

Central Board of School Education

**Marking Scheme 2016**

[Official]

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## **Senior School Certificate Examination**

**March 2016**

**Marking Scheme — Mathematics 65/1/S, ~~65/2/S, 65/3/S~~**

### ***General Instructions:***

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage
2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.
3. Alternative methods are accepted. Proportional marks are to be awarded.
4. In question (s) on differential equations, constant of integration has to be written.
5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.
6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
7. Separate Marking Scheme for all the three sets has been given.
8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/1/S  
**EXPECTED ANSWER/VALUE POINTS**

**SECTION A**

$$1. \text{ Getting } \sin \theta = \frac{\frac{1}{\sqrt{3}}}{\frac{1}{2} \cdot \frac{4}{\sqrt{3}}} = \frac{1}{2} \quad \frac{1}{2}$$

$$\text{Hence } |\vec{a} \cdot \vec{b}| = \frac{1}{2} \cdot \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} = 1 \quad \frac{1}{2}$$

$$2. |\vec{a} - \sqrt{2}\vec{b}|^2 = 1 \Rightarrow \vec{a} \cdot \vec{b} = \frac{1}{\sqrt{2}} \quad \frac{1}{2}$$

$$\therefore \text{ Angle between } \vec{a} \text{ and } \vec{b} = \frac{\pi}{4} \quad \frac{1}{2}$$

$$3. \text{ Writing or using, that given planes are parallel} \quad \frac{1}{2}$$

$$d = \frac{|4+10|}{\sqrt{4+9+36}} = 2 \text{ units} \quad \frac{1}{2}$$

$$4. |AA^T| = |A||A^T| = |A|^2 \quad \frac{1}{2}$$

$$= 25 \quad \frac{1}{2}$$

$$5. \text{ Getting } AB = \begin{pmatrix} 7 & -8 \\ 0 & -10 \end{pmatrix} \quad \frac{1}{2}$$

$$|AB| = -70 \quad \frac{1}{2}$$

$$6. k(2) = -8 \Rightarrow k = -4 \quad \frac{1}{2}$$

$$-4(3) = 4a \Rightarrow a = -3 \quad \frac{1}{2}$$

**SECTION B**

$$7. y = (\sin 2x)^x + \sin^{-1}(\sqrt{3x}) = u + v$$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \frac{1}{2}$$

$$u = (\sin 2x)^x \Rightarrow \log u = x \log \sin 2x \quad \frac{1}{2}$$

$$\frac{1}{u} \frac{du}{dx} = 2x \cdot \cot 2x + \log \sin 2x \quad 1$$

$$\therefore \frac{du}{dx} = (\sin 2x)^x [2x \cot 2x + \log \sin 2x] \quad \frac{1}{2}$$

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-3x}} \frac{\sqrt{3}}{2\sqrt{x}} \quad 1$$

$$\therefore \frac{dy}{dx} = (\sin 2x)^x [2x \cot 2x + \log \sin 2x] + \frac{\sqrt{3}}{2\sqrt{x}\sqrt{1-3x}} \quad \frac{1}{2}$$

**OR**

$$\text{Let } y = \tan^{-1} \left( \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right) \text{ and } z = \cos^{-1} x^2$$

$$z = \cos^{-1} x^2 \Rightarrow x^2 = \cos z \Rightarrow y = \tan^{-1} \left( \frac{\sqrt{1+\cos z} - \sqrt{1-\cos z}}{\sqrt{1+\cos z} + \sqrt{1-\cos z}} \right) \quad 1$$

$$\therefore y = \tan^{-1} \left( \frac{\cos \frac{z}{2} - \sin \frac{z}{2}}{\cos \frac{z}{2} + \sin \frac{z}{2}} \right) = \tan^{-1} \left( \frac{1 - \tan \frac{z}{2}}{1 + \tan \frac{z}{2}} \right) \quad \frac{1}{2} + \frac{1}{2}$$

$$\therefore y = \tan^{-1} \left[ \tan \left( \frac{\pi}{4} - \frac{z}{2} \right) \right] = \frac{\pi}{4} - \frac{z}{2} \quad \frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow \frac{dy}{dz} = -\frac{1}{2} \quad 1$$

8. LHL =  $\lim_{x \rightarrow 0^-} k \cdot \sin \frac{\pi}{2}(x+1) = k \quad 1$

$$\text{RHL} = \lim_{x \rightarrow 0^+} \frac{\tan x(1 - \cos x)}{x^3} \quad 1$$

$$= \lim_{x \rightarrow 0^+} \frac{\tan x}{x} \cdot 2 \left( \frac{\sin x/2}{2x/2} \right)^2 = \frac{1}{2} \quad 1$$

$$\Rightarrow k = \frac{1}{2} \quad 1$$

9. When  $x = am^2$ , we get  $y = \pm am^3 \quad 1$

$$ay^2 = x^3 \Rightarrow 2ay \frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay} \quad 1$$

$$\text{slope of normal} = \mp \frac{2a}{3} \frac{am^3}{a^2 m^4} = \mp \frac{2}{3m} \quad 1$$

$$\therefore \text{Equation of normal is } y \mp am^3 = \mp \frac{2}{3m}(x - am^2) \quad 1$$

[Full marks may be given, if only one value for point, slope and equation is derived]

$$\begin{aligned}
 \text{10. Writing } \int \frac{1 - \sin x}{\sin x(1 + \sin x)} dx &= \int \frac{(1 + \sin x) - 2 \sin x}{\sin x(1 + \sin x)} dx && 1 \\
 &= \int \frac{1}{\sin x} dx - 2 \int \frac{1}{1 + \sin x} dx && 1 \\
 &= \int \operatorname{cosec} x dx - 2 \int \frac{(1 - \sin x)}{\cos^2 x} dx && 1 \\
 &= \log |\operatorname{cosec} x - \cot x| - 2 \int (\sec^2 x - \sec x \tan x) dx && \frac{1}{2} \\
 &= \log |\operatorname{cosec} x - \cot x| - 2(\tan x - \sec x) + C && \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{11. } I &= \int \left[ \log(\log x) + \frac{1}{(\log x)^2} \right] dx \\
 &= \int \log(\log x) \cdot 1 dx + \int \frac{1}{(\log x)^2} dx && 1 \\
 &= \log(\log x) \cdot x - \int \frac{1}{\log x} \cdot \frac{1}{x} dx + \int \frac{1}{(\log x)^2} dx && 2 \\
 &= x \cdot \log(\log x) - \left[ \frac{1}{\log x} \cdot x - \int \frac{-1}{(\log x)^2} \cdot \frac{1}{x} \cdot x dx \right] + \int \frac{1}{(\log x)^2} dx && \frac{1}{2} \\
 &= x \log(\log x) - \frac{x}{\log x} + C && \frac{1}{2}
 \end{aligned}$$

$$\text{12. } I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx \quad \dots(\text{i})$$

$$I = \int_0^{\pi/2} \frac{\sin^2(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)} dx = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x} dx \quad \dots(\text{ii}) \quad 1$$

$$2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx \quad 1$$

$$\Rightarrow I = \frac{1}{2\sqrt{2}} \int_0^{\pi/2} \frac{1}{\cos x \frac{1}{\sqrt{2}} + \sin x \frac{1}{\sqrt{2}}} dx = \frac{1}{2\sqrt{2}} \int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right) dx \quad 1$$

$$= \frac{1}{2\sqrt{2}} \left[ \log \left| \sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right) \right| \right]_0^{\pi/2} \quad \frac{1}{2}$$

$$= \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right| \text{ or } \frac{1}{\sqrt{2}} \log |\sqrt{2} + 1| \quad \frac{1}{2}$$

OR

$$\begin{aligned}
 I &= \int_0^1 \cot^{-1}(1-x+x^2) dx = \int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right) dx && \frac{1}{2} \\
 &= \int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1-x(1-x)}\right) dx = \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1}(1-x) dx && 1 \\
 &= 2 \int_0^1 \tan^{-1} x dx && \frac{1}{2} \\
 &= 2 \left[ \left( \tan^{-1} x \cdot x \right)_0^1 - \int_0^1 \frac{x}{1+x^2} dx \right] && \frac{1}{2} \\
 &= 2 \left[ x \tan^{-1} x - \frac{1}{2} \log |1+x^2| \right]_0^1 && 1 \\
 &= 2 \left[ \frac{\pi}{4} - \frac{1}{2} \log 2 \right] \text{ or } \frac{\pi}{2} - \log 2 && \frac{1}{2}
 \end{aligned}$$

13. The given differential equation can be written as

$$\frac{dy}{dx} - \frac{1}{x+1} y = (x+1)^2 \cdot e^{3x} \quad \frac{1}{2}$$

Here, integrating factor =  $e^{\int -\frac{1}{x+1} dx} = \frac{1}{x+1}$  1

$\therefore$  Solution is  $y \frac{1}{x+1} = \int (x+1) e^{3x} dx$  1

$\therefore \frac{y}{x+1} = (x+1) \frac{e^{3x}}{3} - \frac{e^{3x}}{9} + C$  1 \frac{1}{2}

or  $y = \left[ \frac{1}{3}(x+1)^2 - \frac{x+1}{9} \right] e^{3x} + C(x+1)$

14. From the given differential equation, we can write

$$\frac{dx}{dy} = \frac{2xe^{x/y} - y}{2ye^{x/y}} = \frac{2x/y e^{x/y} - 1}{2e^{x/y}} \quad 1$$

Putting  $\frac{x}{y} = v \Rightarrow \frac{dx}{dy} = v + y \frac{dv}{dy}$  \frac{1}{2}

$\therefore v + y \frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} \Rightarrow y \frac{dv}{dy} = -\frac{1}{2e^v}$  1

$\Rightarrow 2 \int e^v dv = -\int \frac{dy}{y}$  \frac{1}{2}

$\therefore 2e^v + \log |y| = C \Rightarrow 2e^{x/y} + \log |y| = C$  1

15. Let length be  $x$  m and breadth be  $y$  m

$$\therefore (x - 50)(y + 50) = xy \Rightarrow 50x - 50y = 2500 \text{ or } x - y = 50 \quad \frac{1}{2}$$

$$\text{and } (x - 10)(y - 20) = xy - 5300 \Rightarrow 2x + y = 550 \quad \frac{1}{2}$$

$$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 50 \\ 550 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 50 \\ 550 \end{pmatrix} \quad \frac{1}{2} + 1$$

$$\Rightarrow x = \frac{1}{3}(600) = 200 \text{ m, } y = \frac{1}{3}(450) = 150 \text{ m} \quad \frac{1}{2}$$

“Helping the children of his village to learn” (or any other relevant value) 1

16. LHS =  $2 \sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right)$

$$= 2 \tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right) \quad 1$$

$$= \tan^{-1}\left(\frac{2 \cdot \frac{3}{4}}{1 - \frac{9}{16}}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right) \quad 1$$

$$\tan^{-1}\left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24}{7} \cdot \frac{17}{31}}\right) = \tan^{-1}\left(\frac{625}{625}\right) = \tan^{-1}(1) = \frac{\pi}{4} = \text{RHS} \quad 1 + 1$$

**OR**

$$\cos(\tan^{-1} x) = \sin\left(\cot^{-1} \frac{3}{4}\right)$$

$$\Rightarrow \cos\left(\cos^{-1} \frac{1}{\sqrt{1+x^2}}\right) = \sin\left(\sin^{-1} \frac{4}{5}\right) \quad 1 + 1$$

$$\Rightarrow \frac{1}{\sqrt{1+x^2}} = \frac{4}{5} \text{ or } \sqrt{1+x^2} = \frac{5}{4} \quad 1$$

$$\Rightarrow x^2 = \frac{9}{16} \Rightarrow x = \pm \frac{3}{4} \quad 1$$

17. Let  $E_1$  : selecting bag A,  $E_2$  : selecting bag B

$\frac{1}{2}$

A : getting 2 white and 1 red out of 3 drawn (without replacement)

$$\therefore P(E_1) = P(E_2) = \frac{1}{2} \quad \frac{1}{2}$$

$$P\left(\frac{A}{E_1}\right) = \frac{{}^3C_2 \cdot {}^4C_1}{{}^7C_3} = \frac{12}{35} \quad 1$$

$$P\left(\frac{A}{E_2}\right) = \frac{{}^4C_2 \cdot {}^3C_1}{{}^7C_3} = \frac{18}{35} \quad 1$$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2} \cdot \frac{18}{35}}{\frac{1}{2} \cdot \frac{12}{35} + \frac{1}{2} \cdot \frac{18}{35}} = \frac{3}{5} \quad 1$$

18.  $\vec{a} = \vec{b} + \vec{c} \Rightarrow p\hat{i} + q\hat{j} + r\hat{k} = (s+3)\hat{i} + 4\hat{j} + 2\hat{k}$

$$p = s + 3, q = 4, r = 2 \quad 1 \frac{1}{2}$$

$$\text{area} = \frac{1}{2} |\vec{b} \times \vec{c}| = 5\sqrt{6}$$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ s & 3 & 4 \\ 3 & 1 & -2 \end{vmatrix} = -10\hat{i} + (2s+12)\hat{j} + (s-9)\hat{k} \quad \frac{1}{2}$$

$$\therefore 100 + (2s + 12)^2 + (s - 9)^2 = (10\sqrt{6})^2 = 600$$

$$\Rightarrow s^2 + 6s + 55 = 0 \Rightarrow s = -11, p = -8, \text{ or } s = 5, p = 8 \quad 1 + 1$$

19. Equation of plane passing through A, B and C is

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 1 & 0 & -3 \\ 2 & 3 & 1 \end{vmatrix} = 0 \quad 2$$

$$\Rightarrow (x-3)9 - (y-2)7 + (z-1)3 = 0 \Rightarrow 9x - 7y + 3z = 16 \quad \dots(i) \quad 1$$

If A, B, C and D are coplanar, D must lie on (i)

$$\Rightarrow 9\lambda - 35 + 15 - 16 = 0 \Rightarrow \lambda = 4. \quad 1$$

**OR**

Equation of plane, perpendicular to  $\vec{n} = \hat{i} + \hat{j} + 3\hat{k}$  and at a distance  $\frac{4}{\sqrt{11}}$  from origin is

$$\vec{r} \cdot \frac{(\hat{i} + \hat{j} + 3\hat{k})}{\sqrt{11}} = \frac{4}{\sqrt{11}} \text{ or } \vec{r} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 4 \quad \dots(i) \quad 1 \frac{1}{2}$$

Any point on the line  $\vec{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$  is

$$(-1+3\lambda)\hat{i} + (-2+4\lambda)\hat{j} + (-3+3\lambda)\hat{k} \quad \dots(ii) \quad 1$$

If this point is the point of intersection of the plane and the line then,

$$(-1+3\lambda)1 + (-2+4\lambda)1 + (-3+3\lambda)3 = 4$$

$$\Rightarrow \lambda = 1. \quad 1$$

Hence the point of intersection is (2, 2, 0) 1



## SECTION C

20. Let  $x_1, x_2 \in \mathbb{N}$  and  $f(x_1) = f(x_2)$

$$\Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$$

$$\Rightarrow 4(x_1^2 - x_2^2) + 12(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(4x_1 + 4x_2 + 12) = 0$$

$$\Rightarrow x_1 - x_2 = 0 \text{ or } x_1 = x_2 \text{ as } 4x_1 + 4x_2 + 12 \neq 0, x_1, x_2 \in \mathbb{N}$$

$\therefore f$  is a 1-1 function

2

$f: \mathbb{N} \rightarrow \mathbb{S}$  is onto as co-domain = range

1

Hence  $f$  is invertible.

$$y = 4x^2 + 12x + 15 = (2x+3)^2 + 6 \Rightarrow x = \frac{\sqrt{y-6}-3}{2}$$

$$\therefore f^{-1}(y) = \frac{\sqrt{y-6}-3}{2}, y \in \mathbb{S}.$$

2

$$f^{-1}(31) = \frac{\sqrt{31-6}-3}{2} = 1$$

 $\frac{1}{2}$ 

$$f^{-1}(87) = \frac{\sqrt{87-6}-3}{2} = 3$$

 $\frac{1}{2}$ 

21. Let  $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$

$$C_1 \rightarrow C_1 + C_2 - 2C_3 \Rightarrow \Delta = (a^2 + b^2 + c^2) \begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$

1

$$R_1 \rightarrow R_1 - R_2, \text{ and } R_2 \rightarrow R_2 - R_3 \Rightarrow \Delta = (a^2 + b^2 + c^2) \begin{vmatrix} 0 & a^2 - b^2 & c(b-a) \\ 0 & b^2 - c^2 & a(c-b) \\ 1 & c^2 & ab \end{vmatrix}$$

 $1\frac{1}{2}$ 

$$= (a^2 + b^2 + c^2)(a-b)(b-c) \begin{vmatrix} 0 & a+b & -c \\ 0 & b+c & -a \\ 1 & c^2 & ab \end{vmatrix}$$

 $\frac{1}{2}$ 

$$R_2 \rightarrow R_2 - R_1 \Rightarrow \Delta = (a^2 + b^2 + c^2)(a-b)(b-c) \begin{vmatrix} 0 & a+b & -c \\ 0 & c-a & c-a \\ 1 & c^2 & ab \end{vmatrix}$$

1

$$\therefore \Delta = (a^2 + b^2 + c^2)(a-b)(b-c)(c-a) \begin{vmatrix} 0 & a+b & -c \\ 0 & 1 & 1 \\ 1 & c^2 & ab \end{vmatrix}$$

1

$$\text{Expanding by } C_1 \text{ to get } \Delta = (a^2 + b^2 + c^2)(a-b)(b-c)(c-a)(a+b+c)$$

1

**OR**

$$\text{Let } A = IA \therefore \begin{pmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \quad 1$$

$$R_2 \rightarrow R_2 - 2R_3 \Rightarrow \begin{pmatrix} 2 & -1 & 3 \\ 1 & -1 & -5 \\ -3 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} A \quad \frac{1}{2}$$

$$\begin{matrix} R_1 \rightarrow R_1 - 2R_2 \\ R_3 \rightarrow R_3 + 3R_2 \end{matrix} \Rightarrow \begin{pmatrix} 0 & 1 & 13 \\ 1 & -1 & -5 \\ 0 & -1 & -12 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & -2 \\ 0 & 3 & -5 \end{pmatrix} A \quad 1$$

$$R_1 \leftrightarrow R_2 \Rightarrow \begin{pmatrix} 1 & -1 & -5 \\ 0 & 1 & 13 \\ 0 & -1 & -12 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ 1 & -2 & 4 \\ 0 & 3 & -5 \end{pmatrix} A \quad \frac{1}{2}$$

$$\begin{cases} R_1 \rightarrow R_1 + R_2 \\ R_3 \rightarrow R_3 + R_2 \end{cases} \Rightarrow \begin{pmatrix} 1 & 0 & 8 \\ 0 & 1 & 13 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 2 \\ 1 & -2 & 4 \\ 1 & 1 & -1 \end{pmatrix} A \quad 1$$

$$\begin{cases} R_1 \rightarrow R_1 - 8R_3 \\ R_2 \rightarrow R_2 - 13R_3 \end{cases} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{pmatrix} A \quad 1$$

$$\Rightarrow A^{-1} = \begin{pmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{pmatrix} \quad 1$$

22.  $f'(x) = 4x^3 - 24x^2 + 44x - 24$  1

$$= 4(x^3 - 6x^2 + 11x - 6) = 4(x - 1)(x - 2)(x - 3) \quad 1 \frac{1}{2}$$

$$f'(x) = 0 \Rightarrow x = 1, x = 2, x = 3 \quad \frac{1}{2}$$

The intervals are  $(-\infty, 1)$ ,  $(1, 2)$ ,  $(2, 3)$ ,  $(3, \infty)$  1

since  $f'(x) > 0$  in  $(1, 2)$  and  $(3, \infty)$

$\therefore f(x)$  is strictly increasing in  $(1, 2) \cup (3, \infty)$  1

and strictly decreasing in  $(-\infty, 1) \cup (2, 3)$  1

**OR**

$$f(x) = \sec x + 2 \log |\cos x|$$

$$f'(x) = \sec x \tan x - 2 \tan x = \tan x (\sec x - 2) \quad 1$$

$$f'(x) = 0 \Rightarrow \tan x = 0 \text{ or } \sec x = 2$$

$$\Rightarrow x = \pi, x = \frac{\pi}{3}, \frac{5\pi}{3} \quad 1 \frac{1}{2}$$

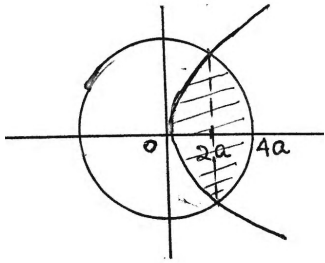
$$f''(x) = \sec x \tan^2 x + (\sec x - 2) \sec^2 x \quad 1$$

$$\left. \begin{aligned} f''(\pi/3) = 6 \text{ (+ve)} &\Rightarrow f(x) \text{ is minimum at } x = \pi/3 \\ f''(\pi) = -3 \text{ (-ve)} &\Rightarrow f(x) \text{ is maximum at } x = \pi \\ f''(5\pi/3) = 6 \text{ (+ve)} &\Rightarrow f(x) \text{ is minimum at } x = 5\pi/3 \end{aligned} \right\} 1 \frac{1}{2}$$

$$\text{Maximum value} = f(\pi) = -1. \quad \frac{1}{2}$$

$$\text{Minimum value} = f(\pi/3) = f(5\pi/3) = 2 - 2 \log 2 \text{ or } 2 + \log (1/4) \quad \frac{1}{2}$$

23.



$$\text{Solving } y^2 = 6ax \text{ and } x^2 + y^2 = 16a^2$$

$$\text{we get } x^2 + 6ax - 16a^2 = 0$$

$$(x + 8a)(x - 2a) = 0$$

$$x = -8a, x = 2a$$

Correct Figure

$$\text{Required area} = 2 \left[ \int_0^{2a} \sqrt{6a}\sqrt{x} \, dx + \int_{2a}^{4a} \sqrt{16a^2 - x^2} \, dx \right] \quad 2$$

$$= 2 \left[ \left( \sqrt{6}\sqrt{a} \frac{2}{3} x^{3/2} \right)_0^{2a} + \left( \frac{x}{2} \sqrt{16a^2 - x^2} + 8a^2 \sin^{-1} \frac{x}{4a} \right)_{2a}^{4a} \right] \quad 1$$

$$= 2 \left[ \frac{8\sqrt{3}a^2}{3} + 8a^2 \frac{\pi}{2} - 2a^2 \sqrt{3} - 8a^2 \frac{\pi}{6} \right]$$

$$= 2 \left[ \frac{2\sqrt{3}a^2}{3} + 8a^2 \frac{\pi}{3} \right] \text{sq. units} \quad 1$$

24. Points on the lines are  $a_1 = (1, -1, 0)$ ,  $a_2 = (0, 2, -1)$ and the direction of lines is  $2\hat{i} - \hat{j} + 3\hat{k}$ let the equation of plane through  $a_1$  be

$$a(x-1) + b(y+1) + c(z) = 0 \quad \dots(\text{i}) \quad \frac{1}{2}$$

$$(0, 2, -1) \text{ lies on it, } \therefore -a + 3b - c = 0 \quad \dots(\text{ii}) \quad 1$$

and  $a, b, c$  are DR's of a line  $\perp$  to the line with DR's 2, -1, 3

$$\therefore 2a - b + 3c = 0 \quad \dots(\text{iii}) \quad 1$$

$$\text{Solving (ii) \& (iii) we get } \frac{a}{8} = \frac{b}{1} = \frac{c}{-5} \quad 1$$

$$\therefore \text{Equation of plane is } 8(x-1) + 1(y+1) - 5z = 0$$

$$\Rightarrow 8x + y - 5z = 7 \quad \dots(\text{iv}) \quad \frac{1}{2}$$

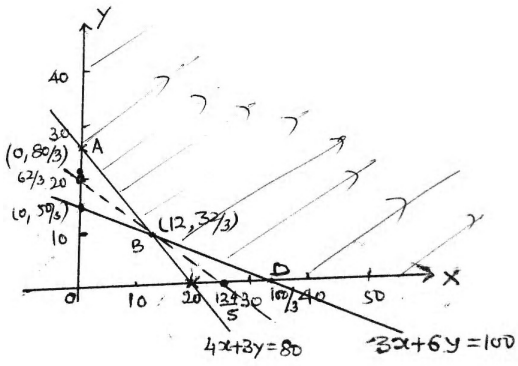
For the line  $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$ , since the point (2, 1, 2) lies on plane (iv)

$$\text{as } 8(2) + 1 - 5(2) = 7 \quad 1$$

$$\text{and } 3(8) + 1(1) + 5(-5) = 25 - 25 = 0$$

 $\therefore$  The plane (iv) contains the given line 1

25.

Let  $x$  units of  $F_1$  and  $y$  units of  $F_2$  be mixed $\therefore$  We have Minimise cost  $(C) = 5x + 6y$ subject to  $4x + 3y \geq 80$  $3x + 6y \geq 100$  $x \geq 0, y \geq 0$ 

Correct Figure

$$C(A) = 160$$

$$C(B) = 60 + 64 = 124$$

$$C(D) = \frac{500}{3}$$

 $5x + 6y \leq 124$  passes through B only $\therefore$  Minimum cost = ₹ 124 $F_1 = 12$  units

$$F_2 = \frac{32}{3} \text{ units}$$

26. Total number of ways =  ${}^6C_3 = 20$ 

X :	1	2	3	4
P(X) :	$\frac{10}{20}$	$\frac{6}{20}$	$\frac{3}{20}$	$\frac{1}{20}$
XP(X) :	$\frac{10}{20}$	$\frac{12}{20}$	$\frac{9}{20}$	$\frac{4}{20}$
$X^2 P(X)$ :	$\frac{10}{20}$	$\frac{24}{20}$	$\frac{27}{20}$	$\frac{16}{20}$

$$\text{Mean} = \sum X P(X) = \frac{35}{20} = \frac{7}{4}$$

$$\text{Variance} = \sum X^2 P(X) - \left[ \sum X P(X) \right]^2 = \frac{77}{20} - \frac{49}{16} = \frac{63}{80}$$