Marking scheme

XII Mathematics

(65/1/1; 65/1/2; 65/1/3)

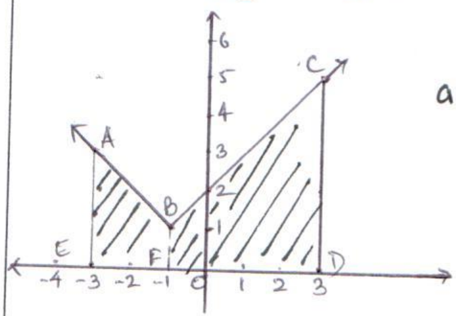
Delhi-2014, Compt.

Q.No.	Value points 65/1/1	Marks
	SECTION A	
1 - 10	1. $\{0,2,4\}$ 2. $\frac{2}{3}$ 3. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 4. $\sqrt{b^2+c^2}$ 5. $3\hat{i}-6\hat{j}+6\hat{k}$	
	6. $40°$ 7. 0 8. $\tan x + \cot x + c$ 9. $-\cot x$ 10. 1	$1\times10=10$
	SECTION B	
11. (i)	sum, difference and product of rational nos. is a unique rational number	$\frac{1}{2}$
	$\therefore$ For each $(a,b) \in S\times S$ there exists unique image $(a+b-ab)$ in S	$\frac{1}{2}$
	$\Rightarrow$ $*$ is a function $\Rightarrow$ $*$ is a binary operation on S	$\frac{1}{2}$
(ii)	$a*b = a+b-ab = b+a-ba = b*a$ $\therefore$ $*$ is commutative	1m
	$a*(b*c) = a*(b+c-bc) = a+(b+c-bc)-a(b+c-bc)$ $= a+b+c-ab-bc-ca+abc$ — ①	
	$(a*b)*c = (a+b-ab)*c = (a+b-ab)+c-(a+b-ab)c$ $= a+b+c-ab-bc-ca+abc$ — ②	$\cdot\cdot 1\frac{1}{2}$m
	From ① & ② $a*(b*c) = (a*b)*c \Rightarrow *$ is Associative.	
12.	$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \cot^{-1}\left(\frac{(\cos\frac{x}{2}+\sin\frac{x}{2})+(\cos\frac{x}{2}-\sin\frac{x}{2})}{(\cos\frac{x}{2}+\sin\frac{x}{2})-(\cos\frac{x}{2}-\sin\frac{x}{2})}\right)\ldots$	2
	$= \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2}$	1+1
	OR	
	$2\tan^{-1}(\cos x) = \tan^{-1}(2\,\text{cosec}\,x) \Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\,\text{cosec}\,x)$ ...	$1\frac{1}{2}$
	$\Rightarrow 2\cot x \cdot \text{cosec}\,x = 2\,\text{cosec}\,x$	1
	$\Rightarrow \cot x = 1$ $\therefore x = \cot^{-1}(1) = \frac{\pi}{4}$	$\frac{1}{2}+1$

Q.No.	Value points 65/1/1	Marks
13.	$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix}$ $\left\{R_1 \to aR_1 \,;\, R_2 \to bR_2 \,;\, R_3 \to cR_3\right\}$	$\frac{1}{2}$
	$= \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$ {Taking (abc) common from C_3}	$\frac{1}{2}$
	$= \begin{vmatrix} a^2 & a^3 & 1 \\ b^2-a^2 & b^3-a^3 & 0 \\ c^2-a^2 & c^3-a^3 & 0 \end{vmatrix}$ $(R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1)$	$1\frac{1}{2}$
	$= (b-a)(c-a)\begin{vmatrix} a^2 & a^3 & 1 \\ b+a & b^2+ab+a^2 & 0 \\ c+a & c^2+ca+a^2 & 0 \end{vmatrix}$ {Taking common $b-a$, $c-a$ from R_2 & R_3 resp.}	$\frac{1}{2}$
	$= (b-a)(c-a)\left(bc(c-b) + a(c^2-b^2)\right)$	
	$= (a-b)(b-c)(c-a)(bc+ca+ab)$	1
14.	$\frac{dy}{dt} = at\sin t$; $\frac{dx}{dt} = at\cos t \Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \tan t$	$\frac{1}{2}+1+1$
	$\frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{dt}{dx} = \frac{\sec^3 t}{at}$ $\therefore \left.\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{4}} = \frac{8\sqrt{2}}{a\pi}$	$1+\frac{1}{2}$
15.	$(x-y)\cdot e^{\frac{x}{x-y}} = a \Rightarrow \log(x-y) + \frac{x}{x-y} = \log a$	1
	Differentiate w.r.t. "x"	
	$\Rightarrow \frac{1-y'}{x-y} + \frac{1\cdot(x-y) - x\cdot(1-y')}{(x-y)^2} = 0$	2
	$\Rightarrow (x-y)(1-y') + x - y - x(1-y') = 0$	
	$\Rightarrow yy' + x - 2y = 0$ or $y\frac{dy}{dx} + x = 2y$	1
16.	let r and s be the radius and the side of the square	
	$\therefore\ 2\pi r + 4s = k \quad \therefore\ s = \frac{k-2\pi r}{4}$ -----	$\frac{1}{2}$
	Sum of their areas, $A = \pi r^2 + s^2 = \pi r^2 + \frac{1}{16}(k-2\pi r)^2$ ---	$\frac{1}{2}$
	$\frac{dA}{dr} = 2\pi r - \frac{\pi}{4}(k-2\pi r)$; $\frac{dA}{dr} = 0 \Rightarrow r = \frac{k}{8+2\pi}$ ---	$1+\frac{1}{2}$
	$\left.\frac{d^2A}{dr^2}\right]_{r=\frac{k}{8+2\pi}} = 2\pi + \frac{\pi^2}{2} > 0$ $\therefore$ Area is least iff $8r + 2\pi r = k$	
	$\Leftrightarrow 8r + 2\pi r = 2\pi r + 4s$	$\frac{1}{2}+1$
	$\Leftrightarrow s = 2r$	
	i.e. side of square is double the radius.	

Q.No.	Value points 65/1/1	Marks
16.	OR	
	let $y=f(x) = x^{3/2}$, $x=4$, $x+\Delta x = 3.968 \therefore \Delta x = -0.032$ ..	2
	$\Delta y = \left.\frac{dy}{dx}\right]_{x=4} \cdot \Delta x = \frac{3}{2} \cdot \left. x^{1/2}\right]_{x=4} \cdot \Delta x = \frac{3}{2} \cdot 2 \cdot (-0.032) = -0.096$ ..	1
	$(3.968)^{3/2} = f(x+\Delta x) = f(x) + \Delta y = 8 - 0.096 = 7.904$ ..	1
17.	$\int x^2 \sin x = -x^2 \cos x + 2\int x \cos x \, dx$	$1\frac{1}{2}$
	$= -x^2\cos x + 2(x\sin x + \cos x)$ ...	$1\frac{1}{2}$
	$\therefore \int_0^{\pi/2} x^2 \sin x \, dx = \{-x^2\cos x + 2(x \sin x + \cos x)\}\Big]_0^{\pi/2} = \pi - 2$ ...	1
	OR	
	$\int (x+3)\sqrt{3-4x-x^2}\,dx = -\frac{1}{2}\int(-2x-4)\sqrt{3-4x-x^2}\,dx + \int\sqrt{(\sqrt{7})^2 - (x+2)^2}\,dx$	2
	$= -\frac{1}{3}(3-4x-x^2)^{3/2} + \frac{x+2}{2}\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + c$	2
18.	$\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x \Rightarrow \frac{1}{1+y}dy = -\frac{\cos x}{2+\sin x}dx$ ...	$\frac{1}{2}$
	Integrating, weget	
	$\log\lvert 1+y\rvert = -\log\lvert 2+\sin x\rvert + \log c$ ...	1
	$\Rightarrow (1+y)(2+\sin x) = c$, Putting $y(0)=1$, weget $c=4$ ...	1
	$\therefore (1+y)(2+\sin x) = 4$ or $y = \frac{2-\sin x}{2+\sin x}$	$\frac{1}{2}$
	$\therefore y\left(\frac{\pi}{2}\right) = \frac{1}{3}$ ...	1
19.	Given differential Equation can be written as:	
	$\frac{dy}{dx} = \frac{x+2y}{x-y} \Rightarrow v + x\frac{dv}{dx} = \frac{1+2v}{1-v}$, where $y = vx$ ..	1
	$\Rightarrow \frac{v-1}{v^2+v+1}dv = -\frac{1}{x}dx$, Integrating both sides	$\frac{1}{2}$
	$\frac{1}{2}\int\frac{2v+1}{v^2+v+1}dv - \frac{3}{2}\int\frac{1}{v^2+v+1}dv = -\log\lvert x\rvert$ ...	$\frac{1}{2}$
	$\Rightarrow \frac{1}{2}\log\lvert v^2+v+1\rvert - \sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) = -\log\lvert x\rvert + c$ ..	$1\frac{1}{2}$
	$\Rightarrow \frac{1}{2}\log\left\lvert\frac{y^2}{x^2} + \frac{y}{x} + 1\right\rvert - \sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}\,x}\right) = -\log\lvert x\rvert + c$	
	or	
	$\log\lvert x^2+xy+y^2\rvert = 2\sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3}\,x}\right) + c_1$ (where $c_1 = 2c$)	$\frac{1}{2}$

Q.No.	Value points 65/1/1	Marks
20.	Equation of line can be written as:	
	$\frac{x+2}{2} = \frac{y-7/2}{3} = \frac{z-5}{-6}$ ----	$1\frac{1}{2}$
	D-ratios of line are 2, 3, −6 $\therefore$ D-cosines are $\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}$ --	$1\frac{1}{2}$
	Vector Equation of line through A(−1, 2, 3) and parallel to given line is	
	$\vec{r} = -\hat{i}+2\hat{j}+3\hat{k}+\lambda(2\hat{i}+3\hat{j}-6\hat{k})$ ---	1
21.	$\vec{a}+\vec{b} = \hat{i}-3\hat{j}+2\hat{k}$; $\vec{b}+\vec{c} = -\hat{i}+2\hat{j}$ ----	1
	$(\vec{a}+\vec{b})\times(\vec{b}+\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ -1 & 2 & 0 \end{vmatrix} = -4\hat{i}-2\hat{j}-\hat{k}$ ---	$1\frac{1}{2}$
	Area of parallelogram $= \frac{1}{2}\left\lvert(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})\right\rvert = \frac{\sqrt{21}}{2}$ sq. units.	$1\frac{1}{2}$
	OR	
	$(\vec{a}+\vec{b})\cdot\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\} = (\vec{a}+\vec{b})\cdot\{\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}+\vec{c}\times\vec{c}\}$ $(\vec{c}\times\vec{c}=\vec{0})$	$\frac{1}{2}$
	$= \vec{a}\cdot(\vec{b}\times\vec{c})+\vec{a}\cdot(\vec{b}\times\vec{a})+\vec{a}\cdot(\vec{c}\times\vec{a}) + \vec{b}\cdot(\vec{b}\times\vec{c})+\vec{b}\cdot(\vec{b}\times\vec{a})+\vec{b}\cdot(\vec{c}\times\vec{a})$	1
	$= \vec{a}\cdot(\vec{b}\times\vec{c})+\vec{a}\cdot(\vec{b}\times\vec{c})$ $\left\{\because \vec{a}\cdot(\vec{b}\times\vec{a}) = \vec{a}\cdot(\vec{c}\times\vec{a})\right.$ --	$\frac{1}{2}$
	$= 2\,\vec{a}\cdot(\vec{b}\times\vec{c})$ —① $= \vec{b}\cdot(\vec{b}\times\vec{c})$ -- $= \vec{b}\cdot(\vec{b}\times\vec{a}) = 0$	$\frac{1}{2}$
	$\vec{a}, \vec{b}, \vec{c}$ are co-planar $\Rightarrow \vec{a}\cdot(\vec{b}\times\vec{c}) = 0$ --	$\frac{1}{2}$
	$\therefore$ From ①, $(\vec{a}+\vec{b})\cdot\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\} = 0$	$\frac{1}{2}$
	$\Rightarrow \vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ are co-planar. ---	$\frac{1}{2}$
22.	let E_1: First ball is red, E_2: First ball is black	
	A: Second ball is red.	
	$P(E_1) = \frac{3}{10}$, $P(E_2) = \frac{7}{10}$, $P(A/E_1) = \frac{2}{9}$, $P(A/E_2) = \frac{3}{9}$	2
	$P(E_1/A) = \dfrac{\frac{3}{10}\cdot\frac{2}{9}}{\frac{3}{10}\cdot\frac{2}{9}+\frac{7}{10}\cdot\frac{3}{9}} = \frac{6}{27} = \frac{2}{9}$	2

Q.No.	Value points 65/1/1	Marks
	SECTION C	
23.	Slope of tangent $= \frac{dy}{dx} = 2x-2$	1
	(i) Tangent parallel to $2x-y+9=0$, $\therefore 2x-2=2$, $x=2, y=7$	$1\frac{1}{2}$
	Equation of tangent through (2,7) and parallel to line is $y-7=2(x-2) \Rightarrow y=2x+3$	1
	(ii) Tangent perpendicular to $5y-15x=13$ $\therefore (2x-2)\cdot 3=-1$ $\therefore x=\frac{5}{6}$, $y=\frac{217}{36}$.	$1\frac{1}{2}$
	Equation of tangent through $(\frac{5}{6}, \frac{217}{36})$ and perpendicular to line is $y-\frac{217}{36}=-\frac{1}{3}\left(x-\frac{5}{6}\right) \Rightarrow y=-\frac{x}{3}+\frac{227}{36}$ or $12x+36y=227$	1
24	 Correct Graph.	$1\frac{1}{2}$
	area(EABCDE) = ar(ABFE) + ar(CBFD) $= \int_{-3}^{-1}(\lvert x+1\rvert+1)\,dx + \int_{-1}^{3}(\lvert x+1\rvert+1)\,dx$	1
	$= \int_{-3}^{-1}(-x)\,dx + \int_{-1}^{3}(x+2)\,dx$	$1\frac{1}{2}$
	$= -\frac{x^2}{2}\Big]_{-3}^{-1} + \frac{(x+2)^2}{2}\Big]_{-1}^{3}$	1
	$= -\frac{1}{2}(1-9)+\frac{1}{2}(25-1) = 16$ sq. units	1
25.	let $x^2=t$ $\therefore \frac{x^2}{(x^2+1)(x^2+4)} = \frac{t}{(t+1)(t+4)} = \frac{A}{t+1}+\frac{B}{t+4}$	1
	Getting, $A=-\frac{1}{3}$, $B=\frac{4}{3}$	2
	$\therefore \int\frac{x^2}{(x^2+1)(x^2+4)}dx = -\frac{1}{3}\int\frac{1}{x^2+1}dx+\frac{4}{3}\int\frac{1}{x^2+4}dx$	
	$= -\frac{1}{3}\tan^{-1}x+\frac{2}{3}\tan^{-1}\frac{x}{2}+c$	3
	OR	
	let, $I=\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$ — (1) $\therefore I=\int_0^{\pi}\frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}dx=\int_0^{\pi}\frac{(\pi-x)\tan x}{\sec x+\tan x}dx$ — (2)	$1\frac{1}{2}$
	Add (1) & (2), $2I=\pi\int_0^{\pi}\frac{\tan x}{\sec x+\tan x}dx = \pi\int_0^{\pi}\tan x(\sec x-\tan x)\,dx$	1
	$= \pi\int_0^{\pi}(\sec x\cdot\tan x-\sec^2 x+1)\,dx$	1
	$= \pi\cdot(\sec x-\tan x+x)\Big]_0^{\pi}$	$1\frac{1}{2}$
	$\therefore 2I=\pi(-1+\pi-1) \Rightarrow I=\frac{\pi}{2}(\pi-2)$ or $\pi\left(\frac{\pi}{2}-1\right)$	1

Q.No.	Value points 65/1/1	Marks

26. let Equation of plane through (1,2,−4) be

$$a(x-1)+b(y-2)+c(z+4)=0 \quad -(1)$$ — 1

The plane is parallel to the given lines

$\therefore \; 2a+3b+6c=0 \; ; \; a+b-c=0$ — 1

Solving: $\dfrac{a}{-9}=\dfrac{b}{8}=\dfrac{c}{-1}=k\text{(say)} \;\therefore\; a=-9k, b=8k, c=-k$ — 1

From (1): $-9k(x-1)+8k(y-2)-k(z+4)=0$

$\therefore$ Equation of plane in cartesian form is

$$9x-8y+z+11=0$$ — 1

vector form of plane is: $\vec{r}\cdot(9\hat{i}-8\hat{j}+\hat{k})=-11$ — 1

Distance of $(9,-8,-10)$ from the plane $=\left|\dfrac{9\cdot 9-8(-8)+1(-10)+11}{\sqrt{81+64+1}}\right|=\sqrt{146}$ — 1

27. let S = Sample space of the Experiment = {HT, HH, T1, T2, T3, T4, T5, T6} — 1

A = Event that die shows a no. greater than 4 = {T5, T6}

B = Event that there is atleast one tail = {HT, T1, T2, T3, T4, T5, T6}

$A\cap B=\{T5, T6\}$

$P(A\cap B)=P(T5)+P(T6)=\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{6}$ — 2

$P(B)=P(HT)+P(T1)+P(T2)+P(T3)+P(T4)+P(T5)+P(T6)$

$=\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{2}\cdot\frac{1}{6}=\frac{3}{4}$ — 2

$P(A/B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{1/6}{3/4}=\dfrac{2}{9}$ — 1

OR

E_1 = Event that 1 occurs ; E_2 = Event that 1 does not occur — 1

A = Event that the man reports that 1 occurs

$P(E_1)=\frac{1}{6}$; $P(E_2)=\frac{5}{6}$; $P(A/E_1)=\frac{3}{5}$; $P(A/E_2)=\frac{2}{5}$. — $2\frac{1}{2}$

$P(E_1/A)=\dfrac{\frac{1}{6}\cdot\frac{3}{5}}{\frac{1}{6}\cdot\frac{3}{5}+\frac{5}{6}\cdot\frac{2}{5}}=\dfrac{3}{13}$. — $2\frac{1}{2}$

Q.No.	Value points 65/1/1	Marks

28. let ₹ x, y and z be invested in saving accounts at the rate 5%, 8% and $8\frac{1}{2}$% respectively. Then the system of equations is

$$x+y+z=7000$$
$$\frac{5x}{100}+\frac{8y}{100}+\frac{17z}{200}=550 \Rightarrow 10x+16y+17z=110000$$
$$x-y=0$$
$1\frac{1}{2}$

Matrix equation is $\begin{bmatrix}1&1&1\\10&16&17\\1&-1&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}7000\\110000\\0\end{bmatrix}$ i.e $A\cdot X=B$ 1

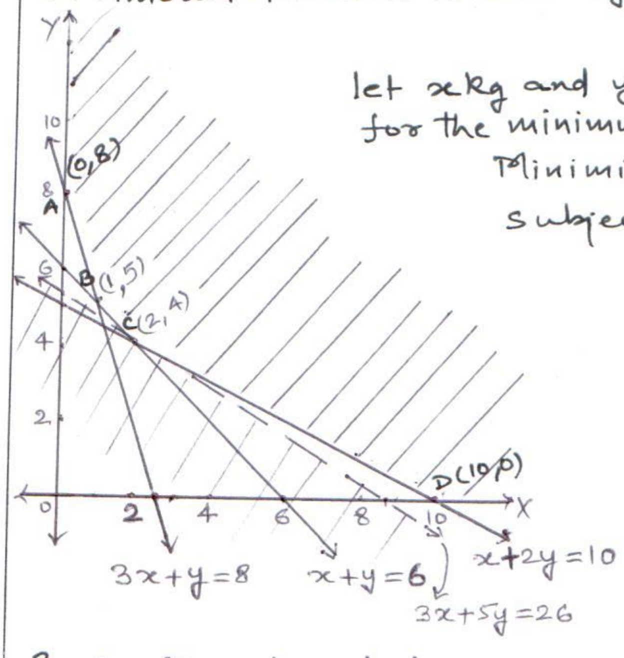
$|A|=1\cdot(0+17)-1(0-17)+1(-10-16)=8\neq 0 \therefore A^{-1}$ exists 1

Co-factors are: $A_{11}=17,\ A_{12}=17,\ A_{13}=-26$
$A_{21}=-1,\ A_{22}=-1,\ A_{23}=2$
$A_{31}=1,\ A_{32}=-7,\ A_{33}=6$ 1

$X=A^{-1}\cdot B \Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{8}\begin{bmatrix}17&-1&1\\17&-1&-7\\-26&2&6\end{bmatrix}\begin{bmatrix}7000\\110000\\0\end{bmatrix}=\begin{bmatrix}1125\\1125\\4750\end{bmatrix}\Rightarrow \begin{matrix}x=1125\\y=1125\\z=4750\end{matrix}$ $1\frac{1}{2}$

$\therefore$ Amount invested in each type of account is ₹1125, ₹1125 and ₹4750 resp.

29.



let x kg and y kg of food X and Y be mixed for the minimum cost of mixture then L.P.P. is

Minimise, $Z=6x+10y$

subject to:
$$x+2y\geq 10$$
$$2x+2y\geq 12 \Rightarrow x+y\geq 6$$
$$3x+y\geq 8$$
$$x,y\geq 0$$
... 2

Correct Graph ... 2

Corner	Value of Z
(0,8)	₹ 80
(1,5)	₹ 56
(2,4)	₹ 52 (Minimum)
(10,0)	₹ 60

Region is unbounded

Z 52 i.e. $6x+10y<52$ or $3x+5y<52$ has no point common with feasible region $\therefore$ the L.P.P has optimum solution at (2,4) and least cost of the mixture = ₹52. 1

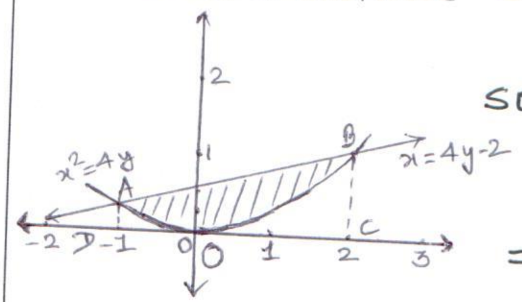
For any one value attached ----- 1 mark

Q.No.	Value points 65/1/2	Marks
	SECTION A	
1 — 10	1. $\frac{\pi}{6}$ 2. $\tan x - \cot x + c$ 3. 1 4. $-\cot x$ 5. $\{0, 2, 4\}$	
	6. $\frac{2}{3}$ 7. 40° 8. 0 9. $\sqrt{b^2+c^2}$ 10. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$1 \times 10 = 10$
	SECTION B	
11.	$R = \{(1,22), (2,20), (3,18), (4,16), (5,14), (6,12), (7,10), (8,8), (9,6), (10,4), (11,2)\}$	2 mark
	Domain $= \{1,2,3,4,5,6,7,8,9,10,11\}$	$\frac{1}{2}$ mark
	Range $= \{2,4,6,8,10,12,14,16,18,20,22\}$.	$\frac{1}{2}$ mark
	$(1,22) \in R$ but $(22,1) \notin R$ $\therefore$ R is not symmetric $\therefore$ R is not an equivalance relation. ...	1 mark
	(Alternatively; student may state that R is not reflexive or transitive)	
12.	Same as Q15 of 65/1/1	
13.	same as Q16 of 65/1/1	
14.	$\frac{dy}{dt} = a\cos t$; $\frac{dx}{dt} = a\left(-\sin t + \frac{\sec^2 t/2}{2\cdot\tan t/2}\right) = \frac{a\cos^2 t}{\sin t}$	$\frac{1}{2}+1$
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = a\cos t \times \frac{\sin t}{a\cos^2 t} = \tan t$	1
	$\frac{d^2y}{dx^2} = \sec^2 t \times \frac{dt}{dx} = \frac{\sec^2 t \times \sin t}{a\cos^2 t} = \frac{1}{a}\sec^3 t \cdot \tan t$	1
	$\therefore \left.\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{3}} = \frac{1}{a}\sec^3\frac{\pi}{3} \cdot \tan\frac{\pi}{3} = \frac{1}{a} \cdot 8 \cdot \sqrt{3} = \frac{8\sqrt{3}}{a}$	$\frac{1}{2}$
15.	Same as Q17 of 65/1/1	
16.	Same as Q19 of 65/1/1	
17.	Same as Q18 of 65/1/1	
18.	Equation of plane is $\frac{x}{-6} + \frac{y}{3} + \frac{z}{4} = 1 \Rightarrow -2x + 4y + 3z = 12$	1
	Normal form of Equation of plane is: $\frac{-2x}{\sqrt{29}} + \frac{4y}{\sqrt{29}} + \frac{3z}{\sqrt{29}} = \frac{12}{\sqrt{29}}$	
	$\therefore$ length of perpendicular from origin to plane $= \frac{12}{\sqrt{29}}$	2

Q.No.	Value points 65/1/2	Marks
19.	Same as Q13 of 65/1/1	
20.	Same as Q21 of 65/1/1	
21.	Same as Q12 of 65/1/1	
22.	S = Sample space of 2 children = $\{(b,b),(b,g),(g,b),(g,g)\}$.	1
	(i) Probability that both are boys given that one of them is a boy $= \frac{1}{3}$	$1\frac{1}{2}$
	(ii) Probability that both are boys given that older child is a boy $= \frac{1}{2}$	$1\frac{1}{2}$
	SECTION C	
23.	$f'(x) = 2x-1$, $f'(x) > 0 \ \forall x \in (\frac{1}{2}, 1)$, $f'(x) < 0 \ \forall x \in (-1, \frac{1}{2})$	1+1+1
	$\therefore f(x)$ is neither increasing nor decreasing in $(-1,1)$ …	1
	$f(x)$ is strictly increasing on $(\frac{1}{2}, 1)$ ….	1
	$f(x)$ is strictly decreasing on $(-1, \frac{1}{2})$ ….	1
24.	Same as Q27 of 65/1/1	
25.	Same as Q24 of 65/1/1	
26.	Same as Q26 of 65/1/1	
27.	Here $a=1$, $b=3$, $nh=2$, $f(x) = 3x^2+1$	
	$\int_1^3 (3x^2+1)\,dx = \lim_{h\to 0} h\left[3h^2(1^2+2^2+\cdots+(n-1)^2) + 6h(1+2+\cdots+(n-1)) + 4n\right]$	2
	$= \lim_{h\to 0}\left[\frac{3(nh-h)(nh)(2nh-h)}{6} + \frac{6(nh-h)(nh)}{2} + 4nh\right]$	2
	$= \lim_{h\to 0}\left[\frac{3(2-h)\cdot 2\cdot(4-h)}{6} + \frac{6(2-h)\cdot 2}{2} + 8\right]$	1
	$= 8+12+8 = 28$	1

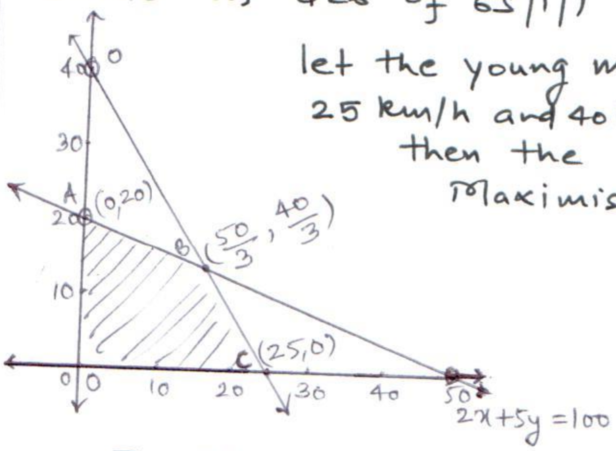
Q.No.	Value points 65/1/2	Marks
27	OR	
	let $\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$	1
	Solving for A, B, C we get $A=-2$, $B=1$, $C=3$	2
	$\therefore \int \frac{x^2+x+1}{(x+1)^2(x+2)}dx = -2\int \frac{1}{x+1}dx + \int \frac{1}{(x+1)^2}dx + 3\int \frac{1}{x+2}dx$	
	$= -2\log\lvert x+1\rvert - \frac{1}{x+1} + 3\log\lvert x+2\rvert + C$	3
28.	Same as Q29 of 65/1/1	
29.	Same as Q28 of 65/1/1	

Q.No.	Value points 65/1/3	Marks
1 – 10	**SECTION A**	
	1. $\frac{1}{\sqrt{3}}$ 2. adj $A = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$ 3. $\{0, 2, 4\}$ 4. 1 5. $\sqrt{b^2+c^2}$	
	6. $3\hat{i} - 6\hat{j} + 6\hat{k}$ 7. $-\cot x$ 8. $\tan x + \cot x + C$ 9. $40°$ 10. 0	$1 \times 10 = 10$
	SECTION B	
11.	$f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \Rightarrow x_1 = x_2$ $\therefore f$ is a one-one function	1
	let $y = f(x) = \frac{x-2}{x-3} \Rightarrow x = \frac{3y-2}{y-1}$ where $y \neq 1$ & $x \neq 3$ $\therefore$ For each $y \in B$ there exists $x \in A$ such that $f(x) = y$ $\Rightarrow f$ is onto	$1\frac{1}{2}$
	f is a one-one and onto function $\Rightarrow f$ is a bijective function	$\frac{1}{2}$
	$f^{-1}: B \to A$ with $f^{-1}(x) = \frac{3x-2}{x-1}$.	1
12.	Same as Q22 of 65/1/1	
13.	Same as Q21 of 65/1/1	
14.	Same as Q12 of 65/1/1	
15	$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & x & x^2 \\ 0 & 1-x^3 & x-x^4 \\ 0 & 0 & 1-x^3 \end{vmatrix}$ $\left\{ \begin{matrix} R_2 \to R_2 - x^2 R_1 \\ ; R_3 \to R_3 - x R_1 \end{matrix} \right.$	3 marks
	Expand by C_1 $= 1 \cdot (1-x^3)^2 = (1-x^3)^2$	1 mark
16.	Same as Q14. of 65/1/1	
17.	Same as Q15 of 65/1/1	
18.	Same as Q19 of 65/1/1	
19.	$\int \frac{5x-2}{1+2x+3x^2} dx = \frac{5}{6} \int \frac{6x+2}{1+2x+3x^2} dx - \frac{11}{3} \int \frac{1}{1+2x+3x^2} dx$	2
	$= \frac{5}{6} \log\lvert 1+2x+3x^2 \rvert - \frac{11}{9} \int \frac{1}{\left(x+\frac{1}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2} dx$	$2\frac{1}{2}$
	$= \frac{5}{6} \log\lvert 1+2x+3x^2 \rvert - \frac{11}{3\sqrt{2}} \tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + C$	$1\frac{1}{2}$

Q.No.	Value points 65/1/3	Marks
19.	$\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16\sin 2x} dx = \frac{1}{16} \int_0^{\pi/4} \frac{\sin x + \cos x}{\left(\frac{5}{4}\right)^2 - (\sin x - \cos x)^2} dx$ ...	2
	OR	
	$= \frac{1}{16} \int_{-1}^{0} \frac{1}{\left(\frac{5}{4}\right)^2 - t^2} dt$ Put $\sin x - \cos x = t$; $(\cos x + \sin x)dx = dt$; Upper limit $= 0$; Lower limit $= -1$	2
	$= \frac{1}{40} \log \left\lvert \frac{5+4t}{5-4t} \right\rvert \Big]_{-1}^{0}$ ...	1
	$= -\frac{1}{40} \log \frac{1}{9}$ or $\frac{1}{20} \log 3$	1
20.	Same as Q16 of 65/1/1	
21.	Same as Q18 of 65/1/1	
22	Here $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$; $\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$; $\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$	1
	$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$; $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} = -9\hat{i} + 3\hat{j} + 9\hat{k}$	1+2
	$(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1) = -27 + 9 + 27 = 9$; $\lvert \vec{b}_1 \times \vec{b}_2 \rvert = \sqrt{171}$	1
	$\therefore$ Shortest distance $= \left\lvert \frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{\lvert \vec{b}_1 \times \vec{b}_2 \rvert} \right\rvert = \frac{9}{\sqrt{171}} = \frac{3}{\sqrt{19}}$ or $\frac{3\sqrt{19}}{19}$	1
	SECTION C	
23.		
	Correct Graph	$1\frac{1}{2}$
	Pts. of intersection $(-1, \frac{1}{4})$, $(2, 1)$ ..	$\frac{1}{2}$
	Req. area is of region (OABO)	
	$= \int_{-1}^{2} \frac{x+2}{4} dx - \int_{-1}^{2} \frac{x^2}{4} dx$	$1\frac{1}{2}$
	$= \frac{1}{4} \left\{ \frac{(x+2)^2}{2} - \frac{x^3}{3} \right\} \Big]_{-1}^{2}$	$1\frac{1}{2}$
	$= \frac{1}{4} \left\{ \frac{16}{3} - \frac{5}{6} \right\} = \frac{27}{24}$ sq. units .. $= \frac{9}{8}$ sq. units	1

Q.No.	Value points 65/1/3	Marks
24.	Same as Q28 of 65/1/1	
25.	Same as Q23 of 65/1/1	
26.	Same as Q25 of 65/1/1	
27.	Same as Q26 of 65/1/1	

28.



let the young man drives x kms & y kms at 25 km/h and 40 km/h speed respectively then the L.P.P. is

Maximise Distance: $Z = x + y$

subject to

$2x + 5y \leq 100$

$\frac{x}{25} + \frac{y}{40} \leq 1$ or $8x + 5y \leq 200$

$x, y \geq 0$ — 2 mark

Correct Graph .... 2 mark

Feasible region is OABC with,

corner	Value of Z
(0,20)	20
$\left(\frac{50}{3}, \frac{40}{3}\right)$	30 (Max.)
(25, 0)	25

$\therefore$ Max distance covered is 30 kms with $\frac{50}{3}$ km at 25 km/h and $\frac{40}{3}$ km at 40 km/h speeds respectively. — 1 mark

Value indicated: Vehicle should be driven at a moderate speed to decrease the pollution .... 1 mark

29. Same as Q27 of 65/1/1

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2 [signature] 5. [signature]
3 [signature]