, Time: 6 minutes)

- **Scoring indicators**

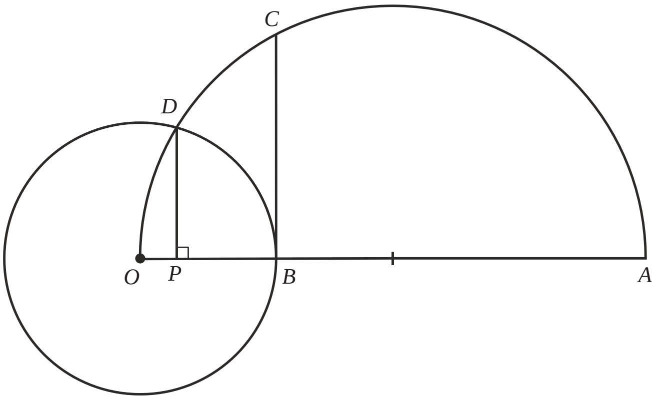
$$OB \times AB = BC^2 \quad (1)$$
$$1 \times 3 = BC^2 = 3 \quad (1)$$
$$BC = \sqrt{3}$$

For writing the angles of ΔOBC are 30^o, 60^o, 90^o. (2)

15. Learning Outcome

- Understanding that any tangent of a circle is perpendicular to the radius at the point of tangency.

In the figure, radius of the circle centred at O is 9 cm. $OA = 15$ cm, Semicircle with diameter OA cuts the circle with centre O at D and BC is a tangent through B.



(1) What is the length of BC?

(2) If the line PD is perpendicular to OA, then what is the length of PD?

(Score: 4, Time: 7 minutes)

- **Scoring indicators**

$$BC^2 = OB \times BA$$
$$= 9 \times 6 = 54$$
$$BC = \sqrt{54} \text{ cm} \qquad (1)$$
$$OP \times OA = r^2$$
$$OP = \frac{9^2}{15} = \frac{81}{15} \qquad (1)$$
$$PD^2 = OP \times PA$$
$$PD^2 = \frac{81}{15} \times \frac{144}{15} \qquad (1)$$
$$PD = \frac{9 \times 12}{15} = \frac{36}{5} = 7.2 \text{ cm} \qquad (1)$$

16. Learning Outcome

- Understanding how to draw tangents through a point on a circle without using the centre.

Draw a circle of radius 4 cm, mark a point *P* on the circle. Draw a tangent through *P* without using centre.

(Score: 3, Time: 3 minutes)

- **Scoring indicators**

For drawing a circle. (1)

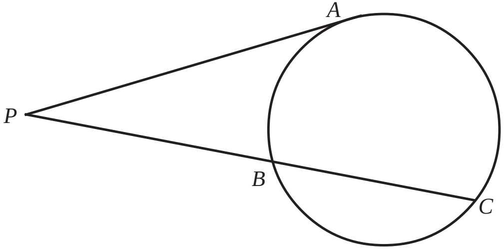
For drawing an arc with centre *P* and cuts the circle at two points, and draw a chord joining these points. (1)

For drawing the tangent. (1)

17. Learning Outcomes

- Realising that the product of an intersecting line and the part of it outside the circle is equal to the square of the tangent.

In the figure, PA is a tangent of the circle. If $PC = 12$ cm, and $PB = 3$ cm, then find the length of PA.



(Score: 3, Time: 5 minutes)

- **Scoring indicators**

$$PB \times PC = PA^2 \qquad (1)$$
$$3 \times 12 = 36 \qquad (1)$$
$$PA = \sqrt{36} = 6 \text{ cm} \qquad (1)$$

18. Learning Outcomes

- Realising that the product of an intersecting line and the part of it outside the circle is equal to the square of the tangent.

Draw a rectangle of one side 7 centimetres and area equal to the area of a square of side 5 centimetres.

(Score: 3, Time: 5 minutes)

- **Scoring indicators**

For drawing square.	(1)
For drawing a circle with one side of the square as a tangent.	(1)
For drawing a chord and extending it upto a total length 7 cm.	(1)
For completing the rectangle.	(1)

19. Learning Outcome

- Realising that the product of an intersecting line and the part of its outside the circle is equal to the square of the tangent.

Length and breadth of a rectangle are 8 cm and 3 cm. Construct a square having the area same as that of the rectangle.

(Score: 4, Time: 7 minutes)

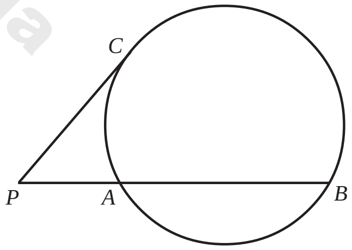
- **Scoring indicators**

For drawing the rectangle.	(1)
For drawing the circle.	(1)
For drawing the tangent.	(1)
For completing the square.	(1)

20. Learning Outcome

- Realising that the product of an intersecting line and the part of it outside the circle is equal to the square of the tangent.

In the figure, $PC = 4$ cm, $AB = 6$ cm Find the length of PA.



(Score: 4, Time: 8 minutes)

- **Scoring indicators**

$PA \times PB = PC^2$	(1)
$PA\,(PA + 6) = 4^2$	
$PA^{2} + 6\,PA = 16$	
$PA^2 + 6\,PA + 9 = 25$	(1)
$(PA + 3)^{2} = 25$	
$PA + 3 = 5$	(1)
$PA = 2$ cm	(1)

21. Learning Outcome

- Realising that a circle can be drawn by touching all the 3 sides of a triangle.

Draw a triangle of sides 6 cm and 8cm angle between them is 70^o and draw its incircle and measure its inradius.

(Score: 5, Time: 9 minutes)

Scoring indicators

For drawing the triangle using the given measures.	(1)
For identifying the centre by drawing angle bisectors of two sides.	(1)
Finding the radius by drawing perpendicular to one side.	(1)
For drawing the circle.	(1)
For measuring the inradius.	(1)

22. Learning Outcome

- Realising that a circle can be drawn by touching all the 3 sides of a triangle.

Consider a triangle of side 6 cm and two angles 80^o and 70^o . Draw an incircle to this triangle. Measure the inradius.

(Score: 5, Time: 10 minutes)

Scoring indicators

Drawing the triangle correctly.	(1)
Drawing the angular bisector of any two angles	(1)
Finding the radius by drawing perpendicular to one side	(1)
Drawing the circle correctly.	(1)
Writing the inradius.	(1)

23. Learning Outcome

- Realising that a circle can be drawn by touching all the 3 sides of a triangle.

Draw a line *AB* of length 8 cm. Draw lines through the end points by marking the angles 110^o and 100^o. Draw a circle touching the three lines and find its radius.

(Score: 5, Time: 10 minutes)

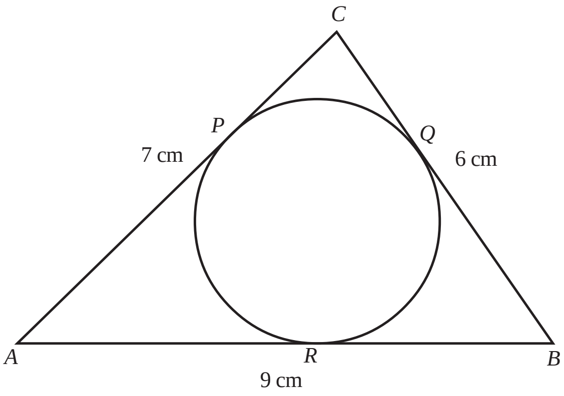
Scoring indicators

For drawing the lines by marking angles.	(1)
For identifying the centre by drawing angle bisectors.	(1)
For finding the radius by drawing perpendicular to one side.	(1)
For drawing the circle.	(1)
For measuring the radius.	(1)

24. Learning Outcome

- Realising that a circle can be drawn by touching all the 3 sides of a triangle.

Sides of the triangle are 6 cm, 7 cm and 9 cm. Find the length of AP, BR and CQ.



(Score: 4, Time: 5 minutes)

- **Scoring indicators**

Perimeter of the triangle: 9 + 7 + 6 = 22

Semi perimeter $= \frac{22}{2} = 11$ (1)

$AP = s - a = 11 - 6 = 5$ (1)

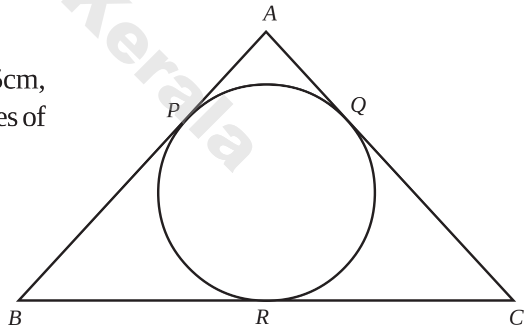
$BR = s - b = 11 - 7 = 4$ (1)

$CQ = s - c = 11 - 9 = 2$ (1)

25. Learning Outcome

- Realising that a circle can be drawn by touching all the 3 sides of a triangle.

In the figure, AP = 4cm, CQ = 2.5cm, BR = 7 cm. Find the length of all sides of the triangle.



(Score: 5, Time: 4 minutes)

- **Scoring indicators**

$AQ = AP = 4$ cm (1)

$CQ = CR = 5$ cm

$BP = BR = 7$ cm (1)

$AB = BP + AP = 7 + 4 = 11$ cm

$AC = AQ + QC = 4 + 5 = 9$ cm (1)

$BC = BR + RC = 7 + 5 = 12$ cm (1)

26. Learning Outcome

- Realising that a circle can be drawn by touching all the 3 sides of a triangle.

What is in radius of the triangle with perimeter 40cm and area 60 sq. centimetres.

(Score: 2, Time: 3 minutes)

■ **Scoring indicators**

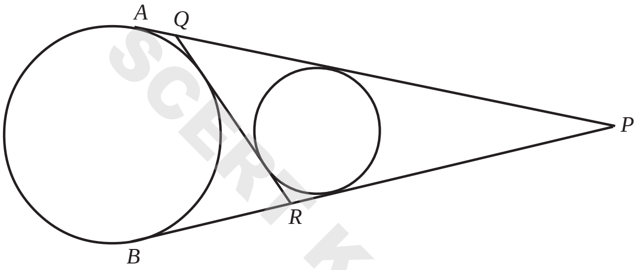
- $r = \frac{A}{S}$ (1)

$r = \frac{60}{20} = 3$ cm (1)

27. Learning Outcome

- Realising that a circle can be drawn by touching all the 3 sides of a triangle.

In triangle *PQR*, *PQ* = 8 cm, *QR* = 9 cm, *PR* = 7 cm.



(1) Find the length of *PA*.

(2) Find the length of *BR*.

(Score: 3, Time: 5 minutes)

■ **Scoring indicators**

PA = *PB* = Semi perimeter of triangle *PRQ*. (1)

$S = \frac{8+9+7}{2} = \frac{24}{2} = 12$ (1)

$BR = S - RP = 12 - 7 = 5$ cm (1)

28. Learning Outcome

- Realising that a circle can be drawn by touching all the 3 sides of a triangle.

Three sides of triangle *ABC* are 21, 20, 13 centimetres.

a) Find the area of the triangle

b) Find the in radius of the triangle

(Score: 4, Time: 7 minutes)

Scoring indicators

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \quad (1)$$

$$= \sqrt{27(27-21)(27-20)(27-13)}$$

$$= \sqrt{27\times6\times7\times14} \quad (1)$$

$$= \sqrt{9\times3\times3\times2\times7\times7\times2}$$

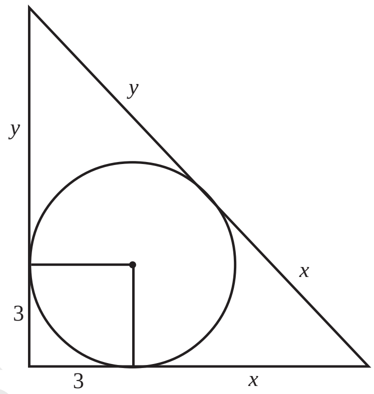
$$= 3\times3\times2\times7 = 126 \text{ sq.cm} \quad (1)$$

$$r = \frac{A}{S} = \frac{126}{27} = \frac{14}{3} = 4\frac{2}{3} \text{ cm} \quad (1)$$

29. Learning Outcome

- Realising that a circle can be drawn by touching all the 3 sides of a triangle.

Hypotenuse of a right triangle is 18 cm and its inradius 3 cm. what is its perimeter? What is its area?



(Score: 3, Time: 5 minutes)

Scoring indicators

$$\text{Perimeter} = 3 + x + x + y + y + 3$$

$$= 3 + 3 + 18 + 18 = 42 \text{ cm} \quad (1)$$

$$\text{Area} = \frac{42}{2} \times 3 = 21 \times 3 = 63 \text{ sq.cm} \quad (1)$$

30. Learning Outcome

- Realising that a circle can be drawn by touching all the 3 sides of a triangle.

Area of a right triangle is 60 sq. centimetres and its inradius 3 cm. what is the length of its hypotenuse?

(Score: 3, Time: 5 minutes)

Scoring indicators

Area = 60 sq.cm, radius = 3

$$\therefore \text{Perimeter} = 2 \times \frac{60}{3} = 40 \text{ cm} \quad (2)$$

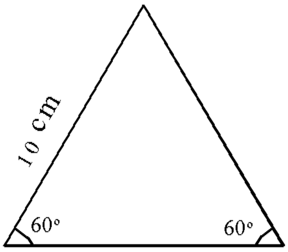
$$\text{Hypotenuse} = \frac{40-6}{2} = 17 \text{ cm} \quad (2)$$

8

SOLIDS

1. Learning Outcome

- Finding the volume of a square pyramid. Understanding the relationship between base edge, height and slant height.



The measurements of the lateral surface of a square pyramid are shown in the figure. Calculate the base edge and slant height of the pyramid.

(Score: 2, Time: 3 minutes)

Scoring indicators

Base edge = 10 cm (1)

Slant height = 5 $\sqrt{3}$ cm (1)

2. Learning Outcome

- Constructing a square pyramid using square sheets and isosceles triangle sheets.

Is it possible to construct a pyramid of base edge 24 cm and lateral edge 13 cm? Justify

(Score : 2 , Time: 3 minutes)

Scoring indicators

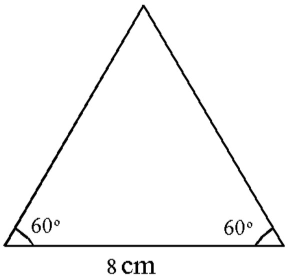
Since slant height is 5 cm, such a pyramid can't be constructed. (1)

Slant height should be greater than half of the base edge. (1)

3. Learning Outcome

- To find out the volume of a square pyramid.

Lateral surface of a square pyramid is shown in the figure. All angles are equal. Find the total length of all edges of the square pyramid. Find the slant height. What is the ratio between height and slant height



(Score: 4, Time: 5 minutes)

■ **Scoring indicators**

Sum of edges $= 8 \times 8 = 64$ cm (1)

Slant height $= 4\sqrt{3}$ cm (1)

$$\text{Height} = \sqrt{(4\sqrt{3})^2 - 4^2}$$

$$= \sqrt{48 - 16}$$

$$= \sqrt{32} = 4\sqrt{2} \text{ cm} \quad (1)$$

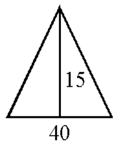
4. Learning Outcome

- Making square pyramids of specified dimensions by cutting out squares and isosceles triangles.

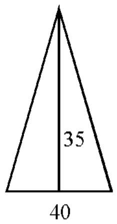
Devika made a square pyramid having base edge 40cm and height 15cm. Unfortunately, one lateral face got separated from the pyramid. Check which figure given below shows the isosceles triangle that got separated.

(Score: 3, Time: 5 minutes)

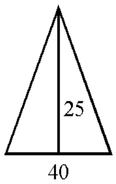
■ **Scoring indicators**



- Square pyramid can't be constructed since slant height should be greater than half the base edge. (1)



- If base edge = 40 and slant height = 35, then height can't be 15 (1)



- Here height of pyramid is 15, so this is the isosceles triangle that got separated (1)

5. Learning Outcome

- Finding the surface area and volume of a square pyramid.

A tent constructed in the form of a square pyramid of base perimeter 80 metres and lateral edge 26 metres.

a) Calculate the slant height of the tent

b) Calculate the area of tarpaulin sheet required to cover the lateral faces of the tent.

(Score: 3, Time: 5 minutes)

Scoring indicators

Base perimeter = 80 m

Base edge = 20 m

Lateral edge = 26 m

a) ∴ Slant height = $\sqrt{26^2 - 10^2}$

$= \sqrt{576} = 24$ m (1)

b) Lateral surface area = $4 \times \frac{1}{2} \times 20 \times 24$

= 960 sq.metre (1)

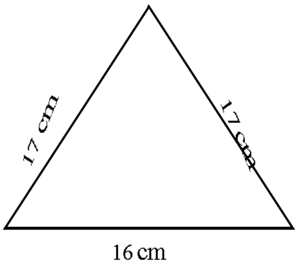
6. Learning Outcome

- Finding the surface area and volume of a square pyramid.

The triangle given in the figure is one lateral face of a square pyramid.

a) Calculate the slant height

b) Find the lateral surface area of the pyramid



(Score: 3, Time: 4 minutes)

Scoring indicators

Slant height = $\sqrt{17^2 - 8^2}$

$= \sqrt{275} = 15$ cm (1)

Lateral surface area $=4 \times \frac{1}{2} \times 15 \times 16$ (1)

= 480 sq.cm (1)

7. Learning Outcome

- Find the surface area and volume of a square pyramid.

A square pyramid is made from a solid cube having edge 30cm. Calculate the surface area of the pyramid.

(Score: 3, Time: 5 minutes)

Scoring indicators

Base edge of the square pyramid = 30 cm

Height = 30 cm

Slant height $= \sqrt{30^2 + 15^2}$

$= \sqrt{900 + 225}$

$= \sqrt{1125} = 15\sqrt{5}$ (1)

Lateral surface area $= 4 \times \frac{1}{2} \times 15\sqrt{2} \times 30$

$= 60 \times 15\sqrt{5}$

$= 900\sqrt{5}$ cm (1)

Total surface area $= 900 + 900\sqrt{5}$

$= 900(1+\sqrt{5})$ cm (1)

8. Learning Outcome

- Finding the surface area and volume of a square pyramid.

The lateral faces of a square pyramid are equilateral triangels. Lateral edge = 20 cm

a) Calculate the slant height
b) Find its surface area
c) Find its volume

(Score: 5, Time: 6 minutes)

■ **Scoring indicators**

a) Slant height $= 10\sqrt{3}$ cm (1)

b) Total surface area $= 20^2 + 4 \times \frac{1}{2} \times 20 \times 10\sqrt{3}$ (1)

$= 400 + 400\sqrt{3}$

$= 400(1 + \sqrt{3})$ cm (1)

c) Height $= (10\sqrt{3})^2 - 10^2$ $= 10\sqrt{2}$ cm (1)

Volume $= \frac{1}{3} 400 \times 10\sqrt{2}$ $= 4000\frac{\sqrt{2}}{3}$ cubic centimetres (1)

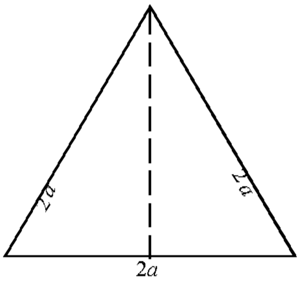
9. Learning Outcome

- Finding the relationship between base edge, slant height, height of a square pyramid of equal edges.

Prove that the ratio between the base edge ,slant height and height of a square pyramid having equal edges is $2 : \sqrt{3} : \sqrt{2}$

(Score: 4, Time: 5 minutes)

■ **Scoring indicators**



Slant height $= \sqrt{3}\ a$ (1)

Height $= \sqrt{\left(\sqrt{3}a\right)^2 - a^2}$ (1)

$= \sqrt{2}\ a$ (1)

a : 1 : h $= 20 : \sqrt{3}a : \sqrt{2}\ a$

$= 2 : \sqrt{3} : \sqrt{2}$ (1)

10. Learning Outcome

- Comparing the measurements of two square pyramids

The ratio between the base edges of two square pyramids is 1:2. The heights are also in the same ratio. If the volume of the first pyramid is 10 cubic centimeters, calculate the volume of the second one.

(Score: 3, Time: 4 minutes)

■ **Scoring indicators**

$v_1 : v_2 = 1 : 8$ (1)

$v_1 : v_2 = \frac{1}{3}\ (2a)^2 \times 2h = 1 : 8$ (1)

$\therefore$ Volume of the second pyramid = 800 cubic centimeters (1)

11. Learning Outcome

- Comparing the volume of two square pyramids

Meera constructed a square pyramid of base edge 10cm and height 6cm. Manu made a square pyramid having base edge 5cm and height 24cm. Find the volume of the pyramids and compare the measurements.

(Score: 3, Time: 4 minutes)

■ **Scoring indicators**

Volume of Meera's pyramid $= \frac{1}{3} \times 10^2 \times 6$

$=$ 200 cubic centimeters (1)

Volume of Manu's pyramid $= \frac{1}{3} \times 5^2 \times 24$

$=$ 200 cubic centimeters (1)

Volumes are equal (1)

12. Learning Outcome

- Computing the dimensions of sectors needed to make cones of specified dimensions.

The central angle of a sector is 288° . If this sector is rolled up to make a cone, find the ratio between the radius and slant height of the cone.

(Score : 4 , Time : 5 minutes)

■ Scoring indicators

$360 \times \frac{4}{5} = 288$ (1)

$\therefore$ Radius of the cone $= \frac{4}{5} \times$ radius of the big circle

$\therefore$ If r is the radius of the circle

Radius of the cone $= \frac{4}{5} r$ (1)

But radius of the circle = Slant height of cone

ie., $l = r$ (1)

$\therefore$ Ratio between the radius of the cone and slant height

$= \frac{4}{5} r : r$

$= \frac{4}{5} : 1$

$= 4 : 5$ (1)

13. Learning Outcome

- Computing the dimensions of sectors needed to make cones of specified dimensions.

The ratio between the radius and slant height of a cone is 2 : 3. Find the central angle of the sector to make the cone.

(Score: 3, Time: 4 minutes)

■ Scoring indicators

Ratio between the radius and slant height 2 : 3 (1)

$\therefore$ Arc length of the sector is equal to $\frac{2}{3}$ part of the circle perimeter (1)

$\therefore$ Central angle of the sector $= 360 \times \frac{2}{3} = 240°$ (1)

14. Learning Outcome

- To find out the curved suface area of a cone

The central angle of a circle is divided in the ratio 2 : 3 to form two sectors. Two cones are made by rolling up the two sectors.

a) Find out the ratio between the base perimeters of the cones.

b) What is the ratio between the curved surface areas

(Score: 3, Time: 6 minutes)

■ **Scoring indicators**

a. Ratio between central angles 2:3 (1)

b. $\therefore$ Base perimeter of the cones will be $\frac{2}{5}$ and $\frac{3}{5}$ parts of the circumference of the circle. (1)

Ratio between the perimeters of the cones

$= \pi r^2 \times \frac{2}{5} : \pi r^2 \times \frac{3}{5} = 2 : 3$ (1)

15. Learning Outcome

- Finding the curved surface area of a cone

Find the ratio between the radius and slant height of a cone by rolling up a sector with central angle 120°. If the curved surface area is 108π, find the radius and slant height of the cone.

(Score: 5, Time: 7 minutes)

■ **Scoring indicators**

Area of the sector with central angle 120° is one third of the area of the circle. (1)

$\therefore$ Ratio between radius and slant height = 1 : 3

Curved surface area = 108π

$\therefore$ Area of the sector = 108π, which (1)

is one third of the area of the circle

$\therefore$ Area of the circle = $108\pi \times 3$

$\pi r^2 = 324\pi$ (1)

$\therefore$ Radius of the circle r = 18 cm

$\therefore$ Slant height = 18 cm (1)

$\therefore$ Radius of the cone = 6 cm (1)

16. Learning Outcome

- Finding the surface area of a cone

A wooden cone is has radius 30cm and height 40cm. Find its slant height. Calculate the cost to paint the face of 10 such cones at the rate of Rs.50/- per square metre.

(Score: 5, Time: 7 minutes)

■ **Scoring indicators**

Base Radius = 30 cm

Height = 40 cm

Slant height $= \sqrt{40^2 + 30^2}$

$= 50$ (1)

Surface area of the cone $= \pi r^2 + \pi r^2 \times \frac{1}{r}$

$= \pi \times 30^2 + \pi \times 30^2 \times \frac{50}{30}$ (1)

$= 900\pi + 1500\pi$ (1)

$= 2400\pi$ (1)

$\therefore$ Total cost to paint 10 cones

$= \frac{2400\pi \times 10 \times 50}{10000}$

$= \frac{2400 \times 3.14 \times 10 \times 50}{10000}$ = 377 Rs (1)

17. Learning Outcome

- Understanding the process to calculate the surface area of a cone.

Two cones are made using two sectors of central angles 60° and 120° of a circle. If the radius of the smaller cone is 5cm

a) Calculate the radius and base area of the smaller cone.
b) Find the surface area of the bigger cone.

(Score: 5, Time: 8 minutes)

■ **Scoring indicators**

Central angle of the small sector = 60°

$\therefore$ Area of the smaller sector $= \frac{1}{6}$ part of the area of the circle

Base radius of cone formed from above sector = 5 cm (1)

Radius of the circle = 5 × 6 = 30 cm

Similarly, area of the sector of central angle 120° $= \frac{1}{3}$ of the area of the circle

$\therefore$ Base radius of the bigger cone = $30 \times \frac{1}{3} = 10$ (1)

Base area of the bigger cone = $\pi \times 10^2 = 100\pi$ (1)

$\therefore$ Curved surface area of the bigger cone = $\pi \times 10 \times 30 = 300\pi$ (1)

$\therefore$ Surface area = $100\pi + 300\pi = 400\pi$ square centimeters (1)

18. Learning Outcome

- Understanding the process to find out the volume of a cone

A sector of central angle 216° and radius 25cm has been rolled up to make a cone. Find the radius, height and volume of the cone.

(Score: 4, Time: 7 minutes)

■ Scoring indicators

Sector is $\frac{3}{5}$ part of the circle.

$\therefore$ Radius of the cone is $\frac{3}{5}$ part of the radius of the circle.

$\therefore$ Radius of the cone $= 25 \times \frac{3}{5} = 15$ cm (1)

$\therefore$ Slant height of the cone = Radius of the circle

= 25cm (1)

$\therefore$ Height $= \sqrt{25^2 - 15^2} = 20$ (1)

$\therefore$ Volume $= \frac{1}{3} \times \pi \times 15^2 \times 20$

$= 1500\pi$ cubic centimetres (1)

19. Learning Outcome

- Find the volume of a cone.

The radius and height of a wax made cylinder are 6cm and 12cm respectively. A cone of same base radius and height has been made from this cylinder by cutting out.

a) Find the volume of cone

b) How many candles with 1cm radius and 12cm height can be made using the remaining wax

(Score: 5, Time: 7 minutes)

■ Scoring indicators

Volume of the cone $= \frac{1}{3}\pi \times 6^2 \times 12$

$= 144\pi$ cubic centimetres (1)

Volume of the cylinder $= \pi r^2 h = 144\pi \times 3$

$= 432\pi$ cubic centimetres (1)

Volume of the remaining wax = 288π cubic centimetres (1)

Volume of one candle = $\pi \times 1^2 \times 12$

= 12π cubic centimetres (1)

$\therefore$ Number of candles = $\frac{288\pi}{12\pi} = 24$ (1)

20. Learning Outcome

- Findign the volume of a cone.

A sector with central angle 288° has been cut off from tin sheet having radius 15 cm. Using the tin sheet a largest conical vessel is made.

a) Find the radius of the vessel.

b) Is this vessel large enough to buy $1\frac{1}{2}$ litres of coconut oil?

(Score: 5, Time: 7 minutes)

Scoring indicator

Since the central angle is 288° the sector is $\frac{4}{5}$ part of the circle.

$\therefore$ Radius of the conical vessel = $15 \times \frac{4}{5} = 12$ cm (1)

Slant height = 15cm (1)

$\therefore$ Height = $\sqrt{15^2 - 12^2}$

= $\sqrt{225 - 144} = 9$ cm (1)

Volume = $\frac{1}{3} \times 3.14 \times 12^2 \times 9$

= 1356.5 cubic centimetres (1)

$\therefore$ The conical vessel can't contain $1\frac{1}{2}$ litres. (1)

21. Learning Outcome

- Finding the surface area of a sphere.

Calculate the radius of a sphere having surface area 144π square centimetres. Find the surface area of the sphere having half the radius of the first sphere.

(Score: 3, Time: 4 minutes)

Scoring indicators

Surface area = 144π

$\therefore 4\pi r^2 = 144\pi$ (1)

$r^2 = 36$

$r = 6$ cm (1)

Surface area of the second sphere = $\frac{144\pi}{4} = 36\pi$ square centimetres (1)

22. Learning Outcome

- Finding the volume of a sphere.

The edge of a cube is 12cm. Find the volume of the largest sphere that can be carved out from it?

(Score: 3, Time: 4 minutes)

Scoring indicators

Diameter of the sphere = 12 cm. (1)

$\therefore$ radius = 6 cm.

$\therefore$ Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 6^3$ (1)

$= 288\pi$ cubic centimeters (1)

23. Learning Outcome

- Comparing the measurements of two hemispheres.

Ratio of the diameters of two hemispheres is 2 : 5

a) Find the ratio of their radii

b) If the surface area of the second hemisphere is 50 square centimetres calculate the surface area of the first hemisphere

(Score: 3, Time: 4 minutes)

Scoring indicators

a. Ratio of the radii = 2 : 5 (1)

b. Ratio of the surface areas of the hemispheres = 4 : 25 (1)

Surface area of the second hemisphere = 50 square centimetre

$\therefore$ Surface area of the first cone = 8 cm (1)

24. Learning Outcome

- Comparing the volumes of hemispheres and cone.

By melting and recasting a metal cone, a hemisphere of same diameter is made.

a) What is the ratio of the height and base diameter of the cone.

b) Which solid is has larger surface area?

(Score: 5, Time: 7 minutes)

Scoring indicators

Volume of the hemisphere $= \frac{2}{3}\pi r^3$

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$\therefore$ cone volume

$$\therefore \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 h \quad (1)$$

$$\therefore h = 2r$$

$$\therefore \text{height} = \text{diameter} \quad (1)$$

$$\therefore \text{Ratio between base diameter and height} = 1 : 1$$

$$\text{Surface area of the hemisphere} = 3\pi r^2 \quad (1)$$

$$\therefore \text{Slant height of cone} = \sqrt{(2r)^2 + r^2}$$

$$= \sqrt{5r^2} = \sqrt{5}\, r$$

$$\therefore \text{Surface area of the cone} = \pi \times r \times \sqrt{5}\, r + \pi r^2 \quad (1)$$

$$= (1 + \sqrt{5})\, \pi r^2 \quad (1)$$

$\therefore$ Surface area of the cone is grater than the hemisphere

25. Learning Outcome

- Comparing the measurements of square pyramid, cone and sphere

Three solids a square pyramid, a cone and a sphere have been carved out from three solid cubes of the same size. Find the volume of each solid.

(Score: 5, Time: 7 minutes)

■ Scoring indicators

$$\text{Volume of square pyramid} = \frac{1}{3}a^3 \quad (1)$$

$$\therefore \text{Volume of cone} = \frac{1}{3}\pi \left(\frac{a}{2}\right)^2 a \quad (1)$$

$$= \frac{1}{3} \times \pi \frac{a^3}{4} = \frac{1}{12}\pi a^3 \quad (1)$$

$$\text{Volume of sphere} = \frac{4}{3}\pi \times \left(\frac{a}{2}\right)^3 \quad (1)$$

$$= \frac{4}{3}\pi \times \frac{a^3}{8}$$

$$= \frac{1}{6}\pi a^3 \quad (1)$$

26. Learning Outcome

- Comparing the measurements of two solids

A metal sphere is melted and recasted into a cone . Both have same radii

a) Find the relationship between the height of the cone and the radius of the sphere.
b) Which solid has greater surface area? Justify

(Score: 5, Time: 7 minutes)

Scoring indicators

If r is the radius of the sphere, then its volume $= \frac{4}{3}pr^3$ (1)

If h is the height of the cone, then its volume $= \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r^3$

$\therefore\ h = 4r$ (1)

Surface area of the sphere $= 4\pi r^2$

Slant height of the cone $= \sqrt{(4r)^2 + r^2} = \sqrt{17r^2}$

$= \sqrt{17}\ r$ (1)

Surface area of the cone $= \pi r^2 + \pi r \sqrt{17}\ r^2$

$= \pi r^2 \left(1 + \sqrt{17}\ r\right)$ (1)

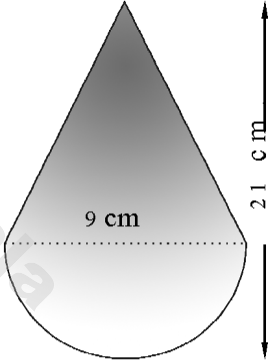
$\therefore$ Surface area of the cone is greater (1)

27. Learning Outcome

- Finding the volume of combination of two different solids

A hemisphere and a cone with same radii are attached to get a solid as given in the figure. Radius of the hemisphere is 9 cm. The height of the two solids together is 21 cm.

a) Find the height of the cone
b) Find the volume of the cone
c) Find the volume of the solids



(Score: 4, Time: 6 minutes)

Scoring indicators

a. Height of the cone = 12 cm (1)

Volume of the sphere $= \frac{2}{3}\pi \times 9^3 = 486\pi$ cubic centimetres (1)

b. Volume of the cone $= \frac{1}{3}\pi \times 9^2 \times 12 = 324\pi$ cubic centimetres (1)

c. Total volume $= 486\pi + 324\pi = 810\pi$ cubic centimetres (1)

28. Learning Outcome

- Finding the volume of solids

A hemisphere and a cone both have same diameter. These two metal solids are joined by putting their bases together. The height of the cone is equal to the diameter of the sphere. This solid is melted and recasted into a sphere of diameter equal to one third of the diameter of the hemisphere.

a) If radius of the hemisphere is 'r', find the volume of the combined solid.

b) Find the number of spheres.

(Score: 5, Time: 7 minutes)

Scoring indicator

Radius of the hemisphere $= r$

Height of the cone $= 2r$ (1)

$\therefore$ Volume of the solid $= \frac{1}{3}\pi r^2 \times 2r + \frac{2}{3}\pi r^3$ (1)

$= \frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3$ (1)

Volume of the sphere $= \frac{4}{3}\pi\left(\frac{r}{3}\right)^3$ (1)

$= \frac{4}{3}\pi\frac{r^3}{27}$

$\therefore$ Number of spheres $= \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi\frac{r^3}{27}} = 27$ (1)

29. Learning Outcome

- Finding the relationship between base edge, height and slant height of a square pyramid

All edges of a square pyramid are equal. Total length of all edges is 96 cm. Find the volume of the square pyramid.

(Score: 3, Time: 4 minutes)

Scoring indicators

- Base edge $= \frac{96}{8} = 12$ cm (1)

$d = a\sqrt{2} = 12\sqrt{2}$

$h = \sqrt{e^2 - \left(\frac{d}{2}\right)^2} = \sqrt{12^2 - \left(6\sqrt{2}\right)^2} = \sqrt{72}$ (1)

$\therefore$ Volume $= \frac{1}{3} \times 12^2 \times \sqrt{72}$

$= 288\sqrt{2}$ cubic centimetres (1)

30. Learning Outcome

- Finding the relationship between base perimeter, slant height and lateral surface area of a square pyramid.

Is it possible to make a square pyramid of slant height 12cm, base perimeter 40cm and lateral surface area 250 square centimetre. Why?

(Score: 3, Time: 3 minutes)

■ Scoring indicators

- Lateral surface area $= \frac{1}{2} \times$ base perimeter $\times$ slant height (1)

  $= \frac{1}{2} \times 12 \times 40 = 240$ square centimetres (1)

So a square pyramid with lateral surface area 250 square centimetres can't be made (1)

31. Learning Outcome

- Finding the relation between height, slant height of a square pyramid.

The lateral faces of a square pyramid are all equilateral triangles. Find the ratio between its height and slant height.

(Score: 3, Time: 4 minutes)

■ Scoring indicators

- Let lateral edge be e, then slant height

$l = \sqrt{e^2 - \frac{e^2}{4}} = \frac{\sqrt{3}}{2}e$ (1)

Height $h = \sqrt{\frac{3}{4}e^2 - \frac{e^2}{4}} = \frac{\sqrt{3}\,e}{2}$ (1)

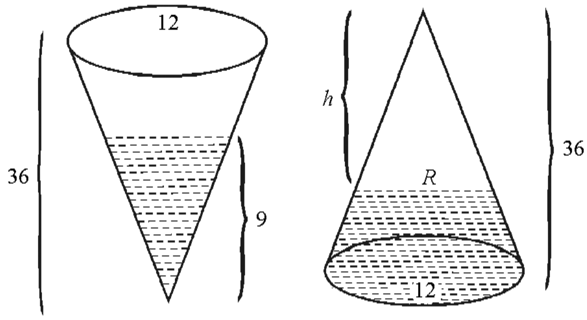
$h : l = \frac{\sqrt{2}}{2}e : \frac{\sqrt{3}}{2}e = \sqrt{2} : \sqrt{3}$ (1)

32. Learning Outcome

- Finding the volume of a cone

A conical vessel of base radius 12cm and height 36cm contains water upto 9cm high. After closing the lid, the vessel is turned up side down as shown. Find the height of the vacant part of the vessel from the water level.

(Score: 5, Time: 8 minutes)



■ Scoring indicators

From the first figure if we take radius of the conical part of water as r

then $= \frac{12}{r} = \frac{36}{9}$

then $r = 3$ cm (1)

In the first figure, volume of the vacant part

$$= \frac{1}{3}\pi \times 12^2 \times 36 - \frac{1}{3}\pi \times 3^2 \times 9$$

$$= 1701\pi \text{ cubic centimetres} \qquad (1)$$

In the second figure, let the height of the vacant part as h and radius R

$$\frac{h}{36} = \frac{R}{12}; \quad \frac{h}{R} = \frac{36}{12}; \quad h = 3R \qquad (1)$$

Volume of the vacant part in the second figure = (1)

volume of the vacant part in the first figure

$$\frac{1}{3}\pi R^2 \times h = 1701\pi$$

$$\frac{1}{3}\pi R^2 \times 3R = 1701\pi$$

$$R = \sqrt[3]{1701}$$

$$h = 3\sqrt[3]{1701} \text{ cm} \qquad (1)$$

33. Learning Outcome

- Finding the relation between radius and slant height of a cone.

A sector is rolled up to make a cone. If the slant height of the cone is two times the radius, what is the central angle of the sector?

(Score: 2, Time: 2 minutes)

Scoring indicators

- $$\frac{\text{central angle}}{360} = \frac{r}{l}$$

$$\frac{\text{central angle}}{360} = \frac{r}{2r} \qquad (1)$$

$$\text{central angle} = \frac{360}{2} = 180^\circ \qquad (1)$$

34. Learning Outcome

- Finding the volume of a cone

The base radius of a cone is two times the height. It is melted and recasted into cones of same height and radius half the height. How many cones can be made ?

(Score: 4, Time: 4 minutes)

Scoring indicators

- $$r = 2h \qquad (1)$$

$$\text{volume} = \frac{1}{3}\pi \times (2h)^2 \times h \qquad (1)$$

$$= \frac{1}{3}\pi \times 4h^2 \times h$$

$$\text{volume of the smaller cone} = \frac{1}{3}\pi \times \left(\frac{h}{2}\right)^2 \times h \qquad (1)$$

$$=\frac{1}{3}\pi\ \frac{h^2}{4}\times h$$

$$\text{Number of cones}=\frac{\frac{1}{3}\pi 4h^2\times h}{\frac{1}{3}\pi\frac{h^2}{4}\times h}$$

$$=16$$ (1)

35. Learning Outcome

- Finding the relation between radius and slant height of a cone.

A circle is cut into 12 sectors. From these one sector is taken and rolled up to form a cone. What is the ratio of the base radius and slant height of the cone?

(Score: 2, Time: 2 minutes)

■ Scoring indicators

- Central angle of sector = $\frac{360}{12}$ = 30°

$$\frac{30}{360} = \frac{r}{l}$$ (1)

$$r : l = 1 : 12$$ (1)

36. Learning Outcome

- Finding the volume of the cone

Angle between the base radius and slant height of a cone is 60°. If its base radius is 8cm, what is its volume?

(Score: 3, Time: 3 minutes)

■ Scoring indicators

- The angles determined by radius, slant height and height are 30°, 60°, 90°. Then their lengths are in the ratio $1 : \sqrt{3} : 2$ (1)

height = $8\sqrt{3}$ cm (1)

volume = $\frac{1}{3}\times\pi\times 8^2\times 8\sqrt{3}$

= $\frac{512\sqrt{3}\pi}{3}$ cubic centimetre = $\frac{512\pi}{\sqrt{3}}$ cubic centimetres (1)

37. Learning Outcome

- Finding the relation between base radius and slant height of the cone.

A semicircle is rolled up to make a cone of radius 12cm. Through the apex, and perpendicular to the base it is cut into two pieces. Show that the new faces so obtained were equilateral triangles.

(Score: 3, Time: 4 minutes)

■ **Scoring indicators**

Among the sides of the new faces, two of them are slant heights and the third side is the diameter of the cone. (1)

When semi circle is rolled upto make a cone

slant height = 2 × base radius = 2 × 12 = 24 cm (1)

Diameter = 24 cm

$\therefore$ the triangle formed is equilateral. (1)

38. Learning Outcome

- Finding the volumes of cone and sphere.

A wooden cone has its slant height two times the radius. If the radius is 6cm, find the volume of the largest sphere that can be carved out of this.

(Score: 5, Time: 4 minutes)

■ **Scoring indicators**

Slant height = 12cm = diameter

Through apex, when the cone is cut perpendicular to the base, we get an equilateral triangle. The inradius of the triangle is the radius of the sphere. (1)

Ratio of the sides of the triangle having angles 30°, 60°, 90° is $1 : \sqrt{3} : 2$ (1)

Side opposite to 60° $= 6$

then side opposite to 30° $= \frac{6}{\sqrt{3}}$ (1)

Hypotenuse $= \frac{2 \times 6}{\sqrt{3}} = \frac{12}{\sqrt{3}}$

Thus, radius of the sphere $= \frac{6}{\sqrt{3}}$

Volume of the sphere $= \frac{4}{3} \times \pi \times \left(\frac{6}{\sqrt{3}}\right)^3$ (1)

$= \frac{96\pi}{\sqrt{3}}$ cubic centimetre (1)

39. Learning Outcome

- Finding the volume of the sphere

A largest sphere is carved out from a cube. If the volume of the sphere is 288π cubic centimetres. Find the volume of the cube.

(Score: 3, Time: 3 minutes)

■ **Scoring indicators**

- $\frac{4}{3}\pi r^3$ = 288π

  r =6 (1)

  ∴ side of one cube = 12 cm (1)

  Volume of the cube = 12^3 = 1728 cubic centimetre (1)

40. Learning Outcome

- Finding the volume of a sphere

Two hemispheres are attached together to get a sphere. Surface area of each hemisphere is 60 sq.cm. Find the surface area of the sphere.

(Score: 3, Time: 4 minutes)

■ **Scoring indicators**

Sum of two curved surface areas of the hemisphere is the surface area of the sphere.

Curved surface area of the hemisphere = $\frac{2}{3}$ of the surface area of the hemisphere

(1)

$= \frac{2}{3} \times 60$

= 40 sq.cm. (1)

∴ Surface area of the sphere = 40 × 2

= 80 sq.cm. (1)

9

GEOMETRY AND ALGEBRA

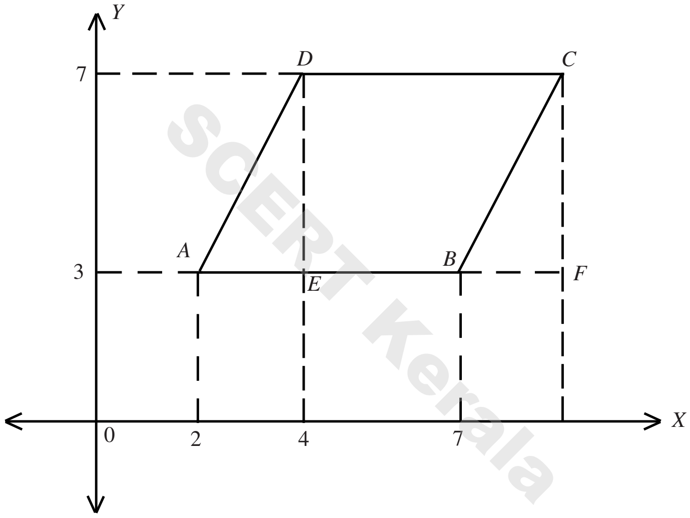
1. Learning Outcome

- Explaining the method to find the coordinates of the vertices of a parallelogram.

In the parallelogram *ABCD*, *A* (2, 3), *B* (7, 3) and *D* (4, 7). Find the coordinates of C

(Score: 4, Time: 6 minutes)

Scoring Indicators



In the figure $\Delta AED, \Delta BFC$

$AD = BC.\ DE = CF$ (1)

$\therefore AE = BF = 2$ unit (1)

$ED = FC = 4$ unit

x coordinate of C= $7 + BF = 7 + AE = 7 + 2 = 9$ (1)

y coordinate of $C = 3 + FC = 3 + ED = 3 + 4 = 7$ (1)

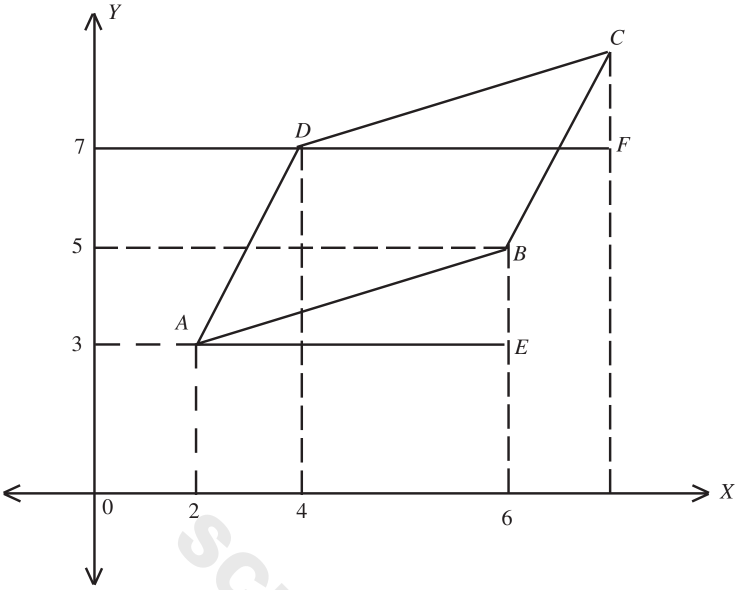
C has the coordinates (9, 7)

2. Learning Outcome

- Explaining the method to find the coordinates of the vertices of a parallelogram.

In parallelogram *ABCD*, *A* (2, 3), *B* (6,5) and *D* (4, 7). Find the coordinates of *C*

(Score: 4, Time: 6 minutes)



- **Scoring Indicators**

Draw *AE, DF* parallel to *x*-axis

Draw *BE, CF* parallel to *y*-axis (1)

In right triangles ΔABE, ΔDCF (1)

$AB = DC$ (1)

$\angle BAE = \angle CDF$

$\angle ABE = \angle DCF$

$\therefore\ AE = DF = 6 - 2 = 4$

$BE = CF = 5 - 3 = 2$

x coordinate of C $= 4 + DF = 4 + 4 = 8$

y coordinate of C $= 7 + CF = 7 + 2 = 9$

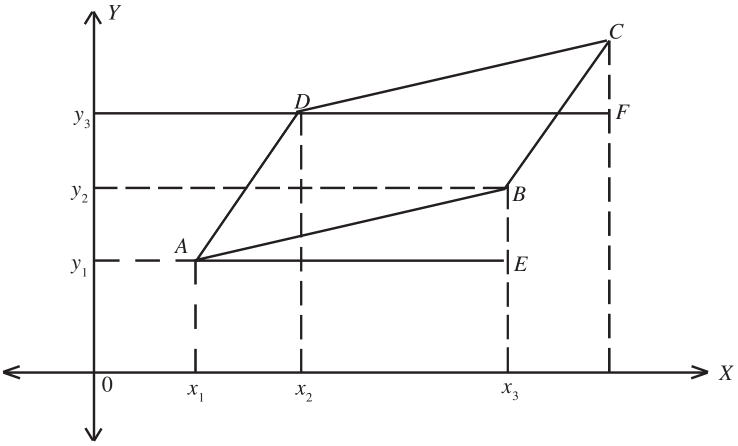
Coordinates of C (8, 9) (1)

3. Learning Outcome

- Explaining the method to find the coordinates of the vertices of a parallelogram.

In parallelogram *ABCD*, *A* (x_1, y_1), *B* (x_2, y_2), *D* (x_3, y_3). Find the coordinates of C

(Score: 4, Time: 6 minutes)



Scoring Indicators

Draw AE, DF parallel to x-axis

Draw BE, CF parallel to y-axis

In right triangles ΔAEB, ΔDFE

$AB = DC$ (1)

$\angle BAE = \angle CDF$ (1)

$\angle ABE = \angle DCF$ (1)

$\therefore AE = DF = x_2 - x_1$

$BE = CF = x_2 - y_1$

x coordinate of C$= x_3 + DF = x_3 + x_2 - x_1$

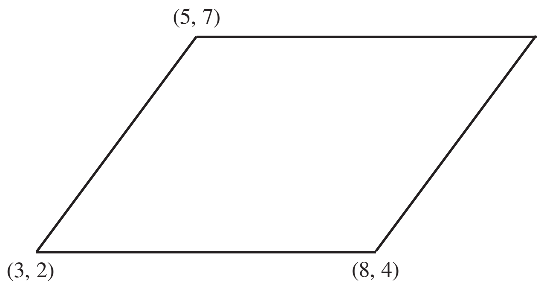
y coordinate of C$= y_3 + CF = y_3 + y_2 - y_1$

C has the coordinates $(x_2 + x_3 - x_1, \ y_2 + y_3 - y_1)$ (1)

4. Learning Outcome

- Explaining the method to find the coordinates of the vertices of a parallelogram.

Find the coordinates of the fourth vertex of the parallelogram shown here



(Score: 2, Time: 6 minutes)

Scoring Indicators

If A (x_1, y_2) ; B (x_2, y_2); D (x_3, y_3) are vertices of a parallelogram ABCD, then C has the coordinates $(x_2 + x_3 - x_1 , y_2 + y_3 - y_1)$ (1)

x coordinate of fourth vertex $= 8 + 5 - 3 = 10$

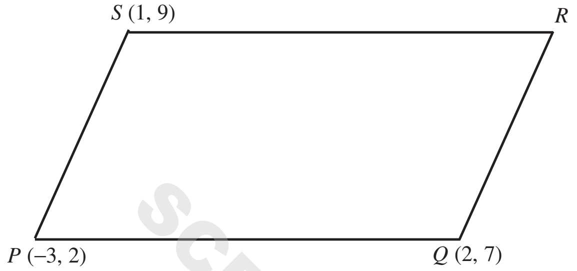
y coordinate of fourth vertex $= 7 + 4 - 2 = 9$

Coordinates of fourth vertex (10, 9) (1)

5. Learning Outcome

- Explaining the method to find the coordinates of the vertices of a parallelogram.

In the parallelogram *PQRS*, *P* (-3, 2) *Q* (2, 7); *S* (1, 9) are three vertices. Find the length of the diagonal *PR*.



(Score: 4, Time: 6 minutes)

Scoring Indicators

x coordinate of R $= 2 + 1 - (^{-}3) = 6$ (1)

y coordinate of R $= 9 + 7 - 2 = 14$ (1)

(6,14) is the coordinate of R

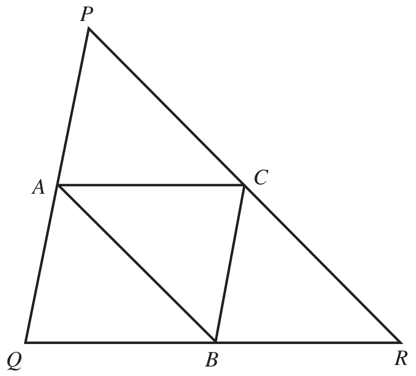
$$PR = \sqrt{((6-(-3))^2 + ((14-2))^2}$$

$$= \sqrt{9^2 + 12^2} = 15 \text{ unit}$$ (1)

6. Learning Outcome

- Explaining the method to find the coordinates of the vertices of a parallelogram.

In the figure *A, B, C* are the midpoints of the sides of Δ *PQR*. If *A, B* and *Q* has the coordinates (3,7), (6,4) and (1,3). Find the coordinates of *P, C* and *R*



(Score: 5, Time: 9 minutes)

- **Scoring Indicators**

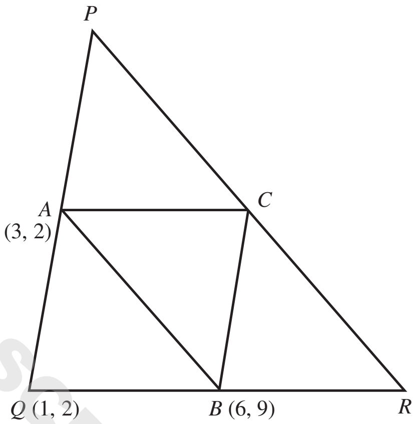
A and *B* are the midpoints of *PQ* and *RQ*

Hence AB is parallel to PC

BC is parallel to PA

PABC is a parallelogram (1)

Similarly, ABRC and AQBC are parallelograms (1)



In the parallelogram C has the coordinates (6 + 3 - 1, 4 + 7 - 2)

Thus coordinates of C is (8,9) (1)

In the parallelogram ABRC, R has the coordinates (8 + 6 - 3, 9 + 4 - 7)

Thus *R* (11, 6) (1)

In the parallelogram *RABC*

P has coordinates (3 + 8 - 6, 7 + 9 - 4)

Thus *P* (5, 12) (1)

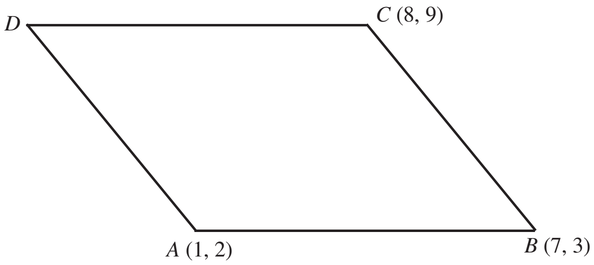
7. Learning Outcome

- Explaining the method to find the coordinates of the vertices of a parallelogram.

In parallelogram ABCD A, B and C has the coordinates (1,2), (7,3) and C (8,9) respectively.

a) Find the coordinates of D

b) Prove that the sum of the squares of the diagonals is equal to the sum of the squares of the sides.



(Score: 5, Time: 9 minutes)

Scoring Indicators

x coordinate of D = $1 + 8 - 7 = 2$

x coordinate of D = $2 + 9 - 3 = 8$

Coordinates of R (2, 8) (1)

$AC^2 = (8 - 1)^2 + (9 - 2)^2 = 98$

$BD^2 = (7 - 2)^2 + (3 - 8)^2 = 50$ (1)

$AB^2 = (7 - 1)^2 + (3 - 2)^2 = 37$

$BC^2 = (8 - 7)^2 + (9 - 3)^2 = 37$

$CD^2 = (8 - 2)^2 + (9 - 8)^2 = 37$ (1)

$AD^2 = (2 - 1)^2 + (8 - 2)^2 = 37$

$AB^2 + BC^2 + CD^2 + AD^2 = 4 \times 37 = 148$

$AC^2 + BD^2 = 98 + 50 = 148$ (1)

$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$ (1)

8. Learning Outcome

- Explaining the method to find the coordinates of the midpoint of a line joining any two points

In quadrilateral ABCD, P,Q,R and S are the midpoints of the sides AB, BC, CD and AD respectively. If P,Q and R has the coordinates, (1,2), (3,-4) and R (7,3), find the coordinates of S.

(Score: 3, Time: 5 minutes)

Scoring Indicators

By joining the midpoints of a quadrilateral we get a parallelogram (1)

x coordinate of S = $1 + 7 - 3 = 5$ (1)

y coordinate of S = $2 + 3 - {}^{-}4 = 9$

The coordinates of S is (5, 9) (1)

9. Learning Outcome

- Explaining the method to find the coordinates of the midpoint of a line joining any two points of a segment.

A (3, 2); B (7, 4) ; C (9, 8) ; D (5, 6) are the vertices of a quadrilateral ABCD. P,Q,R and S are the midpoints of AB, BC, CD and AD respectively.

a) Find the coordinates of P,Q,R and S

b) Show that in quadrilateral PQRS , its perimeter is equal to AC + BD

(Score: 5, Time: 8 minutes)

Scoring Indicators

coordinates of $P \left(\frac{3+7}{2}, \frac{2+4}{2}\right)$ = P (5, 3) (1)

coordinates of Q (8, 6)

coordinates of $R\,(7, 7)$

coordinate of $S\,(4, 4)$ (1)

$PQ = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2}$

$QR = \sqrt{1^2+1^2} = \sqrt{2}$

$RS = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2}$ (1)

$PS = \sqrt{1^2+1^2} = \sqrt{2}$

$PQ + QR + RS + PS = 3\sqrt{2} + \sqrt{2} + 3\sqrt{2} + \sqrt{2} = 8\sqrt{2}$

$AC = \sqrt{6^2+6^2} = 6\sqrt{2}$

$BD = \sqrt{2^2+2^2} = 2\sqrt{2}$ (1)

$AC + BD = 8\sqrt{2}$

$PQ + QR + RS + PS = AC + BD$ (1)

10. Learning Outcome

- Explaining the method to find the coordinates of the midpoint of a line joining any two points of a segment.

A triangle has its vertices $A\,(3, 2)$; $B\,(^-5, ^-4)$, $C\,(7, 8)$. P, Q, R are the midpoints of AB, BC and AC respectively.

a) Find the coordinates of P, Q ,R
b) Find the perimeter of ΔPQR

(Score: 4, Time: 7 minutes)

Scoring Indicators

coordinates of $P(^-1, ^-1)$ (1)
Similarly $Q\,(1, 2)$
$R\,(5, 5)$

$PQ = \sqrt{2^2+3^2} = \sqrt{13}$ (1)

$QR = \sqrt{4^2+3^2} = 5$ (1)

Perimetre of $\Delta PQR = \sqrt{13} + 5 + 6\sqrt{2}$ (1)

11. Learning Outcome

- Explaining the method to find the coordinates of the midpoint of line joining any two points

$A\,(1, 2)$; $B\,(7, 3)$, $C\,(8, 9)$ are the vertices of a parallelogram $ABCD$

(a) Find the coordinates of D
(b) If the diagonals intersect at P, find the coordinates of P
(c) Show that ΔAPB is right angled
(d) Show that $ABCD$ is a rhombus

(Score: 5, Time: 8 minutes)

- **Scoring Indicators**

(a) Coordinates of $D\ (1 + 8 - 7, 2 + 9 - 3)$=(2, 8) (1)

(b) Coordinates of $P\left(\frac{1+8}{2},\frac{2+9}{2}\right)$=(4.5, 5.5) (1)

(c) $AP^2 = (3.5)^2 + (3.5)^2 = 24.5$

$BP^2 = (2.5)^2 + (2.5)^2 = 12.5$ (1)

$AB^2 = 6^2 + 1^2 = 37$

$AD^2 + BD^2 = 24.5 + 12.5 = 37 = AB^2$

$\therefore \Delta$APB is right angled (1)

(d) Since the diagonals are perpendicular to each other, parallelogram is a rhombus. (1)

12. Learning Outcome

- Explaining the method to find the coordinates of the midpoint of line joining any two points

$A\,(1, 2); B\,(7, 4), C\,(5, 10)$ are the vertices of ΔABC. Also P, Q and R are the midpoints of AB, BC and AC respectively

a) Find the coordinates of P, Q and R

b) Prove that $\Delta\, PQR$ is right angled

(Score: 4, Time: 7 minutes)

- **Scoring Indicators**

a) Coordinates of P $\left(\frac{1+7}{2},\frac{2+4}{2}\right) = P\,(4, 3)$ (1)

Coordinates of Q $Q\,(6, 7)$

Coordinates of R $R\,(3, 6)$ (1)

b) $PQ^2 = 2^2 + 4^2 = 20$

$QR^2 = 3^2 + 1^2 = 10$

$PR^2 = 1^2 + 3^2 = 10$ (1)

$PR^2 + QR^2 = PQ^2$ (1)

$\therefore \Delta\, PQR$ is right angled

13. Learning Outcome

- Explaining the method to find the coordinates of the midpoint of line joining any two points

Consider two points $A\,(-2, 3)$; $B\,(6, 9)$

a) Find the coordinates of the centre of the circle with diameter AB. Compute the diameter

b) If C and D are (-3, 5) and (5, -1) , justify whether PQ is a diameter?

c) If P and Q are (5,10) and Q (-1,2), justify whether PQ is a diameter?

(Score: 5, Time: 8 minutes)

- **Scoring Indicators**

a) Coordinates of the midpoint of AB is (2, 6), which is the centre

Diameter = $\sqrt{8^2+6^2}$ = 10 (1)

b) $CD = \sqrt{8^2+6^2} = 10$

Midpoint of CD is (1, 2). This is not the centre of the circle (1)

Hence CD is not the diameter (1)

b) $PQ = \sqrt{6^2+8^2} = 10$ (1)

Mid point *of* PQ is (2, 6), which is the centre of the circle

Hence PQ is the diameter of the circle (1)

14. Learning Outcome

- Explaining the method to find the coordinates of the midpoint of line joining any two points

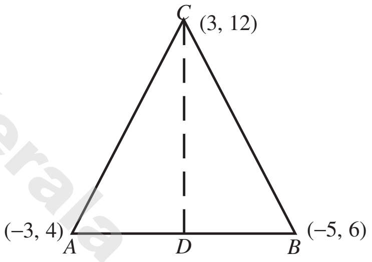
In ΔABC, (-3, 4), (-5, 6), (3, 12) are the coordinates of A,B and C respectively

a) Find the perpendicular distance from C to AB

b) Find the area of ΔABC

(Score: 4, Time: 7 minutes)

- **Scoring Indicators**



$AC = \sqrt{6^2+8^2} = 10$

$BC = \sqrt{8^2+6^2} = 10$ (1)

ΔACB is isosceles

If midpoint of AB is D,

then D (-4, 5) (1)

(a) $CD = \sqrt{7^2+7^2} = 7\sqrt{2}$ (1)

$AB = \sqrt{2^2+2^2} = 2\sqrt{2}$ (1)

(b) Area of Δ ABC $= \frac{1}{2} \times 2\sqrt{2} \times 7\sqrt{2}$

$=14$ (1)

15. Learning Outcome

- Explaining the method to find the coordinates of the midpoint of line joining any two points

A (1, 2) and *B* (7, 5) are two points on a line. *P* and *Q* are other two points on this line. If $AP = PQ = QB$, find the coordinates of *P* and *Q*.

(Score: 4, Time: 6 minutes)

- **Scoring Indicators**

Since $AP = PQ = QB$, $AP : PB = 1 : 2$ (1)

x coordinate of $P= 1 + \frac{1}{3} \times (7 - 1)=1 + 2 = 3$ (1)

y coordinate of $Q= 2 + \frac{1}{3}\ (5- 2)=2 + 1 = 3$

Coordinate of P (3, 3) (1)

$AQ : QB = 2 : 1$

x coordinate of $Q = 1 + \frac{2}{3}\times (7 - 1) = 5$

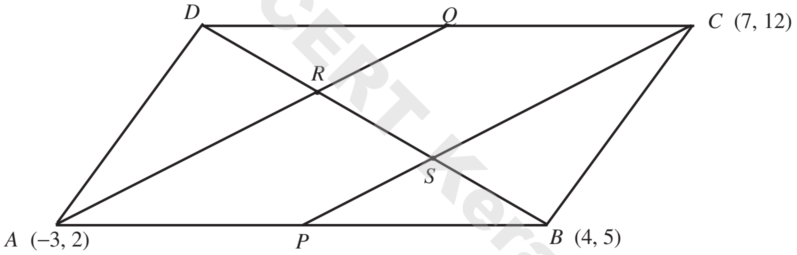
y coordinate of $Q = 2 + \frac{2}{3} \times (5 - 2) = 4$

coordinates of Q (5, 4) (1)

16. Learning Outcomes

- Explaining the method to find the coordinates of the vertices of a parallelogram.
- Explaining the method to find the coordinates of the midpoint of line joining any two points
- Finding the coordinates of a point which divides a line in a specific ratio.

In the parellelogram $ABCD$, $A(^-3, 2)$, $B(4, 5)$, C (7, 12) . P is the midpoint of AB. Q is the midpoint of CD. Two segments AQ and CP cut the diagonal BD at R and S respectively.



a) Find the coordinates of D

b) Find the coordinates of P and Q

c) Find the points of intersection of two diagonals.

d) Find the coordinates of R and S

(Score: 5, Time: 10 minutes)

■ Scoring Indicators

a) Coordinates of D $(7 + {^-3} - 4, 12 + 2 - 5) = (0, 9)$ (1)

b) Coordinates of P $\left(\frac{1}{2}, \frac{7}{2}\right)$

Coordinates of Q $\left(\frac{7}{2}, \frac{21}{2}\right)$ (1)

c) If O is the point of intersection of the diagonals
Then midpoint of AC is O,

Hence O is (2,7) (1)

d) R is the point of intersection of medians of ΔACD

$RD : RO = 2 : 1$

x coordinate of $R = 0 + \frac{2}{3}(2 - 0) = \frac{4}{3}$

y coordinate of $R = 9 + \frac{2}{3}(7 - 9) = \frac{23}{3}$

Coordinates of $R \left(\frac{4}{3}, \frac{23}{3}\right)$

Similarly S is the point of intersection of medians of ΔABC (1)

$OS : SB = 1 : 2$

x coordinate of $S = 2 + \frac{1}{3}(4 - 2) = \frac{8}{3}$

y coordinate of $S = 7 + \frac{1}{3}(5 - 7) = \frac{19}{3}$

Coordinates of $S = \left(\frac{8}{3}, \frac{19}{3}\right)$ (1)

17. Learning Outcome

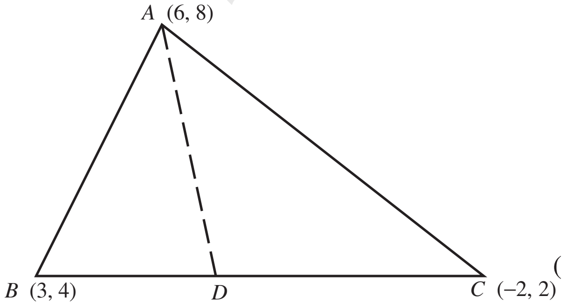
- Finding the coordinates of a point which divides a line in a specific ratio

In a triangle, $A(6, 8)$; $B(3, 4)$; $C(-2, 2)$ are its vertices. The bisector of $\angle A$ cuts BC at D.

a) Find $BD:CD$
b) Find the coordinates of D

(Score: 4, Time: 8 minutes)

Scoring Indicators



$AB = \sqrt{3^2 + 4^2} = 5$

$AC = \sqrt{8^2 + 6^2} = 10$

$\text{AB} : \text{AC} = 5 : 10 = 1 : 2$

$\text{BD} : \text{CD} = 1 : 2$ (1)

x coordinate of D

$= 3 + \frac{1}{3} \times (^-2 - 3) = \frac{4}{3}$ (1)

x coordinate of $D = 4 + \frac{1}{3}(2 - 4) = \frac{10}{3}$ (1)

Coordinates of $D \left(\frac{4}{3}, \frac{10}{3}\right)$ (1)

18. Learning Outcome

- Explaining the method to find the coordinates of the midpoint of line joining any two points
- Finding the coordinates of a point which divides a line in a specific ratio.

A circle is drawn with AB as diameter whose endpoints are $A\,(^-3, 1)$ and $B\,(9, 10)$

a) Find the coordinates of the centre of the circle.

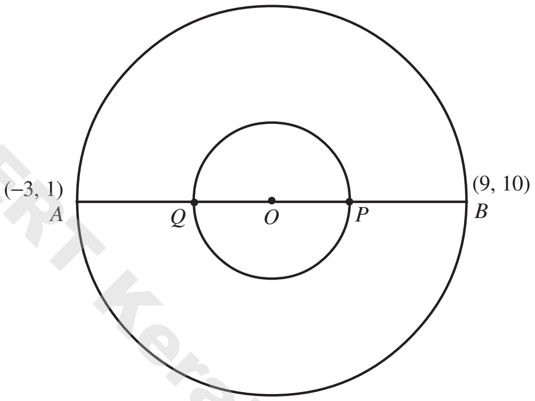
b) If another circle with diameter one third of the above circle is drawn with the same centre, what are the points that the circle cuts *AB*.

(Score: 5, Time: 8 minutes)

Scoring Indicators

a) Coordinates of the centre O

$$\left(\frac{-3+9}{2}, \frac{1+10}{2}\right) = \left(3, \frac{11}{2}\right)$$ (1)



b) $OP : PB = 1 : 2$ (1)

x coordinate of P

$= 3 + \frac{1}{3}\,(9 - 3) = 5$

x coordinate of P (1)

$= \frac{11}{2} + \frac{1}{3}\left(10 - \frac{11}{3}\right) = 7$

Coordinates of P (5, 7) (1)

O $\left(3, \frac{11}{2}\right)$; Which is the midpoint of *PQ*

Since the coordinates of P *is* (5, 7) , Coordinates *of* Q becomes (1, 4) (1)

19. Learning Outcome

- Justifying that the change in *y* coordinates is proportional to the change in *x* coordinates of any two points on a line

Can $A\,(^-3, 7)$, $B\,(0, 2)$, $C\,(2, 8)$ be the vertices of a triangle. Justify your answer. .

(Score: 3, Time: 5 minutesš)

Scoring Indicators

a) Slope of *AB* $= \frac{2-{}^-7}{0-{}^-3} = \frac{9}{3} = 3$ (1)

b) Slope of *BC* $= \frac{8-2}{2-0} = \frac{6}{2} = 3$

Since AB and BC has same slope , A, B and C lie on a line.

20. Learning Outcomes

- Justifying that the change in y coordinates is proportional to the change in x coordinates of any two points on a line
- Explaining the method to find the coordinates of the midpoint of line joining any two points

Consider a quadrilateral *ABCD* with vertices $(3, 5)$, $(9, 7)$, $(5, 9)$, $(7, 11)$ taken in order Show that the quadrilateral obtained by joining the midpoints of *ABCD* is a parallelogram.

(Score: 3, Time: 5 minutes)

Scoring Indicators

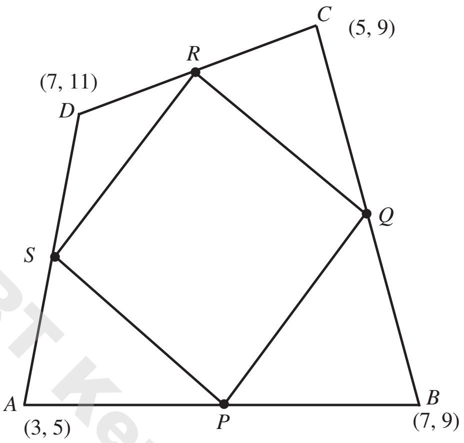
a) Coordinates of the vertices of quadrilateral *PQRS* are

$P\ (6, 6)$

$Q\ (7, 8)$

$R\ (6, 10)$ (1)

$S\ (5, 8)$



Slope of $PQ = \frac{8-6}{7-6} = 2$

Slope of $RS = \frac{10-8}{6-5} = 2$ (1)

RS is parallel to *PQ*

Slope of $QR = \frac{10-8}{6-7} = {}^{-}2$

Slope of $PS = \frac{8-6}{5-6} = {}^{-}2$

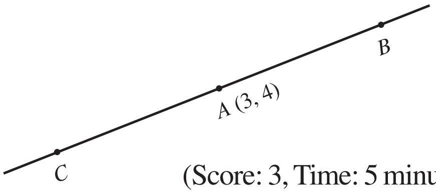
PS is parallel to *QR*

Since opposite sides are parallel, *PQRS* is a parallelogram (1)

21. Learning Outcome

- Explaining the method to find the coordinates of the midpoint of line joining any two points

$A\ (3, 4)$ is a point on a line having slope $\frac{2}{3}$. Write down the coordinates of nearby points on either side of A on this line having x and y coordinates natural numbers.



(Score: 3, Time: 5 minutes)

Scoring Indicators

Let *B* be right side of *A*

x coordinate of $B = 3 + 3 = 6$ (1)

y coordinate of $B = 4 + 2 = 6$

Coordinates of $B = (6, 6)$ (1)

Let *C* be right side of *A*

x coordinate of $C = 3 - 3 = 0$

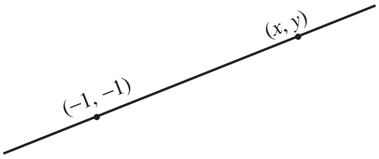
y coordinate of $C = 4 - 2 = 2$ (1)

Coordinate of $C = (0, 2)$

22. Learning Outcome

- Explaining the method to find the coordinates of the midpoint of line joining any two points

Consider a line with slope 3, passing through(3,4). If the x coordinate of a point on the line is 'a'. Show that its y coordinate is (3a-5). Also find the y coordinate of a point on this line whose x coordinate is 2



(Score: 3, Time: 5 minutes)

Scoring Indicators

When (a,y) is a point on the line

$$\text{Slope, } \frac{y-4}{a-3} = 3 \qquad (1)$$

$$y - 4 = 3\,(a-3) = 3a - 9$$

$$y = 3a - 9 + 4$$

$$y = 3a - 5 \qquad (1)$$

$$\text{when } a = 2,\ y = 3 \times 2 - 5 = 6 - 5 = 1 \qquad (1)$$

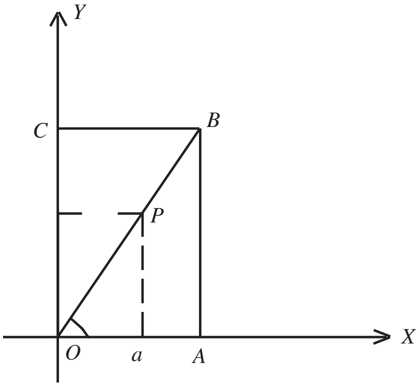
23. Learning Outcome

- Explaining the method to find the coordinates of the midpoint lies between anytwo points of a segment.

The two sides of a rectangle lies on coordinate a axis. Its diagonal makes an angle 60^o with the x-axis. Compute the slope of the diagonal. Show that either x coordinate or y coordinate or both of any point on this line are irrational.

(Score: 4, Time: 8 minutes)

Scoring Indicators



$OABC$ is a rectangle

$\angle AOB = 60^o$

P is a point on the diagonal (1)

x coordinate of $P = a$

y coordinate of $P = \sqrt{3}\,a$ (1)

Coordinates of $P\ (a,\ \sqrt{3}\,a)$ (1)

When 'a' is rational, $\sqrt{3}\,a$ is irrational

That is among a, $\sqrt{3}$ a one should be irrational. When 'a' is irrational $\sqrt{3}$ a will either rational or irrational. This shows that atleast one of the coordinate P is irrational (1)

24. Learning Outcome

- Framing the equation of a line joining two points.

Solving 1 and 2 we get
(-1,2)

($^-$1, $^-$1), (2, 8) are two points on a line

a) Find the slope of the line
b) If (x,y) lies on this line, write down a relation beteen x and y
c) Find the coordinates of the point, which the above line cuts the y axis.

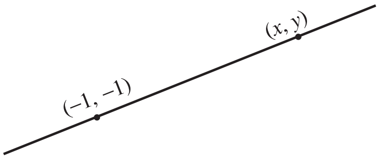
(Score: 4, Time: 7 minutes)

Scoring Indicators

a) Slope $= \frac{8-(^-1)}{2-(^-1)} = \frac{9}{3} = 3$

(1)

b) Let (x,y) be a point on the line



$$\frac{y-(-1)}{x-(-1)} = 3$$ (1)

$$\frac{y+1}{x+1} = 3$$

$$y + 1 = 3x + 3$$

$$y = 3x + 2$$

c) When the line cuts the y axis , x = o

$$y = 3 \times 0 + 2 = 2$$ (1)

The line cuts the y-axis at = (0, 2) (1)

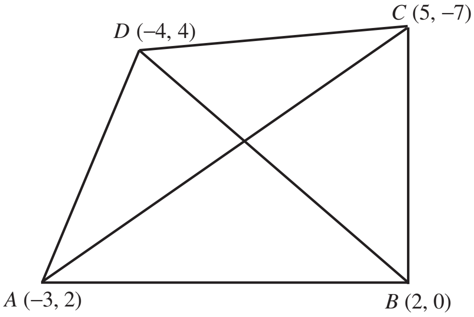
25. Learning Outcomes

- Framing the equation of a line joining two points.
- Finding the coordinates of point of intersection of two lines whose equations are known.

A (-3, 5), B (2, 0), C (-5, -$^-$7), D (-$^-$4, 4) are the vertices of quadrilateral $ABCD$. Find the point of intersection of its diagonals.

(Score: 5, Time: 9 minutes)

Scoring Indicators



(1)

Slope of diagonal AC

$$= \frac{5-{}^-7}{{}^-3-5} = \frac{12}{{}^-8} = \frac{{}^-3}{2}$$

When (x,y) is a point on this diagonal

$$\frac{y-5}{x+3} = \frac{{}^-3}{2}$$

$$2\ (y - 5) = -3\ (x + 3)$$

$$2y + 3x = 1$$ ________ (1)

Slope of diagonal BD

$$= \frac{4-0}{{}^-4-2} = \frac{{}^-4}{6} = \frac{{}^-2}{3}$$ (1)

When (x,y) is a point on this diagonal

$$\frac{y-0}{x-2} = \frac{^{-}2}{3}$$ (1)

$3y = \;\; -2x + 4$

$2x + 3y = 4$ ________ (2)

Solving (1) and (2) we get $x = -1,\; y = 2$

Point of intersection of the diagonals is (-1, 2) (1)

26. Learning Outcome

- Finding the coordinates of point of intersection of two lines whose equation are known.

(a) Find the slope of the line $2y - 3x = 6$

(b) Find the equation of another line, which is parallel to the given line and passes through(3,3)

(Score: 4, Time: 6 minutes)

Scoring Indicators

(a)When $(x_1, y_1), (x_2, y_2)$ are points on the given line

$2y_1 - 3x_1 = 6$

$2y_2 - 3x_2 = 6$ $(\sqrt{8})^2$ (1)

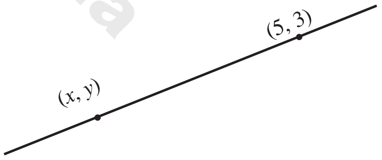
$2(y_1 - y_2) - 3(x_1 - x_2) = 0$

$2(y_1 - y_2) = 3(x_1 - x_2)$

$$= \frac{y_1 - y_2}{x_1 - x_2} = \frac{3}{2}$$

Slope of the line $= \frac{3}{2}$ (1)

(b) When (x,y) is a point on this line



$$\frac{y-3}{x-3} = \frac{3}{2}$$ (1)

$2y - 6 = 3x - 9$

$2y - 3x + 3 = 0$ (1)

27. Learning Outcome

- Framing the equation of a circle with specified centre and radius.

(5, 5) is a point on a circle with centre (1, 2)

a) Find the radius of the circle

b) Find the equation of the circle

(Score: 3, Time: 5 minutes)

Scoring Indicators

(a) Radius of the circle $= \sqrt{4^2 + 3^2} = 5$ unit (1)

(b) If (x,y) is a point on this circle,

$(x-1)^2 + (y-2)^2 = 5^2$ (1)

$x^2 - 2x + 1 + y^2 - 4y + 4 = 25$

$x^2 + y^2 - 2x - 4y - 20 = 0$ (1)

28. Learning Outcome

- Framing the equation of a circle with specified centre and radius.

Consider $A\ (0, 1)$, $B\ ({}^{-}4, 5)$ any two points

a) What is the equation of the circle with diameter AB.

b) Find the coordinates of the point of intersection of the circle and x axis

(Score: 5, Time: 9 minutes)

Scoring Indicators

(a) Centre of the circle $= \left(\frac{0+{}^{-}4}{2},\frac{1+5}{2}\right) = ({}^{-}2, 3)$ (1)

Radius $= \sqrt{2^2+2^2} = \sqrt{8}$ (1)

When (x,y) is a point on this circle,

$(x - {}^{-}2)^2 + (y-3)^2 =$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 8$

$x^2 + y^2 + 4x - 6y + 5 = 0$ (1)

(b) When the circle cuts the x-axis, $y = 0$

$x^2 + 4x + 5 = 0$

$(x+1)(x+4) = 0$ (1)

$x = {}^{-}1, {}^{-}4$

The circle cuts the x-axis at the points $({}^{-}1, 0), ({}^{-}4, 0)$ (1)

29. Learning Outcome

- Finding the centre and radius of circle whose equation is known

Find the centre and radius of the circle $x^2 + y^2 - 6x - 8y + 9 = 0$

(Score: 4, Time: 6 minutes)

Scoring Indicators

$x^2 + y^2 - 6x - 8y + 9 = 0$

$x^2 - 6x + y^2 - 8y + 9 = 0$

$x^2 + 6x + 9 + y^2 - 8y + 16 + 9 = 9 + 16$ (1)

$(x-3)^2 + (y-4)^2 + 9 = 25$ (1)

$(x-3)^2 + (y-4)^2 = 16 = 4^2$

Centre of the circle $(3, 4)$ (1)

Radius of the circle $=$ 4 unit (1)

30. Learning Outcome

- Explaining the properties of a circle whose equation is known

Prove that y axis is a tangent to the circle $x^2 + y^2 - 5x - 6y + 9 = 0$

(Score: 4, Time: 7 minutes)

■ Scoring Indicators

(a) When the circle $x^2 + y^2 - 5x - 6y + 9 = 0$ cuts the y axis

We have $x = 0$ (1)

At the time the equation becomes

$$y^2 - 6y + 9 = 0 \quad (1)$$

$$(y - 3)^2 = 0$$

$$y = 3 \quad (1)$$

This means that the circle touches the *y* axis at (0,3). Then *y* axis becomes tangent to the circle (1)

31. Learning Outcome

- Framing the equation of a line joining two points

A (3, 2), *B* (9, 4), *C* (7, 10) are any two points

a) Find the radius and coordinates of the centre of the circle having diameter AC
b) Write down the equation of a circle with diameter AC
c) Prove that this circle passes through B

(Score: 5, Time: 8 minutes)

■ Scoring Indicators

(a) Coordinates of the centre of the circle is (5,6) (1)

$$\text{Radius} = \sqrt{2^2 + 4^2} = \sqrt{20} \quad (1)$$

(b) When (*x*,*y*) is a point on the circle

$$(x - 5)^2 + (y - 6)^2 = 20 \quad (1)$$

$$x^2 - 10x + 25 + y^2 - 12y + 36 = 20$$

$$x^2 + y^2 - 10x - 12y + 41 = 0,$$ which is the equation of the circle (1)

(c) $$9^2 + 4^2 - 10 \times 9 - 12 \times 4 + 41 = 81 + 16 - 90 - 48 + 41$$

$$= 138 - 138 = 0$$

(9,4) is a point on this circle. (1)

10

POLYNOMIALS

1. Learning Outcome

- Factorisation of polynomials and solution of polynomial equations.

Write the second degree polynomial $p(x) = x^2 + x - 6$ as the product of first degree polynomials. Find also the solution of the equation $p(x) = 0$

(Score: 4, Time: 7 minute)

■ Scoring Indicators

- $p(x) = x^2 + x - 6 \quad = (x - a)(x - b)$

$$= x^2 - (a + b)x + ab \qquad (1)$$

- $a + b = -1, \quad ab = -6$

$$(a - b)^2 = (-1)^2 - 4 \times -6 = 25$$

$$a - b = 5, \qquad (1)$$

$$a = \frac{-1+5}{2} = 2, b = \frac{-1-5}{2} = -3 \qquad (1)$$

$$x^2 + x - 6 = (x - 2)(x + 3)$$

$x^2 + x - 6 = 0, (x - 2)(x + 3) = 0$

$x - 2 = 0, \quad x + 3 = 0$

$x = 2, \quad x = -3$ (1)

2. Learning Outcome

- Factorisation of polynomials and solution of polynomial equations.

For what values of x, the polynomial $2x^2 - 7x - 15$ is equal to zero? Write this polynomial as the product of two first degree polynomials

(Score: 4, Time: 7 minute)

■ Scoring Indicators

- $2x^2 - 7x - 15 = 0$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4 \times 2 \times -15}}{2 \times 2} = \frac{7 \pm \sqrt{49 + 120}}{4} \qquad (1)$$

$\frac{7\pm13}{4}$ $x = 5$ or $x = \frac{-3}{2}$ (1)

$p\,(5) = 0$, $x - 5$ is a factor

$p\ \frac{-3}{2} = 0$, $2x + 3$ is a factor (1)

$2x^2 - 7x - 15 = (x - 5)(2x + 3)$ (1)

3. Learning Outcome

- Factorisation of polynomials and solution of polynomial equations.

Write the polynomial $p\,(x) = x^2 + 4x + 1$ as the product of two first degree polynomials. Find the solution of the equation $p\,(x) = 0$

(Score: 4, Time: 5 minute)

Scoring Indicators

$x^2 + 4x + 1 = (x - a)(x - b) = x^2 - (a + b)\,x + ab$ (1)

$a + b = -4$, $ab = 1$, $a - b = 2\sqrt{3}$

$a = -2 + \sqrt{3}$, $b = -2 - \sqrt{3}$ (1)

$x^2 + 4x + 1 = (x + 2 + \sqrt{3})(x + 2 - \sqrt{3})$ (1)

$x^2 + 4x + 1 = 0 \Rightarrow (x + 2 + \sqrt{3})(x + 2 - \sqrt{3}) = 0$ (1)

$x = -2 - \sqrt{3}$, or $x = -2 + \sqrt{3}$

4. Learning Outcome

- Writing polynomial using factors.

In the polynomial $p\,(x) = x^2 + ax + b$, $p\,(3 + \sqrt{2}) = 0$, $p\,(3 - \sqrt{2}) = 0$, write this polynomial after finding a and b.

(Score: 4, Time: 5 minute)

Scoring Indicators

$p\,(x) = x^2 + ax + b$

$p\,(3 + \sqrt{2}) = 0$, $(x - 3 - \sqrt{2})$ is a factor (1)

$p\,(3 - \sqrt{2}) = 0$, $(x - 3 + \sqrt{2})$ is a factor (1)

$p\,(x) = x^2 + ax + b = (x - 3 - \sqrt{2})(x - 3 + \sqrt{2})$

$= (x - 3)^2 - (\sqrt{2})^2$ (1)

$x^2 + ax + b = x^2 - 6x + 7$ (1)

5. Learning Outcome

- Factorisation of the polynomials

What number should be added to the polynomial $p\,(x) = x^2 + x - 1$, so that $(x - 2)$ is a factor the new polynomial.

(Score: 4, Time: 6 minute)

- **Scoring Indicators**

  - $p(x) = x^2 + x - 1$, remainder $p(2)$ (1)
  - $p(2) = (2)^2 + 2 - 1 = 5$ (1)
  - For $(x - a)$ to become a factor of $p(x)$, $p(2)$ must be equal to zero.
    For $p(2) = 0$ here we have to substract 5 from $p(x)$ (1)
    That is, add -5 to $p(x)$ for $(x$-2) become a factor (1)

6. Learning Outcome

- The idea of a factor of a polynomial

What is the smallest natural number k, for which the polynomial $2x^2 + kx + 6$ can be written as a product of two first degree polynomials. Write down the polynomial using k and express it as the product of two first degree polynomials.

(Score: 4, Time: 8 minute)

- **Scoring Indicators**

  - $p(x) = 2x^2 + kx + 6,\ 2\ \ x^2 + \frac{k}{2}x + 3$

$x^2 + \frac{k}{2}x + 3 = x^2 - (a + b)x + ab$

$a + b = \frac{-k}{2}$, $ab = 3$ (1)

$(a - b)^2 = \frac{k^2}{4} - 12 = \frac{k^2 - 48}{4}$

$k^2 - 48 \geq 0,\ k^2 \geq 48$

The smallest natural of k is 7 or (1)

$p(x) = 2x^2 + 7x + 6$

$x = \frac{-7 \pm 1}{4}$

$x = -2$ or $\frac{-3}{2}$ (1)

$p(x) = (x + 2)(2x + 3)$ (1)

7. Learning Outcome

- The idea of factor, remainder and factorisation of a polynomial

The remainder on dividing $p(x) = x^3 - 5x^2 + kx + 19$ by $(x - 3)$ is -5

a) What is the number k?

b) What is the remainder on dividng $p(x)$ by $(x$-4)?

c) What number should be added to $p(x)$ to get a polynomial for which $(x - 3)$ and $(x - 4)$ are factors.

(Score: 5, Time: 8 minute)

- **Scoring Indicators**

$p(x) = x^3 - 5x^2 + kx + 19$ (1)

Remiander = $p(3) = -5$

$p(3) = (3)^3 - 5(3)^2 + k(3) + 19 = -5$

$= 27 - 45 + 3k + 19 = -5$

$k = \frac{-6}{3} = -2$ (1)

- $p(x) = x^3 - 5x^2 - 2x + 19$

  Remiander = $p(4)$

  $p(4) = (4)^3 - 5\,(4)^2 - 2\,(4) + 19$ (1)

  $= 64 - 80 - 8 + 19$ (1)

  $= -5$
- $p(x) + 5 = x^3 - 5x^2 - 2x + 24$ (1)

8. Learning Outcome

- Factorisation of polynomials, and the idea of factor.

$x^2 - 5x + 6$ is a factor of $x^3 - 5x^2 + ax + b$. Find the numbers a and b.

(Score: 5, Time: 7 minute)

Scoring Indicators

- $p(x) = x^3 - 5x^2 + ax + b$ (1)

  Factor $= x^2 - 5x + 6 = (x - 2)(x - 3)$ (1)

  $x - 2$ is a factor $\therefore p(2) = 0$

  $p(2) = (2)^3 - 5\,(2)^2 + 2a + b = 0$

  $2a + b = 12(1)$

  $x - 3$ is a factor $\therefore p(3) = 0$

  $p(3) = (3)^3 - 5\,(3)^2 + 3a + b = 0$ (1)

  $3a + b = 18$

  $2a + b = 12$ (1)

  $a = 6,\ b = 0$

9. Learning Outcome

- Idea of factor of a polynomial

a) $(x^2 - 1)$ is a factor of the polynomial $ax^3 + bx^2 + cx + d$. Prove that $a + c = 0$ and $b + d = 0$

b) What first degree polynomial should be added to $3x^3 - 7x^2 + 2x + 3$ gives a multiple $(x^2 - 1)$

(Score: 4, Time: 6 minute)

Scoring Indicators

(a) $p(x) = ax^3 + bx^2 + cx + d$ (1)

$(x^2 - 1)$ is a factor $\Rightarrow (x + 1)$, $(x - 1)$ are factors (1)

$(x - 1)$ is a factor $p(1) = 0, \Rightarrow p(1) = a + b + c + d = 0$ .... (1)

$(x + 1)$ is a factor $p(-1) = 0, \Rightarrow p(-1) = -a + b - c + d = 0$ .... (2) (1)

(1) + (2) $\Rightarrow 2b + 2d = 0,\ b + d = 0$

$b + d = 0 \Rightarrow a + c = 0$ (1)

(b) Let the polynomial added is $ax + b$

$3x^3 - 7x^2 + 2x + 3 + ax + b$

$3x^3 - 7x^2 + (2 + a)\,x + 3 + b$

$x^2 - 1$ is a factor $\Rightarrow (3 + 2 + a) = 0 \Rightarrow a = -5;\ (-7 + 3 + b) = 0 \Rightarrow b = 4$

$\therefore\ ax + b = -5x + 4$

Polynomial $3x^3 - 7x^2 + 2x + 3$

10. Learning Outcome

- Method to check whether $(x - a)$ and $(x + a)$ are factors of a polynomial $p(x)$

Check whether $(x + 2)$ and $(x - 5)$ are factors of the polynomial $p(x)$

(Score: 4, Time: 6 minute)

Scoring Indicators

- $p(x) = x^2 + 7x + 10$

$p(-2) = 4 - 14 + 10 = 0$ (1)

$\therefore (x + 2)$ is a factor (1)

$p(5) = (5)^2 + 7\,(5) + 10$ (1)

$= 25 + 35 + 10 \neq 0$ (1)

$\therefore (x - 5)$ is not a factor

11. Learning Outcome

- Calculating the remainder on dividing a polynomial by a first degree polynomial without actual division .

Find the remainders on dividing $9x^3 + 18x^2 - 4x - 10$ by $(3x + 2)$ and $(3x - 2)$. Write a third degree polynomial for which $(3x + 2)$ and $(3x - 2)$ are factors of it.

(Score: 5, Time: 7 minute)

Scoring Indicators

- $p(x) = 9x^3 + 18x^2 - 4x - 10$

The remainder on dividing p(x) by ($3x$+2) is $p\,(\frac{-2}{3})$ (1)

$$p\ \frac{-2}{3} = 9 \times\ \frac{-2}{3}^{\ 3} + 18\ \frac{-2}{3}^{\ 2} - 4\ \frac{-2}{3} - 10$$

$$= 9 \times \frac{-8}{27} + 18 \times \frac{4}{9} + \frac{8}{3} - 10 = -2$$ (1)

$$p\ \frac{2}{3} = 9 \times\ \frac{2}{3}^{\ 2} + 18 \times\ \frac{2}{3}^{\ 2} - 4 \times \frac{2}{3} - 10 = -2$$ (1)

$3x + 2,\ 3x - 2$ are factors of $p(x)+2$ (1)

The required polynomial = $9\,x^3 + 18x^2 - 4x - 8$ (1)

12. Learning Outcome

- Calculating the remainder on dividing a polynomial by a first degree polynomial without actual division.

When dividing $x^2 + ax + b$ by $(x - 2)$ and $(x - 3)$ the remainder is zero. What are the numbers a and b.

(Score: 3, Time: 5 minute)

Scoring Indicators

- $p(x) = x^2 + ax + b = (x - 3)(x - 2)$ (1)

  $x^2 + ax + b = x^2 - 5x + 6$ (1)

  $a = -5, \ b = 6$ (1)

13. Learning Outcome

- Identifying the polynomials that cannot be written as the product of two first degree polynomials.

Prove that the polynomial $x^2 + 4x + 5$ cannot be written as a product of first degree polynomials.

(Score: 3, Time: 5 minute)

Scoring Indicators

- $x^2 + 4x + 5 = (x - a)(x - b) = x^2 - (a + b)x + ab$

  $a + b = -4, \ ab = 5$ (1)

  $(a - b)^2 = (-4)^2 - 4 \times 5 = -4$ (1)

  Thus $x^2 + 4x + 5$ cannot be split as a product of first degree polynomials. (1)

14. Learning Outcome

- Identifying the polynomials that cannot be written as the product of two first degree polynomials.

In the polynomial $p(x) = x^2 + 6x + k$.

a) If k = -10, prove that *p(x)* can be written as the product of two first degree polynomials.

b) If k = 10, prove that *p (x)* cannot be written as the product of two first degree polynomials.

c) What is the largest number 'k' for which *p (x)* can be written as the product of two first degree polynomials

(Score: 5, Time: 7 minute)

Scoring Indicators

a) $x^2 + 6x - 10 = x^2 - (a+b)x + ab$

$a + b = -6, \ ab = -10$ (1)

$(a - b)^2 = 36 + 40 = 76$ (1)

Which is a positive number $\therefore$ we can write *p (x)* as a product two first degree polynomial when k = -10 (1)

b) $x^2 + 6x + 10 = x^2 - (a + b)x + ab.$

$a + b = -6, \ ab = 10.$

$(a - b)^2 = 36 - 40 = -4$

Square of a number can't be negative, hence we cannot split the polynomial as a product of first degree polynomials. (1)

c) $x^2 + 6x + k = x^2 - (a + b)\,x + ab$

$$a + b = -6, \quad ab = k$$

$$(a - b)^2 = 36 - 4k$$

If possible $36 - 4k \geq 0 \Rightarrow k \leq 9$ (1)

15. Learning Outcome

- If p(a) = 0 then (*x*-a) is a factor of the polynomial *p(x)*

$p(x) = x^2 - 4x + 4$

a) Prove that $(x - 2)$ is a factor of $p(x)$

b) Prove that for any number x, $p(x)$ is always non negative

c) Find the number a and b such that $p(a) = p(b)$

(Score: 4, Time: 7 minute)

Scoring Indicators

a) $p(x) = x^2 - 4x + 4$

$p(2) = 2^2 - 4 \times 2 + 4 = 0$

$\therefore$ $(x - 2)$ is a factor. (1)

b) $p(x) = x^2 - 4x + 4 = (x - 2)^2$

Square of any number cannot be negative number

$\therefore p(x)$ is nonnegative (1)

c) $p(a) = p(b) = (a - 2)^2 = (b - 2)^2 \Rightarrow a - 2 = \pm (b - 2)$

$a - 2 = b - 2$ or $a - 2 = 2 - b$ (1)

$a = b$ *or* $a + b = 4$

Number with sum 4 are

example:

a	0	1	2	3	4	5	6	...
b	4	3	2	1	0	–1	–2	...

(1)

16. Learning Outcome

- If (*x*-a) is a factor of *p (x)* then *p(a)* = 0

$(x + a)$ is a factor of $x^3 + ax^2 + 2x + a + 4$. What is the number ‘a’?

(Score: 3, Time: 5 minute)

Scoring Indicators

$P(x) = x^3 + ax^2 + 2x + a + 4$ (1)

$(x + a)$ is a factor \ $p(-a) = 0$ (1)

$p(-a) = (-a)^3 + a(-a)^2 + 2(-a) + a + 4 = 0$

$-2a + a + 4 = 0$ (1)

$a = 4$

17. Learning Outcome

- The remainder on dividing $p(x)$ by $(x - a)$ is $p(a)$

The remainder on dividing $x^3 + ax^2 + 7x + 6$ and $x^3 + 5x^2 + bx + 8$ by $(x - 2)$ is same. Prove that $2a - b = 4$

(Score: 5, Time: 7 minute)

Scoring Indicators

$p(x) = x^3 + ax^2 + 7x + 6,\ \ q(x) = x^3 + 5x^2 + bx + 8$ (1)

Remainder on dividing by $(x - 2)$ are equal to $\Rightarrow p(2) = q(2)$ (1)

$p(2) = (2)^3 + a(2)^2 + 7(2) + 6 \Rightarrow 4a + 28$ (1)

$q(2) = (2)^3 + 5(2)^2 + b(2) + 8 \Rightarrow 2b + 36$ (1)

$4a + 28 = 2b + 36 \Rightarrow 4a - 2b = 8$

$2a - b = 4$ (1)

18. Learning Outcome

- If p(a) = 0 then (x-a) is a factor of the polynomial $p(x)$

Write a third degree polynomial if $p(2 + \sqrt{3}) = 0$, $p(2 - \sqrt{3}) = 0$ and $p(1) = 0$

(Score: 4, Time: 7 minute)

Scoring Indicators

since $p\left(2+\sqrt{3}\right)=0,\ \left(x-2-\sqrt{3}\right)$ is a factor of $p(x)$ (1)

since $p\left(2-\sqrt{3}\right)=0,\ \left(x-2+\sqrt{3}\right)$ is a factor of $p(x)$ (1)

since $p(1)=0,\ (x-1)$ is a factor of $p(x)$

$p(x)=\left(x-2-\sqrt{3}\right)\left(x-2+\sqrt{3}\right)(x-1) = (x-2)^2 - 3\,(x-1) = (x^2 - 4x + 1)(x - 1)$ (1)

$= x^3 - 5x^2 + 5x - 1$ (1)

19. Learning Outcome

- Calculating the remainder on dividing a polynomial by a first degree polynomial without actual division

The remainders on dividing $p(x) = x^3 + 4x^2 + ax + 5$ by $(x - 3)$ and $(x + 2)$ are same number. What is the number a?

(Score: 4, Time: 7 minute)

■ **Scoring Indicators**

$p(x) = x^3 + 4x^2 + ax + 5$

Remainder $p(-2) = p(3)$ (1)

$p(3) = (3)^3 + 4(3)^2 + a(3) + 5 = 3a + 68$ (1)

$p(-2) = (-2)^3 + 4(-2)^2 + a \times -2 + 5 = -2a + 13$ (1)

$3a + 68 = -2a + 13 \qquad a = -11$ (1)

20. Learning Outcome

- If $(x - \mathrm{a})$ and $(x + a)$ are factors of the polynomial $p(x)$ then $p(a)$ and $p(-a)$ are equal to zero

a) $x^2 - 4$ is a factor of $ax^3 + bx^2 + cx + d$. Prove that $4(a - b) = d - c$

b) What first degree polynomial should be added to $2x^3 - 4x^2$ for $(x^2 - 4)$ to be its factor

(Score: 4, Time: 10 minute)

■ **Scoring Indicators**

$p\ (x) = ax^3 + bx^2 + cx + d$

$x^2 - 4 = (x + 2)\ (x - 2)$

$p\ (-2) = -8a + 4b - 2c + d = 0$ ________ (1) (1)

$p(2) = 8a + 4b + 2c + d = 0$ _________ (2) (1)

$(1) + (2) \quad 4b \quad d \quad 0, \quad (1) - (2) \quad 4a \quad c \quad 0$

$4b + d = 4a + c$ (1)

$4(a - b) = d - c$

(b) $q(x) = 2x^3 - 4x^2 + ax + b$

$x^2 - 4$ is a factor $\qquad 4 \times 2 + a = 0, \quad a = -8$

Similarly $4 \times -4 + b = 0 \quad b \quad 16$

$\therefore$ first degree polynomial = $-8x + 16$ (1)

21. Learning Outcome

- If $(x - \mathrm{a})$ and $(x + a)$ are factors of the polynomial $p(x)$ then $p(a)$ and $p(-a)$ are equal to zero

$(x^2 - 9)$ is a factor of $ax^3 + bx^2 + cx + d$ then prove that $9(a - b) = d - c$. Write a third degree polynomial with $(x^2 - 9)$ is a factor

(Score: 4, Time: 7 minute)

- **Scoring Indicators**

$p(x) = ax^3 + bx^2 + cx + d$

$(x^2 - 9)$ is a factor $(x+3), (x-3)$ are factors (1)

$p\,(3) = a(3)^3 + b(3)^2 + c(3) + d = 0$

$27a + 9b + 3c + d = 0$ ________ (1) (1)

$p(-3) = -27a + 9b - 3c + d = 0$ ________ (2) t#(1)

$(1)+(2) \quad 9b \quad d \quad 0 \quad (1)-(2) \quad 9a \quad c \quad 0$

$\therefore 9a - 9b = d - c$

$9(a - b) = d - c$ (1)

11

STATISTICS

1. Learning Outcome

- Recognising the contexts where the mean cannot be used to represent a set of measures

10 households in a neighbourhood are sorted according to their monthly income are given below

16500, 21700, 18600, 21050, 19500

17000, 21000, 18000, 22000, 75000

a) What is the mean income of there 10 families?

b) How many families have monthly income less than the mean income? Prove that in such situation this average is suitable or not?

(Score:3, Time :5 minutes)

Scoring indicators

a) Mean = $\frac{\text{sum}}{\text{number}} = \frac{248000}{10} = 24800$ (1)

b) 9 families have monthly income less than the mean income. (1)

So in this situation this is not a suitable average. (1)

2. Learning Outcome

- Explaining the method to compute the median of a set of measures.

Number of members in 10 families, collected by mathematics club survey are given. Calculate mean, median and explain which is the suitable average?

4, 2, 3, 5, 4, 3, 2, 20, 4, 3

(Score: 3, Time :5 minutes)

Scoring indicators

Mean = 5 (1)

Median = 3.5 (1)

Suitable average median = 3.5 (1)

3. Learning Outcome

- Explaining the method to compute the median of a set of measures.

Weekly wages of 9 persons working in a factory are given find the median

2100, 3500, 2100, 2500, 2800
4900, 2300, 2200, 3300

(Score: 2, Time : 3 minutes)

- **Scoring indicators**

Write the number in order.

2100, 2100, 2200, 2300, 2500, 2800, 3300, 3300, 3500 (1)

Median=2500 (1)

4. Learning Outcome

- Explaining the method to compute the median from a set of measures given as a frequency table.

The table shows the workers doing different jobs in a factory according to their daily wages.

Daily wages (Rs)	Number of workers
225	4
250	7
270	9
300	5
350	3
400	2

Calculate the median daily wage

(Score: 3, Time : 4 minutes)

- **Scoring indicators**

Daily wages(Rs)	Number of workers
up to 225	4
up to 250	11
up to 270	20
up to 300	25
up to 350	28
up to 400	30

(1)

The worker is the 12th position to 20th position have daily wage 270

$$\therefore \text{Median} = \frac{\text{wage of } 15^{\text{th}} \text{ person} + \text{wage of } 16^{\text{th}} \text{ person}}{2} \quad (1)$$

$$= \frac{270+270}{2} = 270 \quad (1)$$

5. Learning Outcome

- Explaining the method to compute the median from a set of measures given as a frequency table.

The table below shows 60 children in a class sorted according to their heights

Height (cm)	**Number of children's**
140 - 145	5
145 - 150	8
150 - 155	12
155 - 160	16
160 - 165	11
165 - 170	5
170 - 175	3

Find the median height?

(Score: 2, Time : 3 minutes)

■ **Scoring indicators**

Height (cm)	**Number of children's**
Below 145	5
Below 150	13
Below 155	25
Below 160	41
Below 165	52
Below 170	57
Below 175	60

$$\text{Median} = \frac{\text{height of } 30^{\text{th}} \text{ child} + \text{height of } 31^{\text{st}} \text{ child}}{2} \qquad (1)$$

$$\text{Height of } 30^{\text{th}} \text{ child} = 155 + \frac{5}{32} + 4\,\frac{5}{16}$$

$$= 155\,\frac{5}{32} + \frac{40}{32}$$

$$= 155\,\frac{5}{32} + \frac{40}{32}$$

$$= 156\,\frac{13}{32}$$

$$\text{Height of } 31^{\text{st}} \text{ child} = 156\,\frac{13}{32} + \frac{5}{16}$$

$$= 156\,\frac{23}{32}$$

$$\text{Median} = \frac{\text{height of } 30^{\text{th}} \text{ child} + \text{height of } 31^{\text{st}} \text{ child}}{2}$$

$$= \frac{156\frac{13}{32} + 156\frac{23}{32}}{2}$$

$$= 156\frac{18}{32} \qquad (1)$$

6. Learning Outcome

- Explaining the method to compute the median from a set of measures given as a frequency table.

The table below shows daily wages of 42 workers in a company

Daily wages (Rs)	Number of workers
0 - 50	3
50 - 100	5
100 - 150	14
150 - 200	12
200 - 250	6
250 - 300	3

Find the median

(Score: 2, Time : 3 minutes)

Scoring indicators

Daily wages (Rs)	Number of workers
below 50	3
below 100	8
below 150	22
below 200	34
below 250	40
below 300	43

Wage of $\frac{43+1}{2}^{\text{th}}$ worker is the median wage. (1)

Wage of 22nd worker = median wage.

Wages of 9th worker to 22nd worker are in an arithmetic sequence with first term $100\frac{50}{28}$ and common difference $\frac{50}{14}$

$$100\frac{50}{28} + 13 \times \frac{50}{14} = 100\frac{50}{28} + \frac{650}{14}$$

$$= 100\frac{50}{28} + \frac{1300}{28}$$

$$= 100 + 48\frac{6}{28}$$

$$= 148\frac{6}{28} \qquad (1)$$

SSLC MODEL QUESTION PAPER 2016-17

MATHEMATICS

Standard: X

Time : 2½ hours
Cool off Time: 15 minutes
Score: 80

Instructions:

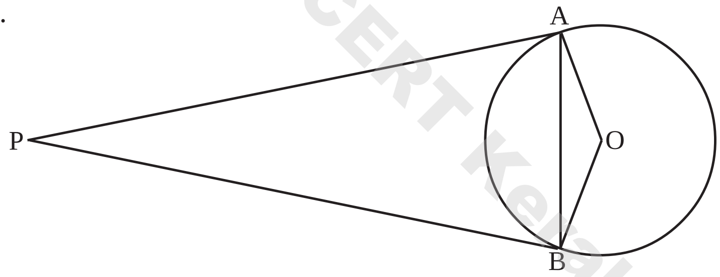
1. Read and understand each question carefully and then only write the answers.
2. Give explanations wherever necessary
3. If there is an 'OR' between any two questions, you may answer only one among them
4. The first 15 minutes is given as "Cool-off Time."You may read and understand the questions during that time.
5. Simiplification using irrational like π, $\sqrt{2}$ $\sqrt{3}$ etc with their approximate values is not required if not specified in the questions.

---

1. Write down an arithmetic sequence with common difference 4 . Can the difference of any two terms of this sequence be 2016? (2)

2. (-3, 4) is a point on a circle with its centre at origin. Does (-4, 2) lie interior to the circle? Why? (2)

3. *PA* and *PB* are two tangents of a circle with its centre at O . If $\angle AOB = 130°$, Find all angles of ΔPAB.



4. $p(x) = x^3 + 6x^2 + 12x + 8$. (2)

   If x is a positive number, can $p(n)$ be zero?

   Among the following, which one is a factor of $p(x)$?

   $[(x - 1), (x - 2), (x + 2), (x - 4)]$ (2)

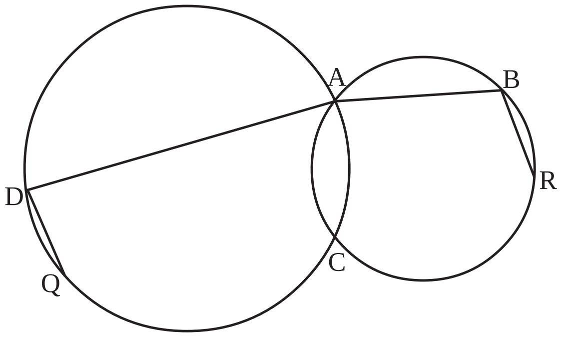
5. The weights of 8 students in a class are given below in kilograms

   37.5, 47.5, 30 ,35, 50, 32.5, 42.5, 45 Find the Mean and Median (2)

6. In the figure, two circles intersect at A and C.

   $\angle ADQ + \angle ABR = 180^0$.

   Prove that Q, C, and R lie on a line. (3)

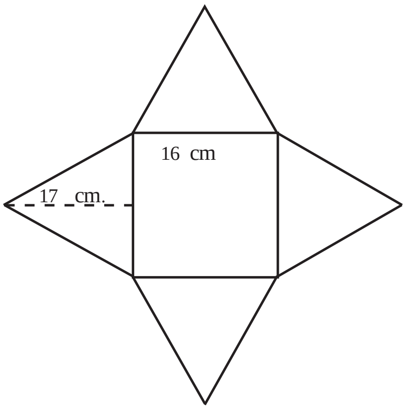


7. The sum of a number and its reciprocal is 4. Find that number (3)

8. A chord makes an angle 35° in a circle of radius 8cm. Find the length of the chord?

   [sin 35° = 0.57, cos 35° = 0.82. tan 35° = 0.7] (3)

9. In the figure, on each side = 16 cm of a square, equal isosceles triangles having height 17 cm are drawn. (3)



The figure is cut out and folded to get a square pyramid. Find its volume

10 **From the following questions only one is to be attended.** (3)

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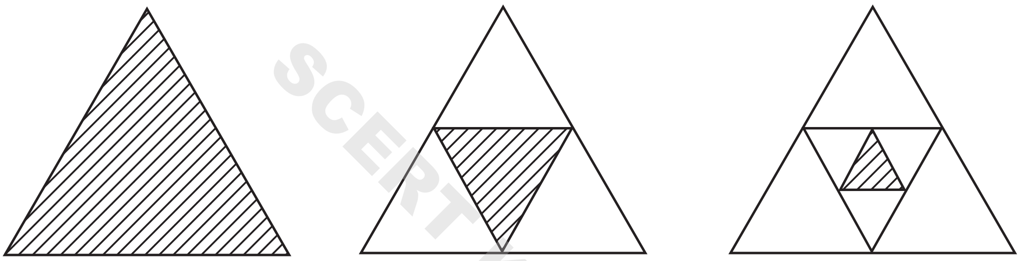


In each figure, the midpoints of the sides of the shaded equilateral triangle are joined and makes the next figure. The area of first equilateral triangle 1 sq.cm.

(a) Write down the sequences of the areas of the shaded portions in the figure.

(b) What is the area of the shaded portion in the 8^{th} figure ?

(c) Write down the algebraic form of this sequence

OR

In an arithmetic sequence, 8 times of its 8th term and 12 times of its 12^{th} term are equal. Find its 20^{th} term?

11. If a dot is put in the incircle of an equilateral triangle without looking into it, (4)

(a) Find the probability of being the dot inside the incircle

(b) Find the probability of being the dot outside the incircle.

12. **From the following questions. Only one is to be attended.**

A line is drawn joining (2, 5) and (8, 9). (4)

(a) Among the following which one is the mid point of this line

[(10, 14); (6, 4); (5, 7); (4, 4)]

(b) Find the radius of the circle having the above line as diameter

(c) Write the equation of the circle

(d) Is (7, 10) a point on this line?

OR

A (3, 2), B (7, 6), C (8, 4) are 3 vertices of a triangle

(a) Find the coordinates of the midpoint of AB.

(b) Find the coordinates of the centroid of triangle

13 (a) What is the sum of first 40 natural numbers? (4)

[(1640; 820; 410; 205)]

(b) What is the sum of first 40 terms of the arithmetic sequances 6, 12, 18... ?

(c) The sum of first 40 terms of an arithmetic sequence with common difference 6 is 5120. Write down the algebraic form of this sequence.

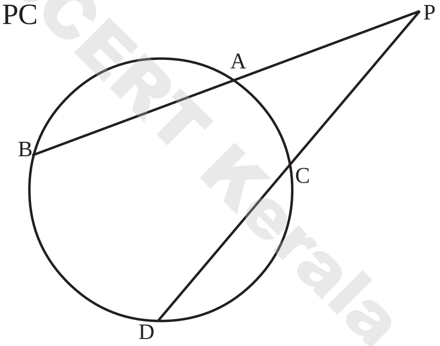
14. Draw a circle of radius 3.5 cm. Construct a triangle with angles $57\frac{1}{2}°$, 65° and the drawn circle as its circumcircle. Measure the sides of the triangle (4)

15. In the figure, BA and DC intersect at P.

AB = 8.5 cm

PA = 5.5 cm

CD = 4 cm. Find the length of PC (4)

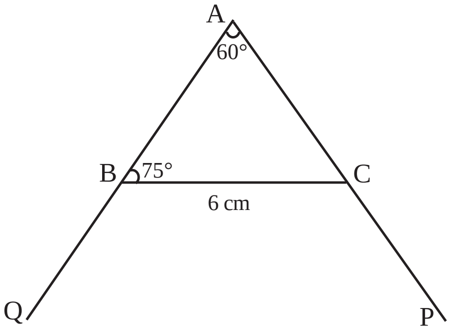


16. *A*(2, 3), *B*(7, 4), *D*(3, 8) are 3 vertices of a parallelogram (4)

(a) Find the coordinates of C

(b) Find the length of diagonals.

17. In ΔABC $BC = 6$ cm, $\angle A = 60°$, $\angle B = 75°$. *AB*, *AC* are extended to get BQ and CP. Construct a circle touching the lines BQ, BC and CP. (4)



18. **From the following questions, only one is to be attended** (4)

What first degree polynomial is added to $(2x^3 + 3x^2)$ to get a polynomial having factor $(x^2 - 4)$

OR

If $x^2 + x + 2 = (x - 2)(x + a) + b$

a) Find a and b

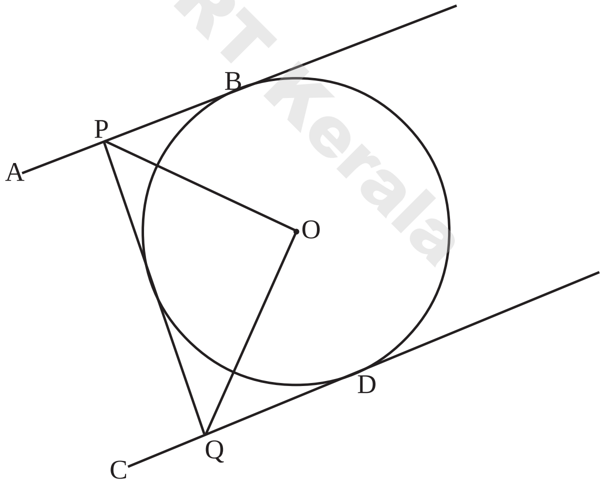
b) What number should be added to get a polynomial having a factor $(x + 3)$

19. Daily income of some families in a locality is tabulated as follows : (4)

Daily Income	**No. of Families**
200 - 300	3
300 - 400	7
400 - 500	10
500 - 600	8
600 - 700	4
700 - 800	3

Calculate the median daily income

20. *AB* and *CD* are two parallel tangents of a circle with centre O. PQ is also a tangent of this circle. Prove that ΔPOQ is right angled. (4)



21. Two children of same height standing on either side of a tower, looks the top of tower at an elevation of 40°, 55°. The distance between the children is 25 metre Height of each one is 1.5 metre. (5)

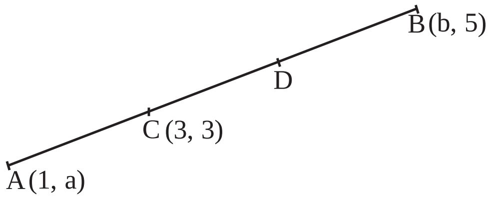
a) Draw a rough figure showing the given measurements

b) Calculate the height of the tower?

[sin 40° = 0.64, sin 55° = 0.82, cos 40° = 0.77, cos 55° = 0.57, tan 40° = 0.84, tan 55° = 1.43]

22. In the figure, the coordinates of *A* and *B* are *A* (1, *a*) and *B* (*b*,5). The points *C* and *D* divide *AB* into

3 equal parts.



a) Find *a* and *b*

b) Find the coordination of *D*

c) Write down the equation of the line (5)

23. From the following questions, only one is to be attended. (5)

If a hemisphere of maximum size is carved out from a cone of base radius 15cm and height 20cm, find its

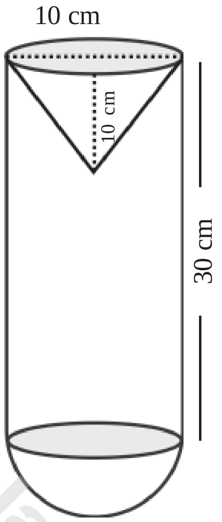
a) Radius b) Volume

OR

On the base of a solid wooden cyclinder of diameter 10 cm and height 30 cm, a hemisphere of same radius is attached.

From the other base of the cycliner, a conical part of diameter and height 10cm is carved out.

Find the volume of this new solid.



SSLC Model Question Paper 2016 -17
Mathematics

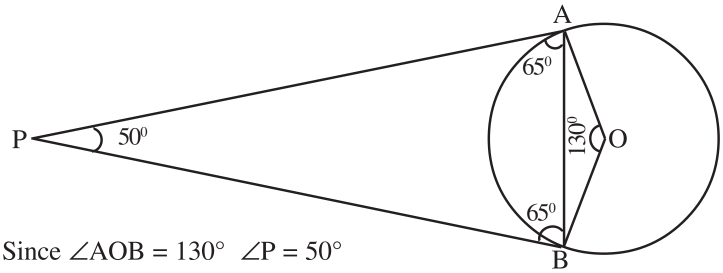
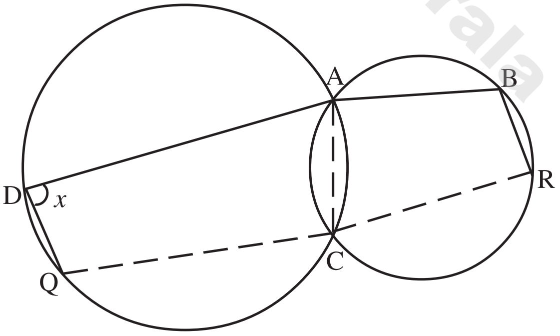
Scoring Indicators

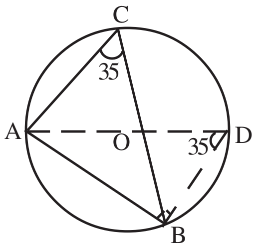
Instructions

- While evaluating the answer scripts the teacher should assess whether the student understands different steps to arrive at the answers. The teacher can give full score, if the student avoids certain steps.
- Full score could be given if the student finds the right answer through right steps that is not included in the evaluation tool. Adequate score should be given if the answer is partially correct
- Do not give much importance to the mistakes in the statements of the answers and the units of measurements.

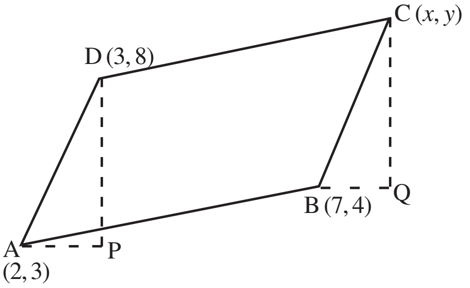
SET - A

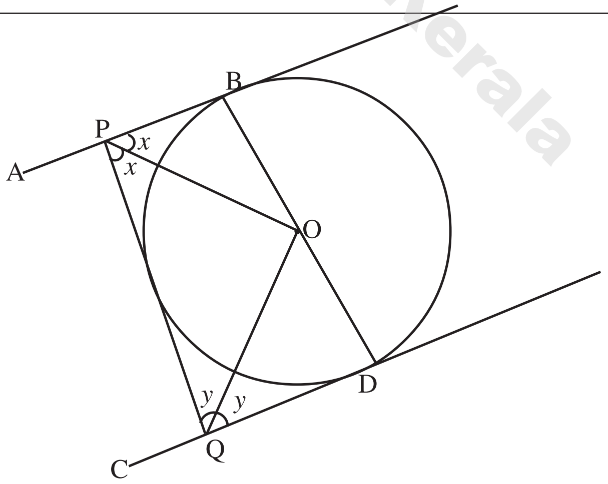
Scoring Indicators

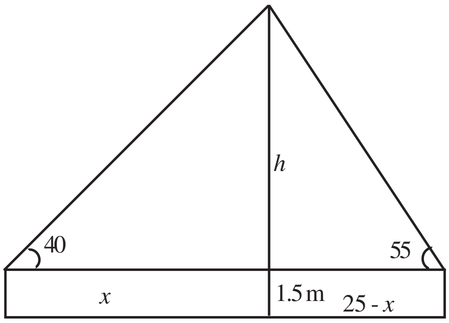
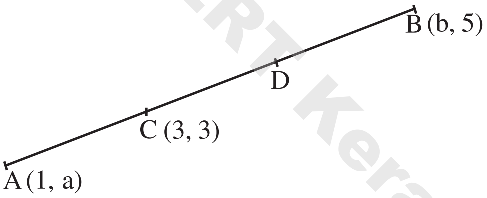
Qn No	Scoring Indicators	Sub Score	Total Score
1	Writing an arithmetic sequence with common difference 4 Being a multiple of 4 The difference of two terms can be 2016.	1 1	2
2	Finding (-4, 2) interior to the circle Justifying the answer	1 1	2
3	 Since $\angle AOB = 130°$ $\angle P = 50°$ Since $PA = PB$ $\angle PAB = \angle PBA = 65°$	1 1	2
4	When x is positive $p(x)$ cannot be zero $p(^{-}2) = 0$, $(x + 2)$ is a factor	1 1	2
5	Mean $= \frac{(37.5+47.5+30+35+50+32.5+42.5+45)}{8} = \frac{320}{8} = 40$ Writing in the ascending order 30, 32.5, 35, 37.5, 42.5, 45, 47.5, 50 Median 40	1 1	2
6	 when AC is drawn ACQD, ACBR are cyclic quadrilaterals $\angle ADQ = \angle ACR$ $\angle ABR = \angle ACQ$ $\angle ADQ + \angle ABR = \angle ACR + \angle ACQ = 180$ Hence Q,C.R lie on a line	1 1 1	3
7	When number $= x$ $x + \frac{1}{x} = 4$ $x^2 - 4x + 1 = 0$ $x^2 - 4x + 4 = 3$ $(x - 2)^2 = 3$	1 1	

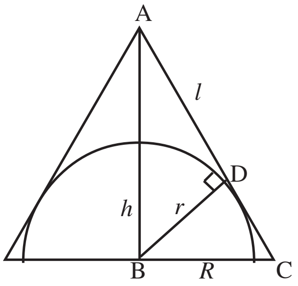
	$x - 2 = \sqrt{3}$ $x = 2 + \sqrt{3}$ Number $2 + \sqrt{3}$ or $\frac{1}{x} = \frac{1}{2+\sqrt{3}} = 2 - \sqrt{3}$	1	3
8	When AD and BD drawn, ΔADB is right $\angle D = 35°$ $\frac{AB}{AD} = \sin 35$ AB = AD sin 35 = 16 sin 35 = 16 ´ 0.57 = 9.12 cm  	1 1 1	3
9	• the base edge = 16cm, slant height = 17cm Using pythaghoras, height = $\sqrt{17^2 - 8^2} = 15$ volume = $\frac{1}{3} \times$ base area $\times$ height $= \frac{1}{3} \times 16^2 \times 15$ = 1280 sq.cm	1 1	3
10	Area of the equilateral triangle formed by joining the m.d. points of the sides is $\frac{1}{4}$ of the first triangle a. $1, \frac{1}{4}, \frac{1}{4^2}, \frac{1}{4^3}$, ... b. 8th term $= \frac{1}{4^7}$ c. Algebraic form $= \frac{1}{4^{n-1}}; n > 1$ $= 1; n = 0$ OR 8 (f + 7d) = 12 (f + 11d) 8f + 56d = 12f + 132d 4f + 76d = 0 f + 19d = 0 20th term = 0	1 1 1 1 1 1	3 3
11	• If circumradius is taken as 'r', circumradius, inradius, half of the side together gives a right triangle with angles 30°, 60°, 90° inradius = $r/2$ Probability of being the dot within the incircle $= \frac{\pi(r/2)^2}{\pi r^2}$ $= \frac{1}{4}$ • Probability of being the dot outside the	1 1 1	4

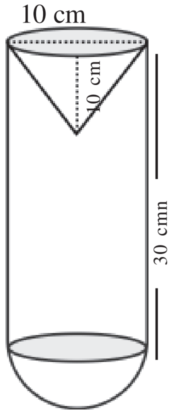
	incircle $= \frac{\pi r^2 - \pi r^2/4}{\pi r^2}$ or $1 - \frac{1}{4} = \frac{3}{4}$	1	
	$= 3/4$		
12	• coordinates of midpoint $= \left(\frac{2+8}{2}, \frac{5+9}{2}\right)$	1	
	Centre = (5, 7)		
	Radius $= \sqrt{(7\text{-}5)^2 + (5-2)^2}$	1	
	$= \sqrt{13}$		
	Let (*x*, y) be a point on the circle. Since centre is		
	(5, 7) we get an equation		
	$(x - 5)^2 + (y\text{-}7)^2 = 13$	1	4
	$x^2 + y^2 - 10x - 14y + 61 = 0$		
	If (7, 10) is a point on the circle		
	We have $(x\text{-}5)^2 + (y\text{-}7)^2 = 13$ icn-bm-I-Ww.		
	$(7\text{-}5)^2 + (10\text{-}7)^2 = 13$		
	Hence (7, 10) is a point on this circle	1	
	OR		
	Coordinates of the mid point of AB $= \left(\frac{3+7}{2}, \frac{2+6}{2}\right)$	1	
	= (5, 4)		
	Centroid will divide CD in the ratio 2 : 1	1	
	$x = 5 + \frac{1}{3}(8\text{-}5) = 5 + 1 = 6$		1
	$y = 4$		
	Centroid (6, 4)	1	
13	a) $1 + 2 + ... + 40 = \frac{40 \times 41}{2} = 820$	1	
	b) The algebraic form of the arithmetic sequence 6, 12, 18, ... 6n		
	sum of first 40 terms = 6 (1+2+......+40)		
	$= 6 \times 820$	1	
	= 4920		4
	c) Difference of first 40 terms of the		
	arthmetic sequence with common difference 6 = 5120 - 4920	1	
	= 200		
	$40 \times 5 = 200$		
	Algebraic form of this sequence = 6n +5	1	
14	Drawing the circle with centre 'O' and radius 3.5cm	1	
	∠AOB = 115°, ∠BOC = 130° By taking the central angles		
	Mark the point A, B, C on the circle	1	
	Drawing ΔADB	1	
	Measuring sides of the triangle	1	4

15	AB = 85 cm., AP = 5.5 cm; PB = 14 cm., CD = 4 cm. PC = x	1	
	then PD = $x + 4$		
	Since, PA × PB = PC × PD	1	
	$x(x + 4) = 77$		
	$x^2 + 4x + 4 = 81$	1	4
	$(x + 2)^2 = 81$		
	$x + 2 = 9 \quad x = 7$		
	PC = 7 cm	1	
16			
	ΔAPD and ΔBQC are equal		
	BP = 1, PD = 5	1	
	then = 1, QC = 5		
	C has the coordinates (8, 9)	1	
	$BD = \sqrt{(7-3)^2 + (4-8)^2} = \sqrt{32} = 4\sqrt{2}$	1	4
	$AC = \sqrt{(8-2)^2 + (9-3)^2} = \sqrt{72} = 6\sqrt{2}$	1	
17	BC = 6 cm., ∠A = 60°, ∠B = 75° drawing the triangle		
	BQ, CP are extending	1	
	Drawing the bisectors of ∠CBQ, ∠PCB centre is determined	1	
	Calculating radius	1	4
	Drawing circle	1	
18	Let the polynomial with $(x^2 - 4)$ as a factor		
	$p(x) = 2x^3 + 3x^2 + ax + b$ since $(x - 2)$, $(x + 2)$ are factors	1	
	$p(2) = 16 + 12 + 2a + b = 0$		
	$2a + b = -28$		
	$p(-2) = -16 + 12 - 2a + b = 0$	1	4
	$2a - b = -4$		
	we get $a = -8$, $b = -12$	1	
	The polynomial to be added $-8x - 12$	1	
	OR		
	$p(x) = x^2 + x + 2 = (x - 2)(x + a) + b$		
	When $x = 2$, $b = 4 + 2 + 2 = 8$	1	
	When $x = 0$, $-2a + b = 2$	1	
	$-2a + 8 = 2$		
	$a = 3$	1	4

	for being $(x + 3)$ as a factor 8 is to be added to $x^2 + x + 2$	1	
	Another Medthod		
	$b = 8$	1	
	$x^2 + x + 2 - 8 = x^2 + x - 6$		
	$x^2 + x - 6 = (x - 2)(x + a)$		4
	$^{-}$-2a = $^{-}$-6, $a = 3$	1	
	When 8 is added $(x + 3)$ is a factor	1	
19	Number of families = 35		
	The income of 18th family is median		
	18^{th} family belongs to 400 - 500		
	The income of 10 families in this group is an arithmetic		
	sequence 405, 415, 425		
	8th term of this sequence is the median (18 - (7 + 3)		
	8th term = 405 + 7 × 10 = 475		
	OR		
	The income of 18th family is the median	1	
	Median belongs 400 - 500	1	
	The income of families in that group 405, 415, 425....an arithmetics		
	sequence	1	4
	Median is the 8th term,		
	Which is 405 + 7 × 10 = 475	1	
20			
	Since AB, CD, PQ are tangents, PO, OQ are bisection of		
	∠BPQ, ∠DQP	1	
	If we take ∠BPQ = 2x, ∠DQP = 2y		
	and since AB, CD are parallel	1	
	$2x + 2y = 180$	1	4
	$x + y = 90$		
	So ∠O = 90	1	
	ΔOPQ is a right triangle		

21			
	Drawing the rough figure	1	
	$h = x \tan 40° = 0.84\,x$		
	$h = (25 - x) \tan 55°$	1	
	$= (25 - x)\,1.43$		
	$1.43\,x + 0.84\,x = 25 \times 1.43$	1	
	$x = \frac{25 \times 1.43}{2.27}$		5
	$= 15.75$ metre	1	
	$h = 15.75 \times 0.84$		
	$= 13.23$	1	
	Height of the tower $= 13.23 + 1.5 = 14.73$ metre		
22			
	C divides AB in the ratio 1:2		
	then $3 = a + \frac{1}{3}(5-a)$		
	$9 = 2a + 5$	1	
	$a = 2$		
	Similarly $3 = 1 + \frac{1}{3}(b-1)$		
	$b = 7$	1	
	D is the midpoint of the line joining C (3, 3), B (7, 5)		
	then D has the coordinates $\frac{3+7}{2}, \frac{5+3}{2} = (5, 4)$	1	5
	slope of the line $= \frac{3-2}{3-1} = \frac{1}{2}$	1	
	Equation of the line		
	$\frac{y-3}{x-3} = \frac{1}{2}$		
	$2y - 6 = x - 3$	1	
	$x - 2y + 3 = 0$		

<table>
<tr><td>23</td><td>Radius of the cone is R and Radius of the largest hemisphere is r.


ΔABD and ΔABC are similar
$$\frac{h}{l} = \frac{r}{R}$$
$$r = \frac{hR}{l}$$
$h = 20,\ R = 15\quad l = 25$
Radius of the semi circle
$$r = \frac{20 \times 15}{25} = 12$$
$$\text{Volume} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times 12^3$$
$$= 1152\,\pi \text{ cubic centimetre}$$</td><td></td><td>5</td></tr>
<tr><td></td><td>Drawing the rough figure</td><td>1</td><td></td></tr>
<tr><td></td><td>ΔABD and ΔABCare similar</td><td>1</td><td></td></tr>
<tr><td></td><td>Slant height of the cone</td><td>1</td><td></td></tr>
<tr><td></td><td>Using proportionality, radius of the hemisphere is calculated</td><td>1</td><td></td></tr>
<tr><td></td><td>Finding the volume</td><td>1</td><td></td></tr>
<tr><td></td><td>OR</td><td></td><td></td></tr>
<tr><td></td><td>volume of the wooden cylinder
$= \pi \times 5^2 \times 30 = 750\,\pi$ cubic centimetre</td><td>1</td><td></td></tr>
<tr><td></td><td>Volume of the hemisphere
$= \frac{2}{3}\pi \times 5^3 = \frac{250}{3}\,\pi$ cubic centimetre</td><td>1</td><td></td></tr>
<tr><td></td><td>Volume of the cone carved out</td><td>5</td><td></td></tr>
<tr><td></td><td>$= \frac{1}{3}\pi \times 5^2 \times 10 = \frac{250}{3}\,\pi$ cubic centimetre</td><td>1</td><td></td></tr>
<tr><td></td><td>Volume of the new solid $= 750\,\pi + \frac{250}{3}\pi - \frac{250}{3}\pi$</td><td>1</td><td></td></tr>
<tr><td></td><td>$= 750\,\pi$ cubic centimetre

</td><td>1</td><td></td></tr>
</table>

WEIGHTAGE TO UNITS

Sl. No.	Unit	Score	Score %
1	Arithematic Sequences	9	11.25
2	Circles	7	8.75
3	Mathematics of chance	4	5
4	Second degree equations	7	8.75
5	Trigonometry	8	10
6	Co-ordinates	6	7.5
7	Tangents	10	12.5
8	Solids	8	10
9	Geometry and Algebra	9	11.25
10	Polynomials	6	7.5
11	Statistics	6	7.5
		80	**100**

BLUE PRINT

Sl. No.	Unit	Unit				Total	
		Very Short Answer		Descriptive			
		No.of Questions	Score	No.of Questions	Score	No.of Questions	Score
1	Arithematic Sequences	2 × 1	2	3 × 1(1) 4 × 1	3(3) 4	3(1)	9(3)
2	Circles			3 × 1 4 × 1	3 4	2	7
3	Mathematics of chance			4 × 1	4	1	4
4	Second degree equations			3 × 1 4 × 1	3 4	2	7
5	Trignometry			3 × 1 5 × 1	3 5	2	8
6	Coordinates	2 × 1	2	4 × 1	4	2	6
7	Tangents	2 × 1	2	4 × 2	8	3	10
8	Solids			3 × 1 5 × 1(1)	3 5(5)	2(1)	8(5)
9	Geometry and Algebra			4 × 1(1) 5 × 1	4(4) 5	2(1)	9(4)
10	Polynomials	2 × 1	2	4 × 1(1)	4(4)	2(1)	6(4)
11	Statistics	2 × 1	2	4 × 1	4	2	6
		5	10	18(4)	70(16)	23(4)	80(16)

Question -wise Analysis

Qn. No	Unit	LO	Learning process	Type of question	Score	Time (mts)
1	Arithmetic sequence	4	Able to classify using known concepts	Very short answer	2	3
2	Coordinates	6	Able to classify using known concepts	Very short answer	2	4
3	Tangents	3	Apply the acquired knowledge	Very short answer	2	3
4	Polynomials	2	Deciding the value	Very short answer	2	3
5	Statitics	2	Forming the concept of finding relations	Very short answer	3	3
6	Circles	2	Finding the logical relationship	Descriptive type	3	6
7	Second degree equations	2	Deciding the value	Descriptive type	3	6
8	Trignometry	2	Apply the acquired knowledge	Descriptive type	3	5
9	Solids	3	Analyzing logically	Descriptive type	3	6
10	Arithmetic sequence	1	Forming the concept by finding relations	Descriptive type	4	6
11	Mathematics of chance	2	Applying the acquired knowledge	Descriptive type	4	6
12	Geometry and Algebra	1, 3	Making conclusion through logical analysis	Descriptive type	4	8
13	Arithmetic sequence	4	Forming the concept by finding relations	Descriptive type	4	7
14	Circles	1	Illustrating by intuition	Descriptive type	4	7
15	Second degree equations	3	Deciding the value	Descriptive type	4	8
16	Coordinates	3	Illustrating by intuition	Descriptive type	4	7
17	Tangents	4	Illustrating by intuition	Descriptive type	4	7
18	Polynomials	2	Finding the logical relationship	Descriptive type	4	8
19	Statistics	3	Deciding the value	Descriptive type	4	7
20	Tangents	3	Finding the logical relationship	Descriptive type	4	6
21	Trigonometry	5	Applying the acquired concept judiciously	Descriptive type	5	8
22	Geometry and Algebra	2	Illustrating by intuition	Descriptive type	5	7
23	Solids	6, 7	Making conclusion through logical analysis	Descriptive type	5	9
	Total				**80**	**140**

SSLC MODEL QUESTION PAPER 2016-17

MATHEMATICS

Standard: X **Time : 2½ hours**

Cool off Time : 15 minutes

Score : 80

Instructions:

1. Read and understand each question carefully and then only write the answers.
2. Give explanations wherever necessary
3. If there is an 'OR' between any two questions, you may answer only one among them
4. The first 15 minutes is given as "Cool-off Time."You may read and understand the questions during that time.
5. Simiplification using irrational like π, $\sqrt{2}$ $\sqrt{3}$ etc with their approximate values is not required if not specified in the questions.

---

1. The scores of some students in an examination are given below

   34, 44, 32, 41, 38, 46, 45 (2)

   Find the mean and median of the scores

2. The difference of two terms of an arithemetic sequence whose common difference is a natural number is 105. Can 9 be its common difference? Why? (2)

3. Is it possible to draw a rectangle with perimeter 48cm and area 150 sq.cm? Why? (3)

4. Two sides of a parallelogram are 12cm and 8cm. Their included angle is $63^{o.}$ (3)

   a) Find the distance between the bigger sides

   b) Find the area of the parallelogram.

   [sin $63^0 = 0.9$, cos $63^0 = 0.45$, tan $63^0 = 1.96$]

5. **From the following questions, only one is to be attended.** (3)

   The sequence obtained by multiplying the consecutive terms of a arithmetic sequence 1,4,7,10..... is 4,28..........

   a) Is the new sequence an arithmetic sequence?

   b) Write down the algebraic form of this sequence

   OR

   Consider the arithmetic sequence 7,9,11.....

   a) Write down the algebraic form of the sum of this sequence.

   b) Which number is added to the sum of first consecutive terms of this sequence to get a perfect square.

6. The side of an equilateral triangle is 10 unit, whose one side is on the *x* axis and one vertex at the origin. Find the coordinates of other two vertices of the triangle. (3)

7. The sum of first two terms of the arithmetic sequence 8,15,22........ is 23, which is not a term of this sequence (4)

    a) What is its algebraic form? [(6*n* + 2 ; 7*n* + 1; 8*n* - 1; 8*n*)].

    b) Can the sum of any two consecutive terms of this sequence be a term of this sequence? Why?

    c) Can the sum of any two terms of this sequence be a term of this sequence? Why?

8. Draw a square of side 6cm and construct a rectangle of same area and with oneside 7cm. (4)

9. In 10A class, there are 20 boys and 20 girls and in 10B class , 15 boys and 25 girls. For participating in a quiz competition, if one student from each class is selected, what is the probability of (4)

    a) Selecting both students boys

    b) Selecting only one girl

    c) Selecting at least one girl

10. A sector of central angle 216^o is cut out from a circular sheet of 20 cm . It is rolled up to form a conical vessel of largest size. (4)

    a) Find the radius of the vessel

    b) Find the height of the vessel

    c) Is this vessel sufficient for filling 2½ litres? Why?

11. Monthly income of employees of a company is given in the following table (4)

Income	**Number of employess**
15000 - 15500	4
15500 - 16000	12
16000 - 16500	25
16500 - 17000	20
17000 - 17500	9
17500 - 18000	5

Find the median of the monthly income

12. $A(3,2)$, $B(9,10)$ and $C(4,2)$ are three vertices of a triangle ABC (4)

    a. Which is the midpoint of AB? [(6, 8) ; (12, 12) ; (6,6) ; (3, 3)]

    b. Write down the equation of the circle with diameter AB

    c. Show that $\angle ACB$ is greater than 90^o

13. From the following questions, only one is to be attened (4)

When the sun is at an elevation of 55^o, the shadow of a tree is 8 metres. What would be the length of the shadow, when the sun is at an elevation of 35^o

[$\sin 55^0 = 0.82$, $\cos 55^0 = 0.57$, $\tan 55^0 = 1.43$, $\sin 35^0 = .57$, $\cos 35^0 = 0.82$ $\tan 35^0 = 0.7$]

OR

Can we cut out a triangle of sides 8 centimetre and its opposite angle 37^o from a circular sheet of diameter 12 centimetre? [$\sin 37^0 = 0.6$; $\cos 37^0 = 0.8$, $\tan 37^0 = 0.75$]

14. Draw a circle of radius 3.5 centimetres. Construct a triangle having two angles 50^o and 70^o whose sides touch the drawn circle. (4)

15. A circle drawn with centre $A(15, 5)$ cuts the x axis at $B(3,0)$ (4)

a) Find the radius of the circle.

b) If C is the point of intersection of the circle with x axis, find the coordinates of C.

c) Find the perimeter of trinangle ABC

16. P is a point 5 centimetre distant from the centre O of a circle. The chord AB drawn through P has length 25 cm. If PA = 9 cm, (4)

a) What is PB?

b) If the radius of the circle is taken as r, find the shortest distance from P to the circle? Find the maximum distance also?

17. The product of the terms at the same position of two arithmetic sequences 5, 7, 9, . . . and 3, 6, 9, . . is 357. (4)

a) Write down the algebraic form of the sequences

b) By taking term position as n, frame a second degree equation

c) Find the terms to get a product 357.

18. From the following questions, only one is to be attended.

$p(x) = x^3 + 2x^2 - 5x - 6$ (5)

a) The remainder when p(x) is divided by (x-2) is [2, 8, 6, 0]

b) What is the polynomial obtained as quotient?

c) Express $p(x)$ as a product of 3 first degree polynomials

OR

$P(x) = 2x^3 - 11x^2 + Kx - 6$

When $P(x)$ is divided by (x-1), remainder obtained is 2

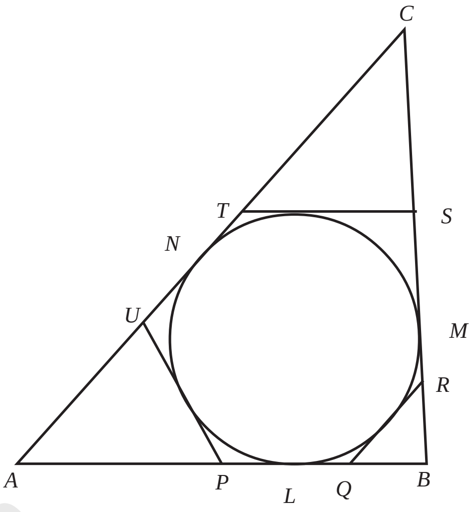
a) Find K?

b) Is $2x - 1$ a factor of p(x)?

c) Is $x^2 - 5x + 6$ a factor of p(x)?

19. From the following questions, only one is to be attended.

In the figure, the incircle of triangle ABC touches the sides of the triangle at L, M and N. QR, ST and UP are other three tangents. (5)



(a) Prove that AL is half the perimeter of ΔAPU

(b) The perimeters of ΔAPU, ΔBQR and ΔCST are 16 cm, 12 cm and 14 cm respectively. Find the lengths of the sides of ΔABC

(c) Find the area of the ΔABC

OR

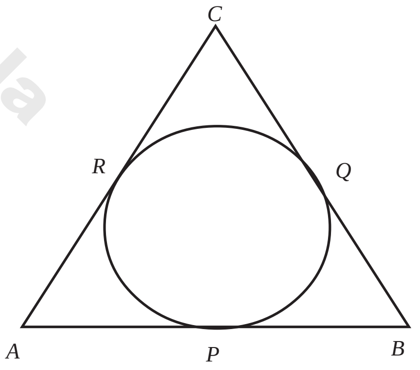
The incircle of ΔABC, touches its 3 sides at P, Q and R as shown

AP = 7 cm. *BQ* = 6 cm. *CR* = 8 cm.

(a) Finds the sides of the ΔABC

(b) Find the area of the triangle

(c) Calculate its inradius



20. A solid sphere is cut into two hemispheres, from one a largest cone and from the other a largest square pyramid is cutout. Find the ratio of the volumes of the cone and the square pyramid (5)

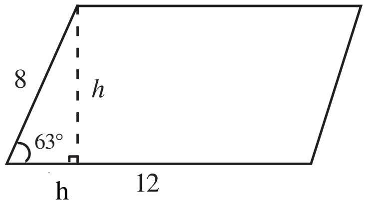
21. *A* (2,3) and *B* (11, 9) are two points on a circle

(a) Find the slope of the line

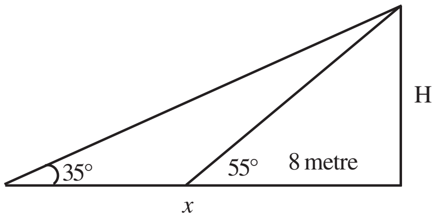
(b) Find the equation of the line

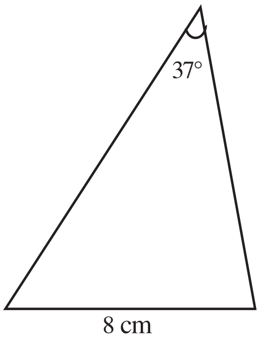
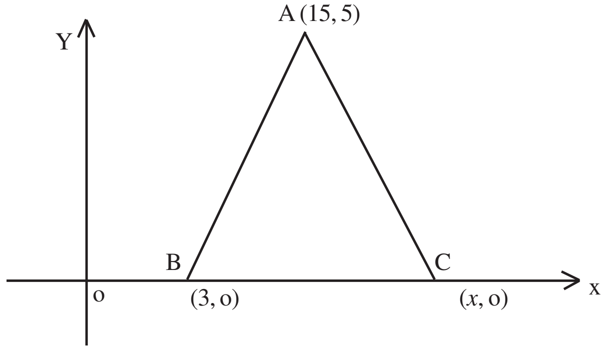
(c) Find the coordinates at two positions of the point C satisfying BC = 2 AC (5)

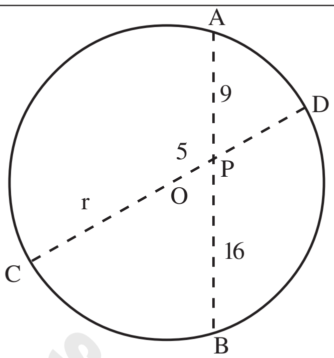
Scoring Indicators

Qn No	Scoring Indicators	Sub Score	Total Score
1	Mean $= \frac{34+44+32+41+38+46+45}{7}$		
	$= \frac{280}{7} = 40$	1	2
	Median :		
	32, 34, 38, 41, 44, 45, 46		
	Median = 41	1	
2	Common difference cannot be 9	1	
	Because 105 is not a multiple of 9	1	2
3	Sides of the rectangle x, $24 - x$		
	$x(24-x) = 150$	1	
	$x^2 - 24x = -150$		
	$(x-12)^2 = -150+144 = -6$	1	3
	The square of a number can't be negative. so we cannot draw rectangle	1	
	OR		
	For the rectangles having same perimeter, square has the largest area	1	
	For the perimeter 48cm, largest = $\left(\frac{48}{4}\right)^2$ = 144, which is less than 150	2	3
	So area cannot be 150 sq.cm		
4		1	
	a) Sin $63^0 = \frac{h}{8}$		
	h $= 8 \times 0.9 = 7.2$ cm	1	3
	b) Area $= 12 \times 7.2$		
	$= 86.4$ sq.cm	1	
5	a) 4, 28, 70, ... is not an arithmetic sequence	1	
	b) For the arithmetic sequence 1, 4, 7, 10, ...		
	n^{th} term = 3n - 2		3
	$(n+1)^{th}$ term = 3n-2 + 3 = 3n+1	1	
	4, 28, 70, ... has the algebraic form (3n-2) (3n+1)	1	

	Or		
	7, 9, 11, ... has the algebraic form		
	$\frac{2n(n+1)}{2} + 5n = n^2 + 6n$	1+1	3
	By adding $\left(\frac{6}{2}\right)^2 = 9$ to $n^2 + 6n$, we get a perfect square	1	
6	One vertex of the equilateral triangle(0, 0)	1	
	The vertex on the *x* axis(10, 0)	1	3
	Third vertex is $(5, 5\sqrt{3})$	1	
7	a) The algebraic term of 8, 15, 22, ... is 7n+1 . Terms of this sequence are 1 added to multiples of 7.	1	
	b) Sum of two consecutive terms		
	= (7n+1) + (7n+8) = 14 n+9		
	= 14n+7+2, which	1	
	is 2 added to multiples of 7. So the sum cannot be a term of this sequence		4
	c) Let any two terms be 7n+1, 7m+1		
	Sum= 7n+1 + 7m+1 = 7 (m+n) + 2, which 2 added to multiples of 7,	1	
	Hence sum cannot be a term	1	
8	Draw ABCD by taking AB= 6cm	1	
	Mark E by extending DA so that AE = 7		
	Extending BA and mark F by taking AF = 6	1	4
	Draw the circumcircle of ΔBEF, which cuts AD at G	1	
	Draw rectangle of length AE and breadth AG	1	
9	Total student pairs in the context = 40 × 40 = 1600	1	
	a) No. of pairs having both boys = 20 × 15 = 300		
	Probability = $\frac{300}{1600} = \frac{3}{16}$	1	
	b) No. of pairs having one girl = 20 × 15 + 25 × 20		4
	= 800		
	Probability = $\frac{800}{1600} = \frac{1}{2}$	1	
	c) No. of pairs having atleast one girl = 20 × 25 + 20 × 15 + 20 × 25		
	= 500 + 300 + 500 = 1300		
	Probability = $\frac{1300}{1600} = \frac{13}{16}$	1	

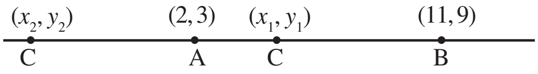
10	a) Radius of the cone $= \frac{216}{360} \times 20$	1	
	$= 12$ cm		
	b) Slant height $= 20$ cm		
	Height $= \sqrt{20^2 - 12^2}$	1	
	$= 16$ cm		
	c) Volume of the vessel $= \frac{1}{3} \times 3.14 \times 12^2 \times 16$	1	4
	$= 2411.52$ cubic centimetre		
	$= 2.412$ litre		
	Vessel is not sufficient for filling 2½ litres.	1	
11	No. of workers = 75	1	
	Median is the income of 38th worker, who belongs to 16000 - 16500 class. The income of 25 workers 16010, 16030, 16050, ... is an arithemetic sequence.	1	4
	38 - 4 - 12 = 22 nd term of this sequence is the median income	1	
	$x_{22} = 16010 + 21 \times 20$	1	
	$= 16430$		
12	a) Midpoint of AB $= \left(\frac{9+3}{2}, \frac{10+2}{2}\right) = (6, 6)$	1	
	Centre of the circle $= (6, 6)$		
	Radius $= \sqrt{(6-3)^2 + (6-2)^2} = 5$	1	
	If (x,y) is a point on this line		
	$(x - 6)^2 + (y - 6)^2 = 5^2$		4
	$x^2 + y^2 - 12x - 12y + 47 = 0$		
	b) OC $= \sqrt{2^2 + 4^2}$	1	
	$= \sqrt{20} < 5$		
	C is interior to this circle	1	
	$\therefore \angle ACB > 90^\circ$		
13		1	
	$\tan 55^\circ = \frac{H}{8}$	1	
	$1.43 = \frac{H}{8}$		

	$H = 11.44$ metre	1	
	$\tan 35^{\circ} = \frac{H}{x}$		
	$x = \frac{11.44}{0.7}$		
	$= 16.34$ metrecm	1	
	OR		
			
	Diametre of the circumcircle of the triangle		
	$= \frac{8}{\text{Sin } 37^{\circ}}$	1	
	$= \frac{8}{0.6} = 13.3$ cm	1	4
	To get a triangle mentioned in the question, diameter of the circle		
	should be 13.3cm	1	
	Hence triangle cannot be cut out	1	
14	Draw a circle of radius 3.5cm with centre O	1	
	By taking central angles $\angle AOB = 130^{\circ}$ & $\angle BOC = 110^{\circ}$ .Mark A,	1	4
	B and C on the circle	1	
	Draw ΔABC	1	
15			
	a) Radius $= \sqrt{12^2+5^2} = 13$ unit	1	
	b) $\sqrt{(x-15)^2+5^2} = 13$		
	$(x-15)^2 = 144$		
	$x-15 = 12$	1	
	$x = 27$		

	Coordinates of C = (27, 0)	1	
	OR		
	Draw AD perpendicular to BC		
	Coordinates of D(15, 0)		4
	BD = 15 - 3 = 12 ; DC = 12		
	Coordinates of C (15 + 12, 0) = (27, 0)		
	c) AB = 13 ; AC = 13; BC = 27-3 = 24		
	Perimeter of ΔABC = 13 + 13 + 24	1	
	= 50 unit		
16			
	a) PB = 25 - 9 = 16 cm	1	
	b) PD = r - 5	1	
	PC = r + 5		
	c) PC × PD = PA × PB	1	4
	(r + 5) (r - 5) = 9 × 16		
	$r^2 - 25 = 144$		
	r = 13 cm	1	
17	a) 5, 7, 9, ... ; $x_n = 2n + 3$		
	3, 6, 9, ... ; $x_n = 3n$	1	
	b) 3n (2n + 3) = 357		
	$2n^2 + 3n = 119$		
	$n = \frac{-3 \pm \sqrt{9 + 4 \times 2 \times 119}}{4} = \frac{-3 \pm 31}{4}$ n being natural number		4
	we take, $n = \frac{-3+31}{4} = 7$	2	
	c) $X_n = 2n + 3$; $X_7 = 17$		
	$X_n = 3n$; $X_7 = 21$	1	
18	Quotient $= ax^2 + bx + c$		
	a) $x^3 + 2x^2 + 5x - 6 = (x - 2)(ax^2 + bx + c)$	1	
	$ax^3 = x^3 \Rightarrow a = 1$		
	$bx^2 - 2ax^2 = 2x \Rightarrow b - 2a = 2$		
	$\Rightarrow b = 2 + 2a = 4$	1	
	$-2c = -6 \Rightarrow c = 3$		
	$ax^2 + bx + c = x^2 + 4x + 3$	1	5
	b) $x^2 + 4x + 3 = 0$		
	$x = -1, -3$	1	

	$x^2 + 4x + 3 = (x + 1)(x + 3)$		
	$p(x) = (x + 1)(x + 3)(x - 2)$	1	
	OR		
	a) p(1) = 2		
	$2 - 11 + k - 6 = 2$	1	
	$k = 17$		
	b) $p\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right)^3 - 11 \times \left(\frac{1}{2}\right)^2 + 17 \times \frac{1}{2} - 6$		
	$= \frac{1}{4} - \frac{11}{4} + \frac{17}{2} - 6 = 0$		
	$(2x - 1)$ is a factor p(x)	1	
c)	$2x^3 - 11x^2 + 17x - 6 = (2x - 1)(ax^2 + bx + c)$		
	$2ax^3 = 2x^3 \Rightarrow a = 1$	1	
	$(2b - a)x^2 = {}^-11x^2 \Rightarrow 2b - a = {}^-11$		5
	$\Rightarrow b = {}^-5$	1	
	${}^-c = {}^-6 \Rightarrow c = 6$		
	$ax^2 + bx + c = x^2 - 5x + 6$		
	$x^2 - 5x + 6$ is a factor p(x)	1	
	OR		
	$x^2 - 5x + 6 = (x - 2)(x - 3)$	1	
	$P(2) = 2 \times 2^3 - 11 \times 2^2 + 17 \times 2 - 6$		
	$= 16 - 44 + 34 - 6 = 0$, thus $x - 2$ is a factor p(x)	1	
	$p(3) = 2 \times 3^3 - 11 \times 3^2 + 17 \times 3 - 6$		
	$= 54 - 99 + 51 - 6 = 0$, thus $(x - 3)$ is a factor p(x)		
	Hence $x^2 - 5x + 6$ is a factor p(x)	1	
19	a) Let the tangent touches the circle at D		
	AP + PU + AU=AP + PD + UD + AU		
	=AP + PL + UN + AU		
	=AL + AN		
	=2AL		
	$\frac{1}{2}$(AP + PU + AU)=AL	1	
	b) AB = AL + BL		
	$= \frac{1}{2} \times$ perimeter of ΔAPU + $\frac{1}{2} \times$ perimeter of ΔBQR	1	
	$= \frac{1}{2} \times 16 + \frac{1}{2} \times 12 = 14$ cm		
	BC= $\frac{1}{2} \times 12 + \frac{1}{2} \times 14 = 13$ cm		
	AC = $\frac{1}{2} \times 14 + \frac{1}{2} \times 18 = 15$ cm	1	5
	c) $s = \frac{14 + 13 + 15}{2} = 21$		
	s - a = 21 - 13 = 8		
	s - b = 21 - 15 = 6		
	s - c = 21 - 14 = 7	1	

	Area $= \sqrt{21\times 8\times 6\times 7}$		
	$=\sqrt{7\times 3\times 4\times 2\times 3\times 2\times 7}$		
	$=7\times 3\ \times 2\ \times 2$		
	=84 sq.cm	1	
	OR		
	a) AP = 7 cm.; AR = 7 cm		
	BQ = 6 cm.; BP = 6 cm	1	
	CR = 8 cm.; CQ = 8 cm		
	AB = 7 + 6 = 13 cm		
	BC = 6 + 8 = 14 cm	1	
	AC = 7 + 8 = 15 cm		
	b) $s = \frac{13+14+15}{2} = 21$		
	s - a = 21 - 13 = 8		
	s - b = 21 - 14 = 7		
	s - c = 21 - 13 = 6	1	5
	Area $= \sqrt{21\times 8\times 7\times 6}$		
	=84 sq.cm	1	
	c) Inradius $= \frac{\text{Area}}{\frac{1}{2}\text{perimeter}}$		
	$=\frac{84}{21} = 4$ cm	1	
20	If the radius of the sphere is r		
	Base diagonal of the pyramid = $2r$	1	
	Base edge $= \frac{2r}{\sqrt{2}} = \sqrt{2}r$	1	
	Height $= r$		
	Volume $= \frac{1}{3}\times\left(\sqrt{2}r\right)^2\times r = \frac{2}{3}\mathrm{r}^3$	1	
	Height of the cone $= r$		
	Radius $= r$		
	Volume $= \frac{1}{3}\pi r^2 r$		5
	$= \frac{\pi}{3}r^3$	1	
	Ratio fo the volumes $= \frac{\pi}{3}r^3 : \frac{2}{3}r^3$		
	$= \pi : 2$	1	

21	a) Slope $= \frac{9-3}{11-2} = \frac{6}{9} = \frac{2}{3}$	1	
	b) $\frac{y-3}{x-2} = \frac{2}{3}$ $3y - 9 = 2x - 4$ $3y = 2x + 5$	1	
	c) 	1	
	C has two positions When C lies between A and B $x_1 = \frac{1}{3}(1 \times 11 + 2 \times 2) = 5$ $y_1 = \frac{1}{3}(1 \times 9 + 2 \times 3) = 5$ Position of C is (5, 5)	1	5
	c, When C lies left of A, the midpoint of BC is A Let $C(x_2, y_2)$ $x_2 = 2 \times 2 - 11 = {}^-7$ $y_2 = 2 \times 3 - 9 = {}^-3$ Position of C is $({}^-7, {}^-3)$	1	

Weightage to Units

Sl.No	Units	Score	Score %
1	Arithmetic sequence	9	11.25
2	Circles	8	10
3	Mathmetics of chance	4	5
4	Second degree equation	7	8.75
5	Trignometry	7	8.75
6	Coordinates	7	8.75
7	Tangents	9	11.25
8	Solids	9	11.25
9	Geometry and Algebra	9	11.25
10	Polynomials	5	6.25
11	Statistics	6	7.5
		80	100

Blue print

Sl.No	Units	Unit				Total	
		Very short answer		Descriptive type			
		Number of questions	Score	Number of questions	Score	Number of questions	Score
1	Arithmetic sequence	2 × 1	2	3 × 1(1) 4 × 1	3(3) 4	3(1)	9(3)
2	Circles			4 × 2	8	2	8
3	Mathmetics of chance			4 × 1	4	1	4
4	Second degree equation			3 × 1 4 × 1	3 4	2	7
5	Trignometry			3 × 1 4 × 1(1)	3 4(4)	2(1)	7(4)
6	Coordinates			3 × 1 4 × 1	3 4	2	7
7	Tangents			4 × 1 5 × 1(1)	4 5(5)	2(1)	9(5)
8	Solids			4 × 1 5 × 1	4 5	2	9
9	Geometry and Algebra			4 × 1 5 × 1	4 5	2	9
10	Polynomials			5 × 1(1)	5(5)	1(1)	5(5)
11	Statistics	2 × 1	2	4 × 1	4	2	6
		2	4	19(4)		21(4)	80(17)

Note: The number shown in brackets denotes the choice.

Question - wise Analysis

Qn. No	Unit	Learning Outcomes	Learning Process	Type of question	Score	Time
1	11	1	Forming the concept by finding relations	Very short answer	2	3
2	1	3	Finding similarity and its converse	Very short answer	2	4
3	4	2	Determining the value	Descriptive type	3	5
4	5	4	Applying the known concept	Descriptive type	3	6
5	1	3	Finding the logical reason	Descriptive type	3	5
6	6	3	Illustrating by intuition	Descriptive type	3	5
7	1	1	Finding the logical reasoning relation	Descriptive type	4	7
8	2	5	Illustrating by intuition	Descriptive type	4	8
9	3	3	Applying the known concept	Descriptive type	4	8
10	8	4	Analysing experience logically	Descriptive type	4	7
11	11	3	Determining the value	Descriptive type	4	7
12	9	3	Forming the concept by identifying the relation	Descriptive type	4	7
13	5	5	Ascertaining the value	Descriptive type	4	6
14	7	4	Illustrating by intuition	Descriptive type	4	8
15	6	3	Determining the value through illustrating by intuition	Descriptive type	4	7
16	2	2	Solving by finding logical reasoning	Descriptive type	4	7
17	4	3	Determining the value	Descriptive type	4	8
18	10	1, 2	Finding the solution through observing logical reasoning relation	Descriptive type	5	10
19	7	4	Finding the solution through observing logical reasoning relation	Descriptive type	5	9
20	8	3, 6, 7	Forming new concept by observing logical reasoning relation	Descriptive type	5	10
21	9	2	Illustrating by intuition , ascertaining value	Descriptive type	5	8
			Total		**80**	**145**

First Terminal Evaluation - 2016
MATHEMATICS

Standard: X

Time: 2½ hours
Score: 80

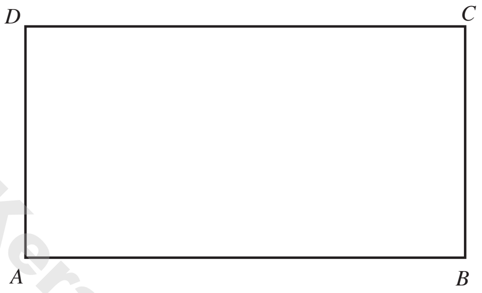
Instructions

1. The first 15 minutes is given as 'cool off time'. You may read and understand the questions during this time.
2. Answer all the questions.
3. If there is an ***OR*** between any two questions, you may answer one among them.
4. Simplification using irrationals like π, $\sqrt{2}$, $\sqrt{3}$ etc., with their approximate values is not required if not specified in the question.

---

1. Algebraic form of an arithmetic sequence is $7n + 3$. What is its common difference? What is its 16th term? (2)

2. *ABCD* is a rectangle. If a circle is drawn with *AB* as diameter, will it pass through *C*? Justify your answer.



(2)

3. Paper slips with numbers 1 to 50 are kept in a box. If a slip is taken without looking into it,
   i) What is the probability of getting an even number?
   ii) What is the probability of getting a multiple of 3 or 7?
   iii) What is the probability of getting a prime number? (3)

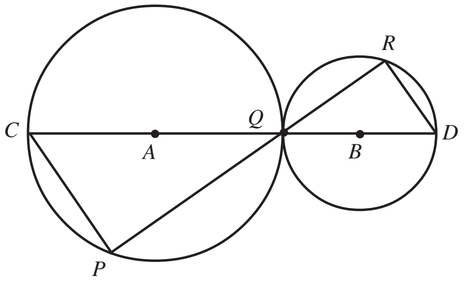
4. The product of two alternate odd numbers is 621. What are the numbers? (3)

5. The common difference of an arithmetic sequence is 8 and its 7th term is 45. Find the 12th term and the position of 285 in the sequence. (3)

6. A circle is drawn with the hypotenuse of a right triangle as its diameter. Is the third vertex of the triangle lie outside the circle or on the circle or inside the circle? If circles are drawn with perpendicular sides as diameters, what can we say about the position of the third vertex? (3)

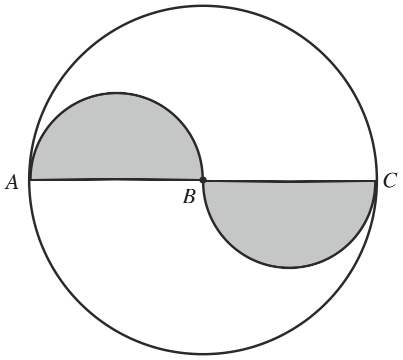
7. The algebraic form of the sum of an arithmetic sequence is $4n^2 + 5n$. Write the sequence. (3)

8. In the figure, *CQ* and *QD* are diameters of the circles with centres at *A* and *B* respectively. Prove that DR is parallel to *PC*.



(3)

9. In the figure, *B* is the centre of the large circle. Shaded portions are two semicircles. If a point is marked in the figure without looking into it:
   - i) What is the probability of the point being marked in the shaded portion?
   - ii) What is the probability of the point being marked outside the shaded portion?



(3)

OR

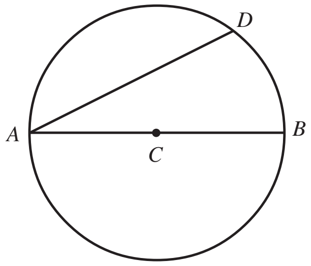
Two dice with face numbers 1 to 6 are rolled together.
   - i) What is the probability of getting same numbers?
   - ii) What is the probability of getting a sum 8?

10. Consider the arithmetic sequence −193, −186, −179, ...
   - i) Is '0' a term in the sequence? Justify.
   - ii) How many terms in this sequence are negative? (3)

11. Construct a square of area 10 square centimetres. (4)

12. 6, 8, 10, ... and 9, 12, 15, ... are two arithmetic sequences. The product of two terms in the same positions of the sequences is 726.
   - i) Write the algebraic form of both the sequences.
   - ii) Find the terms whose product is 726? (4)

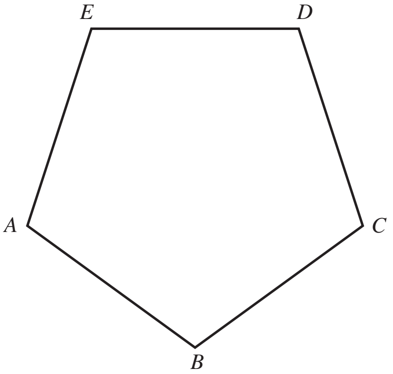
13. In the figure, *C* is the centre of the circle with diameter *AB*. *D* is a point on the circle. Prove that the circle with diameter *AC* passes through the midpoint of *AD*.



(4)

OR

In the figure, *ABCDE* is a regular pentagon.



i) If a circle is drawn with *AC* as the diameter, where will be the position of *B*; inside, outside or on the circle?

i) Can we draw a circle passing through the points *A*, *C*, *D* and *E*? Why?

14. The 6th term of an arithmetic sequence having common difference 4 is 29.
    - i) Write the algebraic form of the sequence.
    - ii) What is the sum of the first 20 natural numbers?
    - iii) What is the sum of the first 20 terms of the above sequence? (4)

15. Construct a triangle with two angles 50^o and 65^o and circumradius 3 centimetres. Write the length of the sides of the triangle. (4)

16. Is there any perfect square in the arithmetic sequence 8, 14, 20, ...? Justify your answer. (4)

17. 25 girls and 20 boys are studying in Standard 10 A. 20 girls and 15 boys are in 10 B. One student from each division is to be selected for a competition.
    - i) What is the probability of getting both are girls?
    - ii) What is the probability of getting at least one girl? (4)

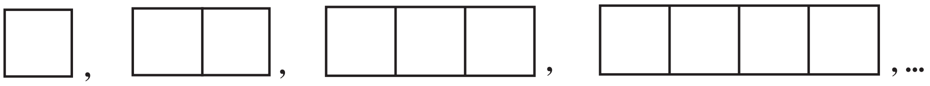
18. $x + 3, 3x - 1, 4x, \ldots$ is an arithmetic sequence.
    - i) Find x?
    - ii) Can 2016 be the difference of any two terms of this sequence? Justify your answer.(4)

OR

3, 10, 17, ... and 54, 58, 62, ... are two arithmetic sequences. Many common terms occur on both the sequences. If a number occur at the same position of both the sequences, find that number?

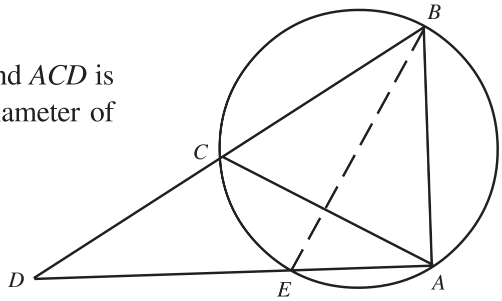
19. Draw a rectangle of length 5 centimetres and breadth 4 centimetres. Construct a new rectangle having the same area and one of its sides as 6 centimetres. (5)

20. Look at the pattern of squares made by sticks of same length.



    - i) Write the sequence of the number of sticks in each figure.
    - ii) Write the sequence of number of rectangles including squares in each figure.
    - iii) Write the algebraic form of both the sequences. (5)

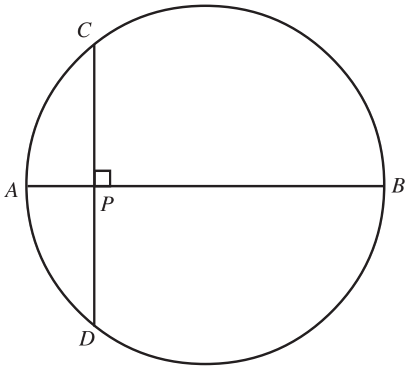
21. In the figure, *ABC* is an equilateral triangle and *ACD* is an isosceles triangle. Prove that *BE* is the diameter of the circle.



(5)

OR

In the figure, *AB* is a diameter of the circle and *CD* is perpendicular to *AB*.



If *CD* = 18 centi- metres and AP = 3 centimetres.

i) What is the diameter?

ii) Can the length of any chord through *P* be a natural number? Justify your answer.

22. Some polygons of 9 sides are drawn having angles in arithmetic sequence.

i) If one angle of all the polygons are the same, what is that angle?

ii) Is it possible to draw such a polygon having 9 sides with the least angle 100^o? Justify your answer. (5)

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Scoring Indicators (Class X)
Set A

Qn No	Scoring Indicator	Score Details	Total Score
1.	Finding,common difference as 7	1	
	Finding 16th term as 105	1	2
2.	Since $\angle ACB < 90^o$, circle does not pass through C.	1 + 1	2
3.	Total Outcomes = 30		
	i) Favourable Outcomes = 15		
	Probability = $\frac{15}{30} = \frac{1}{2}$	1	
	ii) Favourable Outcomes = 13		
	Probability = $\frac{13}{30}$	1	
	iii) Probability = $\frac{10}{30} = \frac{1}{3}$	1	3
4.	If x is an odd number, then alternate odd number = $x + 4$	1	
	$x(x + 4) = 621$		
	$x^2 + 4x + 4 = 625$	1	
	$(x + 2)^2 = 625$		
	$x = 23, \ x + 4 = 27$	1	3
5.	12th term = 7th term + 5 × common difference		
	= $45 + 5 \times 8 = 85$	1	
	12th term + n × common difference = 285		
	$85 + 8n = 285$		
	$n = 25$	1	
	The term position of 285 = $12 + 25 = 37$	1	3
6.	The angle inside a semi circle is right angle		
	A circle drawn with hypotenus as diameter will pass through		
	the right angled vertex	1 + 1	
	The angles subtented by the perpendicular side on the hypotenese		
	will be less than 90^0.	1	3
7.	Common difference = $4 \times 2 = 8$	1	
	first Term = $4 + 5 = 9$	1	
	Sequence : 9, 17, 25, ...	1	3
8.	The angle in a semi circle is right angle	1	
	$\angle CPQ = \angle DRQ = 90^o$ the line which makes equal angles		
	to a line are parallel	1	
	CP, DR are parallel	1	3

Qn No	Scoring Indicator	Score Details	Total Score
9.	If the radius of semicircle is *r*, then the radius of larger circle is 2*r* Total outcomes = $\pi \times (2r)^2 = 4\pi r^2$	1	
	i) Favourable outcomes = $2 \times \frac{\pi r^2}{2} = \pi r^2$ Probability = $\frac{\pi r^2}{4\pi r^2} = \frac{1}{4}$	1	
	ii) Probability of not in the shaded region $= \frac{3\pi r^2}{4\pi r^2} = \frac{3}{4}$ [or $1 - \frac{1}{4} = \frac{3}{4}$]	1	3
	OR Total Outcomes = 6 × 6 = 36	1	
	i) Favourable outcomes = 6 Probability = $\frac{6}{36} = \frac{1}{6}$	1	
	ii) Favourable outcomes = 5 Probability= $\frac{5}{36}$	1	3
10.	Position of '0' = $\frac{0-(-193)}{7} + 1 = \frac{193}{7} + 1$	1	
	$= 28\frac{4}{7}$ '0' is not a term in the sequence 1 there are 28 terms before 0 That is 28 negative terms	1	3
11.	• Drawing *AB* = 7 cm. (*AB* = 11cm.)	1	
	• Mark *P* in *AB* such that *PA* = 5 cm. *PB* = 2 cm.. (*PA* = 10, *PB* = 1)		
	• Drawing a semi circle with *AB* as diameter	1	
	• Draw line perpendicular *AB* passing through *P*, which meets the semi circle at C	1	
	• Draw square with *PC* as a side	1	4
12.	(i) The algebraic expression of 6, 8, 10, ... is $x_n = 2n + 4$ The algebraic expression of 9, 12, 15, ... is $x_n = 3n + 6$ The terms at the same position be taken as the n^{th} the term	1	
	(ii) The Product of n^{th} terms of the above sequence is 726 $(2n + 4)(3n + 6) = 726$	1	
	$2(n + 2)\,3(n + 2) = 726$ $6(n + 2)^2 = 726$	1	
	$(n + 2)^2 = 121$ $n = 9$ Then the terms are 22, 23 OR	1	4
	$(2n + 4)(3n + 6) = 6n^2 + 24n + 24 = 726$ $\Rightarrow n^2 + 4n + 4 = 121$ by solving this quadratic we can find '*n*'		

Qn No	Scoring Indicator	Score Details	Total Score
13.	• Circle with diameter AC meet AD at P	1	
	• Draw PC	1	
	• Since the angle in a semi circle is right angle, $\angle APC = 90^{o}$	1	
	• Since CP, is perpendicular to AD, P is the midpoint of AD	1	4
	OR		
	• Since $ABCD$ regular pentagon, $\angle B = 108^{o}$	1	
	• Since $\angle B > 90^{o}$, B will be inside a circle with diameter AC	1	
	• Since ΔABC is an isosceles triangle , $\angle BAC = 36^{o}$	1	
	• $\angle CAE = 108^{o} - 36^{o} = 72^{o}$		
	• $\angle CAE + \angle CDE = 72^{o} + 108^{o} = 180^{o}$		
	• $ACDE$ is a cyclic quadrilateral	1	4
14.	(i) x_n =6th term + $(n - 6)$ common difference		
	$= 29 + (n - 6)\,4 = 4n + 5$	1	
	(ii) $1 + 2 + 3 + ... + 20 = \frac{20 \times 21}{2} = 210$	1	
	(iii) Sum of 20 terms $= 4 \times 210 + 5 \times 20$	1	
	$= 940$	1	4
15.	• Draw a circle with centre O and radius 3 cm	1	
	• Mark A, B, C in the circle such that $\angle AOB = 100^{o}$; $\angle BOC = 130^{o}$	1	
	• Draw ΔABC	1	
	• Measure the sides	1	4
16.	The algebraic expression of 8, 14, 20, ... is $x_n = 6n + 2$ when the terms in the sequence are divided by 6 we get remainder 2.	1	
	Any integer will be of the form $6n, 6n \pm 1, 6n \pm 2, 6n \pm 3$		
	$(6n)^2 = 36n^2$, When divided by 6, remainder 0		
	$(6n \pm 1)^2 = 36n^2 \pm 12n + 1$ When divided by 6, remainder 1	1	
	$(6n \pm 2)^2 = 36n^2 \pm 24n + 4$ When divided by 6, remainder 4		
	$(6n \pm 3)^2 = 36n^2 \pm 36n + 9$ When divided by 6, remainder 3	1	
	When perfects squares are divided by 6 we get remainders 0, 1, 3, 4		
	Since the terms of the sequence 8, 14, 20, ... when divided by 6 remainder is 2, there are no perfect squares in the given sequence.	1	4

Qn No	Scoring Indicator	Score Details	Total Score
17.	Total outcomes $= (25 + 20)(20 + 15)$		
	$= 1575$	1	
	(i) Favourable outcomes $= 25 \times 20 = 500$		
	Probability $= \frac{500}{1575} = \frac{20}{63}$	1	
	(ii) Favourable outcomes$= 25 \times 20 + 25 \times 15 + 20 \times 20$	1	
	Probability $= \frac{1275}{1575} = \frac{53}{63}$	1	4
18.	(i) The sum of first and third term will be two times the middle term	1	
	$2(3x - 1) = x + 3 + 4x$		
	$6x - 2 = 5x + 3$		
	$x = 5$	1	
	Sequence 8, 14, 20, ...		
	(ii) Difference between two terms will be the multiple of common difference $2016 = 6 \times 336$ (is a multiple of common difference 6)	1	
	Therefore difference between two terms can be 2016	1	4
	OR		
	The algebraic expression of 3, 10, 17, ... is $x_n = 7n - 4$	1	
	The algebraic expression of 54, 58, 62, ...is $x_n = 4n + 50$	1	
	If the nth term of two sequences are same,		
	$7n - 4 = 4n + 50$	1	
	$3n = 54$		
	$n = 18$		
	$n = 18; 7n - 4 = 7 \times 18 - 4$	1	4
	$= 122$		
19.	• Draw a square $ABCD$ with $AB = 5$ cm , $AD = 4$ cm		
	• Extend AB so that $BE = 1$ cm.	1	
	• Extend DA so that $AF = AE = 6$ cm	1	
	• Extend BA so that $AG = 4$ cm	1	
	• Draw the circumcircle of ΔBFG, which meet AD at P.	1	
	• Draw a rectangle with sides AE and AP	1	5
20.	i) 4, 7, 10, ...	1	
	ii) $1, \quad 1 + 2, \quad 1 + 2 + 3, \quad 1 + 2 + 3 + 4, ...$		
	$1, \quad 3, \quad 6, \quad 10$	1	
	iii) $4, \quad 4 + 3, \quad 4 + 2 \times 3, \quad 4 + 3 \times 3, ...$		
	$x_n = 4 + (n - 1)3 = 3n + 1$	1	
	$1, \quad 1 + 2 \quad 1 + 2 + 3, \quad 1 + 2 + 3 + 4, ...$	1	
	$x_n = 1 + 2 + 3 + 4 + ... + n$		
	$= \frac{n(n+1)}{2}$	1	5

Qn No	Scoring Indicator	Score Details	Total Score
21.	Since ΔACB is equilateral,		
	$\angle ACB = 60^o$	1	
	$\angle ACD = 120^o$	1	
	Since ΔACD is isosceles		
	$\angle CAD = 30^o$	1	
	$\angle BAD = \angle BAC + \angle CAD$		
	$= 60^o + 30^o = 90^o$	1	
	Since angle in a semi circle is right angle, BE diameter	1	5
	OR		
	(i) $PA \times PB = PC^2$	1	
	$PB = \frac{81}{3} = 27$ cm		
	$AB = 3 + 27 = 30$ cm	1	
	(ii) $PA \times PB = 3 \times 27 = 81$		
	If EF is a chord through P		
	$PE \times PF = 81$	1	
	If the length of EF will be a natural number then $PE = 3$cm		
	$PF = 27$ cm or $PE = 1$ cm $PF = 81$ cm		
	$PE = 1$ cm, $PF = 81$ cm then $EF = 82$cm, not possible	1	
	$PE = 3$ cm $PF = 27$ cm then $EF = 30$ cm		
	Therefore there is no diameter passing through P other than AB.	1	5
22.	Sum of angles $= 7 \times 180^o$		
	(i) 5th term $= \frac{7 \times 180}{9}$		
	$= 140^o$	1	
	The 5th term of all arithmetic sequences is 140^o	1	
	(ii) If the smallest angle is 100^o, then first term = 100		
	First term $+ 4 \times$ common difference = 5th term		
	$100 + 4d = 140$		
	$d = 10$	1	
	9th term = 5th term + $4d$		
	$= 140 + 4 \times 10 = 180$		
	The angle of a polygon can't be 180^o	1	5

First Terminal Evaluation - 2016
MATHEMATICS

Standard: X **Time: 2½ hours**
Score: 80

Instructions

1. The first 15 minutes is given as ‘cool off time’. You may read and understand the questions during this time.
2. Answer all the questions.
3. If there is an ***OR*** between any two questions, you may answer one among them.
4. Simplification using irrationals like π, $\sqrt{2}$, $\sqrt{3}$ etc., with their approximate values is not required if not specified in the question.

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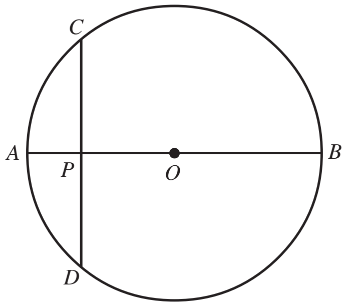
1. 98 is a term of the arithmetic sequence having common difference 7. Is 2016 a term of this sequence? Why? (2)

2. One angle of a triangle is 130^o. If we draw a circle with diameter as the opposite side of this angle, examine whether the vertex is inside, outside or on the circle. (2)

3. i) What is the sum of first 20 natural numbers?
   ii) The algebraic form of an arithmetic sequence is $6n + 5$. Find the sum of the first 20 terms of this sequence. (3)

4. 12 balls are in a box. 5 among them are blue and others are black. Without looking into the box, one ball is taken.
   i) Find the probability of getting a blue ball?
   ii) After putting one blue and one black ball into the box, a ball is taken. Is the probability of getting a blue ball increases or decreases? Justify your answer. (3)

5. In the figure, AB is a diameter of the circle and the chord CD is perpendicular to AB. If $CD = 4\sqrt{5}$ centimetres and $PA = 2$ centimetres, find AB.

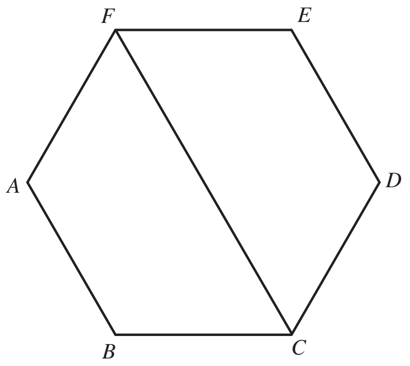


(3)

6. Consider the arithmetic sequence 171, 167, 163, ...
   i) Is ‘0’ a term of this sequence? Justify.
   ii) How many positive terms are in this sequence? (3)

7. The difference of the length of perpendicular sides of a right triangle is 10 centimetres. Its area is 72 square centimetres. Find the length of the perpendicular sides. (3)

8. Examine whether the two quadrilaterals obtained by joining the vertices *F* and *C* of a regular hexagon *ABCDEF* are cyclic or not? Why?



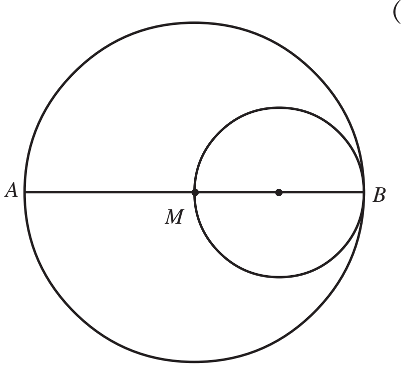
(3)

9. 10th term of an arithmetic sequence is 82. If its common difference is 8, find the position of the term 250 in the sequence? (3)

10. In the figure *M* is the centre of the larger circle. A smaller circle is drawn with diameter as the radius of the larger circle as shown in the figure. Without looking in to the figure, a point is marked.

What is the probability that the point is inside the smaller circle?

What is the probability that the point is outside the smaller circle?



(3)

OR

i) How many distinct 3 digit numbers can be written using the digits 4, 6 and 9 without repeating the digits?

ii) What is the probability that the numbers are odd numbers?

iii) What is the probability that the numbers are even numbers?

11. Consider the arithmetic sequence 10, 17, 24, ...

i) What is its algebraic form?

ii) Prove that there is no perfect square in this sequence. (4)

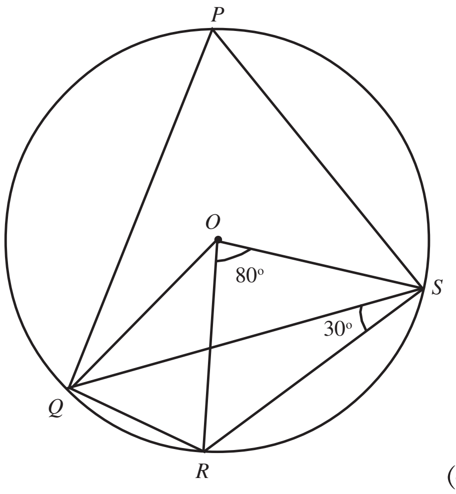
12. In the figure *P*, *Q*, *R* and *S* are the points on the circle with centre at *O*. If $\angle ROS = 80^o$ and $\angle QSR = 30^o$; compute the following angles.

i) $\angle OSQ$ = ...

ii) $\angle SQR$ = ...

iii) $\angle P$ = ...

iv) $\angle QOR$ = ...



(4)

13. All the terms of an arithmetic sequence with common difference 4 are positive numbers. The product of two consecutive terms of this sequence is the same as their sum.
    - i) If one term is x, what is the next term?
    - ii) Calculate those terms. (4)

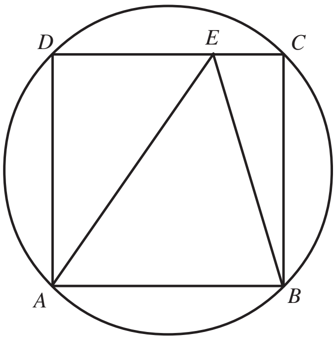
14. Construct a triangle with two angles 45° and 60° and its circumradius 3.5 centimetres. Measure the sides of this triangle. (4)

15. In an arithmetic sequence, m times the n^{th} term is equal to n times the m^{th} term. Prove that its first term and common difference are equal. (4)

OR

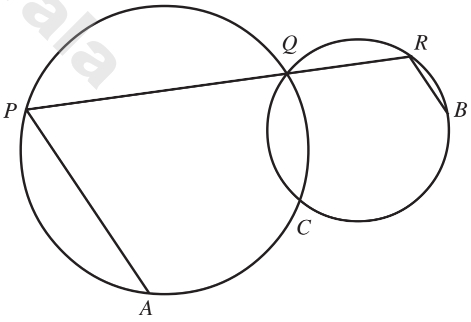
The algebraic form of an arithmetic sequence is $7n + 3$.
- i) What is the remainder when each term of this sequence is divided by 7?
- ii) How many numbers are there in between 100 and 300 in this sequence?

16. In the figure, *ABCD* is a square. Without looking a point is marked in the figure.
    - i) Find the probability that the marked point lies inside the square?
    - ii) Find the probability that the point lies in triangle *ABE*?



(4)

17. Two circles are intersecting at Q and C as shown in the figure. RB is parallel to PA. Prove that the points A, C, B lie on the same line.



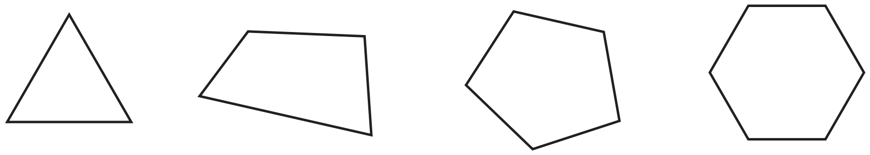
(4)

OR

Prove that the quadrilateral obtained by joining any two alternate vertices of a regular pentagon is cyclic.

18. Construct a square of area 12 square centimetres. (4)

19. Polygons like triangle, quadrilateral, pentagon, hexagon, are drawn as shown below by increasing the number of sides one at a time.



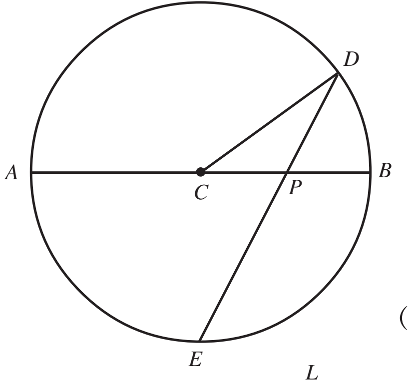
i) Write the sequence of the sum of the angles of each polygon.

ii) Write down the number of possible diagonals in each polygon as a sequence.

iii) Write down the algebraic form of the above two sequences. (5)

20. In the figure, *C* is the centre of the circle and *AB*, its diameter. ΔPDC is an isosceles triangle.

Prove that $AB^2 = 4PD \times DE$.



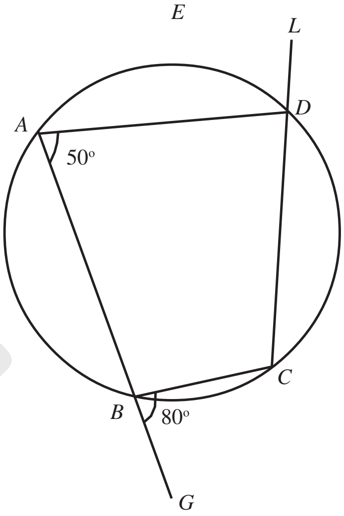
(5)

OR

In the figure, quadrilateral *ABCD* is cyclic.
$\angle GBC = 80^o$; $\angle A = 50^o$

i) Compute the other angles of the quadrilateral. Find also $\angle ADL$?

ii) Prove that the sum of the exterior angles at opposite vertices of a cyclic quadrilateral is 180^o.



21. Observe the number pattern made using the terms of the arithmetic sequence 3, 7, 11, ...

3			
7	11		
15	19	23	
27	31	35	39

....
....

i) Write the next two lines.

ii) Which term of the arithmetic sequence 3, 7, 11, ... is the last number of the 15th row of the above pattern?

iii) Find the first and last numbers in the 15th line? (5)

22. Draw a rectangle having length 7 centimetres and breadth 3 centimetres. Construct another rectangle having the same area and one side 8 centimetres. (5)

*****

Scoring Indicators

Mathematics (Class X) Set B

Q. No.	Scoring Indicators	Score details	Total Score
1.	$98 = 7 \times 14$; 98, a multiple of 7.		
	$2016 = 7 \times 288$; 2016, a multiple of 7.	1	
	2016; is a term of the sequence	1	2
2.	If the angle made by the diameter at a point on the circle		
	is greater than 90^0, then the point will be inside the circle,	1	
	therefore the point making angle 130^o will be inside the circle	1	2
3.	i) $1 + 2 + 3 + ... + 20 = \frac{20 \times 21}{2} = 210$	1	
	ii) sum of 20 terms $= 6\ (1 + 2 + ... + 20) + (5 + 5 + ... + 5)$	1	
	$= 6 \times 210 + 5 \times 20$		
	$= 1360$	1	3
4.	i) Total Outcomes = 12		
	Favourable Outcomes = 5		
	Probability = $\frac{5}{12}$ 1		
	ii) Total Outcomes = 12 + 2 = 14		
	Favourable Outcomes = 6		
	Probability = $\frac{6}{14}$ 1		
	$\frac{5}{12} < \frac{6}{14}$		
	Probability Increases	1	3
5.	$PA \times PB = PC^2$	1	
	$2 \times PB = (2\sqrt{5})^2 = 20$	1	
	$PB = 10$ cm		
	$AB = 12$ cm	1	3
6.	i) The Position of zero $= \frac{0-171}{-4} + 1$		
	$= 43\frac{3}{4}$		
	0 is not a term in the sequence	1	
	ii) Upto 0, there are 43 number, all are positive.	1	
	The numbers after this are negative		
	Therefore there are 43 positive terms	1	3
7.	Let x be the length of one of the perpendicular sides		
	The second perpendicular side $= x - 10$		
	Area, $\frac{1}{2}x\ (x - 10) = 72$	1	
	$x^2 - 10x = 144$		
	$(x - 5)^2 = 169$	1	
	$x = 18$		
	Perpendicular sides are : 18cm, 8 cm	1	3

Q. No.	Scoring Indicators	Score details	Total Score
8.	The angle of a regular hexagon is 120^o.	1	
	$\angle AFE$, $\angle BCD$ have the common by bisector CF		
	$\angle AFC = \angle BCF = 60^o$	1	
	$\angle ABC = 120^o$		
	$\angle AFC + \angle ABC = 180^o$	1	3
	$ABCF$ is cyclic		
9.	10^{th} term + n. common difference $= 250$	1	
	$82 + 8n = 250$		
	$n = \frac{168}{8} = 21$	1	
	The position of 250 $= 10 + 21 = 31$	1	3
10.	Radius of the small circle $= r$		
	Radius of the big circle $= 2r$		
	Area of big circle $= 4\pi r^2$		
	Total Outcomes $= 4\pi r^2$	1	
	Favourable Outcomes $= \pi r^2$		
	Probability $= \frac{\pi r^2}{4\pi r^2} = \frac{1}{4}$	1	
	Probability that the point is outside the smaller circle		
	$= \frac{3}{4}$	1	3
	OR		
	i) No of 3 digit numbers $= 2 \times 3 = 6$	1	
	ii) Favourable Outcomes $= 2$		
	Probability $= \frac{2}{6} = \frac{1}{3}$	1	
	iii) Probability that the numbers are even $= \frac{2}{3}$	1	3
11.	i) $x_n = dn + f - d$		
	$= 7n + 3$	1	
	ii) when the terms of the sequence are divided by 7,		
	the remainder is 3		
	The terms are $7n, 7n \pm 1, 7n \pm 2, 7n \pm 3$		
	$(7n)^2 = 49n^2$, the remainder is 0, when divided by 7		
	$(7n \pm 1)^2 = 49n^2 \pm 14n \pm 1$; the remainder is 1, when divided by 7	1	
	$(7n \pm 2)^2 = 49n^2 \pm 28n \pm 4$; the remainder is 4, when divided by 7		
	$(7n \pm 3)^2 = 49n^2 \pm 42n \pm 9$; the remainder is 2, when divided by 7	1	
	When the perfect squares are divided by 7,		
	the remainder will be 0, 1, 2, 4	1	4
	Here the remainder is 3, no perfect square will be a term		
	of the given sequence.		

Q. No.	Scoring Indicators	Score details	Total Score
12.	i) $\angle OSQ = 50 - 30 = 20^o$	1	
	ii) $\angle SQR = 40^o$	1	
	iii) $\angle QRS = 180 - (40 + 30) = 110^o$		
	$\angle P = 70^o$	1	
	iv) $\angle QOR = 2 \times 30 = 60^o$	1	4
13.	(i) $x, x + 4$, be the terms		1
	(ii) $x(x + 4) = x + x + 4$	1	
	$x^2 + 4x = 2x + 4$		
	$x^2 + 2x = 4$		
	$(x + 1)^2 = 5$	1	
	$x = \sqrt{5} - 1$		
	$\sqrt{5} - 1, \sqrt{5} + 3$ are the terms	1	4
14.	Draw a circle with radius 3.5 cm.	1	
	Mark the points A, B, C so that		
	$\angle AOB = 90^o; \angle BOC = 120^o$	1	
	Draw ΔABC	1	
	Measure the sides AB, BC, AC	1	4
15.	First term $= f$, Common difference $= d$		
	$x_m = dm + f - d$	1	
	$nx_m = dnm + fn - dn$	1	
	$x_n = dn + f - d$		
	$mx_n = dnm + fm - dm$	1	
	$nx_m = mx_n$, then $dnm + fn - dn = dnm + fm - dm$		
	$f(n - m) = d(n - m)$		
	$f = d$	1	4
	OR		
	i) $x_n = 7n + 3$		
	Remainder is 3, when divided by 7	1	
	ii) 14 42		
	$7 \,\vert\, 100 + 1 = 101$ $7 \,\vert\, 300 - 3 = 297$		
	98 98		
	2 + 1 = 3 20		
	16	1	
	6 − 3 = 3		
	101, 108, 115, ..., 297	1	
	No. of terms = 29	1	4

Q. No.	Scoring Indicators	Score details	Total Score
16.	$AB = a$		
	Area of the square $= a^2$		
	Radius of the circle $= \frac{a}{\sqrt{2}}$	1	
	Area of the circle $= \frac{\pi a^2}{2}$	1	
	i) Probability that the point lies inside the square $= \frac{2}{\pi}$	1	
	ii) Area of Δ ABE $= \frac{a^2}{2}$		
	Probability $= \frac{1}{\pi}$	1	4
17.	Join AC, QC, BC		
	$\angle P = x^o$, $\angle R = 180 - x$	1	
	$\angle ACQ = 180 - x^o$	1	
	$\angle BCQ = x$		
	$\angle ACQ + \angle BCQ = 180 - x + x = 180$	1	
	A, C, B lie on a line	1	4
	OR		
	$ABCDE$ is a regular pentagon		
	Join AC, $\angle ABC = 108^o$, $\angle BAC = \angle BCA = 36^o$		
	In quadrilateral $ACDE$		
	$\angle EAC = 108^o - 36^o = 72^o$		
	$\angle CDE = 108^o$		
	$\angle EAC + \angle CDE = 72 + 108 = 180^o$		
18.	Draw $AB = 7$ cm ($AB = 8$ cm . or $AB = 13$ cm)	1	
	Mark the point P So that $PA = 4$ cm, $PB = 3$ cm		
	($PA = 6$ cm, $PB = 2$ cm, $PA = 12$ cm, $PB = 1$ cm)		
	Draw a semicircle with diameter AB	1	
	Draw PC perpendicular to AB	1	
	Draw a square with side PC	1	4
19.	i) 180, 360, 540, 720, ...	1	
	ii) 0, 2, 5, 9, ...	1	
	iii) 180, 360, 540, 720, ..., $x_n = 180n$	1	
	0, 2, 5, 9, ..., $x_n = \frac{(n+2)(n-1)}{2}$	2	4

Q. No.	Scoring Indicators	Score details	Total Score
20.			
	$PA \times PB = PD \times PE$	1	
	$(CA + PC)(CB - PC) = PD(ED - PD)$	1	
	$CA^2 - PC^2 = PD . DE - PD^2$		
	$CA^2 = PC^2 + PD . DE - PD^2$	1	
	$CA^2 = PD \times DE$ [$\because PC = PD$]	1	
	$\frac{AB}{2}^{2} = PD \times DE$		
	$(AB)^2 = 4 PD \times DE$	1	5
	OR		
	i) $\angle ABC = 100^\circ$	1	
	$\angle ADC = 80^\circ$		
	$\angle ADL = 100^\circ$	1	
	ii) $\angle CBG = x^\circ; \angle ABC = 180 - x^\circ$		
	$\angle ADC = x^\circ$	1	
	$\angle ADL = 180 - x^\circ$	1	
	$\angle CBG + \angle ADL = 180^\circ$	1	5
21.	i) 43 47 51 55 59		
	63 67 71 75 79 83	1	
	ii) Last term of the second row, 1 + 2 = third term		
	Last term of the third row 1 + 2 + 3 = sixth term	1	
	Last term of the fifteenth row = 1 + 2 + 3 + ... + 15		
	= 120th term	1	
	The 120th term of the sequence 3, 7, 11, ... is		
	$3 + 119 \times 4 = 3 + 476 = 479$	1	
	First term of 15th row = $479 - 14 \times 4$		
	$= 479 - 56 = 423$	1	5
22.	Draw the rectangle with *AB* = 7 cm *BC* = 3 cm		
	Draw *ABCD*	1	
	Draw *AE so that BE* = 1 cm (*AE* = 8 cm)		
	Extend *DA,* mark *F* so that *AF* = *AE* = 8 cm	1	
	Extend *BA* so that *AG* = 3 cm	1	
	Draw the circumcircle of ΔBFG which meet AD at P.	1	
	Draw the rectangle with sides *AE, AP*	1	5

First Terminal Evaluation - 2016
MATHEMATICS

Standard: X **Time: 2½ hours**
Score: 80

Instructions

1. The first 15 minutes is given as 'cool off time'. You may read and understand the questions during this time.
2. Answer all the questions.
3. If there is an ***OR*** between any two questions, you may answer one among them.
4. Simplification using irrationals like π, $\sqrt{2}$, $\sqrt{3}$ etc., with their approximate values is not required if not specified in the question.

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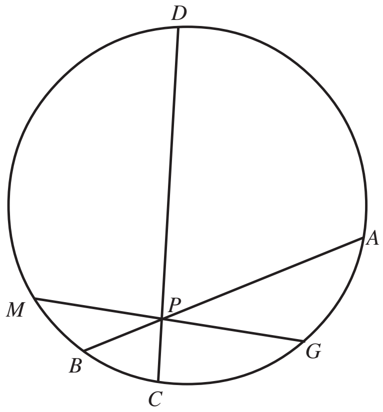
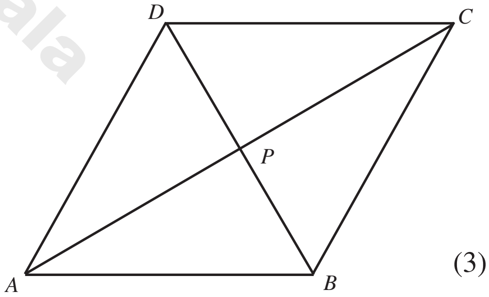
1. In the figure, chords AB, CD and MG are intersecting at P. PA = 12 centimetres, BP = 6 centimetres, PD = 18 centimetres, PM = 7.2 centimetres. Find the length of PG and PC. (2)



2. Algebraic form of an arithmetic sequence is $6n + 3$. Can the sum of a few terms of the sequence be 2017. Justify your answer. (2)

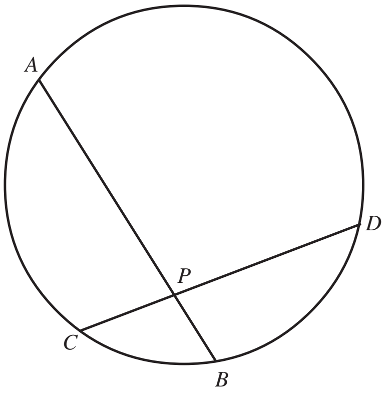
3. In the figure, $ABCD$ is a rhombus. Can the circle with diameter AB pass through any vertex of ΔCPD? Justify your answer. (3)



4. A box contains 10 black beeds and 17 white beeds. Another box contains 11 black beeds and 17 white beeds. Without looking a beed is drawn from each box.
   i) What is the probability of getting a black beed from the first box?
   ii) What is the probability of getting a black beed from the second box?
   iii) Which box gives more chance to get a white beed? Why? (3)

5. 9, 15, 21, ... is an arithmetic sequence.
   i) Write the algebraic form of the sequence.
   ii) Write the algebraic form of the sum. (3)

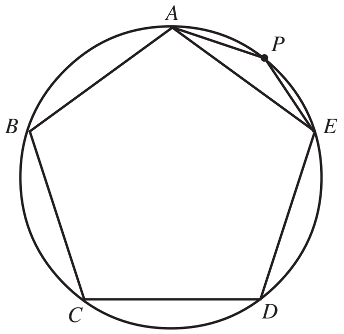
6. In the figure, chords *AB* and *CD* are intersecting at *P*. *CP* = 3 centimetres, *CD* = 11 centimetres, *AB* = 14 centimetres. Find the length of *AP*.



(3)

7. Algebraic form of an arithmetic sequence is $\frac{3}{8}n + 2$. Write the sequence of whole numbers occuring in the sequence. What is the peculiarity of the sequence? (3)

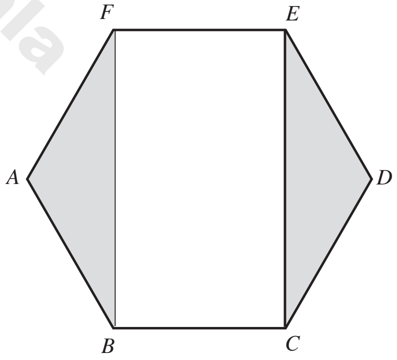
8. *ABCDE* is a reqular pentagon and its vertices are on a circle. *P* is a point on the circle. Calculate $\angle APE$?



(3)

9. Common difference of an arithmetic sequence is 7 and its 8th term is 50. What is its 18th term? Can 1947 a term of this sequence? Why? (3)

10. In the figure *ABCDEF* is a regular hexagon. Without looking a point is marked.
    - i) What is the probability of the point to be inside the shaded portion?
    - ii) What is the probability of the point to be in the rectangle *FBCE?*



(3)

OR

Two dice, numbered 1 to 6 are rolled together.
- i) What is the probability of getting the product of the numbers 36?
- ii) What is the probability of getting the product a prime number?

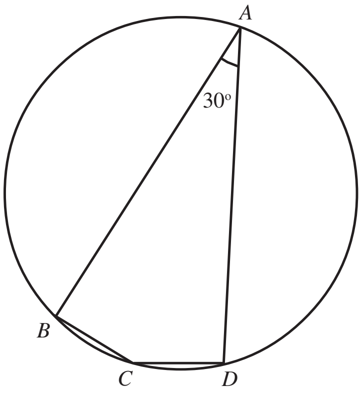
11. Construct a triangle with two angles 40^o and 65^o and circumradius 3 centimetres. Measure the sides of the triangle. (4)

12. Algebraic form of an arithmetic sequence is $4n + 3$. Prove that square of any natural number cannot be a term of this sequence. (4)

13. Sides of a right triangle are in an arithmetic sequence. If the length of the larger perpendicular side is 6 centimetres, find the other two sides of the triangle. (4)

14. Construct a square of area 11 square centimetres. (4)

OR



In the figure $\angle A = 30^{\circ}$.

i) Find $\angle C$?

ii) With the help of this figure construct angles of measures 75° and $37\frac{1}{2}^{\circ}$.

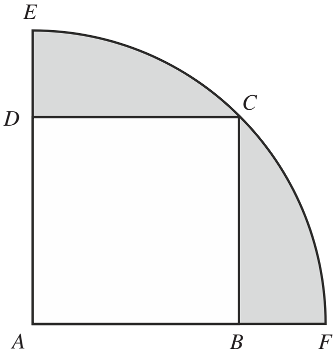
15. From the following expressions, identify the algebraic form of the sum of *n* terms of an arithmetic sequence. (4)

i) $2n^2 + \frac{1}{3n}$

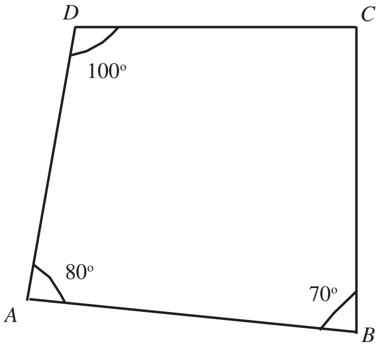
ii) $2n^2 + 3n$

iii) $2n^2 + 3n + 1$

Write the algebraic form of that sequence.



16. In the figure, *ABCD* is a square and a circular portion is drawn with centre at *A* and radius *AE*. Without looking into the figure if a point is marked in it, what is the proabability of the point to be in the square? What is the probability of the point to be in the shaded portion? (4)



17. Is the circumcircle of triangle *ABD* passes through the vertex *C* of quadrilateral *ABCD*? Why? (4)

18. Can whole numbers occur in the arithmetic sequnce $\frac{15}{4}, \frac{27}{4}, \frac{39}{4}, \ldots$? Why? (4)

OR

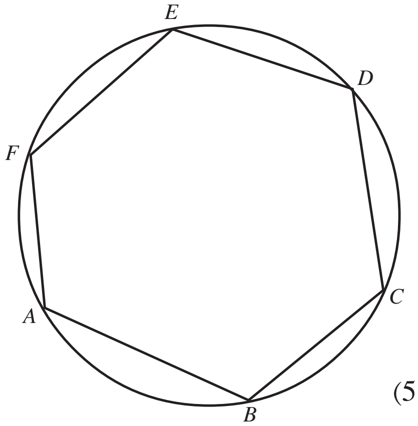
How many natural numbers between 300 and 700 gives remainder 1 when divided by 6. Also find their sum.

19. Draw a rectangle of sides 7 centimetres and 4 centimetres. Construct another rectangle of one side 6 centimetres and having the same area. (5)

20. An object moves from a point A with speed 5m/sec and in a certain direction. The speed of the object is increasing at the rate of 4m/sec. (5)
    - i) Write the sequence of the speed at the end of each second.
    - ii) Write the total distance travelled by the object at the end of each second as a sequence.
    - iii) Write the algebraic form of both the sequences. (5)

21. *ABCDEF* is a hexagon with its vertices on a circle.

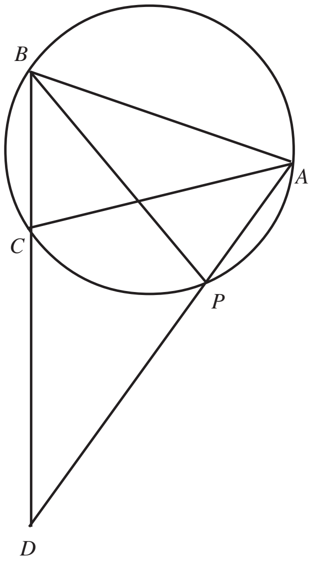
    Prove that $\angle A + \angle C + \angle E = \angle B + \angle D + \angle F$



(5)

OR

In the figure $AB = AC$. The bisector of $\angle B$ intersects the circle at P. BC and AP are extended to meet at D. Prove that $CA = CD$.



22. Observe the pattern formed using the terms of the arithmetic sequence 5, 8, 11, ...

```
                    5
               8         11
          14        17        20
     23        26        29        32
..........................................................
..........................................................
```

- i) Write the next two lines.
- ii) Which term of the arithmetic sequence 5, 8, 11, ... is the last number of the tenth row of the above pattern.
- iii) Find the first and the last numbers in the 10th row of the above pattern. (5)

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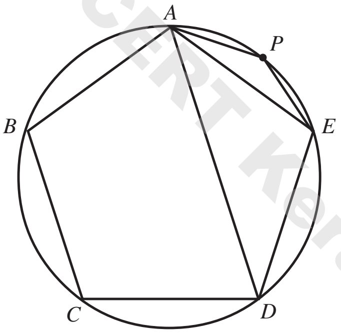
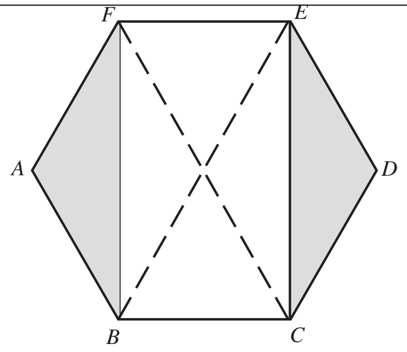
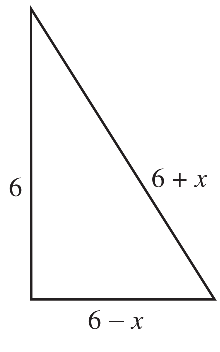
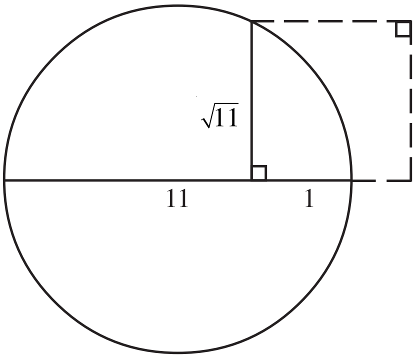
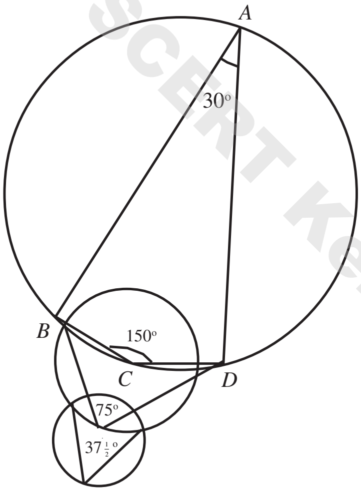
Scoring Indicators (Class X)
Mathematics Set C

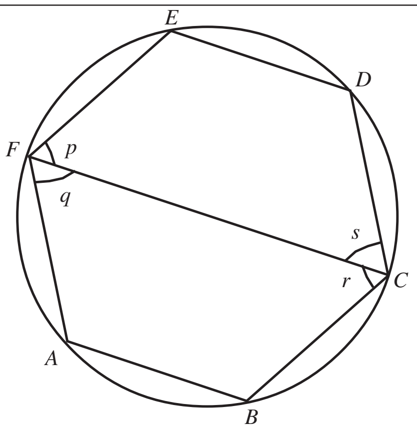
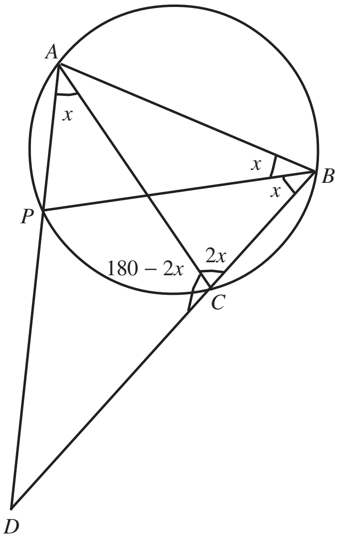
Qn No.	Scoring Indicators	Sub Score	Total Score
1.	If two chords $AB,\ CD$ of a circle cut at P then		
	$PA \times PB = PC \times PD$		
	$\Rightarrow PC \times PD = PA \times PB = 12\times6$	1	
	$= PG \times PM = 12\times6$		2
	Finding		
	$PC = 4$		
	$PG = 10$	1	
2.	Understanding that every term in the arthmetic sequence with		
	$6n+3$ is a multiple of 3	1	
	Sum of multiples of 3 will be a multiple 3.	1	2
	But 2017 is not a multiple of 3		
3.	Understanding the diagonals AC & BD of the rhombus $ABCD$ are	1	
	perpendicular to each other $\angle APB = 90^o$	1	
	Then P is a point on the circle with diameter AB		
	The circle passess to through the point P of the triangle CPD	1	3
4.	i) Probability of obtaining black beeds in the first		
	box = $\frac{10}{25} = \frac{2}{5}$	$\frac{1}{2}$	
	ii) Probability of obtaining black beeds in the second		
	box = $\frac{11}{28}$	$\frac{1}{2}$	
	iii) Probability of getting white beed from the first	1	
	and second boxes are $\frac{15}{25}$ and $\frac{17}{28}$.		
	$\frac{17}{28}$ greater than $\frac{15}{25}$		
	It is letter to take white beed from the second box	1	3
5.	i) Algebraic form = $6n + 3$	1	
	ii) Algebraic form of the sum		
	$= 6 \times \frac{n(n+1)}{2} + 3n$		
	$= 3n^2 + 3n + 3n$		
	$= 3n^2 + 6n$	2	3

Qn No.	**Scoring Indicators**	**Sub Score**	**Total Score**
6.	If $CP = 3$, $PD = 8$ then		
	$AP = x$, $BP = 14-x$	1	
	$CP \times PD = AP \times BP$		
	$3 \times 8 = x(14 - x)$	1	
	$x^2 - 14x + 24 = 0$		
	$x^2 - 14x + 49 = 25$		
	$(x - 7)^2 = 25$	1	3
	$x - 7 = 5$		
	$x = 12$		
	therefore $AP = 12$		
	OR		
	Find the numbers with sum *14* and product *24* which are 2,12	3	
7.	n is a multiple of 8, then $\frac{3}{8}n + 2$ is an integer	1	
	If $n = 8, 16, 24, 32, ...$ then the sequence is 5, 8, 11, 14, ...	1	
	This is an *AP* with common difference 3	1	3
8.			
	Join *AD* then we can find $\angle EDA = 36^\circ$	1	
	APED is cyclic	1	
	$\angle APE = 180^\circ - 36^\circ = 144^\circ$	1	
	OR		
	The central angle of arc *APE* is 72°		
	$\angle APE = \frac{360 - 72}{2} = 144^\circ$		
9.	If we add 10 times common difference to the 8th term, we get		
	the 18th term	1	
	$18^{th} = 50 + 10 \times 7 = 120$	1	
	The difference of two terms will be the multiple of the common		
	common difference	1	3
	1947 - 50 = 1897, which is a multiple of 7.		
	1947 is a term of the sequence.		

Qn No.	Scoring Indicators	Sub Score	Total Score
10.			
	The areas of shaded region and non shaded region are $\frac{2}{6}$ and $\frac{4}{6}$ of the area of the hexagon	2	
	Finding the probability i) $\frac{2}{6} = \frac{1}{3}$		
	ii) $\frac{4}{6} = \frac{2}{3}$	1	3
	OR		
	Probability of getting the product 36 is $\frac{1}{36}$	1	
	There are 6 pairs with their product as prime numbers	1	
	Probability of getting a prime number = $\frac{1}{6}$	1	3
11.	Draw a circle with radius 3 cm.	1	
	Mark the central angles	1	
	Draw the triangle	2	4
12.	If we divide the terms of the sequence by 4, remainder is 3	1	
	For perfect squares, the remainder will be 0, 1	1	
	There are no perfect square in the given sequence	2	4
13.			
	Take the sides of the right triangle as $6 - x, 6, 6 + x$	1	
	$6^2 + (6 - x)^2 = (6 + x)^2$	1	
	The sides are $4\frac{1}{2}, 6, 7\frac{1}{2}$	2	4

Qn No.	Scoring Indicators	Sub Score	Total Score
14.	Draw a circle with diameter 12 cm	1	
	Draw a line of length $\sqrt{11}$ cm.	2	
	Draw a square of side $\sqrt{11}$ cm	1	4
			
	OR		
			
	$\angle C = 180^\circ - 30^\circ = 50^\circ$	1	
	Draw a figure and mark $\angle C$ and $\angle A$	1	
	Draw a circle with centre C and construct an angle of 75°	1	
	Draw a circle with centre at angle 75° and construct an angle of $37\frac{1}{2}^\circ$.	1	4
15.	Stating the algebraic expression of an arithmetic sequence of the form $an^2 + bn$.	1	
	$2n^2 + 3n$ is in the above form	1	
	Finding the common difference, first term and the algebraic expression of the sequence.	2	4

Qn No.	Scoring Indicators	Sub Score	Total Score
16.	In the figure $AF = AC = AE = r$, then	1	
	Area of square = $\frac{r^2}{2}$; area of sector = $\frac{\pi r^2}{4}$		
	Probability of point inside square = $\frac{2}{\pi}$.	1	
	Probability of point on the shaded region = $\frac{\pi - 2}{\pi}$	1	4
17.	Finding $\angle C = 110^o$	1	
	$\angle A + \angle C = 190 > 180^o$	1	
	Stating that the circumcircle of the ΔABD will not pass		
	through *C*.	2	4
	or $\angle B + \angle D = 170^{o,}$ $\therefore ABCD$ is not cyclic		
18.	Stating all the numberators are odd numbers.	2	
	Any odd number can't be completely divisible by 4.	2	4
	OR		
	Finding first term as 301, last term as 697 and common		
	difference 6	1	
	Finding the number of terms as 67	1	
	Finding the sum as 18463	2	4
19.	Drawing a rectangle *ABCD* with sides 7 cm, 4 cm		
	$AB = 7$ cm, $BC = 4$ cm	1	
	Extending *AB* to get $AE = 11$ cm		
	Extending *CB* to get $BF = 6$cm	1	
	Draw a circle passing through *A*, *E*, *F* an then mark a point G		
	so that $AB \times BE = BF \times BG$	2	
	Drawing a rectangle with sides *BF*, *BG*.	1	5
20.	Sequence of speed 9, 13, 17, ...	1	
	Finding $u = 5, a = 4$ using the sequence of distance equation	1	
	$s = ut + \frac{1}{2}at^2$	1	
	Finding sequence of distance as 7, 18, 33, ...	1	
	Understanding algebraic expression of speed as the algebraic expression of the sequence 9, 13, 17, ...		
	writing the algebraic form as $s_n = 5n + 2n^2$, the expression of distance.	1	5

Qn No.	Scoring Indicators	Sub Score	Total Score
21.			
	Drawing FC and constructing two cyclic quadrilaterals	1	
	Sum of opposite angles of a cyclic quadrilateral is 180^{o}		
	Using this concept, $q + \angle B = 180^{o}$ $\angle A + r = 180^{o}$ $p + \angle D = 180^{o}$ $\angle E + s = 180^{o}$	2	
	Rearranging we get $p + q + \angle D + \angle B = 180^{o} + 180^{o} = 360^{o}$ $\angle F + \angle D + \angle E = 360$ $s + r + \angle A + \angle E = 180^{o} + 180^{o} = 360^{o}$ $\angle A + \angle C + \angle E = 360$ Finding $\angle A + \angle C + \angle E = \angle B + \angle D + \angle F$	2	5
	OR		
			
	Since $AB = AC$ then, $\angle ABC = \angle ACB.$	1	
	Since BP bisects $\angle B$ and $\angle ACB = 2x$		
	then $\angle CBP = \angle PBA = x$	1	

Qn No.	Scoring Indicators	Sub Score	Total Score
	$\angle PAC = \angle PBC = x$, (Central angles of same arc)	1	
	Using the statement linear pairs are supplementary		
	Finding $\angle ACD = 180 - 2x$	1	
	$\angle D = 180 - (180 - 2x + x) = x$		
	Showing ΔACD is an equilateral triangle.	1	5
	$AC = CD$		
22.	i) Writing next two rows	1	
	ii) Showing the position of last number in each row as		
	are 1, 3, 6, 10, ...	1	
	iii) Understanding this as the sum of natural numbers.		
	Position of last number in the 10th row		
	$= 1 + 2 + ... + 10 = \frac{10\times11}{2} = 55$	1	
	Last number in the 10th row = $5 + 54 \times 3 = 167$	1	
	First number in the 10th row $167 - 9 \times 3 = 140$	1	5